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Estimation of predictive hydrological uncertainty using quantile regression: examples from the national flood forecasting system (England and Wales)

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Abstract

In this paper, a technique is presented for assessing the predictive uncertainty of rainfall-runoff and hydraulic forecasts that conditions forecasts uncertainty on the forecasted value itself, based on retrospective quantile regression of hindcasted water level forecasts and forecast errors. To test the robustness of the method, a number of retrospective forecasts for different catchments across England and Wales having different size and hydrological characteristics have been used to derive in a probabilistic sense the relation between simulated values of discharges and water levels, and matching errors. From this study, we can conclude that using quantile regression for estimating forecast errors conditional on the forecasted water levels provides an extremely simple, efficient and robust means for uncertainty estimation of deterministic forecasts.

1 Introduction

Real-time flood forecasting, warning and response systems (often referred to simply as “flood warning systems”) aim to give property owners, floodplain residents and responsible authorities time to respond to a flood threat before a critical threshold is exceeded, thus allowing for mitigation of adverse consequences. As such, they constitute a non-structural flood risk management measure. Extending the forecasting lead time allows for time allocation for mitigating actions. A reliable assessment of certainty of predicted events in a real-time context safeguards operational users from issuing false alarms and institutional decision-makers from calling for unwarranted action. Real-time flood forecasting systems are currently operational in many parts of the world, including England and Wales where the National Flood Forecasting System (NFFS) is used by the Environment Agency (Werner et al., 2009).

Following the 2007 summer floods in England and Wales, the Pitt Review (Pitt, 2008) recommended that “The Met Office and the Environment Agency should issue warnings against a lower threshold of probability to increase preparation lead times for emer-

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agency responders". This implies that the Environment Agency shift its flood forecasting operations from a deterministic to a probabilistic approach.

While flood warning systems have the potential to significantly reduce flood risk, uncertainty in water level forecasts may cause imperfect flood warning system performance. This uncertainty has multiple causes including uncertain future boundary conditions of precipitation, evaporation and temperature from numerical weather prediction models, imperfect model schematisations, parameter uncertainty and unknown initial states or imperfect estimates thereof. A wide body of research over the past decades has attempted to reduce these individual sources of uncertainty. This research includes reducing, characterisation, assessment and modeling of parametric uncertainty, deterministic state updating or data assimilation (Serban and Askew, 1991; Moore, 2007), error correction (Broersen and Weerts, 2005; Shamseldin and O'Connor, 2001), ensemble data assimilation (Clark et al., 2008; Seo et al., 2009; Weerts et al., 2010b), multimodel ensemble prediction and various probabilistic post-processing techniques (Reggiani and Weerts, 2008; Reggiani et al., 2009; Seo et al., 2006; Wood and Schaake, 2008; Krzysztofowicz and Maranzano, 2004).

While this research has led to considerable progress in reducing uncertainties related to these sources, there will always be a residual uncertainty that cannot be fully eliminated. Although this uncertainty in flood forecasting is widely recognised, many if not most of the existing flood warning systems are based on deterministic forecasts, implying a certain, accurate prediction of water levels. In contrast, probability forecasts explicitly estimate predictive uncertainty about future flows or water levels.

For a number of reasons, the move from deterministic forecasting to probability forecasting constitutes an improvement to flood warning systems. First of all, hydrological forecasts are inherently uncertain. Deterministic forecasting alone suggests certainty which is unwarranted, whereas probabilistic forecasts explicitly show the certitude of a prediction. Communicating that level of certitude then allows for the decision to be made by a decision maker rather than a decision being implicitly taken by forecasters. Probability forecasts can then be used to take a risk-based decision, where

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the consequences of possible outcomes can be weighted by their probability of occurrence function (Todini, 2007; Raiffa and Schlaifer, 2000) Also, depending on these consequences, decision makers can set a threshold of probability against which to decide, thus choosing an appropriate balance between false alarms and missed floods.

Montanari and Brath (2004) report three approaches for estimating predictive uncertainty. The first option is that the model used for forecasting may be structured as a probability model that generates probability distributions. A second option is to estimate predictive uncertainty by analysing the statistical properties of the forecast error series (that is, the difference between the prediction and the observation). The third option is to use simulation and re-sampling techniques, thus applying Monte Carlo analyses. When choosing an approach to be implemented in an operational, real-time flood forecasting system, computational efficiency and data availability are issues that need to be taken into account.

The present paper proposes the use of “Quantile Regression” (Koenker and Bassett, 1978; Koenker, 2005) as a method to estimate predictive uncertainty. Quantile regression, as applied in this study, aims to assess the relationship between the hydrological forecast and the associated forecast error. In contrast with “classical” regression methods, quantile regression does not optimise on the mean of the dependent variable (the forecast error) but rather on the quantiles (e.g. the median). By thus estimating quantiles, an estimate of the full probability distribution of the forecast error may be achieved. This probability distribution may serve as an estimate for predictive hydrological uncertainty.

The applicability of quantile regression is demonstrated by applying it on a number of catchments in England and Wales. The catchments vary in size and in dominant hydrological processes. A stand-alone version of the National Flood Forecasting System (NFFS, Werner et al., 2009, 2004) was adapted to serve as a prototype of the probabilistic forecasting system.

This paper first describes the theory of quantile regression and its application to flood forecasting. Subsequently, the application to the NFFS catchments is described. This

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is followed by the results and discussion section showing verification metrics and, the paper ends with conclusions.

2 Material and methods

2.1 Uncertainty estimation of water level forecasts using quantile regression

With progressing lead time, many sources of uncertainty impact the accuracy of forecasts, different uncertainty components dominating at different lead times. In an operational setting, forward modelling of all these uncertainties can be infeasible because it requires many data (e.g. meteorological ensemble forecasts) or many model runs (e.g., Beven, 2006).

The stochastic approach used in this study, estimates effective uncertainty due to all uncertainty sources. The approach is implemented as a post-processor on a deterministic forecast. We estimate the probability distribution of the forecast error at different lead times, by conditioning the forecast error on the predicted value itself. Once this distribution is known, it can be efficiently imposed on forecasted values as a post-processor. We estimate the relation between the probability distribution of the errors and the forecasted values at a given lead time, by means of quantile regression. Quantile regression is a method for estimating conditional quantiles (Koenker, 2005; Koenker and Basset, 1978; Koenker and Hallock, 2001). This requires conditioning of the quantile regression relationships on a calibration dataset of forecasted values and associated errors at the lead time of interest. To keep the methodology as objective as possible, the degrees of freedom of the quantile regression relationships is kept minimal by using a linear regression for each quantile of interest. Prior to linear quantile regression, the heteroscedasticity of the error process, typically associated with rainfall-runoff or hydraulic models, is taken care of by making the training population of forecasted values and associated errors Gaussian. For each lead time of interest, a different set of quantile regression relationships needs to be derived. This

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is because the magnitude of the effective forecast error increases with lead time. Below, the method to derive the quantile regressions is formulated in detail. The method can be followed when a large enough sample of hindcasted water levels or flows and concurrent observed values is available.

5 **2.1.1 Transformation of forecast errors from the original domain to the Gaussian domain**

Let us denote the process of flow or water levels with a certain lead time as follows:

$$s(t) = \hat{s}(t) + e(t) \tag{1}$$

10 where s is the real process of river flow or water levels at time t , \hat{s} is the forecasted value and e is the error estimate, all at a certain time t and with a certain lead time (not explicitly denoted here). In practice, we approximate the process s by collecting a population of observations and \hat{s} by simulating a population of hindcasted values with the lead time of interest, at concurrent time stamps. These estimates can be provided by (a combination of) hydrological and hydraulic models. Let us assume
15 that the error may be estimated by means of a probabilistic error model based on the following functional form:

$$e(t) = f[\hat{s}(t)] \tag{2}$$

Once this relation is found, it allows an user to apply the error model in any case, without any additional data requirements besides the estimate of the process \hat{s} . This is convenient in an operational context, where the availability of additional data besides s to estimate uncertainty is not trivial. Now let us assume that the error structure is both ergodic and stationary (i.e. no significant changes in the hydrological or hydraulic processes or measurements have taken place). There is no warrant however, that the error structure is homoscedastic. In fact, residual time series of rainfall-runoff models
20 are known to be heteroscedastic and non-linear in nature. In order derive an objective and reliable probabilistic relation as given in Eq. (2) a transformation of the regressor

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and the concurrent associated errors to the Gaussian domain is applied through the normal quantile transform (NQT). The NQT (van der Waerden, 1952, 1953a,b) is a non-parametric method to map a variable, having any distribution, to a Gaussian distribution and has been described for hydrological applications by Kelly and Krzysztofowicz (1997). Applications of the NQT in error estimation have been performed by Krzysztofowicz and Maranzano (2004) and by Montanari and Brath (2004). In detail, the plotting positions of the cumulative distribution function of the available samples are associated with their counterparts in the Gaussian domain. The samples in the Gaussian domain can be discretely described by the inverse, Q^{-1} of the normal distribution

$$F(s_N) = Q^{-1}F(s) \quad (3)$$

or for individual samples

$$s_{\text{NQT}}(t) = Q^{-1}(\text{Pr}[s \leq s(t)]) \quad (4)$$

where s_{NQT} is the Gaussian-transformed discharge or water level. Equation (4) describes the NQT of s . The same can be done for the error series, which results in a discrete population of errors in the Gaussian domain here denoted as e_{NQT} . To apply the inverse of the NQT, i.e. to convert any value in between the sampled points of s_{NQT} or e_{NQT} to associated values s and e , we use linear interpolation for points within the domain covered by the populations. If values are sought outside this domain linear extrapolation is applied on a number of points in the tails of the distribution. Equation (1) can now be applied in the normal domain,

$$s_{\text{NQT}}(t) = \hat{s}_{\text{NQT}}(t) + e_{\text{NQT}}(t) \quad (5)$$

where the subscript “NQT” refers to variables, transformed to the Gaussian domain.

2.1.2 Quantile regression in the Gaussian domain

If one assumes that the residuals of a relation between errors and forecasted values (such as defined in Eq. 2) are Gaussian, an estimate of the sample mean of the relation

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can be defined as the solution to the problem of minimizing a sum of squared residuals. This approach has been followed by Montanari and Brath (2004) who consequently estimate the variance of the relation to estimate effective uncertainty. If Gaussianity of the residuals of Eq. (2) cannot be assumed, one can turn to the estimation of the sample median and other quantiles instead. This approach has been chosen in this study by using quantile regression, which does not make any assumptions about the nature of the residuals of the forecast – error relationship. The quantile regression methodology is further explained below based on the Gaussian samples of e_{NQT} and s_{NQT} .

The sample median (as opposed to the sample mean) can be estimated by minimizing the sum of absolute residuals (as opposed to squared residuals). Minimizing a sum of asymmetrically weighted absolute residuals (by giving different weights to positive and negative residuals) can yield other (in fact, any) quantiles besides the median (for more details, see Koenker and Hallock, 2001). Applying this to the sampled values of s_{NQT} and e_{NQT} at a certain lead time of interest, this can be formulated as

$$\min \sum_{t=1}^n \rho_{\tau} [e_{\text{NQT},\tau}(t) - \hat{e}_{\text{NQT},\tau}(t, \hat{s}_{\text{NQT}}(t))] \quad (6)$$

with $\hat{e}_{\text{NQT},\tau}$ the estimate of the Gaussian-transformed error at a certain quantile interval τ and ρ_{τ} the weighting function that pushes $\hat{e}_{\text{NQT},\tau}$ to its associated quantile location. In case $\tau=0.5$ this would yield the unconditional median. Other values of τ can be used to determine other quantiles.

We assume that estimates of the conditional quantiles in the Gaussian domain can be described by the linear equation

$$\hat{e}_{\text{NQT},\tau}(t) = a_{\tau} \hat{s}_{\text{NQT}}(t) + b_{\tau} \quad (7)$$

where a_{τ} and b_{τ} are the regression constants, valid for the lead time of interest. After substitution of Eq. (7) into Eq. (6), a_{τ} and b_{τ} can be optimised efficiently by linear programming (Koenker, 2005; Koenker and Hallock, 2001).

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2.1.3 Imposing the error models in operational forecasts

To describe the distribution of errors, conditional on the forecasted values, quantile regression functionalities within the R-package *quantreg* (Koenker, 2010) were used. To transform quantile estimates within the Gaussian domain, back to the untransformed domain, linear interpolation has been used to connect the combinations of NQT transformed simulated values \hat{S}_{NQT} and estimated error quantiles $\hat{e}_{\text{NQT},\tau}$ in the Gaussian domain, to their counterparts in the untransformed domain, using the following relationship:

$$\hat{S}_{\tau}(t) = \hat{S}(t) + \text{NQT}^{-1} [a_{\tau} \hat{S}_{\text{NQT}}(t) + b_{\tau}] \quad (8)$$

This yields calibrated discrete quantile relationships in the untransformed domain, which can be imposed on any forecasted value by means of linear interpolation, or if forecasted values are found outside the domain of the calibration dataset, with linear extrapolation.

The quantile error models can be derived at several lead times. The derived error models can consequently be applied in an operational context. An experimental module setup for imposing the error models in the standalone version of NFFS has been developed in the statistical computing language R-statistics (<http://www.r-project.org/>). This R based module can be executed from within Delft-FEWS (Weerts et al., 2010a).

2.2 Case study descriptions

Figure 1 shows the locations of the case study areas relative to the coast line of England and Wales. The catchment in the North is the Upper Calder, the middle part is the Upper Severn with multiple interconnected catchments and the two catchments South are used in the Ravensbourne case study.

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2.2.1 North East Region, Upper Calder, Todmorden (147 km²)

The Upper Calder catchment, draining an area of 147 km² to the river gauging station at Mytholmroyd, just upstream of Caldene Bridge. It is underlain by Carboniferous rocks of Millstone Grit and Coal Measures, with the former predominating in the high moorland areas. The river and its tributaries flow through steep and relatively narrow valleys. About 18% (26 km²) of the area drains to reservoirs. Typically these are reservoirs for direct water supply, releasing only compensation flows unless spilling at times of flood. The natural flow regime has also been modified by various channel improvements and flood defences, including schemes at Todmorden and Mytholmroyd. Todmorden is a forecast location for the Upper Calder with forecast location Todmorden is part of the the NorthEast (England, Environment Agency, NorthEast). The Upper Calder is a fast responding catchment and is modelled with a PDM rainfall runoff model (Moore, 1997). The PDM model is forced by the input created by a snow model.

2.2.2 Midlands Region, Upper Severn, various locations and sizes (150–1000 km²)

The River Severn rises in the Cambrian mountains at Plynlimon at a height of 741 m a.o.d. and flows northeastwards through Llanidloes, Newtown and Welshpool before meeting the Vyrnwy tributary upstream of Shrewsbury. The valley is wide and flat in this confluence area, with a considerable extent of floodplain.

The river then flows through Montford to Shrewsbury, and is joined at Montford Bridge by the River Perry which flows from the Oswestry area to the North. The lowermost point in the Upper Severn catchment is defined by Midlands Region as the gauge at Welshbridge in Shrewsbury. There are also significant areas of floodplain in the reach from Shrawardine, upstream from Montford, to Welshbridge. Average annual rainfall can exceed 2500 mm in the Cambrian mountains in the upper reaches of the Severn, and in Snowdonia National Park, in the upper reaches of the Vyrnwy catchment. The catchment area to Welshbridge is approximately 2284 km², including 778 km² for the

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Vyrnwy catchment to the flow gauge at Llanymynech. The main reservoirs in the Upper Severn catchment are Lake Vyrnwy in the upper reaches of the Vyrnwy, and the Clywedog Reservoir in the upper reaches of the Severn. There are no significant flow control structures in the remainder of the Upper Severn catchment. The Upper Severn catchment is represented in NFFS by a combination of MCRM rainfall-runoff models (Bailey and Dobson, 1981; Wallingford Water, 1994) and DODO routing models (Wallingford Water, 1994).

2.2.3 Thames Region, Ravensbourne, two locations (32 and 68 km²)

The River Ravensbourne and tributaries drain highly urbanised areas from the south of London towards Lewisham before joining the Thames at Deptford Creek. The total Ravensbourne catchment area is approximately 180 km². The more slowly responding rural part of the Ravensbourne South Branch catchment makes up around 55 km², which does not contribute significantly to flood events and is therefore generally discarded from analyses of flood hydrology (as verified by historic calibration data over many events).

The remaining 125 km² is highly urbanised and has a very rapid response to rainfall and, due to the large proportion of paved surfaces, there is very little hydrological memory (antecedent storage). As a result, hydrographs throughout the catchment often rise steeply from baseflow to threshold levels in around 30 min (sometimes less), and fall again almost as rapidly at the upstream locations, whilst locations lower in the catchment take slightly longer to recede (due to later arrival of upstream contributions). The NFFS integrated catchment model for Ravensbourne (Thames Region) comprises 16 TCM models (Greenfield, 1984; Wilby et al., 1994) providing inputs to an ISIS hydrodynamic model.

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2.3 Data used in the case studies

For both the calibration and validation only operational available data are used. The data available from the archive consist of RTS data (observed level data, rain gauge data, air temperature and catchment average rainfall data), Radar Actuals, Radar Forecast, and Numerical Weather Prediction data (all three from UKMO) and is available from 2006 onwards. The operational forecasts of river discharges and water levels benefit from the availability of operationally observed values. These are propagated into the model by means of data assimilation, prior to forecasting.

The data was splitted in a calibration data set (2006 and 2007) and a validation data set (2008 and 2009). Both the calibration and validation datasets contain several major flood events. Only the validation data set for the Thames Region contains few minor, medium and major events for the period 2008 and 2009.

3 Results and discussion

3.1 Derivation of forecast error models

The quantile regression methodology to describe the predictive uncertainty as described in Sect. 2 has been applied on several forecast locations in the National Flood Forecasting System (NFFS). In the calibration phase, the forecast error models are derived. Figure 2 shows an example of the calibration for the Upper Severn forecast location Welshbridge (2077) in the Gaussian domain. Figure 3 shows the example of the derived forecast error model in the untransformed domain. The effect of the NQT on the derived 50% and 90% quantiles is evident.

When fitting the different quantiles during calibration it was sometimes found that the different quantiles crossed at low water levels. This is an artefact of the fitting procedure and this problem has been overcome by using a fixed error model below certain water levels as is visible in Fig. 2.

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After deriving these distributions for all forecast locations and at all lead times used in the case studies, it can subsequently be applied in NFFS in real-time or hindcast mode. For all case studies, the 5%, 25%, 50%, 75%, 95% quantiles have been derived and used during the validation.

3.2 Validation of forecast error models

Below, the application of quantile regression to provide a probabilistic forecast is presented for the 3 case studies, representing a wide variety of hydrological conditions, and catchment sizes.

3.2.1 North East, Upper Calder

Figure 3 show the validation of the quantile regression method for the January 2008 events for 4 leadtimes (2, 6, 12 and 24 h). The observation fall most of the time within the 90% confidence interval. It is also clear that the uncertainty increases with lead-times as expected. One can also see that the uncertainty increases and decreases depending on the forecasted water level and lead time. In Fig. 3, it is also visible that some observed values lie outside the confidence interval. For a 90% confidence interval, and for each forecasted water level, this should happen only 10% of the time. Because the sample size at high forecasted and observed water level in both the calibration and validation phase is limited (only few major and minor events, see for example Fig. 2) it is very difficult to make strong statements of the performance at higher water levels. Unfortunately, this is the area of main interest.

Figure 4 shows the validation results for the January 2009 events. The forecasted water level for January 2009 for longer leadtimes are not as good as in January 2008 (as can be seen by the 50% estimate). There are several false alarms forecasted especially at longer leadtimes. The reason for this is not yet clear. The wider confidence bounds at the peak values also indicate that false alarms, to some degree, were present in the calibration data set. However, it is also possible that it could possibly be

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related to changes in numerical weather prediction products used for the forecast.

Both Figs. 4 and 5 show that the quantile regression method tested here can not correct “bad” forecasts. Surprisingly, the confidence intervals seem to be reasonably accurate even though they were only estimated from a hindcast over a small period (2006–2007). The quantile regression method more or less provides the confidence interval where one can expect the observation. The performance is further illustrated by Table 1 which show the percentage of the observations within the confidence intervals at various lead times.

3.2.2 Midlands region, Upper Severn

The case study on the Upper Severn using quantile regression is focused on several locations (see Table 2). These locations consists of both upstream (modelled with MCRM) and downstream (modelled using DODO) forecast locations and were chosen to show how quantile regression can be used to derive predictive confidence intervals in a end-to-end forecasting system. Figure 6 shows the validation for the flood events of November 2009 at Welshbridge (2077) at 12, 24, 36 and 48 leadtime. This figure shows how quantile regression can give estimates of the confidence intervals during flood events. Validation results for the other locations are given in Table 2. Again, the results for most locations are promising as most percentage of observations within a respective confidence intervals correspond well with the theoretical values of 50% and 90%. This was further confirmed by Quantile-Quantile plots and sharpness plots (not shown here). Table 2 shows poor performance at Caersws and to some degree at Welshpool, possibly related to operational water management (e.g. weirs, reservoirs).

3.2.3 Thames region, Ravensbourne

Figure 7 shows the validation results of the method for the February 2009 event for Manor House Gardes (3489TH). Most of the time the observations fall within the 90% confidence interval. It is also clear that the uncertainty increases with lead times as ex-

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pected. One can also see that the uncertainty increases and decreases depending on the forecasted level. The performance is further illustrated by Table 3 which show the percentage of the observations within the confidence intervals at various lead times. Table 3 shows that for these two locations the derived forecast error models are maybe
 5 somewhat to wide as too many observations lie within the 50% and 90% confidence limits. However, this could be the results that the period 2008 and 2009 was not completely similar to the calibration period 2006 and 2007. In 2008 and 2009, there were relatively few small, medium and major flood events for the whole of Thames region. This should be further tested with a longer record that corresponds more to the cali-
 10 bration period.

4 Conclusions

A method to provide predictive uncertainty estimates of water level or flow forecasts is presented. The developed method aims at characterising the distribution function of the water level or flow forecast error conditioned on the value of the predicted water
 15 level or flow by means of quantile regression relationships at quantiles of interest. The method therefore does not consider the independent sources of uncertainty but instead considers the effective uncertainty of the forecast process only, which can be a result of input or output uncertainty, model structural uncertainty or parameter uncertainty. The quantile error relationships are estimated in the Gaussian domain. To this end, both
 20 the available forecast population and error population are made Gaussian by means of a Normal Quantile Transform. Several sets of quantile regression relationships may be derived at specific lead times that are of interest to the user. The quantile regression relationships can straightforwardly be implemented as a post-processor in an operational forecasting system.

25 The method was tested by deriving quantile regression relations for several lead times using a calibration hindcast set and consequently predicting forecast errors of water levels using an independent hindcast set in three case studies in a standalone

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version of the National Flood Forecasting System of England and Wales. The three case studies across England and Wales, contain a variety of catchments (size and behaviour) and hydrological models. From the results, we can conclude that the quantile regression method to provide predictive uncertainty estimates of water level forecasts is promising. We showed that the derived quantile regression relationships predict the error quantiles satisfactorily at all lead times. Moreover, the developed method is simple to apply, requires very few assumptions, and is easy to understand by both scientists and forecasters.

However, there are also a few limitations. The method, like all postprocessing methods, requires a long (homogeneous) calibration and validation set. In the case studies, only short calibration and validation sets were available, containing only a few extreme events. This compromised error descriptions at higher predicted water levels, which could only be made through extrapolation of the derived quantile regressed relations. A validation period without real extreme events (e.g. Ravensbourne case study) indicated that currently available data records may cause problems with deriving and testing the method.

Furthermore, homogeneity of data sets can be an issue (observed in particular in the Upper Calder case study). Such inhomogeneities may be caused by a number of factors, such as (a) changes in the hydraulics/hydrology of the river/catchment (in fact, in this case the hydrological/hydraulic model itself needs re-calibration) and (b) changes in the forecasting system changes considerably (e.g. by adaptation of state updating procedures or models used) or (c) changes in external data sources used to drive the forecasting system (e.g. meteorological models or external observed boundary conditions). Like any other statistical postprocessing method, the developed method requires recalibration if such inhomogeneities occur.

If these limitations are considered or taken care of by the user, the developed method can be straightforwardly employed in operational forecasting because (a) the required data is always available in the operational context and (b) the methodology requires little computation time.

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Table 1. Percentage of observations within respective confidence intervals for period January 2008–July 2009 for forecast location Todmorden (North East region).

Location id & name	Confidence interval	2 h	4 h	6 h	12 h	18 h	24 h
TODMDN1	25–75%	61.8	65.5	66.4	48.0	41.5	39.9
Todmorden	5–95%	89.2	90.2	90.6	89.7	90.1	90.4

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Table 2. Percentage of observations within respective confidence intervals for period January 2008–November 2009 for forecast locations Upper Severn (Midlands region).

Location id & name	Confidence interval	12 h	24 h	36 h	48 h
2003	25–75%	64.9	67.1	68.4	66.9
Vyrnwy weir	5–95%	92.2	92.3	92.3	91.1
2005	25–75%	53.3	49.5	52.9	60.3
Montford	5–95%	91.9	87.9	87.4	87.9
2014	25–75%	69.1	66.8	65.0	75.3
Abermule	5–95%	93.6	93.2	94.1	95.5
2020	25–75%	46.9	47.2	48.0	54.6
Yeaton	5–95%	88.0	84.8	85.8	90.9
2025 Rhos Y	25–75%	52.4	50.8	54.6	52.9
Phentref	5–95%	89.9	89.1	90.3	93.9
2028	25–75%	49.9	50.7	56.1	65.9
Llanymymech	5–95%	92.6	90.6	88.1	90.9
2038	25–75%	47.4	48.0	53.6	59.3
Llandlowel	5–95%	91.1	88.3	85.3	87.1
2072	25–75%	48.4	45.7	54.7	58.7
Llanidloes	5–95%	83.6	89.2	90.2	95.1
2074	25–75%	23.2	18.1	10.9	16.9
Caersws	5–95%	46.1	42.4	31.6	29.4
2076	25–75%	59.4	53.0	52.8	48.6
Meiford	5–95%	93.6	93.4	93.0	95.9
2077	25–75%	53.9	52.6	54.5	61.1
Welshbridge	5–95%	91.4	90.7	88.4	86.9
2109	25–75%	36.8	40.6	42.3	47.2
Bryntail	5–95%	86.6	89.9	89.8	91.2
2156	25–75%	41.4	47.3	53.3	60.2
Pont Robert	5–95%	88.6	89.7	89.4	90.7
2159	25–75%	40.9	40.2	44.5	45.1
Lanerfyl	5–95%	82.8	83.1	80.9	81.7
2175	25–75%	51.5	53.4	59.4	65.8
Crew Green	5–95%	91.7	91.8	88.9	90.0
2176	25–75%	54.6	51.1	48.6	59.9
Buttington	5–95%	87.4	90.6	90.2	92.7
2638	25–75%	49.9	48.7	43.2	39.6
Welshpool	5–95%	84.4	83.4	83.4	61.9

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Table 3. Percentage of observations within respective confidence intervals for period January 2008–July 2009 for forecast locations Ravensbourne (Thames Region).

Location id & name	Confidence interval	3 h	6 h	9 h	12 h	15 h	18 h	21 h	24 h
3470TH	25–75%	56.4	51.1	44.4	41.2	42.5	42.5	42.8	56.4
Catford Hill	5–95%	96.7	96.2	95.5	95.4	94.3	93.5	92.3	96.7
3489TH	25–75%	65.1	58.7	56.2	55.4	55.7	55.1	55.0	65.1
Manor House Gardens	5–95%	96.0	94.8	94.5	94.4	94	93.8	94.2	96

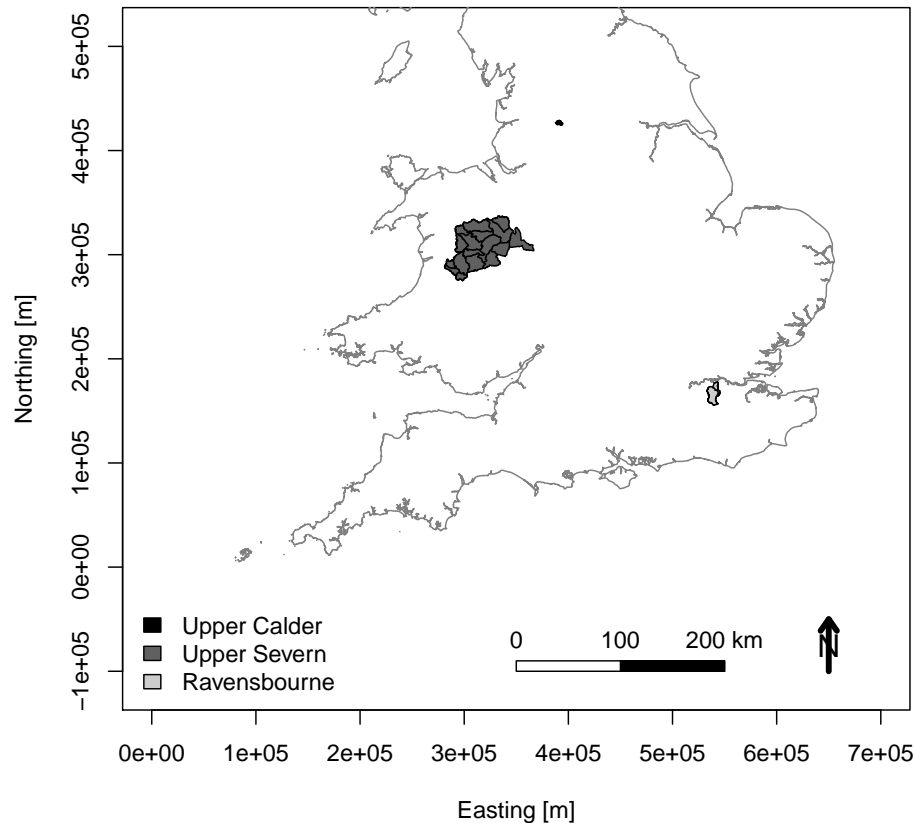


Fig. 1. Overview of the locations of the case study areas relative to the coastline of England and Wales. At the top the catchment of the Upper Calder, in the middle the Upper Severn with multiple interconnected catchments, and at the bottom the two Ravensbourne catchments.

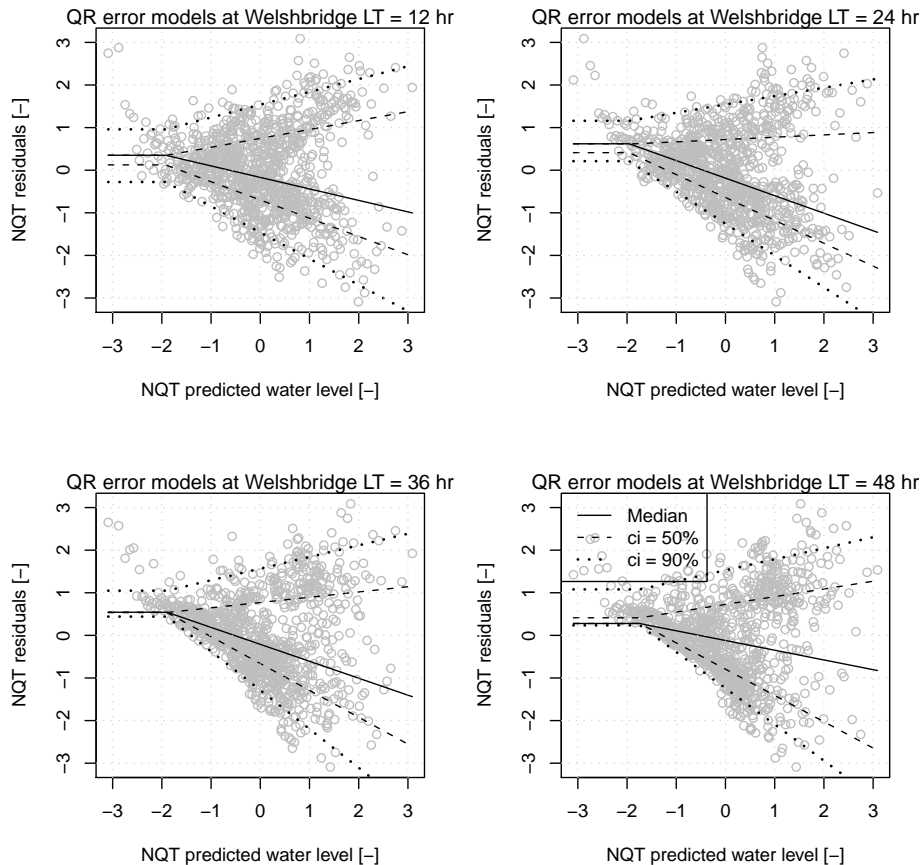


Fig. 2. Transformed forecast errors vs. transformed forecasted water levels (both in Gaussian domain) together with the derived quantile regressions for the different confidence levels.

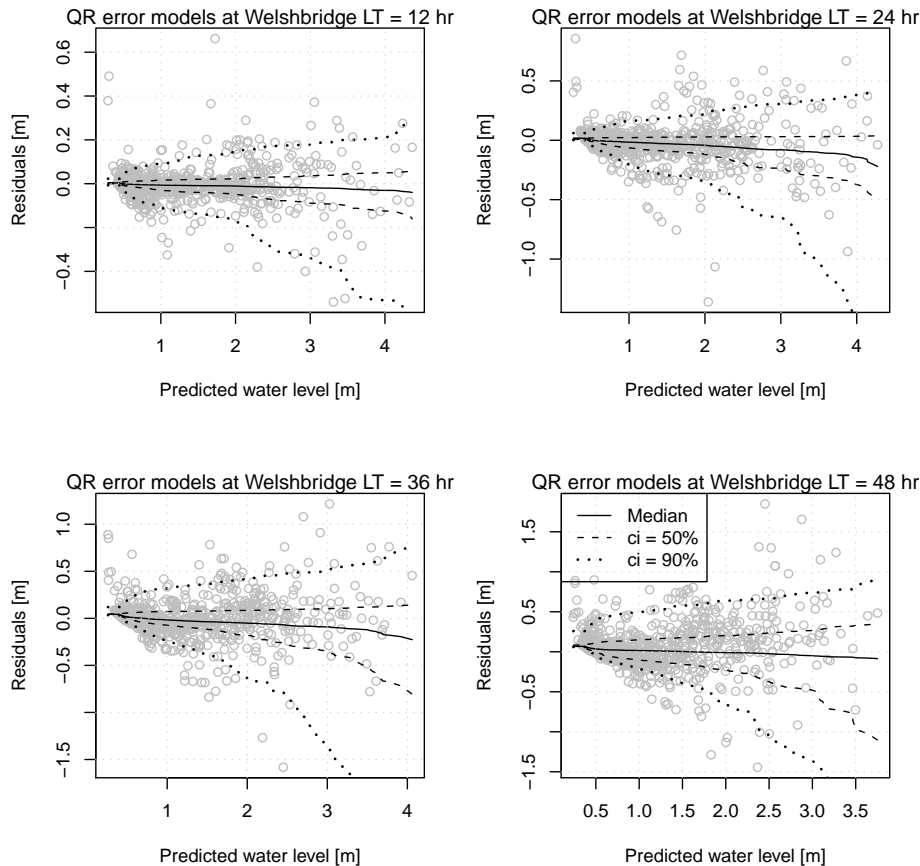


Fig. 3. Forecast errors vs. forecasted water levels together with the derived quantile regressions for the different confidence levels.

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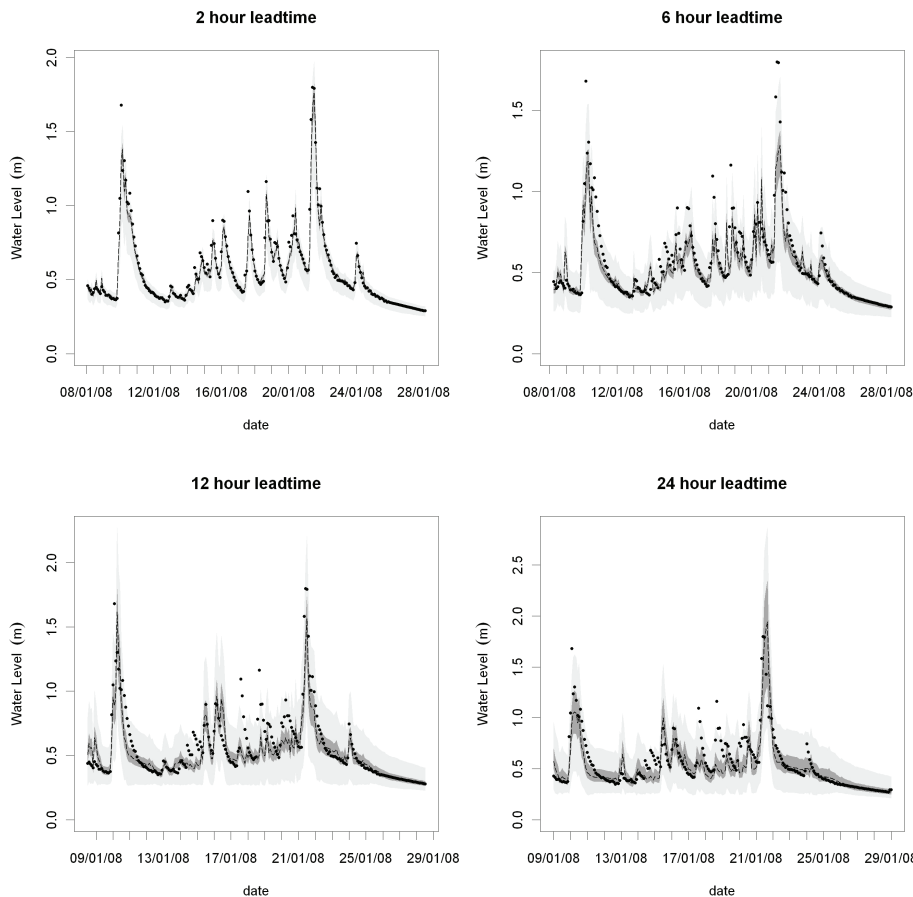


Fig. 4. Validation of quantile regression method for water level forecasts at Todmorden for the January 2008 events for 2, 6, 12, and 24 h leadtimes. The darkgrey area represents the 50% confidence interval and the lightgrey area represents the 90% confidence interval, the black dashed line the 50% estimate, and the black dots the observations.

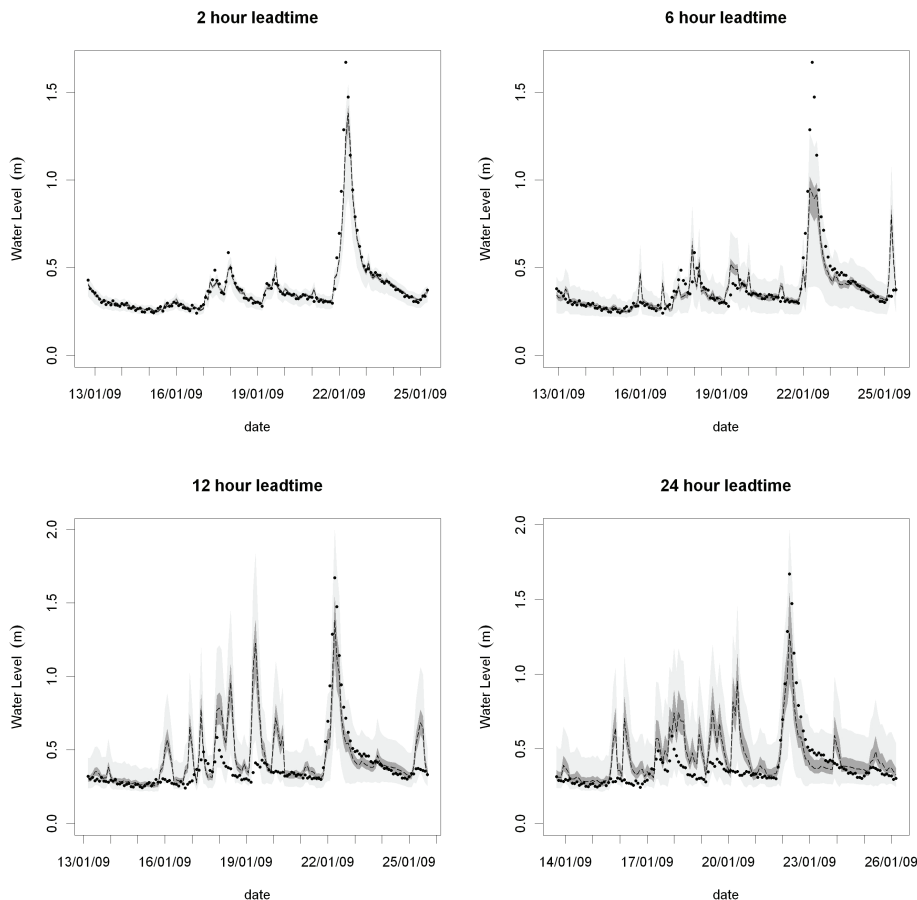


Fig. 5. Validation of quantile regression method for water level forecasts at Todmorden for the January 2009 events for 2, 6, 12 and 24 h leadtimes. The darkgrey area represents the 50% confidence interval and the lightgrey area represents the 90% confidence interval, the black dashed line the 50% estimate, and the black dots the observations.

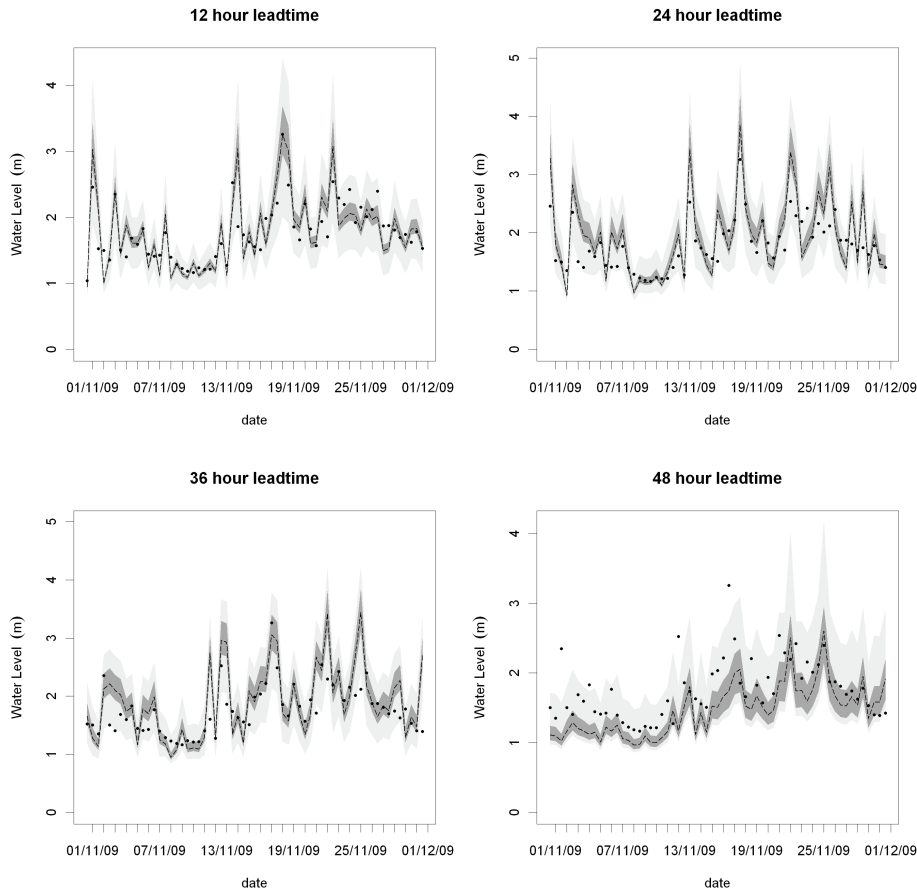


Fig. 6. Validation of quantile regression method for water level forecasts at Welshbridge (2077) for the November 2009 events for 12, 24, 36 and 48 h leadtimes. The darkgrey area represents the 50% confidence interval and the lightgrey area represents the 90% confidence interval, the black dashed line the 50% estimate, and the black dots the observations.

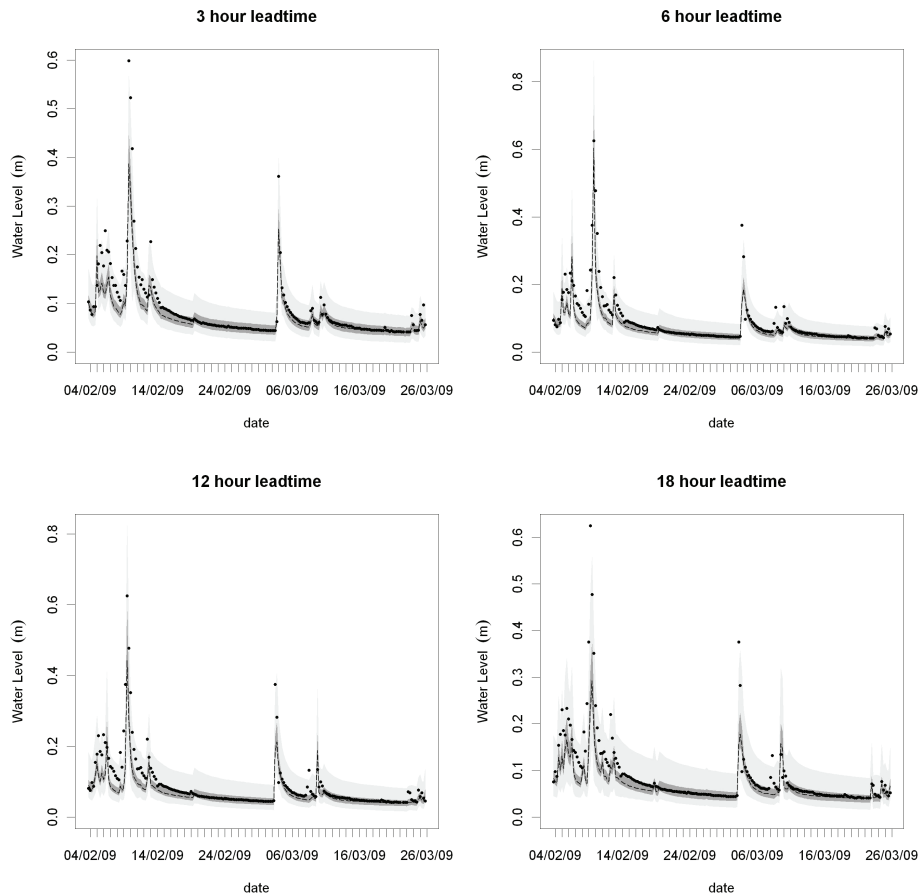


Fig. 7. Validation of quantile regression method for water level forecasts at Maner House Garden for the February 2009 event for 3, 6, 12 and 18h lead times. The darkgrey area represents the 50% confidence interval and the lightgrey area represents the 90% confidence interval, the black dashed line the 50% estimate, and the black dots the observations.