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Discussions

Prediction of direct runoff hydrographs utilizing stochastic network models: a case study in South Korea

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Abstract

In this study, we combine stochastic network models that reproduce the actual width function and the width function based instantaneous unit hydrograph (WFIUH) that directly makes use of a width function and converts it into runoff hydrographs. We evaluated the stochastic network models in terms of reproducing the actual width function and also the robustness of the semi-distributed model (WFIUH) in application to a test watershed in South Korea. The stochastic network model has an advantage that it replicates width functions of actual river networks, whereas the WFIUH has an advantage that the parameter values are physically determined, which can be potentially advantageous in prediction of ungauged basins. This study demonstrates that the combination of the Gibbsian model and the WFIUH is able to reproduce runoff hydrographs not just for the case of uniform rainfall over the test catchment but also for moving storms. Therefore, results of this study indicate that the impact of spatial and temporal rainfall variation on runoff hydrographs can be evaluated by the suggested approach in ungauged basins even without detailed knowledge of river networks. Once the regional similarity in river network configuration is identified, the proposed approach can be potentially utilized to estimate the runoff hydrographs for ungauged basins.

1 Introduction

Prediction in ungauged basins (PUB) has been a long-standing topic in hydrology, which aims at accurate simulation of a catchment without any observation and, hence, without model calibration. The assessment of future impact of changes such as climate change or land use changes on hydrologic responses is also solved partly in the same way of the ungauged basin problem because future responses under different condition cannot be gaged (Beven, 2012). In essence, the ungauged basins requires the development of new predictive approaches that are based on a deep understanding of hydrologic function at multiple space–time scales and also they encourage us to go beyond an immediate problem-solving needs and to pursue knowledge and understanding of natural processes (PUB Science Plan, 2003). In this regard, the implication and importance of the ungauged basin problem cannot be emphasized enough in hydrology. However, most catchments around the world still do not have runoff measurements, although runoff information is needed almost everywhere for the purpose of water-related risk management such as flood and drought mitigation, water resources management such as development, distribution and maintenance of water supply systems, guidelines for land development and so forth (Blöschl et al., 2013).

In spite of substantial improvement in the PUB, there is no common agreement that the PUB is solvable by a regionalization strategy or any other advanced theories (Beven, 2012). Making predictions in ungauged basins has primarily focused on regionalization methods, which tie hydrologic or physical characteristics of watersheds mainly with runoff characteristics or model parameters. Then, assuming such characteristics of an ungauged basin are known, the runoff characteristics of the ungauged basin are estimated based on regression equations or other regionalization schemes. In most cases, regionalization methods have been developed for the estimation of the characteristics of flood frequency distributions (Fleming and Franz, 1971; Lamb, 1999; Blazkova and Beven, 2002), flow duration curves (Holmes et al., 2002; Castellarin et al., 2007; Li et al., 2010), and the parameters of hydrological models (Nash, 1960; Abdulla and Lettenmaier, 1997; Fernandez et al., 2000; Heuvelmans et al., 2006; Boughton and Chiew, 2007; Bastola et al., 2008; Hundecha et al., 2008; Wallner et al., 2008) at ungauged sites. Especially, Oudin et al. (2008) compared three regionalization scheme, namely, spatial proximity, physical similarity, and regression scheme in France and showed that spatial proximity showed best results while the regression approach showed the least compared to other schemes, but all regionalization results were far behind the results with full calibration.

In this paper, our research interests are focused on the river network of an ungauged basin. The idea is coupling a synthetic width function obtained from stochastic network models and a runoff-rainfall model utilizing the width function in a direct manner to

estimate hydrographs of an ungauged basin. Seo and Schmidt (2014) showed that the network characteristics of urban drainage networks in terms of the width function can be regenerated by a stochastic network model, which is referred to as the Gibbsian model proposed by Troutman and Karlinger (1992). In addition, we introduce a geomorphologic rainfall–runoff models that directly utilizes the width function proposed by Kirkby (1976), Mesa and Mifflin (1986), and Naden (1992). They have proposed formulations of the geomorphologic Instantaneous Unit Hydrograph (IUH) based on the width function of a basin coupled with various routing procedures, which was later denoted as a WFIUH by Franchini and O'Connell (1996). The WFIUH is different from the well-known Geomorphologic Instantaneous Unit Hydrograph (GIUH) (Rodriguez-Iturbe and Valdes, 1979), which is based on Horton's geomorphologic laws and the Strahler ordering scheme (Strahler, 1957). Instead, it utilizes the width function obtained from a river network directly and converts it to a hydrologic response function. The hydrologic response of a basin should be closely linked to the width function (Gupta and Waymire, 1983) and grouping channel segments such as Strahler ordering scheme can result in loss of information about this response from the width function (Troutman and Karlinger, 1985). The width function approach is considerably simpler than the GIUH approach because it emphasizes the metric representation of the basin instead of the topologic one (Di Lazzaro, 2009). Moreover, the hydraulic parameters of the WFIUH are physically consistent, while the GIUH velocity parameter lacks physical interpretation (Franchini and O'Connell, 1996).

The Gibbsian model is based on the state of the network in terms of sinuosity and imposing different probability at each state. The model has a control over the overall sinuosity of the network (Troutman and Karlinger, 1992). The control is depending on the value of a parameter, β. They estimated β for 40 river networks in Montana, of which average catchment slope is 16.73 % and found that average β is order of 10^0. In contrast, Seo and Schmidt (2012) applied the Gibbsian model to urban drainage networks in Chicago areas and found that urban drainage network has a wide range of β from 10^{-2} to 10^2. These results imply that river networks are more efficient in terms of sinuosity and the drainage time compared to artificial drainage networks, which is counterintuitive because, typically, man-made drainage systems have been considered to be more efficient in terms of drainage time. Small roughness of the conveyance system has been often regarded as the main reason for the increased peak discharge and decreased arrival time to the outlet of the urban drainage system. However, in terms of network efficiency and total drainage time of a drainage network, artificial drainage network can be less efficient compared with river networks, which is the result of an evolution through geological time scales to discharge water more efficiently in nature.

In application of the WFIUH, the observed width function is coupled with the advection–diffusion equation (Mesa and Mifflin, 1986; Naden, 1992). The WFIUH is a semi-distributed model with two parameters: kinematic celerity, c, and diffusion coefficient, D. It should be noted that these parameters are dependent on the geomorphic characteristics of local slope and discharge, which implies that, at least, the order of magnitude of these quantities is physically determined (Franchini and O'Connell, 1996). Due to the nature of a semi-distributed model, it is necessary to define a representative cross-section of a catchment. However, it has not been discussed enough to suggest a guideline to apply the WFIUH. If the Gibbsian model, combined with the WFIUH, can reproduce the observed width function of an ungauged basin, it also enables us to contribute the prediction of runoff hydrographs in an ungauged basin because the width function is closely linked to hydrologic response of a basin. Moreover, hydrological similarity in terms of network configuration can help runoff prediction in ungauged basins (Blöschl et al., 2013). As mentioned earlier, the overall sinuosity of the Gibbsian model is represented by the value of a parameter, β. Troutman and Karlinger found the average parameter value is 10^0 in Montana, which shows that the similarity in terms of β can be assessed, and consequently these efforts would contribute to the PUB.

In this regard, this paper suggests an approach that couples a synthetic width function obtained from the Gibbsian model and the WFIUH for accurate simulation of direct

runoff hydrographs in an ungauged basin especially for the purpose of flood estimation and direct runoff hydrographs. We applied the suggested approach in a test watershed in South Korea and evaluated the possibility of the suggested approach in ungauged basins for prediction. The basic idea of this study is to combine stochastic network models that reproduce the width function of a watershed and a semi-distributed hydrologic model that directly utilizes the width function and converts it to a runoff hydrograph. First, it is necessary to examine the ability of the stochastic network models that reproduce the actual width function and also the robustness of the semi-distributed model in application to a test watershed. Then, we demonstrate that the combination of the two models is able to contribute to the PUH and discuss the implication of the results.

2 Methodology

2.1 The Gibbsian model

Three stochastic network models are considered in this study – the uniform model, the Scheidegger model and the Gibbsian model to generate networks with specified conditions. The uniform model (or a random walk model) is defined on a lattice with flow being allowed in every direction with equal probability but still flows are not allowed to cross over existing walk, which is a self-avoiding property. To generate random walk model, first, it starts from the outlet point and proceed to the upstream with equal probabilities until every single point in the whole domain is visited. The Scheidegger model (1967a, b) is a directed self-avoiding random walk model on a lattice with flow from each point being allowed in only two directions, each with equal probability.

Troutman and Kalinger (1992) proposed the Gibbsian model, which is based on Gibbs' distribution. The idea is to define a Markov chain with the spanning trees S as the state space. To draw an analogy with the initial tree which will be obtained with the uniform distribution (random walk model), let the elements of S to be points of a graph, and define the lines in this graph as follows. The analogy means what can be obtained with minimum changes to original state (tree) here. Let two points, s_1 and s_2, be adjacent if one may be obtained from the other. To do this, it is needed to randomly select a point in s_1 and define a new direction from that point, which is going to be a new spanning tree, s_2. Then the transition probability from s_1 to s_2 can be defined as follows (Troutman and Karlinger, 1992):

$$R_{s_1 s_2} \begin{cases} r^{-1}\min\{1, e^{-\beta[H(s_2)-H(s_1)]}\} & s_2 \in N(s_1) \\ 1-\sum_{s\in N(s_1)} R_{s_1 s} & s_2 = s_1 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

where $N(s_1)$ is the set of trees adjacent to s_1, and r, the maximum degree of the points in S, is

$$r = \max_{s\in S} |N(s)| \tag{2}$$

The degree here means the maximum number of direction it can go. It may be shown that the Markov chain with this transition probability has a stationary distribution given as Gibbs' distribution (Troutman and Karlinger, 1992).

$$P_\beta\{s\} = [C(\beta)]^{-1} e^{-\beta H(s)} \tag{3}$$

where s belongs to S, β is a parameter, $C(\beta)$ is a normalization factor defined to make the probabilities sum to 1.

$$C(\beta) = \sum_{t\in S} e^{-\beta H(t)} \tag{4}$$

$H(s)$ is measuring the sinuosity of a given spanning tree, s.

$$H(s) = \Xi(s) - \Xi(B) = \sum_{v\in V(B)} d_s(v) - \sum_{v\in V(B)} d_B(v) \tag{5}$$

where v is a point of a subgraph B and d_s is the distance to the outlet along s from v while d_B is the shortest distance to the outlet not along s from v. During every generation of adjacent spanning trees, it is checked if there exists any loop inside the network.

The procedure used in this paper to generate a network given a value of β is as follows (Barndorff-Nielsen, 1998): first, start from a uniform network generated by the uniform model, s_1, and randomly select a point, v, in the network and assign a new flow direction from v to generate neighboring network, s_2. Here, the uniform model (Leopold and Langbein, 1962; Karlinger and Troutman, 1989) is a stochastic network model assuming uniform probability for all directions in the generation of a network. Second, check whether the new network, s_2, has any loop inside the network. If it does not have any loop, draw a random value x between 0 and 1 and check that x is less than $e^{-\beta[\Delta H]}$ where ΔH is equal to the change in sinuosity between s_1 and s_2 ($H(s_2) - H(s_1)$). If this holds, then take the s_2 as a new network. Third, let s_2 be equal to s_1 and repeat the above steps sufficiently large times until the resulting tree has a distribution close to the stationary Gibbs' distribution.

2.2 The width function based IUH (WFIUH) coupled with the Gibbsian model

When transforming the distance into time with constant flow velocity, the unit hydrograph for a generic watershed can be easily obtained from discretizing the width function. However, volume of an actual channel and the deviation in time flowing along a route are non-negligible. The drainage network is composed of each inflow infused from the sides and the outlet connected to the basin at a distance x. The network can be considered as a series of each channel links with length of Δx. In the case of a semi-infinite uniform channel fed by inflow at the upstream end ($x = 0$), the routing function can be derived from the advection–diffusion equation of flow perturbation:

$$\frac{\partial Q_\mathrm{p}}{\partial t} = D\frac{\partial^2 Q_\mathrm{p}}{\partial x^2} - c\frac{\partial Q_\mathrm{p}}{\partial x} \tag{6}$$

where Q_p is flow perturbation. The solution of the advection–diffusion equation is given as follows under the boundary condition, $Q_p(0,t) = \delta(t)$, $Q_p(x,0) = 0$ and $Q_p(\infty,\ t) = 0$, and the constant coefficients D and c which are diffusion coefficient and celerity, respectively (Van de Nes, 1973; Naden, 1992; Franchini and O'Connel, 1996; Da Ros and Borga, 1997):

$$u(x,t) = \frac{x}{\sqrt{4\pi D t^3}} \exp\left[-\frac{(x-ct)^2}{4Dt}\right] \tag{7}$$

where $u(x,t)$ is the impulse response of the advection–diffusion equation, i.e. when an instantaneous upstream impulse $\delta(t)$ is introduced, the time of the discharge at a distance x from the upstream is over (Naden, 1992; Franchini and O'Connel, 1996; Da Ros and Borga, 1997). From the unit impulse response, $u(x,t)$ in Eq. (7), an IUH of a catchment can be defined as

$$h(t) = \int_0^\infty W(x)u(x,t)\mathrm{d}x \tag{8}$$

where $W(x)$ is the width function.

$W(x)$ in Eq. (8) can be substituted by $W'(x)$ which is a synthetic width function obtained by simulation results of the Gibbsian model. Then, for a discrete distance interval, Eq. (8) can be rewritten as

$$\hat{h}(t) = \sum_{i=1}^{n} \frac{i\Delta x}{\sqrt{4\pi D t^3}} W'(i\Delta x) \exp\left[-\frac{(i\Delta x - ct)^2}{4Dt}\right] \Delta x \tag{9}$$

where $W'(i\Delta x)$ which is a synthetic width function. The IUH defined as Eq. (9) can be regarded as a response function depending on spatial and temporal variation of rainfall for each time step. Due to a stochastic nature of the Gibbsian model, one hundred synthetic width functions were obtained from the simulation of networks.

2.3 Study area and application of the WFIUH

The test watershed, the Chungju Dam watershed, is located in South Korea, of which area is 6648 km^2 (Fig. 1). The construction of the Chungju Dam was completed in 1985 for multiple purposes of water supply, power generation, and flood control in Han River, the longest river in South Korea. The volume of the dam reservoir is up to 2.75 billion m^3. The mean annual precipitation of the watershed is 1359.5 mm and the mean annual temperature is 9.4 °C. Most of the test watershed is covered with forests (82.2 %) and lest of the watershed is mostly covered by farmlands (11.6 %). The watershed is located in a mountainous region and the mean catchment slope is 36.9 %.

Figure 2 shows the reconstructed river networks of the Chungju Dam watershed on a 24 × 28 lattice and the corresponding width function. Here, the width function is defined as the catchment area at a distance from the outlet (Moussa, 2008). The width function and the area function can be differently defined based on channelization (Lashermes and Foufoula-Georgiou, 2007) but the width function basically represents the distance-area function (Lee and Delleur, 1976). For a given flow distance from the outlet, the width function can be defined as number of grid points in this study. Then, the width function is normalized by the total number within the watershed boundary.

The application of the WFIUH is based on grids (24 × 28) with four flow directions. The reconstructed river networks and the corresponding width function is directly used to obtain runoff hydrographs (Eq. 8). Assuming a wide rectangular channel cross-sectional geometry, the channel bottom slope is most important to determine the value of parameters. The celerity and the diffusion coefficient for a wide rectangular channel is given as follows (Franchini and O'Connell, 1996):

$$c = \frac{3}{2} \times \frac{1}{n} R^{2/3} S_0^{1/2} \tag{10}$$

$$D = \frac{c y_0}{3 S_0} \tag{11}$$

where, n is Manning's roughness coefficient, R is hydraulic radius, and y is initial depth. The celerity of the Chungju Dam watershed is determined as 1.91 $\mathrm{m\,s^{-1}}$, and the diffusion coefficient is obtained as $5.3 \times 10^3\,\mathrm{m^2\,s^{-1}}$ using the mean channel bottom slope over the watershed. The values of the both parameters are within the range of parameter values suggested by Franchini and O'Connell (1996) for natural watersheds: 10^0 and 10^3 for celerity and diffusion coefficient, respectively. Figure 3 shows the resulting runoff hydrographs obtained by the WFIUH to the test watershed for three rainstorm events from 1999 and 2004. Figure 3 compares the obtained runoff hydrographs from the WFIUH with the observed flows and also the runoff hydrographs from other runoff model, HEC-1, of which parameters were optimized from two other events from 1999 and 2004. The result shows that the WFIUH successfully reproduces the runoff hydrographs of the test watershed compared with the observed data and also with the results from HEC-1. It should be noted that the results from HEC-1 were obtained from calibrated model. In contrast, the parameter values of the WFIUH is relatively stable, robust and determined physically.

3 Results and discussion

3.1 Generation of networks

Two items are needed for generation of networks using stochastic network models. One is the location of the outlet and the other is the boundary of a watershed. Simulation is performed on a 24 × 28 lattice within the watershed boundary with the one outlet. Figure 4 shows the a realization of the Scheidegger model and the uniform model of the river network for the test watershed. The outlet of the test watershed is located at the far left of the lattice. The channel width at each point represents the maximum amount of discharge (peak flows) at an uniform instantaneous injection of rainfall and is normalized by the discharge at the outlet (the peak flow at the outlet is 1 and it is between 0 and 1 for other parts). The result shows that the uniform model (Fig. 4a)

is more sinuous than the Scheidegger model (Fig. 4f). The magnitude of the peak flow at the outlet itself is higher on the Scheidegger model compared to the uniform model (Seo and Schmidt, 2012). However, the results indicate that the maximum peak flows are observed not just at the outlet but also other parts of the mainstream on the uniform model as shown in Fig. 4. In contrast, although the magnitude of the peak flow is greater than the uniform model, the maximum peak flow is only constrained to the outlet on the Scheidegger model. The comparison between the flow distribution of the Scheidegger and the uniform model implies the need to consider the spatial distribution of peak flows inside the drainage network because the spatial distribution of peak flows is directly connected to the risk of the corresponding drainage system.

The Scheidegger and the uniform model are related to the Gibbsian model in that the Scheidegger model can be represented as one extreme of the Gibbsian model when β tends to infinity and the uniform model can be also represented as the other extreme of the Gibbsian model when β tends to zero. The realizations of the Gibbsian model are shown in Fig. 4b–e for the different β values of 10^{-4}, 10^{-2}, 10^{-1}, and 10^0, respectively. The result in Fig. 4 illustrates that the river network becomes less sinuous as β increases. When $\beta = 10^{-4}$, the Gibbsian network is closer to the uniform model, whereas it is closer to the Scheidegger model when $\beta = 10^2$.

The result shows that the uniform model (Fig. 3a) is highly sinuous compared with the Scheidegger model (Fig. 3f). The magnitude of the peak flow at the outlet itself is higher on the Scheidegger model compared to the uniform model (Seo and Schmidt, 2012). However, the results indicate that the maximum peak flows are observed not just at the outlet but also other parts of the mainstream on the uniform model as shown in Fig. 3. In contrast, although the magnitude of the peak flow is greater than the uniform model, the maximum peak flow is only constrained to the outlet on the Scheidegger model. The comparison between the flow distribution of the Scheidegger and the uniform model suggests the need to consider the spatial distribution of peak flows inside the drainage network because the spatial distribution of peak flows is directly related to the risk of the corresponding drainage system (Seo et al., 2014).

Figure 5 shows the bifurcation ratio (R_B) calculated for the simulated networks from the Gibbsian model, the Scheidegger model, and the uniform model. The results shows that the mean R_B tends to increase as β increases for the Gibbsian model; $R_B = 2.14$ for the uniform model, 2.21–3.30 for Gibbs' model, and 2.71 for the Scheidegger model). It should be noted that the bifurcation ratio is maximized for the Gibbsian model when β is equal to 10^2 as shown in Fig. 5. Menabe et al. (2001) investigated the scaling exponent of the peak flow and peak of the width function for a Random Self-similar Channel Network (RSN) with different bifurcation ratios using the linear storage-discharge approximation. They showed that the scaling exponent of the peak flows is similar to that of the width function for the average Shreve model ($R_B = 4$), whereas the scaling exponent of the peak flows behaves different from the peak of the width function for the range of R_B between 4.2 and 4.7. Therefore, the behavior of peak flow and peak of the width function should be assessed separately with care in this study.

Figure 6a compares the averaged width function for each stochastic network model with the actual width function from the test watershed of the Chungju Dam. The Gibbsian models with $\beta = 10^{-2}$ and 10 are given in black dash and grey dot, respectively. The result shows that the actual width function (A) is far from the Scheidegger model (S) or the Gibbsian model with higher beta (10^0) whereas, it is close to the uniform model (U) and the Gibbsian model with lower beta (10^{-2}). Figure 6b depicts the Nash–Sutcliffe model efficiency coefficient (E) between the actual width function and the averaged width function from each stochastic network model. The result indicates that the model efficiency decreases as β increases for the test catchment. When $\beta = 10^{-4}$, the model efficient is to 0.65, but it decreases to -0.08 when $\beta = 10^3$. The result from model efficiency coefficient indicates that the uniform model and the Gibbsian model with lower β ($= 10^{-4}$) generates the closest width function to the actual one.

3.2 Combination of the Gibbsian and the WFIUH for prediction purposes

As mentioned in the introduction, this study aims to combine stochastic network models that reproduces the width function of a watershed and a semi-distributed hydrologic model that directly utilizes the width function and converts it to a runoff hydrograph. In previous section, we examined the ability of the stochastic network models that reproduce the actual width function and also the robustness of the semi-distributed model in application to a test watershed. In this section, we make an attempt to demonstrate that the combination of the two models is able to contribute to the PUB. Once it is demonstrated that the stochastic network model is utilized to reproduce the runoff hydrographs, the subsequent request will be finding a regional characteristics in terms of river network configuration.

As shown in Eq. (9), the width function from the actual river network, $W(x)$ can be easily substituted by $W'(x)$, which is a synthetic width function obtained by simulation results of the Gibbsian model. Due to a stochastic nature of the Gibbsian model, one hundred synthetic width functions were obtained from the simulation of networks. The grey part Fig. 7 shows the upper and lower quartile of the runoff hydrographs obtained from one hundred synthetic width function obtained from the Gibbsian model with different values of β for a rainstorm event on 1 August in 1999 in the test watershed. As shown in the previous section, the Gibbsian model with lower $\beta (= 10^{-4})$ generates the closest width function to the actual one, which is consistent in terms of runoff hydrograph as shown in Fig. 7a. As β increases the runoff hydrographs becomes narrower and the peak grows higher compared to observed flows.

To evaluate how close the averaged runoff hydrograph to the observed hydrographs quantitatively, the Nash–Sutcliffe model efficient coefficient is once more calculated between them for three rainstorm events (Fig. 8). Again, the results indicate consistently that the Gibbsian model with lower β generates the closest runoff hydrographs compared with observed hydrographs. However, for the rainstorm event of 1 August 1999, the result indicates that the model efficiency is highest (0.87) when $\beta = 10^{-3}$, which is different from the case of width function; the Gibbsian with $\beta = 10^{-4}$ produced the closest width function to the actual one. For other two rainstorm events (19 September 1999 and 19 June 2004), the model efficiencies have maximum values of 0.71 and 0.64, respectively when $\beta = 10^{-4}$, which is consistent with the behavior of the synthetic width function. As mentioned earlier, the behavior of the width function and the runoff hydrographs do not coincide with each other unconditionally. In general, it is obvious that the Gibbsian model with lower β produces closest width function as well as resulting runoff hydrographs compared with observation.

3.3 Spatial and temporal variation of rainfall: rainstorm movement

This section additionally introduces synthetic moving storms to evaluate that the suggested approach is able to reproduce runoff hydrographs not just for the case of uniform rainfall but also for the case of moving storms for the test watershed. Once it is evaluated, the impact of spatial and temporal rainfall distribution on runoff hydrographs can be potentially assessed by the suggested approach in ungauged basins even without detailed knowledge of river networks. The hypothetical moving storm's shape is a narrow band, of which width (the lengthscale parallel to the storm direction) is same with the grid size of 4 km and the lengthscale perpendicular to the storm direction is wide enough to cover the entire watershed with a rainfall intensity of 1 mm hr^{-1}.

Figure 9 depicts runoff hydrographs from synthetic width functions of one hundred simulations depending on β for a unit instantaneous rainfall, which is uniform throughout the test watershed. The grey area in Fig. 9 illustrates the range between the lower and upper quartile from the simulation results, whereas the black dot represents the averaged hydrograph compared with the hydrograph from the actual width function (solid). As β increases, the result shows that the peak increases and the shape of hydrographs becomes narrower compared with the actual width function as shown in Fig. 9. Figure 10 illustes the same results for a moving storm, which is moving upstream depending on β. The result shows that the peak is decreased and the rising limb of the

resulting hydrographs starts earlier compared with the uniform rainfall results shown in Fig. 9.

In order to evaluate the proposed approach more quantitatively, Fig. 11 depicts Nash–Sutcliffe efficiency of averaged runoff hydrographs from synthetic width functions compared with the hydrograph of the actual network for a uniform rainfall, and a storm moving south, north, east, and west. Figure 11 additionally shows the ratio of peak flows when synthetic width functions are used (Q_{ps}) and when actual network is used (Q_{pa}). The result shows that the combination of the Gibbsian model and the WFIUH sucessfully regenerates the runoff hydrographs in case of moving storms. For example, Fig. 11d indicates that the Nash–Sutcliffe coefficient is up to 0.98 and the peak ratio is also 0.98 when the Gibbsian model ($\beta = 10^{-4}$) is utilized instead of the real river network to obtain runoff hydrographs. In Fig. 11e, the Nash–Sucliffe (0.87) and the peak ratio (0.70) are lowest for a rainstorm moving west among the moving storm cases, but the suggested approach still captures the main characteristics of runoff hydrographs resulting from temporal and spatial rainfall variation due to storm movement. Therefore, these results imply that the impact of spatial and temporal rainfall distribution on runoff hydrographs can be potentially assessed by the suggested approach in ungauged basins even without detailed knowledge of river networks.

3.4 Sensitivity of the parameters depending on geometry

As mentioned earlier, the width function approach is considerably simpler than the GIUH approach because it emphasizes the metric representation of the basin instead of the topologic one. Moreover, the hydraulic parameters of the WFIUH are physically consistent, while the GIUH velocity parameter lacks physical interpretation (Franchini and O'Connell, 1996). Figure 12 illustrates the changes in Nash–Sucliffe model efficient coefficient (E) between runoff hydrographs using synthetic width functions ($\beta = 10^{-4}$) compared with observed flows for the event on 1 August 1999 depending on the changes of parameter values. The results shows that the runoff hydrographs are more sensitive to the celerity (c) than the diffusion doefficient (D). This study applied the mean channel bottom slope over the watershed to determine the parameter values, but a more detailed analysis is expected in the future to build a general consensus about proper methodology for application of the WFIUH. Moreover, it should be noted that the sensitivity analysis for grid sizes and the process involving effective rainfall are excluded in this study and should be considered in future studies more in detail.

4 Conclusions

In this paper, we suggested an approach that combines a synthetic width function obtained from the stochastic network model and the WFIUH especially for the purpose of flood estimation and direct runoff hydrographs. We applied the suggested approach in a test watershed in South Korea and evaluated the possibility of the suggested approach in ungauged basins for prediction. The original intend of this study is to combine stochastic network models that reproduce the width function of a watershed and a semi-distributed hydrologic model that directly utilizes the width function and converts it to a runoff hydrograph. We demonstrated the ability of the stochastic network models that reproduce the actual width function and also the robustness of the semi-distributed model in application to a test watershed in South Korea.

Additional analysis for moving storm effects revealed that the proposed approach is able to assess the impact of spatial and temporal rainfall distribution on runoff hydrographs even without detailed knowledge of river networks. The proposed approach is beneficial especially in prediction of ungauged basins because the stochastic network model has advantage that it reproduces width functions of actual river networks, whereas the WFIUH has advantages that the parameter values are physically determined, which can be advantageous in prediction of ungauged basins. Once the regional similarity in river network configuration is identified, the suggested approach can be potentially utilized to estimate the runoff hydrographs for ungauged basins.

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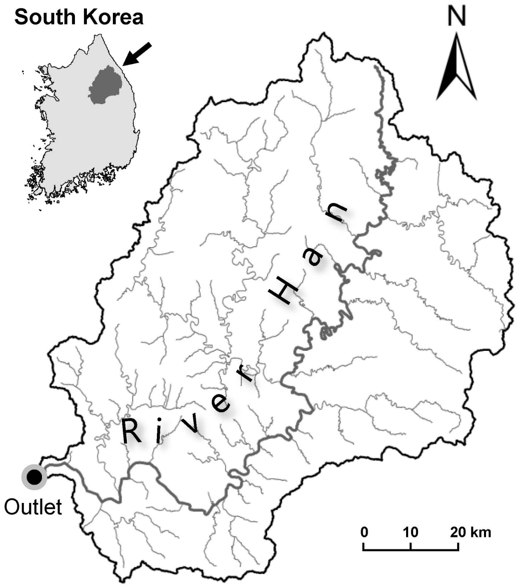

Figure 1. Test watershed (the Chungju Dam watershed) in South Korea.

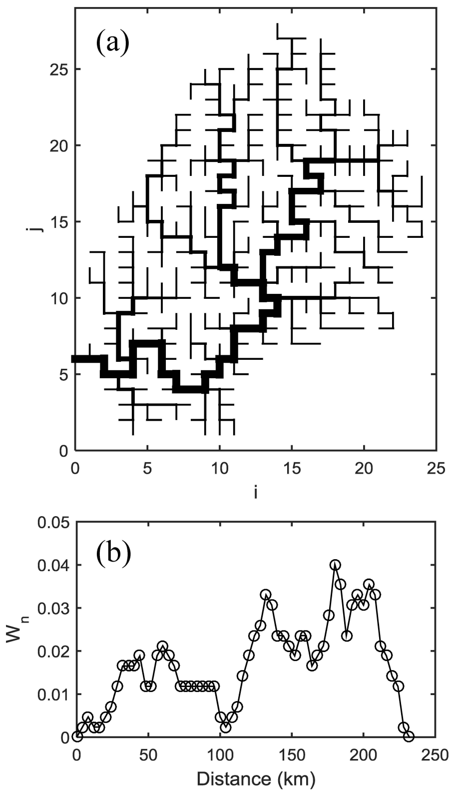

Figure 2. (a) Reconstructed river network on a lattice and **(b)** the corresponding normalized width function of the test watershed.

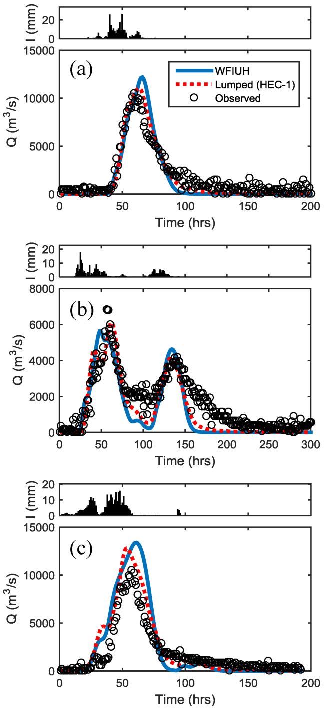

Figure 3. Application of the WFIUH to the test watershed for rainstorm events of **(a)** 1 August 1999, **(b)** 19 September 1999, and **(c)** 19 June 2004.

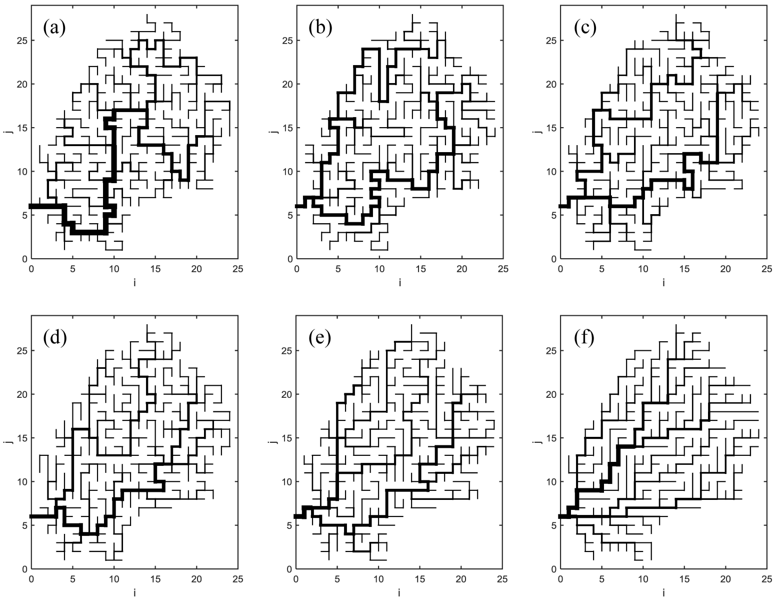

Figure 4. Realization of networks using stochastic network models: **(a)** the uniform model, **(b)** the Gibbsian ($\beta = 10^{-4}$), **(c)** the Gibbsian ($\beta = 10^{-2}$), **(d)** the Gibbsian ($\beta = 10^{-1}$), **(e)** the Gibbsian ($\beta = 10^{0}$), **(f)** the Scheidegger model.

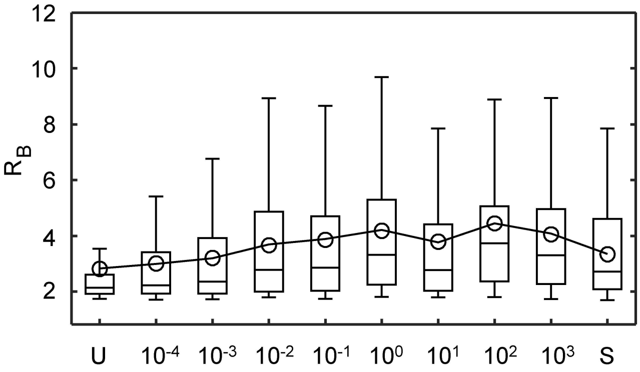

Figure 5. Bifurcation ratio (R_B) of the uniform model (U), the Gibbsian model (with β from 10^{-4} to 10^{3}), and the Scheidegger model (S) simulated for the test watershed.

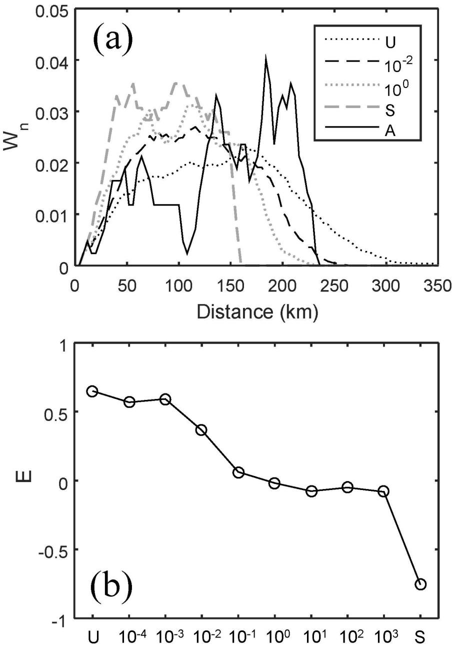

Figure 6. (a) Averaged width functions of the uniform model (U), the Gibbsian model (with β from 10^{-4} to 10^{3}), the Scheidegger model (S), and the actual width function (A) of the test watershed; **(b)** Nash–Sutcliffe efficiency coefficient (E) between actual and averaged width functions.

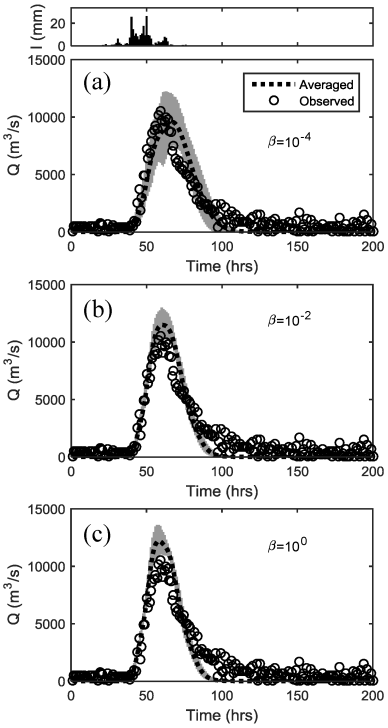

Figure 7. Runoff hydrographs from synthetic width functions of 100 simulations depending on β for the rainfall event of 1 August 1999.

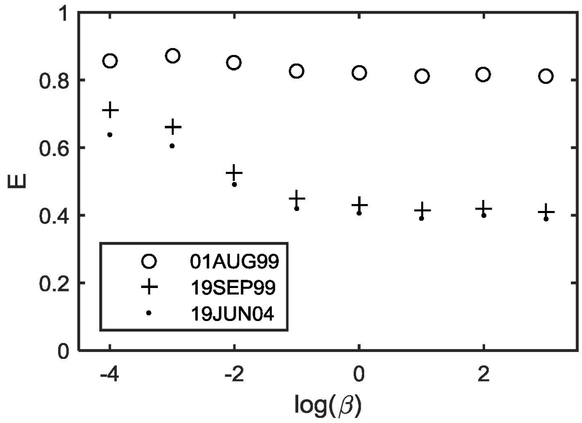

Figure 8. Nash–Sutcliffe efficiency coefficient (E) between observed runoff hydrograph and averaged runoff hydrographs from synthetic width functions for three historical rainfall events.

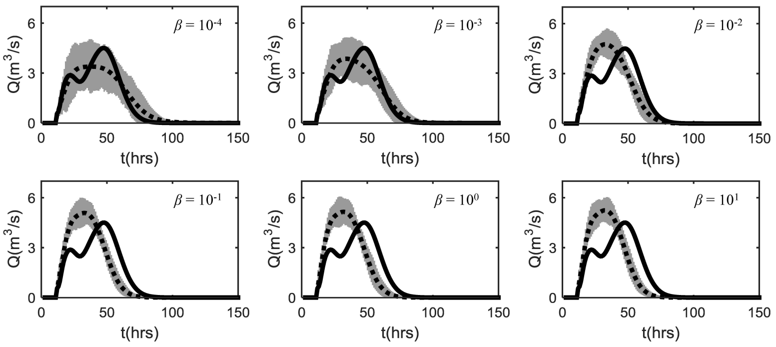

Figure 9. Runoff hydrographs from synthetic width functions of one hundred simulations depending on β for a unit instantaneous rainfall (uniform throughout the watershed).

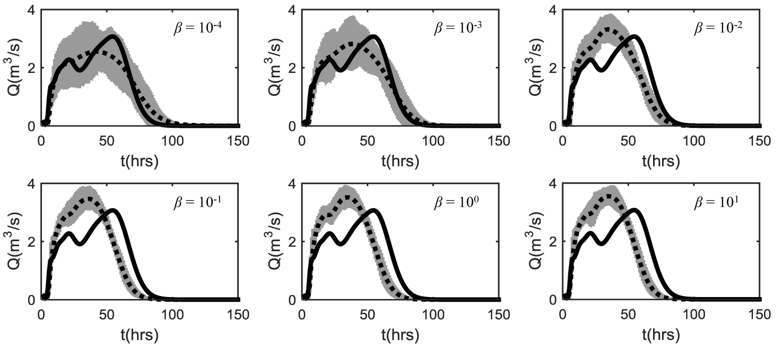

Figure 10. Runoff hydrographs from synthetic width functions depending on β for a rainstorm moving north (a hypothetical storm band moving upstream).

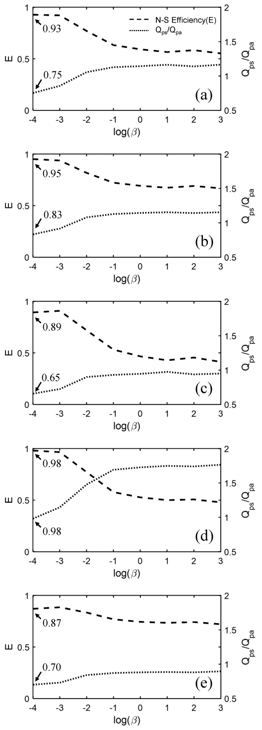

Figure 11. Nash–Sutcliffe efficiency of averaged runoff hydrographs from synthetic width functions compared with the hydrograph of the actual network for **(a)** a uniform rainfall, and a storm moving **(b)** south, **(c)** north, **(d)** east, and **(e)** west.

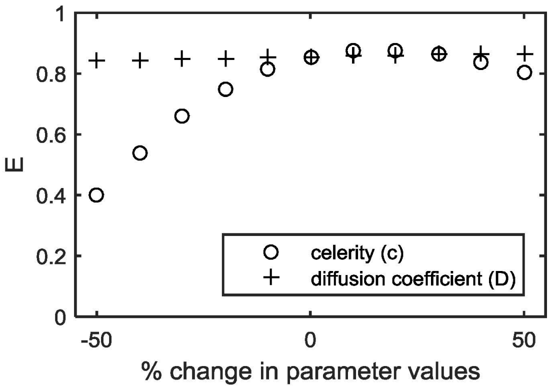

Figure 12. Changes in Nash–Sucliffe model efficient coefficient (E) between runoff hydrographs using synthetic width functions ($\beta = 10^{-4}$) compared with observed flows for the event on 1 August 1999 depending on the changes of parameter values.