



Accounting for
dependencies in
regionalized
signatures

S. Almeida et al.

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Accounting for dependencies in regionalized signatures for predictions in ungauged catchments

S. Almeida^{1,2}, N. Le Vine², N. McIntyre^{2,3}, T. Wagener^{1,4}, and W. Buytaert²

¹Department of Civil Engineering, University of Bristol, Bristol, UK

²Department of Civil and Environmental Engineering, Imperial College London, London, UK

³Centre for Water in the Minerals Industry, Sustainable Minerals Institute, The University of Queensland, Brisbane, Australia

⁴Cabot Institute, University of Bristol, Bristol, UK

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Correspondence to: S. Almeida (susana.almeida@bristol.ac.uk)

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Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures



Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



Abstract

A recurrent problem in hydrology is the absence of streamflow data to calibrate rainfall-runoff models. A commonly used approach in such circumstances conditions model parameters on regionalized response signatures. While several different signatures are often available to be included in this process, an outstanding challenge is the selection of signatures that provide useful and complementary information. Different signatures do not necessarily provide independent information, and this has led to signatures being omitted or included on a subjective basis. This paper presents a method that accounts for the inter-signature error correlation structure so that regional information is neither neglected nor double-counted when multiple signatures are included. Using 84 catchments from the MOPEX database, observed signatures are regressed against physical and climatic catchment attributes. The derived relationships are then utilized to assess the joint probability distribution of the signature regionalization errors that is subsequently used in a Bayesian procedure to condition a rainfall-runoff model. The results show that the consideration of the inter-signature error structure may improve predictions when the error correlations are strong. However, other uncertainties such as model structure and observational error may outweigh the importance of these correlations. Further, these other uncertainties cause some signatures to appear repeatedly to be disinformative.

1 Introduction

In many areas of the world the absence of past observational streamflow time series to calibrate rainfall-runoff models limits the ability to apply such models reliably to predict streamflow and inform effective water resources management. Whilst a large and increasing number of regions across the world are insufficiently gauged (Mishra and Coulibaly, 2009), there are also many highly monitored catchments (Gupta et al., 2014). Transferring the knowledge gained in data-rich areas to ungauged catchments -

HESSD

12, 5389–5426, 2015

Accounting for dependencies in regionalized signatures

S. Almeida et al.

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures



Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



our initial lack of knowledge. We follow Almeida et al. (2013) and express our initial state of indifference in the same space that we seek information about – the signature space. This is different from the common assumption of a uniform distribution on the rainfall-runoff model parameter space (e.g. Yadav et al., 2007), and is shown to avoid the problem of bias in predictions (Almeida et al., 2013). As it is usually not possible to sample directly from the uniform signature prior distributions, an importance sampling is utilized to approximate the distributions numerically (Doucet et al., 2000).

2.2.2 Likelihood function approximation

The likelihood functions are defined using joint distributions of respective signature errors obtained from the regionalization model. Errors introduced by the regionalization procedure may come from at least five sources. First, errors are introduced by the fact that the regression model is estimated using a specific sample of catchments rather than the entire population; second, differences may exist between the observed and the true value of the response signature due, for example, to factors such as the discharge record length and time period of record used in the computation (Kennard et al., 2010); third, errors are present due to errors in the catchment properties data; fourth, errors exist due to the incomplete set of catchment properties used as explanatory variables in the regression equations; and, fifth, they exist due to the assumed linear regression structure. It is assumed that the total error model for the regionalized signature(s) s^* can be estimated using the following procedure:

1. Considering all available gauged catchments stepwise regression is applied to each signature independently to determine the predictors to include. The predictors are then fixed for the remaining steps.
2. Considering all available gauged catchments, one catchment is left out and the remaining are used in the fitting of the regression models for each signature.

Accounting for dependencies in regionalized signatures

S. Almeida et al.

[Title Page](#)

[Abstract](#)

[Introduction](#)

[Conclusions](#)

[References](#)

[Tables](#)

[Figures](#)



[Back](#)

[Close](#)

[Full Screen / Esc](#)

[Printer-friendly Version](#)

[Interactive Discussion](#)



3. The regression models obtained in Step 2 are used to estimate the signature values for the omitted catchment.
4. The error for each signature is calculated for the omitted catchment by comparing the regionalized and observed signature values.
5. The process is repeated for all catchments.
6. A parametric joint probability distribution is fitted to all the computed errors. Furthermore, the errors are tested for independence that allows (approximately) factorizing a joint distribution into a product of marginal distributions.

The resultant error distribution defines the likelihood function L in Eq. (2). The main assumption here is that the potentially complex nature of errors in the set of signature values can be usefully represented by the fitted error distributions.

2.2.3 Synthetic case and likelihood functions

To avoid masking the potential value of the regionalized signatures with model structure and observational errors, a “perfect model” is first employed. This involves using the pre-selected rainfall-runoff model and the observed forcing data to generate the “observed” catchment signatures. The Nash–Sutcliffe criteria (NSE) (Nash and Sutcliffe, 1970) optimal parameter set is taken to generate a “perfect model” streamflow time series for each catchment. To produce regionalized signature analogues in this case, two types of imposed errors are introduced to these “observed” signatures. The first error type is characterized by a range of standard deviations (1, 5, 10 and 20 % of the signature value range observed over all catchments used in this study) and a range of inter-signature error correlations (Pearson correlation coefficients equal to 0, 0.25, 0.50, 0.75 and 0.90). This allows the sensitivity of the results to the regionalization quality and the regionalization errors’ correlations to be evaluated. The second error type is set to be equal to the observation-based likelihood function (Sect. 2.2.2). These

nature errors are largely uncorrelated, whilst others are strongly correlated (see also Sect. 3.1).

RR reflects the amount of precipitation that becomes streamflow over a certain area and time. It is determined as the ratio of catchment's outlet streamflow and catchment average precipitation over the 10 years used in this study. BFI gives the proportion of streamflow that is considered to be base flow. A simple one-parameter single-pass digital filter method is used to derive BFI (Arnold and Allen, 1999). SE provides a measure of the sensitivity of streamflow to changes in precipitation (Sankarasubramanian et al., 2001). It is calculated as a median of the inter-annual variation in total annual streamflow to the inter-annual variation in total annual precipitation ratios normalized by the long-term runoff ratio (Sawicz et al., 2011; Sankarasubramanian et al., 2001). SFDC gives an indication of the streamflow variability and is calculated as the slope of the flow duration curve between the 33 and 66 % flow exceedance values in a semi-log scale (Sawicz et al., 2011). HPC reflects aspects of the high flow regime and catchment flashiness, and is calculated as the average number of events per year that exceed three times the median daily flow (Clausen and Biggs, 2000; Yadav et al., 2007).

2.3.3 Rainfall-runoff model choice

The probability distributed moisture (PDM) model (Moore, 2007) together with two parallel linear routing stores and a simple snow model (Hock, 2003) is selected with two major motivations (for a detailed description of the model see Kollat et al., 2012; Almeida, 2014). First, this type of model has been shown to have a suitable complexity for modelling daily rainfall-runoff over a large sample of the MOPEX catchments (Wagener and McIntyre, 2012). Second, the model has been successfully applied in other regionalization studies across a wide range of climate and physiographic conditions, for example Calver et al. (1999), Lamb and Kay (2004), McIntyre et al. (2005), Young (2006), and De Vleeschouwer and Pauwels (2013). Even though other model structures may be better suited for some specific catchments, it is prohibitively difficult to vary model structure between catchments and no single model structure will ever

Accounting for dependencies in regionalized signatures

S. Almeida et al.

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures



Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



be best for all catchments (Lidén and Harlin, 2000; Clark et al., 2008; van Werkhoven et al., 2008). Consequently, the selected model structure is believed to be a sufficient choice for the purposes of this paper. Most importantly, the general framework is independent of the rainfall-runoff model choice.

2.4 Posterior distribution and performance assessment

Employing Bayes' law (Eq. 2), the rainfall-runoff model is conditioned on different combinations of signatures: (1) assuming independence between the signature regionalization errors (setting the correlation values to zero in the joint probability function); and (2) accounting for the inter-signature error correlations (using the estimated covariance in the joint probability function).

Two metrics are used to assess the effectiveness of the parameter conditioning procedure: (1) the Bayes factor (Jeffreys, 1961) to assess convergence of the parameter posteriors to known parameter values; (2) the probabilistic Nash–Sutcliffe efficiency (Bulygina et al., 2009) to assess convergence of the flow ensembles to the observed flows.

The Bayes factor BF is defined as the ratio between two marginal distributions of the data \mathbf{y} (e.g. observed streamflow time series) for two competing hypotheses (H_1 and H_2) (Kass and Raftery, 1995) (more detail is given in Appendix A):

$$\text{BF} = \frac{\rho(\mathbf{y}|H_1)}{\rho(\mathbf{y}|H_2)} \quad (3)$$

Thus, to test the impact of representing the error correlations, the hypothesis H_1 corresponds to the inter-signature errors being treated as correlated, while the hypothesis H_2 corresponds to the inter-signature errors assumed to be independent. If the resulting Bayes factor is greater than 1, there is more support for hypothesis H_1 , and the inter-signature error correlation is worth considering.

When using synthetic streamflow data (“perfect model” approach), with the streamflow time series generated by a pre-selected parameter set, $\rho(\mathbf{y}|H)$ in Eq. (3) can be

distributions, shown on the Fig. 1 diagonal, are approximated using histograms, and parameters of normal distributions are fitted using the method of moments. The univariate Kolmogorov-Smirnov test shows that the marginal distribution normality cannot be rejected at the 95 % confidence level for each of the five signatures. The off-diagonal shows the regionalization errors for different signature pairs (lower off-diagonal), the corresponding correlation coefficient values and their statistical significance (upper off-diagonal). The joint error distributions are approximated using multivariate normal distributions that are fitted using estimates of the marginal normal distribution parameters and the inter-signature error correlations. These marginal and joint distributions define the likelihood functions in Eq. (2). Note that Fig. 1 represents the regionalization errors based on all 84 catchments. Meanwhile, the jack-knife procedure (see Sect. 2.4) utilized in the performance assessment employs only 83 catchments at a time.

3.2 The impact of inter-signature error correlations (Pairs of signatures)

This section considers the role of inter-signature error correlation on model parameter estimation when pairs of signatures are used. First, different imposed error variances and correlations together with synthetic streamflow data are employed to test the impact of inter-signature error correlation without the impact of model structural error. Then, the results obtained using the observation-based error structure, for both synthetic and observed data streamflow, are analyzed.

3.2.1 Synthetic streamflow data (Imposed likelihoods)

Synthetic streamflow data are generated as described in Sect. 2.2.3, and the imposed likelihood functions are defined as described in Sect. 2.2.3. The imposed likelihoods are considered to have standard deviations equal to 1, 5, 10, 20 % of the signature value range observed over all catchments. A comparison of the imposed error structures under the different levels of variance and the observed error structure is given in

Accounting for dependencies in regionalized signatures

S. Almeida et al.

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures



Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



Table 2. Furthermore, different inter-signature error correlations are also tested, namely 0 (linear independence), 0.25, 0.50, 0.75 and 0.90.

Ten possible pairs of the five response signatures are used in parameter conditioning, and the median Bayes factor, calculated over the 84 MOPEX catchments, is calculated for each pair. The Bayes factor (Eq. 3) compares the two following hypothesis: H_1) the inter-signature error correlation is to be taken into account, and H_2) the errors between the different sources of information can be assumed independent. The Bayes factor is found to be relatively insensitive to the selection of response signature pairs (Kruskal–Wallis test). Table 3 summarizes the 95 % pooled confidence intervals for the median Bayes factor across all catchments and across all 10 signature pairs, for each choice of the likelihood (i.e. 20 likelihoods). This provides reference values indicative of the error interdependency importance in model regionalization depending on the signature pair correlations and marginal distribution variances. As it would be expected, the median Bayes factor is equal to 1 when signatures errors are not correlated (i.e. $\rho = 0$). However, as correlations between signatures errors increase the median Bayes factor increases noticeably. This suggests that considering error correlations allocates higher likelihoods to parameter sets that capture a considered signature pair. Furthermore, the results shown in Table 3 also imply that the median Bayes factor is relatively insensitive to the precision with which the signatures are regionalized.

3.2.2 Synthetic and observed streamflow data (Observation-based likelihoods)

Figure 2 shows the distribution of the Bayes factor values obtained across the 84 catchments for each of the 10 possible different pairs of signatures, when the observation-based error structure is used for each catchment. Figure 2a shows the results for the observed streamflow data with regionalized signatures calculated from the derived regressions; Fig. 2b shows the results for the synthetic streamflow data with regionalized signatures calculated by adding noise to the exact signature values. The Tukey boxplots in red correspond to pairs of signatures whose errors are statistically significantly correlated (see Fig. 1). The upper whisker represents the upper quartile plus one and

Accounting for dependencies in regionalized signatures

S. Almeida et al.

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures



Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



a half times the interquartile range, and the lower whisker represents the lower quartile minus one and a half times the interquartile range. The matrix below Fig. 2b shows the pairs of signatures used.

The signature pair [SFDC, HPC] shows the strongest correlation between errors ($\rho = 0.65$, Fig. 1). A likelihood function with a standard deviation equal to 10% of the observed signature ranges and $\rho = 0.75$ in Table 3 is comparable to the observation-based likelihood of the pair [SFDC, HPC] (Table 2), with Table 3 indicating [1.45, 1.53] as a 95% confidence interval for the median Bayes factor. However, a median Bayes factor of 2.17 is obtained for the observed streamflow data (Fig. 2a). Similar differences are found for the other pairs of signatures, although the comparison with the reference table (Table 3) becomes challenging, as the individual signatures have not been regionalized necessarily with similar quality. On the other hand, Fig. 2b shows that the Bayes factors for the synthetic study (when there is no model structural error) are consistent with the values provided in the look-up Table 3. The difference between the median Bayes factor for the two cases is likely to be caused by the model structure error, or may be related to the location of the NSE-optimal in the parameter space.

Nevertheless, it is clear from Fig. 2 that those pairs of signatures whose errors are significantly correlated (i.e. [SFDC, HPC], [BFI, HPC], [BFI, SFDC] and [BFI, SE]) have wider interquartile ranges. Furthermore, the pair of signatures with the strongest correlation between errors [SFDC, HPC] presents the greatest interquartile range. Therefore the inclusion of significant correlations in the likelihood function matters, but whether or not it is beneficial to conditioning the parameters seems to depend on the interplay between model structure error, parameter space and likelihood function. Only strong correlations (as in the [SFDC, HPC] case) can be expected to result in a median Bayes factor clearly above 1.

HESSD

12, 5389–5426, 2015

Accounting for dependencies in regionalized signatures

S. Almeida et al.

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures



Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



3.3 The impact of inter-signature error correlations (Multiple signatures)

Multiple signatures are used for parameter constraining and flow prediction. The information value of multiple signatures and its dependence on inter-signature error correlations is explored in this section.

3.3.1 Synthetic streamflow data (Observation-based likelihood)

Figure 3 shows Bayes factors derived for the synthetic streamflow data (generated using the NSE-optimal parameter set) when the observation-based likelihood is used. The Bayes factor considers $p(\cdot|H_2)$ to be the prior parameter distribution, and $p(\cdot|H_1)$ to be one of the parameter posteriors that includes or ignores the inter-signature error correlations. Figure 3 summarizes the variability in the Bayes factor for the different combinations of signatures for all 84 catchments. Boxplots are color coded by the total number of signatures combined, when the inter-signatures error correlation is considered in the likelihood function definition. The grey dashed boxplots correspond to the results obtained assuming that the inter-signature errors are independent when defining the likelihood function. Although the colored boxplots visually seem to have higher values than the grey dashed boxplots, these differences are not statistically significant at a 95 % confidence level (Kolmogorov–Smirnov two-sided tests).

To better evaluate whether the incorporation of additional sources of information improves parameter identification, one-sided Kolmogorov–Smirnov tests are applied between any combination of certain signatures (e.g. [SE, SFDC]) and any other combination that contains the same signatures and a new one (e.g. [SE, SFDC, HPC]). It is found that adding more signatures improves parameter identification in 82.5 % of the cases (66 out of 80 cases) at a 95 % confidence level).

Figure 4 summarizes the variability in the analog Nash–Sutcliffe efficiency measure NSEprob for different combinations of signatures for all 84 catchments. The colored boxplots correspond to the results obtained when the inter-signature error correlations are considered in the likelihood definition, and the grey dashed boxplots correspond to

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures



Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



HESSD

12, 5389–5426, 2015

Accounting for dependencies in regionalized signatures

S. Almeida et al.

[Title Page](#)

[Abstract](#)

[Introduction](#)

[Conclusions](#)

[References](#)

[Tables](#)

[Figures](#)



[Back](#)

[Close](#)

[Full Screen / Esc](#)

[Printer-friendly Version](#)

[Interactive Discussion](#)



dent information. Although the proposed framework can be applied to any number of signatures, the limited sample size (i.e. number of gauged catchments available) can have an impact on the definition of the likelihood distribution. For this specific study 83 samples were available to define that distribution. When a single response signature is used to condition the hydrological model this sample size is likely to be sufficient to confidently judge whether the normal distribution assumption is sufficient. However, when moving to multidimensional problems, in which various signatures may be used simultaneously to condition the hydrological model, it is increasingly difficult to judge the adequacy of any multivariate parametric distribution and to judge which catchments are outliers. This implies that as more signatures are used simultaneously in the conditioning of the hydrological model, the more gauged catchments should be used to define the likelihood function. As stressed by Gupta et al. (2014), large samples are of great importance to support statistical regionalization of uncertainty estimates, and this is particularly the case if dependencies between information sources are to be specified.

While the work presented in this paper addresses a number of issues associated with model regionalization, it is important to highlight some additional areas for future research. An important source of uncertainty comes from model structure error (Gupta et al., 1998; Kuczera et al., 2006). The conditioning framework suggested here is independent of the selected model, and, in principle, Figs. 5 and 6 could be created by using the model structure that is considered suitable for each catchment rather than using a model structure that we consider good for generalizing. Further research is needed to diagnose the relative importance of different model structures in various climate regimes and for different catchment characteristics (Clark et al., 2008; Hrachowitz et al., 2013). This is crucial to both identifying the most appropriate model structure for an ungauged location and quantifying the uncertainty in the model structure that should be integrated into the likelihood, thus allowing virtually any model choice. Similarly, other sources of uncertainty, namely observational error (e.g. rainfall error), should ideally be evaluated and integrated into the likelihood function. By accounting for all the

from a set of uncertain signatures and determining how many and which signatures should be used given their error dependencies has not been extensively studied.

The method suggested in this paper allows the specification of a signature error structure. A common reason for not including large numbers of signatures in regionalization studies is the potential for under-estimation of uncertainty due to duplication of information. This study helps to justify the inclusion of larger sets of signatures in the regionalization procedure if their error correlations are formally accounted for and thus enables a more complete use of all available information. The results show that adding response signatures to constrain the hydrological model, while accounting for inter-signature error correlations, can contribute to a stronger identification of the optimum parameter set when the error correlations between different sources of information are strong. Furthermore, the results show that assuming independency of errors does not result in significant deterioration in model performance, unless the error correlation is very strong. The results also show that the effect of error correlations is likely to be overwhelmed by model structure and observation errors. The method suggested here can therefore become more relevant if observational and structural errors are reduced. In addition, it is illustrated that using more signatures, with and without considering their error correlations, may lead to deterioration in performance. In our case, there were particular problems when adding the slope of the flow duration curve and/or the high pulse count. As this is likely to be specific to the rainfall-runoff model used, the selected performance criteria and the set of catchments, it is recommended that the disinformative information sources are identified as part of any regionalization study, in a similar manner as has been done here.

Appendix A: The Bayes factor

When evaluating the impact of inter-signature error correlations on model parameter identification, results are assessed in terms of Bayes factor (Jeffreys, 1961). This form of assessment is preferred to the most commonly used QQ plots (Laio and Tamea,

Accounting for dependencies in regionalized signatures

S. Almeida et al.

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures



Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



Accounting for dependencies in regionalized signatures

S. Almeida et al.

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

⏪

⏩

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



2007), due to the particular nature of the problem under analysis. When signature(s) (either regionalized for the case of an ungauged catchment, or derived from actual observations for the case of gauged catchments) is employed to reduce uncertainty beyond what is possible by defining the priors on model parameters, QQ plots may not be the most effective form of assessment. Although response signatures are measures of theoretically relevant system process behaviors (Gupta et al., 2008; Wagener et al., 2007), they reflect fragmented knowledge as different signatures capture different catchment processes. Consequently, the quantiles of observed flows are not conditioned to follow a uniform distribution, as QQ plots assess. Rather, quantiles of response signatures should follow this condition (for all catchments considered – Almeida et al., 2013). Therefore, an alternative performance measure that more adequately reflects the aim of this particular application (i.e. the reproduction of certain aspects of the hydrograph) is used. The Bayes factor BF is particularly relevant in the current context as it allows comparison of predictions based on two competing theories (Jeffreys, 1961). It is defined as the ratio between the marginal distributions of the data \mathbf{y} for the two hypotheses (H_1 and H_2) being compared (Kass and Raftery, 1995):

$$\text{BF} = \frac{\rho(\mathbf{y}|H_1)}{\rho(\mathbf{y}|H_2)} \quad (\text{A1})$$

When the two hypotheses are equally likely a priori, the Bayes factor is the posterior odds in favor of H_1 (Kass and Raftery, 1995). In other words, a value of BF greater than 1 means that H_1 is more strongly supported by the data than H_2 . For example, a Bayes factor equal to 2 implies that H_1 is favored over H_2 with 2 : 1 odds given the evidence provided by the data.

For a given hypothesis H , parameterized by model parameter set Θ , the marginal density $\rho(\mathbf{y}|H)$ represents the likelihood of the data and it is given by

$$\rho(\mathbf{y}|H) = \int \rho(\mathbf{y}|\Theta, H) \rho(\Theta|H) d\Theta \quad (\text{A2})$$

where $p(\mathbf{y}|\Theta, H)$ is the conditional density function given parameters Θ under hypothesis H and $p(\Theta|H)$ is the distribution of parameters under H . Hypothesis H may represent different model and parameter distributions. In this paper, the same model structure is considered. However, different parameter distributions are used in Eq. (A2) to enable prediction comparison based on two theories about parameter distributions.

The above integral can be numerically approximated as,

$$\int p(\mathbf{y}|\Theta, H)p(\Theta|H)d\Theta \approx \frac{1}{N} \sum_{i=1}^N p(\mathbf{y}|\Theta^{(i)}, H)p(\Theta^{(i)}|H) \quad (\text{A3})$$

where $\Theta^{(i)}$ is the i th of N draws from $p(\cdot|\Theta)$, and N is the size of the Monte Carlo sample (in this paper N is equal to 10 000).

In a “perfect model” study, data \mathbf{y} are generated by a model with parameter set Θ^* , so that there is no model structural or observational error. This means that $p(\mathbf{y}|\Theta^{(i)}, H)$ is always equal to zero, except when $\Theta^{(i)} = \Theta^*$. Mathematically this is expressed as $p(\mathbf{y}|\Theta^{(i)}, H) = \delta_{\Theta^{(i)}=\Theta^*}$, where δ is the Dirac delta function. Therefore Eq. (A3) is equal to $1/N$ times $p(\Theta^{(i)} = \Theta^*|H)$ and the Bayes factor is given by

$$\text{BF} = \frac{\frac{1}{N} \sum_{i=1}^N \delta_{\Theta^{(i)}=\Theta^*} p(\Theta^{(i)}|H_1)}{\frac{1}{N} \sum_{i=1}^N \delta_{\Theta^{(i)}=\Theta^*} p(\Theta^{(i)}|H_2)} = \frac{p(\Theta^{(i)} = \Theta^*|H_1)}{p(\Theta^{(i)} = \Theta^*|H_2)} \quad (\text{A4})$$

While other choices can be made, two cases are considered in this paper. First, the two distributions in Eq. (A4) are posterior distributions, but with different assumptions about the likelihood functions. Given that we are particularly interested in evaluating the impact of considering the inter-signature error correlations versus ignoring them, H_1 will correspond to the joint likelihood defined such that inter-signature error correlations are considered, while H_2 corresponds to the likelihood when inter-signature error correlations are ignored. For the Bayes factor defined in this way, a value greater than 1 supports the idea that considering inter-signature error correlations contributes to an

Accounting for dependencies in regionalized signatures

S. Almeida et al.

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

⏪

⏩

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



Accounting for dependencies in regionalized signatures

S. Almeida et al.

[Title Page](#)

[Abstract](#)

[Introduction](#)

[Conclusions](#)

[References](#)

[Tables](#)

[Figures](#)

[⏪](#)

[⏩](#)

[◀](#)

[▶](#)

[Back](#)

[Close](#)

[Full Screen / Esc](#)

[Printer-friendly Version](#)

[Interactive Discussion](#)



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Accounting for dependencies in regionalized signatures

S. Almeida et al.

[Title Page](#)

[Abstract](#)

[Introduction](#)

[Conclusions](#)

[References](#)

[Tables](#)

[Figures](#)

[⏪](#)

[⏩](#)

[◀](#)

[▶](#)

[Back](#)

[Close](#)

[Full Screen / Esc](#)

[Printer-friendly Version](#)

[Interactive Discussion](#)



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Accounting for dependencies in regionalized signatures

S. Almeida et al.

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

⏪

⏩

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



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Accounting for dependencies in regionalized signatures

S. Almeida et al.

[Title Page](#)

[Abstract](#)

[Introduction](#)

[Conclusions](#)

[References](#)

[Tables](#)

[Figures](#)

[⏪](#)

[⏩](#)

[◀](#)

[▶](#)

[Back](#)

[Close](#)

[Full Screen / Esc](#)

[Printer-friendly Version](#)

[Interactive Discussion](#)



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HESSD

12, 5389–5426, 2015

Accounting for dependencies in regionalized signatures

S. Almeida et al.

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures



Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



HESSD

12, 5389–5426, 2015

Accounting for dependencies in regionalized signatures

S. Almeida et al.

Table 2. Tested variance values for the data-based and imposed error structures.

	Observed error structure	1 % observed signature ranges	5 % observed signature ranges	10 % observed signature ranges	20 % observed signature ranges
RR residuals	0.054 ²	0.005 ²	0.027 ²	0.055 ²	0.109 ²
BFI residuals	0.044 ²	0.006 ²	0.030 ²	0.060 ²	0.121 ²
SE residuals	0.635 ²	0.023 ²	0.116 ²	0.232 ²	0.464 ²
SFDC residuals	0.006 ²	0.0005 ²	0.002 ²	0.005 ²	0.010 ²
HPC residuals	10.687 ²	0.977 ²	4.883 ²	9.767 ²	19.533 ²

[Title Page](#)[Abstract](#)[Introduction](#)[Conclusions](#)[References](#)[Tables](#)[Figures](#)[Back](#)[Close](#)[Full Screen / Esc](#)[Printer-friendly Version](#)[Interactive Discussion](#)

Accounting for dependencies in regionalized signatures

S. Almeida et al.

Table 3. Reference table showing the 95% confidence interval for the median Bayes factor. The correlation coefficient ρ and the standard deviation of the marginal distributions σ are shown.

		σ			
		1%	5%	10%	20%
ρ	0	1	1	1	1
	0.25	1.01–1.03	1.03–1.04	1.02–1.04	1.04–1.05
	0.50	1.09–1.15	1.16–1.19	1.14–1.17	1.14–1.18
	0.75	1.41–1.51	1.50–1.57	1.45–1.53	1.40–1.49
	0.90	1.94–2.11	2.11–2.32	2.12–2.26	2.20–2.34

[Title Page](#)
[Abstract](#)
[Introduction](#)
[Conclusions](#)
[References](#)
[Tables](#)
[Figures](#)

[Back](#)
[Close](#)
[Full Screen / Esc](#)
[Printer-friendly Version](#)
[Interactive Discussion](#)


Accounting for dependencies in regionalized signatures

S. Almeida et al.

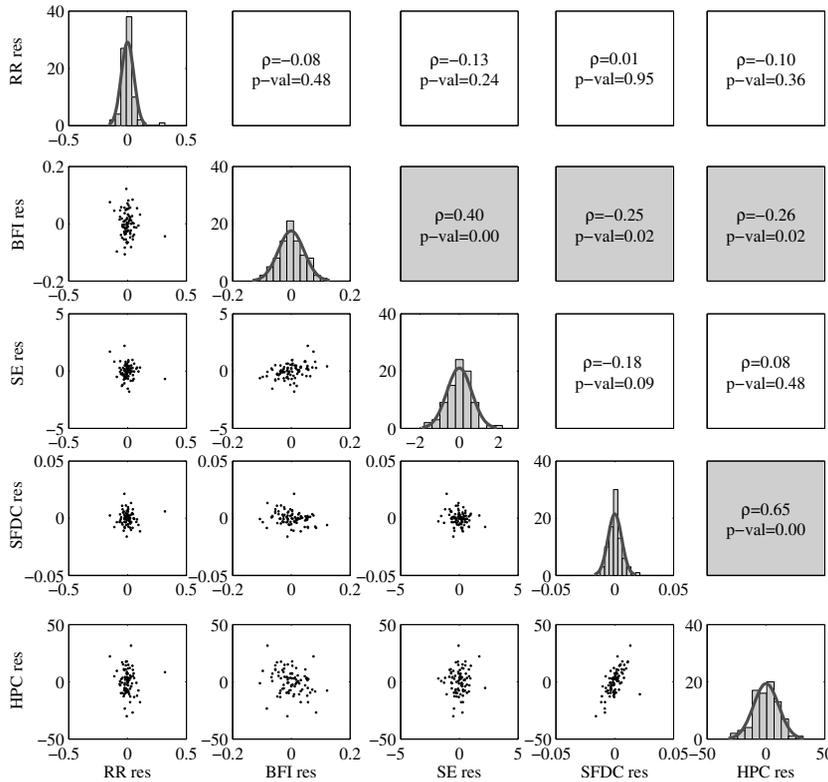


Figure 1. Distribution of individual signature residuals (res) are approximated as histograms and normal distributions. The scatterplots and correlation coefficients (ρ) show correlation between the signature residuals.

[Title Page](#)
[Abstract](#) [Introduction](#)
[Conclusions](#) [References](#)
[Tables](#) [Figures](#)
⏪ ⏩
◀ ▶
[Back](#) [Close](#)
[Full Screen / Esc](#)
[Printer-friendly Version](#)
[Interactive Discussion](#)



Accounting for dependencies in regionalized signatures

S. Almeida et al.

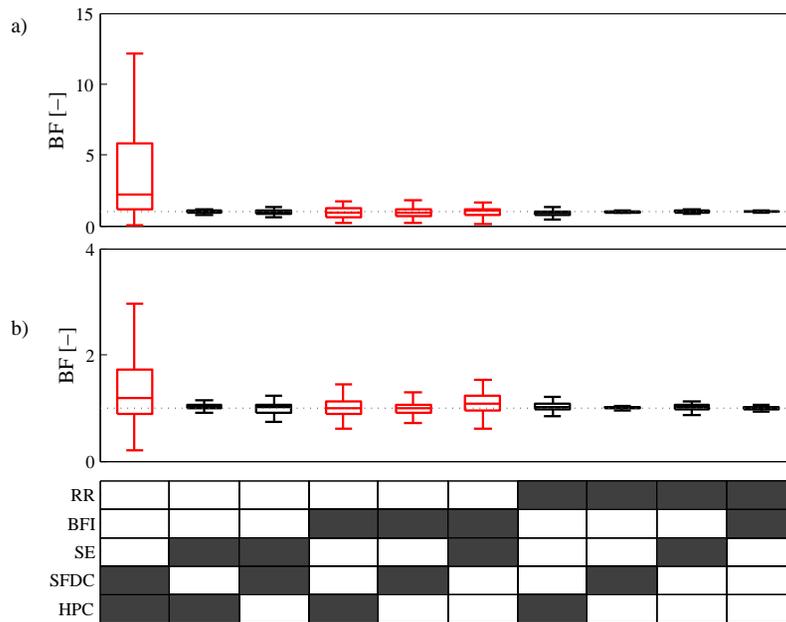


Figure 2. The Bayes factor for the 10 pairs of signatures over the 84 catchments when the observation-based error structure is used with **(a)** observed streamflow data, **(b)** synthetic streamflow data. The upper whisker represents the upper quartile plus one and a half times the interquartile range, and the lower whisker represents the lower quartile minus one and a half times the interquartile range. The dashed line represents $BF = 1$.

[Title Page](#)

[Abstract](#)

[Introduction](#)

[Conclusions](#)

[References](#)

[Tables](#)

[Figures](#)

[⏪](#)

[⏩](#)

[⏴](#)

[⏵](#)

[Back](#)

[Close](#)

[Full Screen / Esc](#)

[Printer-friendly Version](#)

[Interactive Discussion](#)



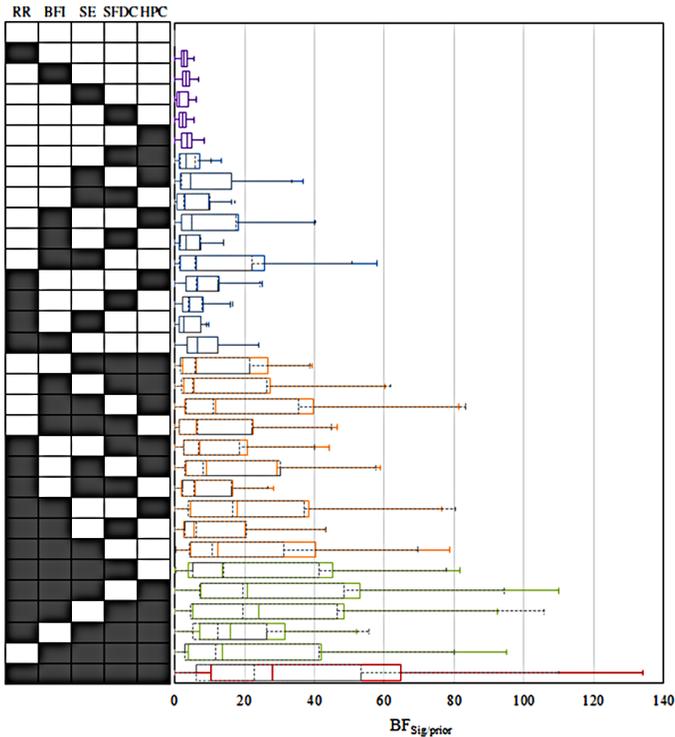


Figure 3. Boxplots representing the distribution of the Bayes factor for each combination of signatures for synthetic streamflow data. The colored boxplots correspond to the results obtained when inter-signature error correlations are considered in the likelihood function, whereas the grey dashed boxplots correspond to the results obtained assuming that the inter-signature errors are independent.

Accounting for dependencies in regionalized signatures

S. Almeida et al.

[Title Page](#)
[Abstract](#) [Introduction](#)
[Conclusions](#) [References](#)
[Tables](#) [Figures](#)
[◀](#) [▶](#)
[◀](#) [▶](#)
[Back](#) [Close](#)
[Full Screen / Esc](#)
[Printer-friendly Version](#)
[Interactive Discussion](#)



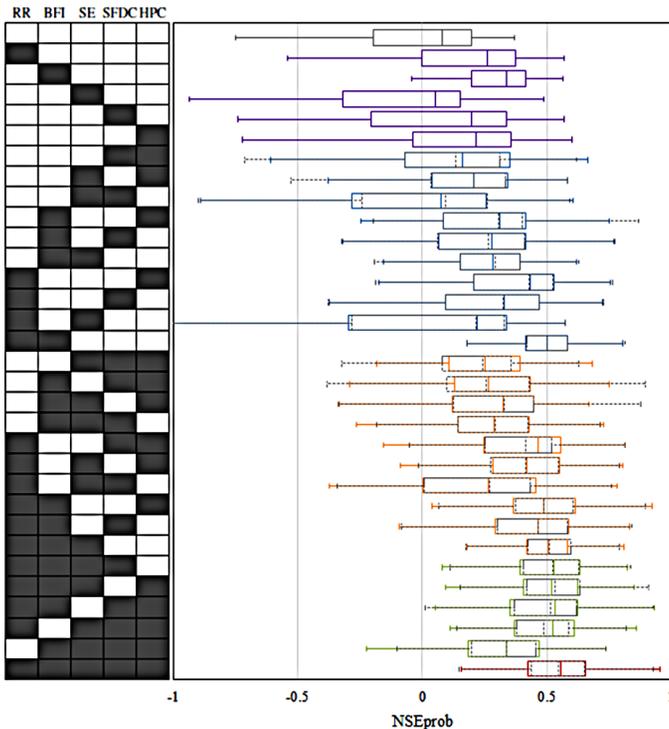


Figure 4. Boxplots representing the distribution of NSEprob values for each combination of signatures for synthetic streamflow data. The colored boxplots correspond to the results obtained when inter-signature error correlations are considered in the likelihood function, whereas the grey dashed boxplots correspond to the results obtained assuming that the inter-signature errors are independent.

Accounting for dependencies in regionalized signatures

S. Almeida et al.

[Title Page](#)

[Abstract](#)

[Introduction](#)

[Conclusions](#)

[References](#)

[Tables](#)

[Figures](#)

[⏪](#)

[⏩](#)

[◀](#)

[▶](#)

[Back](#)

[Close](#)

[Full Screen / Esc](#)

[Printer-friendly Version](#)

[Interactive Discussion](#)



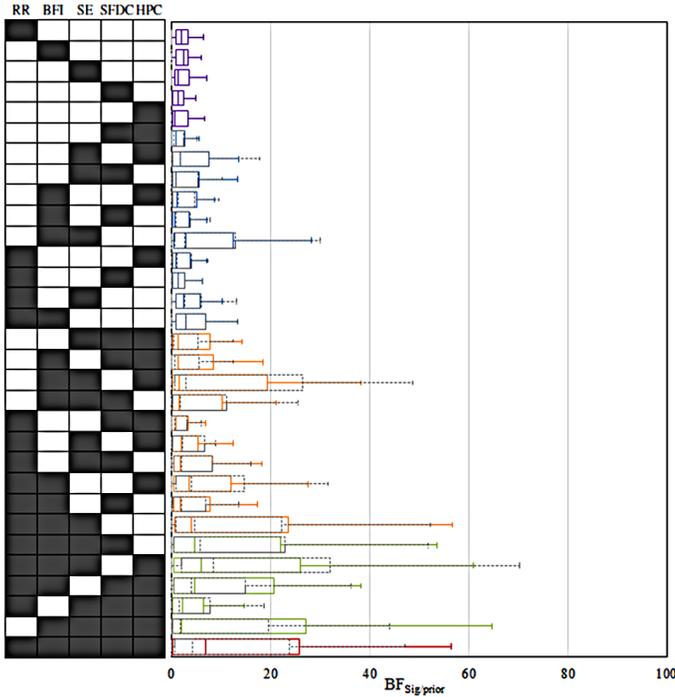


Figure 5. Boxplots representing the distribution of the Bayes factor for each combination of signatures for observed streamflow data. The colored boxplots correspond to the results obtained when inter-signature error correlations are considered in the likelihood function, whereas the grey dashed boxplots correspond to the results obtained assuming that the inter-signature errors are independent.

Accounting for dependencies in regionalized signatures

S. Almeida et al.

[Title Page](#)

[Abstract](#)

[Introduction](#)

[Conclusions](#)

[References](#)

[Tables](#)

[Figures](#)

[⏪](#)

[⏩](#)

[◀](#)

[▶](#)

[Back](#)

[Close](#)

[Full Screen / Esc](#)

[Printer-friendly Version](#)

[Interactive Discussion](#)



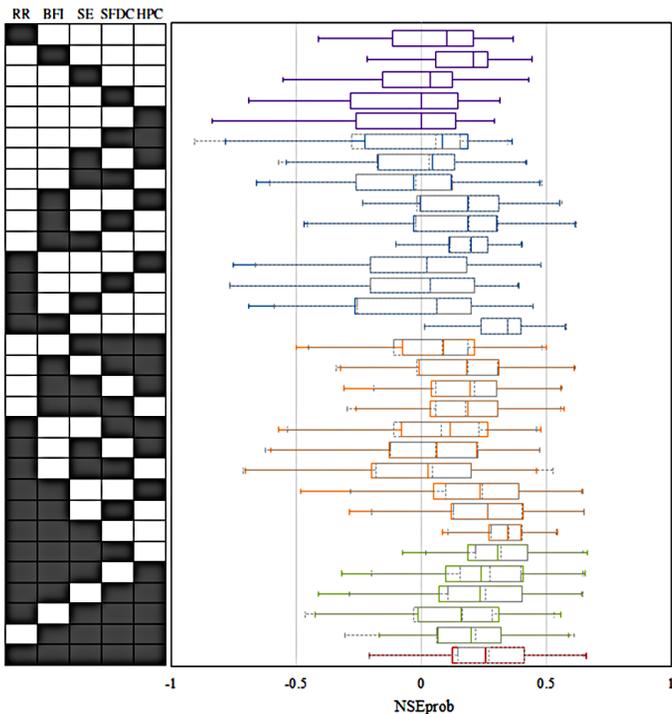


Figure 6. Boxplots representing the distribution of NSEprob values for each combination of signatures for observed streamflow data. The colored boxplots correspond to the results obtained when inter-signature error correlations are considered in the likelihood function, whereas the grey dashed boxplots correspond to the results obtained assuming that the inter-signature errors are independent.

Accounting for dependencies in regionalized signatures

S. Almeida et al.

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

◀

▶

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion

