Spectral induced polarization measurements for environmental purposes and predicting the hydraulic conductivity in sandy aquifers

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Abstract

Low-frequency field and laboratory induced polarization measurements are carried out to characterize the hydrogeological conditions at Schillerslage test site in Germany. The laboratory spectral induced polarization (SIP) data are analyzed to derive an empirical relationship for predicting the hydraulic conductivity ($K$) in the field scale. On the other hand, the results from SIP sounding and profiling field data indicate that the method identifies the lithological layers with sufficient resolution to achieve our objectives. Two main Quaternary groundwater aquifers separated by a till layer can be well differentiated. Furthermore, the phase images are also capable of monitoring thin peat layers within the sandy groundwater aquifer. However, the field results show limitations of decreasing resolution with depth and/or low data coverage. Similarly, the SIP laboratory results show a certain shift in SIP response due to different compaction and sorting of the samples. The overall results obtained show that the integration of field and laboratory SIP measurements is an efficient tool to avoid a hydrogeological misinterpretation.

In particular, two significant but weak correlations between individual real resistivities ($\rho'$) and relaxation times ($\tau$), based on a Debye decomposition (DD) model, with measured $K$ are found for the upper groundwater aquifer. While the maximum relaxation time ($\tau_{\text{max}}$) and logarithmically weighted average relaxation time ($\tau_{\text{lw}}$) show a better relation with $K$ values than the median value $\tau_{50}$, however, the single relationships are weak. A combined power law relation between individual $\rho'$ and/or $\tau$ with $K$ is developed with an expression of $A \cdot (\rho')^B \cdot (\tau_{\text{lw}})^C$, where $A$, $B$ and $C$ are determined using a least-squares fit between the measured and predicted $K$. The suggested approach with the calculated coefficients of the first aquifer is applied for the second one. The results indicate a good correlation with the measured $K$ and prove to be superior to single phase angle models as the Börner or Slater models.
1 Introduction

A key prerequisite for reliable prediction of groundwater movements is the hydraulic conductivity ($K$). The pumping test, grain size or coring sleeve analyses have traditionally been the standard methods used to evaluate the hydraulic parameters of subsurface material for characterizing groundwater aquifers. These methods are expensive, slow and often unavailable due to disturbance during drilling to get the borehole samples. Partly for these reasons, electrical methods are playing an increasingly important role in predicting the aquifer hydraulic parameters (e.g., Zisser et al., 2010a; Khalil, 2012).

The relationship between direct current (DC) resistivity $\rho$ and $K$ is first considered as this makes up the bulk of the pertinent literatures published over the last 30 yr (e.g., Heigold et al., 1979; Niwas and Singhal, 1985). Empirical and semi-empirical relationships have been established in terms of power law relations between various aquifer parameters and those obtained by resistivity measurements (e.g., Attwa et al., 2009). Because the hydraulic parameters depend on the porosity and the geometry of the pore space, $K$ cannot be uniquely determined by DC resistivity alone without further assumptions (Hördt et al., 2007).

The induced polarization (IP) of non-metallic minerals is generally referred to as interface or membrane polarization (Marshall and Madden, 1959; Vinegar and Waxman, 1984). In the absence of metallic conductors, spectral induced polarization (SIP) phenomena are commonly associated with polarization effects related to an electrochemical double layer (EDL), which describes the organization of ionic charges at the interface between solid and fluid (Revil and Florsch, 2010). The EDL provides the conceptual background for the electrochemical processes considered to be responsible for a large amount of the observed SIP response (e.g., Leroy et al., 2008; Revil and Cosenza, 2010). Accordingly, both ohmic (i.e. amplitude) and capacitive (i.e. phase) parts of interface polarization contain additional information about the textural characteristics of sedimentary rocks (e.g., Kemna, 2000; Blaschek and Hördt, 2009).
By measuring chargeability or, equivalently, the phase shift between current and voltage, information about the pore space can be gained. Previous workers have shown that the IP mechanisms can be very sensitive to changes in lithology and pore fluid chemistry (e.g., Pelton et al., 1978; Weller et al., 2010b; Skold et al., 2011). Environmental examples for the successful use of SIP include the detection of clay units (e.g., Hördt et al., 2009; Attwa et al., 2011; Attwa and Günther, 2012), the detection of both organic and inorganic contaminants (e.g., Olhoeft, 1984, 1985, 1992; Chen et al., 2008, 2012; Abdel Aal et al., 2009, 2010) and hydraulic conductivity estimation (e.g., Tong and Tao, 2007). Although the SIP method offers potential for subsurface structure and process characterization, in particular in hydrogeophysical and biogeophysical studies (Kemna et al., 2012), it is not as widely used as other electrical methods (e.g., DC resistivity), and its full potential has yet to be realized.

Recently, hydrogeophysical research of the IP method has shown that SIP data can be correlated with physical properties of the pore space in non-metallic soils and rocks, such as the specific surface area ($S_{por}$) and $K$. There is a growing interest in the use of spectral induced polarization (SIP) for a wide range of environmental applications, in particular those focused on hydrogeological investigations. A clear link between hydrogeological properties and IP/SIP parameters has been empirically documented by various studies (e.g., Binley et al., 2005; Hördt et al., 2007). However, recent studies by Binley et al. (2010), Kruschwitz et al. (2010) and Titov et al. (2010) also point out that the detailed nature and origin of such linkages still lies ahead. Börner et al. (1996) Slater and Lesmes (2002) introduced models to estimate $K$ from IP measurements. Both models require a direct link between the imaginary conductivity and $K$. Thus, the absence of such a relationship indicates that the imaginary conductivity is not related to the effective hydraulic length scale (e.g., Zisser et al., 2010b). Hence, these models seem to be of limited applicability.

Laboratory SIP investigations covering a wide frequency range provide more information on the spectral behavior of conductivity amplitude and phase shift. There are no universal physically based models which describe the frequency-dependent complex
conductivity response of sediments and, consequently, macroscopic representations have been adopted in the literature (Binley et al., 2005). The SIP response can be described as a superposition of relaxation processes, which might reflect grain-size distribution (Lesmes and Morgan, 2001). The induced polarization decay contains much more information than single frequency or total chargeability data that are often used in traditional IP measurements. Significant progress has been made over the last decade in the understanding of the microscopic mechanisms of IP; however, integrated mechanistic models involving different possible polarization processes at the grain/pore scale are still lacking (Kemna et al., 2012).

Various conceptual and theoretical relaxation models exist to describe the SIP mechanism within the rocks and the relaxation process after terminating the current. Different approaches have been reported for the computation of the relaxation time ($\tau$) distribution. One of the most popular models is defined as Debye decomposition (DD) (Nordsiek and Weller, 2008). The DD can be applied to any measured spectral data sets independent whether the spectra exhibit constant phase, by the same performance as Cole-Cole or other behavior (Weller et al., 2010b) presents. Binley et al. (2005) use the mean relaxation time ($\tau_m$) from a Cole-Cole model of the complex resistivity to estimate the permeability of sandstone. Their measurements reveal evidence of a relationship between $\tau$ and a dominant pore throat size.

The main objective of this paper is to use SIP field and laboratory data to predict the $K$ of unconsolidated sandy aquifers at the hydrogeological test site Schillerslage, Germany. A practical empirical relation will be introduced to calculate $K$. The format of this paper is as follows: (a) SIP field (sounding and profiling) and laboratory data will be acquired for describing the geological and hydrogeological characteristics of the subsurface Quaternary aquifers. (b) Field and laboratory SIP data are used to evaluate various published approaches to predict $K$ values. The Debye decomposition model (Nordsiek and Weller, 2008) is applied to determine the $\tau$ from available laboratory SIP data. (c) The important use of combination of electrical resistivity ($\rho'$) and $\tau$ for computing $K$ is emphasized. Finally, the calculated $K$ from field and laboratory data...
using various approaches are compared with those measured from the pumping test and coring sleeve information.

2 Material and methods

The spectral induced polarization (SIP) technique is a complex multi-frequency extension of the DC resistivity. Alternating currents of various frequencies are injected and the phase shift between voltage and current signals is measured in addition to the resistance (amplitude ratio). If the porous medium consists of clay-free, uniformly sized particles, the IP decay curve would consist of a single exponent decay of relaxation time constant, $\tau$, (Vinegar and Waxman, 1984, 1987, 1988) with

$$\tau = \frac{L^2}{D}$$

(1)

where $D$ is the diffusion coefficient and $L$ is the relaxation distance along the direction of the applied electric field. Then the pore radius, $R$, is given by

$$R = (D\tau)^{0.5}$$

(2)

The estimated geoelectrical properties of earth materials may be equally represented by complex electrical resistivity ($\rho^*$),

$$\rho^* = \rho' + i\rho''$$

(3)

where $\rho^*$, $\rho'$ and $\rho''$ are the complex resistivity, real resistivity part and the imaginary part of $\rho^*$, respectively, and $\varphi$ is the phase angle of $\rho^*$ that can be also written as follows:

$$\varphi = \arctan(\rho''/\rho').$$

(4)
Traditionally, the real part of resistivity (i.e. DC resistivity) has been used to estimate \( K \) values for unconsolidated aquifers. For example, positive or negative linear log–log relationships between \( K \) and \( \rho' \) have been explained by Purvance and Andricevic (2000) and Attwa (2012), using

\[
K = a(\rho')^b. \tag{5}
\]

To apply Eq. (5) it would be necessary to estimate \( a \) and \( b \) based on the comparison of geoelectrical measurements with measured \( K \) values (e.g., pumping tests or grain size analyses). For sandy-clayey soils the correlation of \( K \) with \( \rho' \) is directly proportional on a large scale, but sometimes on a local scale the correlation is inversely proportional (Mazac et al., 1990; Attwa, 2012).

There exists a substantial volume of literatures demonstrating a power law dependence of \( \rho' \) on \( S_{\text{por}} \) (e.g., Börner et al., 1996; Slater and Lesmes 2002; Slater et al., 2006; Weller et al., 2010b). Börner et al. (1996) observed that the phase shift (\( \varphi \)) for most of their samples was particularly constant over a broad frequency range and suggested an empirical relation to estimate \( K \) from data for one frequency only, preferably around 1 Hz. Consequently, the basic assumption of this model is that the measured phase shift of complex conductivity is independent of measurement frequency. However, for most rocks, this assumption is not valid. The proposed model for a constant-phase-angle (CPA) to calculate \( K \) (in \( \text{m s}^{-1} \)) is,

\[
K = \frac{a}{F(S_{\text{por}})^c} \tag{6}
\]

where \( a \) and \( c \) exponents are adjustable parameters, \( F \) is the formation factor and \( S_{\text{por}} \) is the specific surface area. Whereas Pape et al. (1987) suggested \( a = 0.00475 \) for consolidated sediments, Börner et al. (1996) formulated a proportionality for unconsolidated sediments, using \( a = 1 \) instead. Here, we work with \( a = 0.00475 \), which was also recommended by Weller and Börner (1996) and Hördt et al. (2009) for unconsolidated sediments. The exponent \( c \) was found to be in the range \( 2.8 < c < 4.6 \) (Börner et al., 1996).
1996), depending on the method of $K$ measurements. Börner et al. (1996) presented a modification to Archie’s law to calculate the $F$ as

$$F = \frac{\sigma_w}{\sigma' - \sigma''/l}$$  \hspace{1cm} (7)

where $\sigma_w$ is the pore fluid conductivity. Börner et al. (1996) suggested to use $l = 0.1$ in unconsolidated sediments. $S_{\text{por}}$ was calculated after Weller et al. (2010b) for unconsolidated sandy aquifer as

$$S_{\text{por}} = \left(\frac{\sigma''}{0.0101}\right)^{1/0.916}$$ \hspace{1cm} (8)

where $S_{\text{por}}$ is in $\mu$m$^{-1}$ and $\sigma''$ in mS m$^{-1}$.

The second approach was suggested by Slater and Lesmes (2002) using the CPA model. Slater and Lesmes (2002) model is based on a relationship of the imaginary conductivity part with the diameter of the grain corresponding to the smallest 10% portion of the sample (i.e. $d_{10}$). Furthermore, this approach is based on the inverse linear relation between $K$ and $\sigma''$ around 1 Hz as

$$K = \frac{m}{(\sigma'')^n}$$ \hspace{1cm} (9)

where, $\sigma''$ is given in $\mu$S m$^{-1}$. $m$ and $n$ are adjustable parameters with $m = (2.0 \pm 0.3) \times 10^{-4}$; $n = 1.1 \pm 0.2$, and $K$ is in m s$^{-1}$. It is noticed that the exponent from Slater and Lesmes model (2002) is smaller than in the model of Börner et al. (1996); hereinafter these models are referred to as “Slater model” and “Börner model”, respectively.

It is generally accepted that $\tau$ (tau) is controlled by the diffusion process (Revil and Cosenza, 2010) in the electrochemical double layer (EDL). A power law relationship between some relaxation time constants and permeability ($k$) has been reported in
published data sets (Binley et al., 2005; Kemna et al., 2005; Tong et al., 2006a,b; Koch et al., 2011):

\[ k = a \tau^b \]  \hspace{1cm} (10)

where \( a \) and \( b \) are formation-specific parameters.

Zisser et al. (2010b) established a power law relationship between the mean relaxation time (\( \tau_m \)) and \( S_{por} \). They proved also that the relationship between the median relaxation time (\( \tau_{50} \)) and \( S_{por} \) is stronger than that between \( \tau_m \) and \( S_{por} \). Accordingly, in presence of a weak relation between the \( \tau \) and the permeability and/or \( K \), this relation cannot be applied.

Weller et al. (2010a) presented an empirical power law relation to predict \( k \) as a function of conductivity \( \sigma'' \), chargeability \( M \) and \( \tau_m \), which are derived from DD,

\[ k = a \cdot (\rho')^b \cdot M^c \cdot (\tau_m)^d. \]  \hspace{1cm} (11)

The four empirical parameters \( a \), \( b \), \( c \) and \( d \) are determined by a multivariate regression analysis.

3 Geological overview and measurement strategy

The test site Schillerslage is located northeast of Hannover and shows a typical geological structure for the Quaternary sediment basin of Northern Germany. Based on borehole information, two sandy aquifers separated by a fine-grained till layer can be recognized (Fig. 1, right). The upper aquifer, down to a depth of about 12 m, consists of medium to partly coarse sand and thin interbedded peat layers. The aquiclude till layer with high clay content is, in general, 12 m to 16 m deep and varies also in thickness. The lower aquifer (i.e. the second one) of 5 m thickness consists of slightly limy medium grey sand and it is overlying cretaceous marls.
The SIP measurements were started by measuring a sounding curve using a Schlumberger configuration with AB/2 values from 1.5 m to 125 m and MN/2 values of 0.5 m and 5 m. Additionally, a short SIP profile (P1) was measured crossing the borehole ENG_03 and the center of sounding curve (see Fig. 1). The IP profile P1 was measured using 21 electrodes and 2 m electrode spacing. The P1 profile was followed by measuring a long profile (P2) by using 36 remote units to characterize lateral heterogeneity. Moreover, electrode spacing of 7 m was used to acquire the 2-D data cross the P2 profile (Fig. 1, right).

SIP laboratory measurements were done on 33 core samples at different depths (from 0.65 m to 23.15 m) of wells ENG_03 and ENG_08 (close to ENG_03). Then, direct K measurements were carried out by Sass (2010) using coring sleeves (16 samples) and grain size analysis (13 samples), for the upper and lower aquifers, respectively. The coring sleeves were carried out on whole cores (8 cm diameter and 100 cm length) and soil samples (6 cm diameter and 4 cm length). For grain size analysis, we used equations after Kozeny (1927) and Beyer (1964) to derive \( K \) and found that the differences are insignificant. Both field and laboratory SIP measurements were conducted to calculate \( K \) values using various approaches. Nine laboratory SIP samples located at the same depth of those used for coring sleeves were used to derive an empirical relationship for predicting \( K \) values. The inverted field (sounding and profiling) and measured laboratory SIP data were correlated and evaluated to assess the differences in \( K \) values.

The SIP measurements (resistivity amplitude and phase angle) are measured over a wide range of frequencies. To minimize induction effects, a dipole-dipole configuration was used to acquire the profile SIP data. The field and laboratory SIP measurements were carried out using SIP256C and SIP Fuchs equipment, respectively, by Radic Research (Radic et al., 1998; Radic, 2004). The four-electrode device SIP Fuchs measures the complex resistivity over almost seven decades of frequency (from 0.0457 Hz to 12 kHz). On the other hand, the multi-electrode instrument SIP256C measures the complex resistivity over more than three decades (from 0.625 Hz to 1 kHz). In both
instruments, the remote units register the voltage or current measurements, digitize the data and transfer them to controlling PC through a fiber topics cable to avoid interference of the current supply cables with the voltage measurement. Acquisition of very low-frequency data is limited by the data acquisition time. For one profile, total measuring time (was about 2 h) and can easily become excessive if even lower frequencies (< 0.625 Hz) are required, even though the system efficiently measures voltages at all receiver channels simultaneously.

4 Inversion/results and interpretation

A Gauss–Newton algorithm using smoothness constraint with fixed regularization was chosen for sounding and profiling inversions, which were done by using DC1dinv and DC2DInvRes (Günther, 2004), respectively. The 2-D inversion algorithm was implemented for a global regularization scheme using a first-order smoothness constraint (Günther, 2004). As introduced by Constable et al. (1987), we used different weights for horizontal and vertical model boundaries such that $a_x = \lambda$ and $a_z = \lambda w_z$. The regularization parameter ($\lambda$) is a weight that adjust the degree of importance of the model smoothness constraints versus the data misfit. A small value of $\lambda$ (or $w_z$) produces a highly structured model with huge parameter contrasts, explaining data well, whereas a big value will not be able to fit the data but provides a smooth model.

While the apparent resistivity/conductivity amplitudes are almost constant over frequency, the phase shows inductive or capacitive coupling at frequencies starting at about 10 Hz. To avoid distortion by the coupling effect, only low frequency data were used in the inversion process. The 2-D processing included the rejection of data points with bad quality from the measured data, i.e. for which the stacking error was above 1 %. In addition, 3 % error was added to the stacking error to account for systematic error components in absence of reciprocal data.
4.1 Field inversion results/interpretation

Figure 2 shows the inversion results of the IP sounding at 366 mHz. A good fit between the measured and theoretical resistivity data can be observed (Fig. 2a). On the other hand, a bad comparison can be observed at AB/2 higher than 50 m (Fig. 2b, right). This can be attributed to the electromagnetic coupling which can be noted by abrupt changes in the phase angle readings.

The inversion results of resistivity amplitudes show a good comparison with the borehole data (Fig. 2c, left). A low-resistivity layer (< 50 Ωm) can be noticed between the two aquifers corresponding to the till layer. At about 23 m depth, another low resistivity layer can be observed corresponding to the Cretaceous marl layer. These low resistivity layers are characterized by high phase angle values (Fig. 2d, left).

Figure 3 shows the inversion results of the profile P1. Low frequency (0.625 Hz) SIP data were used in the inversion process. For the 2-D inversion, the regularization anisotropy was set to \( w_z = a_z/a_x = 0.01 \) in order to achieve predominantly layered structures. It is clear from the resistivity amplitude (Fig. 3a) that the resistivity decreases with depth. In comparison with the borehole data, the heterogeneities of thin layers are not well defined from the resistivity amplitude image. On the contrast, within the upper sandy aquifer thin layers with high phases (> 8 mrad) and sharp boundaries can be well noticed from the conductivity phase model (Fig. 3b), which corresponds to the peat layers. At about 13 m depth, there is a sharp boundary between low (< 4 mrad) and high phase values (> 13 mrad) which corresponds to the boundary between the second sandy aquifer and the till layer. The first sandy aquifer of low phase values coincidences with the borehole data.

Figure 3c represents the coverage plot for the model of profile P1 (for location see Fig. 1). According to Günther (2004), the zones of higher coverage values indicate that the model can be reliably derived from the data. Consequently, the maximum depth of sensitive area is about 12 m (Fig. 3c) and beyond this depth the data will lose the
capability to resolve the heterogeneity between sedimentary layers. Accordingly, the heterogeneity within the upper aquifer can well be recognized (Fig. 3b).

The inversion results of the other longest 2-D profile P2 showed no great differences to P1. Similar to the P1 profile, we chose the 0.625 Hz data for presentation and further discussion. The regularization parameter of z-weight ($\omega_z$) was set to $a_z/a_x = 0.01$. The resistivity amplitude $|\rho|$ image exhibits a predominantly layered structure with lateral variations within these layers (Fig. 4a). The phase image of the complex electrical resistivity is shown in Fig. (4b). It exhibits a clear layering in comparison with the resistivity amplitude sections. Based on the coverage plot of the measured data, the maximum depth of the reliably inverted model is 30 m (Fig. 4c).

The high resistivity (>300 $\Omega$m) complex of 5.6 m thickness (Fig. 4a) can be differentiated into three layers in the phase image (Fig. 4b); a low phase layer (<4 mrad), which corresponds to sand, between two high phase (>8 mrad) layers corresponding to peats. Because of the high data coverage at 4.1 m depth (Fig. 4c), the peat layer can well be recognized in comparison with the inversion results of profile P1 (Fig. 3b). The soil layer is followed by a medium resistivity layer of ~18 m thickness, which can correspond to the sandy aquifer. This layer overlays a low resistivity layer (<40 $\Omega$m) corresponding to Cretaceous marl. The heterogeneity within the sandy aquifers cannot be recognized. On the contrast, the phase shift image (Fig. 4b) represents the medium resistivity layer in the form of two layers; the upper low phase layer (<4 mrad) corresponding to the first sandy aquifer, which is followed by a high phase layer (>13 mrad), corresponding to the till layer. The lower sandy aquifer cannot be differentiated from the upper till. This can be attributed to the presence of a high phase layer (till) above the lower aquifer decreasing the resolution with increasing depth and/or to the low data coverage with increasing depth (Fig. 4c). At about 23.6 m depth, this high phase layer is followed by the low phase layer (<7 mrad) corresponding to the Cretaceous marl. This layer cannot be recognized at the eastern part of the profile, which can be attributed to the bad data coverage (Fig. 4c).
4.2 Laboratory measurements and model comparison

In the laboratory, the samples were filled into a transparent sample holder (Fig. 5). The sample has to be saturated in the sample holder, which has to be fixed between two chambers that are filled with water. The water conductivity was 450 µS cm$^{-1}$, which is similar to the groundwater conductivity. Sample holder and chambers were put into a conditioning cabinet in order to guarantee constant temperatures for every measurement (25°C ± 0.1°C). It is important to use short cables and to keep their position constant, so that changes in between measurements are avoided. The reproducibility of the measured response was measured by repeating the entire sample preparation and measurement procedure at least twice. Examples of IP spectra in terms of resistivity amplitude and phase angle for three samples (at depth of 2.8 m, 8.82 m and 11.8 m) are shown in Fig. 6. The $|\rho|$ curves show relatively constant values. The $\phi$ are almost constant over a limited range of frequencies (up to 100 Hz) and show inductive or capacitive coupling at higher frequencies indicated by a log-log increase of $\phi$ with frequency ($f$).

Figure 7 shows the borehole information (ENG_03) and SIP measurements (field and laboratory data) at the intersection point ($x = 154$ m, see Fig. 3). Figure 7 indicates that both SIP sounding and profiling models show the general features of the subsurface: High resistivity ($\geq 200$ $\Omega$m) in the unsaturated zone in the first 1 m to 2 m, high phase ($\geq 10$ mrad) in till, marl and clay and low phases for the upper sandy aquifer ($< 5$ mrad). In the 2-D resistivity model, the resistivities of the upper aquifer and the till layer are in the same order of amplitude. In spite of good correspondence of resistivity amplitudes between IP 2-D inversion results and laboratory data, the sounding curve inversion results show slightly lower resistivity values. In both models, thin peat layers around 2 m depth are detected by high phase values, which coincide with laboratory data. On the other hand, the 2-D phase model shows the peat layer at 4.1 m with higher phase values than sounding model and laboratory data. Laboratory resistivity data show slightly lower values in the second aquifer than sounding model and they are...
Hydraulic conductivity \((K)\) estimation

To estimate \(K\) from IP results, three different approaches were applied using Eqs. (5), (6) and (9). These approaches are meaningful in the saturated zone only (i.e. below 2 m). For calibration and the sake of clearness, the derivation of \(K\) values was focused on the laboratory data for the upper aquifer. Then, the \(K\) will be predicted for the second aquifer based on the derived \(K\) of the upper aquifer. In order to apply Eq. (5), a linear relationship should be achieved with a good correlation coefficient \((R^2)\) between the logarithms of aquifer resistivity \(\rho'\) and hydraulic conductivity \(K\). We calculated the real part of resistivity \(\rho'\) using Eq. (4). Figure 8 shows log-log plot of \(\rho'\) (at 1.5 Hz) against the measured \(K\) using coring sleeves of the nine core samples (green solid circles). While there is a general trend of increasing \(\rho'\) with increasing \(K\), the relationship is weak \((R^2 = 0.4)\). Consequently, the estimation of \(K\) from individual \(\rho'\) seems to be of limited applicability.

Both Börner and Slater models were applied to predict \(K\) from the laboratory data and field inversion results in comparison with the measured \(K\) values. Note that the adjustable parameters for each model were derived from the correlation with the measured \(K\) values of the first aquifer and then they were applied for the second aquifer. The inversion results of IP field data (Fig. 7) at the intersection point \((X = 154 \text{ m})\) were used to calculate \(K\) using both models. The Börner model Eq. (6) was applied using \(\sigma_w = 0.015 \text{ S} \text{ m}^{-1}\) and standard values of \(a = 0.00475\) and \(l = 0.1\), as explained above. The exponent \(c = 2.8\) gave the best least-squares fit for the measured \(K\) values below the groundwater level. \(S_{por}\) was calculated using the Weller et al. (2010b) approach.
The second approach for $K$ estimation after Slater and Lesmes (2002) was applied using $m = 2.3 \times 10^{-4}$ and $n = 0.9$.

Figure 9 shows the applicability of both Börner and Slater models on the laboratory data in comparison with the measured $K$ values from coring sleeves and grain size analyses. The Börner approach represents bigger variations ($\sim 1 \times 10^{-7} \text{ to } \sim 1 \times 10^{-2} \text{ ms}^{-1}$), while the $K$ estimation after the Slater model is varying in a smaller range ($1 \times 10^{-6} \text{ to } 1 \times 10^{-4} \text{ ms}^{-1}$). Focusing on the sandy aquifers, it is clear that great differences can be noticed between the results of both approaches for the first aquifer. For the first aquifer, the calculated $K$ values from Börner model are, in general, higher than the measured $K$ values. On the contrast, a good correspondence can be observed between the results of the two approaches in the second aquifer. In general, the calculated $K$ values from Slater model are lower than the measured $K$ values.

The calculated $K$ values from the field inverted data (sounding and profiling) show a wide range using both Börner and Slater models. Figure 10 shows that the estimation of $K$ values from sounding data is better than from 2-D data, in comparison with the measured $K$ values. The $K$ values using the Börner model for the first aquifer of sounding or profiling data are underestimated. On the other hand, the estimated $K$ values from sounding data show a good correspondence with the measured $K$ for the second aquifer (from 17 m to 23 m depth) using the Börner model. It is clear that the calculated $K$ values from sounding and profiling data for the till layer are smaller than the measured $K$ values. Also, at 23 m depth the decreasing $K$ values from both sounding and profiling data are likely to be caused by the Cretaceous marl.

Based on the above results, it is clear that the use of a CPA model, i.e. a single frequency data as representative to the spectral response, values is not valid to predict $K$. Consequently, the spectra were investigated using the DD to calculate the following parameters: DC resistivity $\rho$, total chargeability ($M$) and different representative relaxation times (i.e. maximum relaxation time $\tau_{\text{max}}$; median relaxation time $\tau_{50}$ and logarithmically weighted average relaxation time $\tau_{\text{lw}}$). Note that DD was applied after removing the EM coupling effect fitting a double Cole-Cole model (Pelton et al., 1978),
i.e. one Cole-Cole term is associated to the coupling and another to the phase anomaly. Figure 11a shows an example of raw data (amplitude and phase) of a core sample at 6.68 m depth. Figure 11b shows the correction of phase values after electromagnetic removal. In addition, Fig. 11d reflects a variation in spectral chargeability with relaxation time (τ) by using DD fitting (Fig. 11c). For this sample, the minimum and maximum values of τ are $1 \times 10^{-4}$ s and 1 s, respectively, and the used smoothness factor ($\lambda$) is 150.

For our samples of the first aquifer, the relation of τ (i.e. $\tau_{\text{max}}$, $\tau_{50}$ and $\tau_{\text{lw}}$) and the measured $K$ was studied. While there is a general trend of increasing $K$ with τ, the relationships are weak. Figure 12 shows an example of the $\tau$–$K$ relationship using different values of τ. Although the $\tau_{\text{max}}$–$K$, $\tau_{\text{lw}}$–$K$ relations show a better correlation than $\tau_{50}$–$K$, the correlation coefficients are still low ($R^2 = 0.3$ and 0.22, respectively). Correspondingly, the use of individual $\tau$ to predict $K$ using a power law relation will not be applicable. Moreover, the total of nine measured $K$ values is not enough to apply the multivariate power law relation after Weller et al. (2010a).

### 5.1 A new approach

The hydraulic and electrical conductivity of an aquifer depend on several factors; such as pore-size distribution, grain size distribution and fluid salinity. For our samples from the first aquifer, which has a homogenous fluid resistivity, we consider the $\rho'$ as a measure of grain size distribution, the aquifer resistivity could be well correlated with the $K$ values of the aquifer (Eq. 5). Similarly, if we consider $\tau$ as a measure of the pore radius and pore size characteristics (Eqs. 1 and 2), a link between $\tau$ and $K$ can be considered. The Debye decomposition of IP spectra yields a characteristic relaxation time $\tau$. These can be expressed in the form of $K = A(\rho')^B(\tau)^C$. Accordingly, the $K$ can be calculated using

$$K = A(\rho')^B(\tau)^C$$  \hspace{1cm} (12)
where the $A$ coefficient and the $B$ and $C$ exponents are determined by a regression analysis of the logarithmic quantities $\tau$ and $K$. This methodology was adopted for the estimation of the $K$ when there is a general trend of increasing or decreasing both individual $\rho'$ and $\tau$ with $K$.

5.2 Correlation studies and testing methodology

Either in the cases of $\rho'$–$K$ or $\tau$–$K$ relationships, the correlations are weak but the there are a general increase of $\rho$ and $\tau$ with the measured $K$ values (Figs. 8 and 12, respectively). Equation (13) was applied to the lab data collected from the upper aquifer (1st). Then, the coefficients $A$ and the exponents ($B$ and $C$) were used to predict the $K$ values of the lower aquifer (i.e. the second one). The $\rho'\tau$–$K$ relation was studied using $\tau_{\text{max}}$, $\tau_{50}$ and $\tau_{\text{lw}}$. A strong correlation coefficient ($R^2 = 0.81$) was achieved between $\rho'\tau_{\text{lw}}$ and the measured $K$ (Fig. 13) for nine samples of the first aquifer. A multivariate power law relationship ($A(\rho')^B(\tau_{\text{lw}})^C$) was examined and it was observed that the exponents $B$ and $C$ are nearly equal; $K = 0.00012(\rho')^{1.08}(\tau_{\text{lw}})^{0.8}$. However, due to the limited number of data points we fixed them to be equal (i.e. $K = A(\rho'\tau)^B$) and thus decreasing the number of unknowns. $A$ and $B$ were calculated for the upper aquifer and they were 0.0006 and 0.73, respectively. Based on these results, Eq. (13) was applied for the all core samples of both aquifers using the deduced coefficient $A$ and exponent $B$. Figure 9 shows the calculated $K$ values using Eq. (13) in comparison with the measured $K$ values and the calculated $K$ values from single frequency approaches. In comparison with Eqs. (6) and (9), Eq. (13) shows the lowest mean-squared logarithmic error of fit,

$$
\varepsilon^2 = \frac{1}{N} \sum_{i=1}^{N} [\ln K_m(i) - \ln K_c(i)]^2, \quad \delta = e^\varepsilon
$$

where $N$ is the number of samples (in this paper it is 29) $K_m(i)$ and $K_c(i)$ are the measured and computed $K$ by conventional measurements and various approaches,
respectively, and δ is the error factor. For Börner and Slater models, the δ values were 3.5 and 3.6, respectively. On the other hand, the δ was 1.4 for proposed approach (Eq. 13). Accordingly, Eq. (13) is at least applicable at this test site (Fig. 9).

6 Discussion

Based on the inversion results of both sounding and profile P1, it is clear that the upper aquifer can be well defined but we still have limitations in imaging the lower one. These limitations include an electromagnetic coupling with increasing current electrode spacing of the sounding and the penetration depth of the 2-D imaging. In general, our field inversion results indicate that the upper and lower boundaries of the upper aquifer (1st) can be well defined from the phase values. The upper boundary of the lower sandy aquifer (i.e. the second one) appears deeper than in the borehole, which can be attributed to the observed misfit between the raw and fitted phase angle data (Fig. 2, right). Also, there is a difference of the phase behavior of the peat layer between the laboratory, sounding and profiling results (Fig. 7, right). Zanetti et al. (2011) and Ponziani et al. (2011) reported high phase values for buried tree root samples using laboratory experiments. They showed that a regular distribution of the pore size in diffuse porous woods leads to a stronger polarization effect. Because the peat layer also contains plant remains, the 2-D IP profiles show a high polarization effect compared to laboratory and sounding results.

The inspection of both sensitivity models and the inversion results of IP data are efficient to evaluate the reliability of the interpretation. The coverage model demonstrates that the reliable results concentrate up to 16.5 m maximum depth, which is only sufficient to cover the upper sandy aquifer. Because we have a bad data coverage and electromagnetic coupling effects with increasing electrode spacing, the second aquifer cannot be imaged well. Accordingly, accurate K values of the lower aquifer from the IP field data cannot be expected. Figure (7) shows slight differences between field data inversion results and laboratory data up to the till layer (∼13 m depth). Koch et al. (2011)
stated that changes in compaction and sorting of samples cause a certain shift in the SIP response but did not fundamentally alter the overall picture. Accordingly, the modification of pore space and grain size characteristics of original samples through compaction of preparing the lap sample will change the SIP response, while the chemistry of the saturation pore fluid is kept constant.

Regarding the above results, the proposed IP–$K$ relationships based on single frequency models appear inappropriate for the unconsolidated heterogeneous sediments under investigation. Clearly, there are limitations of using both Börner and Slater models. In order to apply the single frequency models (Eqs. 6 and 9) of both Börner et al. (1996) and Slater and Lesmes (2002), certain conditions are required; (a) a constant phase angle over a wide frequency range, (b) a direct relation between $S_{\text{por}}$ and $\sigma''$ and (c) an inverse relation between the measured $K$ and $\sigma''$. Figure 6 shows noticeable differences in phase angle with frequencies and it would appear inappropriate to use a single frequency complex measure as a representative of IP spectra. The correlation between $S_{\text{por}}$ and $\sigma''$ cannot be proved because $S_{\text{por}}$ values were not measured (i.e. by BET measurements). The use of Eq. (8) after Weller et al. (2010b), which was derived from an extensive sample database, shows that there is not necessarily a correlation between $S_{\text{por}}$ and $\sigma''$. Moreover, Fig. 8 shows that the relationship between $\sigma''$ and $K$ is weak and not linear, although a general decrease of $\sigma''$ with $K$ can be observed. The existence of laboratory data over two decades could be the reason for this weak relation. Therefore, the models of Börner and Slater seem to be of limited applicability at this test site. Compared with the overall lithology and measured $K$ values, the results from the Slater model are appear more realistic than the Börner model, but, in general, the predicted $K$ values are too small.

Since phase spectra of unconsolidated sediments typically show frequency dependence, it is difficult to obtain a constant phase angle behavior over a limited frequency range for inhomogeneous unconsolidated aquifers; multiple frequency data are required to study the spectral behavior and to derive the $K$ values. Our laboratory measurements indicate that for inhomogeneous sandy aquifers the direct relation between
$K$ and $\tau$ is not applicable. Independently of the soil texture, soil compaction has an influence on both the conductive properties (Seladji et al., 2010) and the polarization response (Koch et al., 2011) of porous media. Accordingly, the weak correlation between $K$ and $\tau$ can be attributed to the compaction and sieving of the core samples through preparing the laboratory measurements (Koch et al., 2011). Consequently, other physical parameters, which show a relation with the measured $K$, are required to support the $K–\tau$ relationship. Since the aquifer resistivities show a positive direct relation with the measured $K$ values, a multiplication of $\tau$ and aquifer resistivities shows a good power law relationship to estimate the $K$. In agreement with Zisser et al. (2010b), the logarithmically weighted average relaxation time shows the superior results to predict the $K$ compared with the median relaxation time used by Weller et al. (2010a).

7 Conclusions

The goal of this study was to use SIP field and laboratory measurements to characterize the hydrogeological conditions of Schillerslage test site, Germany. The SIP method provided a tool for imaging subsurface hydrogeological parameters of unconsolidated inhomogeneous sediments. In our particular case, the IP inversion results indicated that the understanding of SIP signatures from laboratory data can now be achieved in the field applications. The results here suggested that SIP is clearly able to differentiate lithological heterogeneities within unconsolidated sediments. The combination of field and laboratory IP measurements was recommended to evaluate the geological situation and the accuracy of the interpretation (e.g., changes in compaction and sorting of samples resulted in a certain shift in the SIP response between laboratory and field inversion results). The mentioned recommendation provided a means to (i) overcome the spectral limitations resulting from EM coupling effects, which mask the SIP response at higher frequency, and (ii) to examine the influence of physical and chemical properties of laboratory samples on SIP response.
Hydraulic conductivity estimates showed that single frequency models to predict $K$ were inappropriate according to our investigations. However, Slater and Lesmes (2002) model appeared more realistic results than Börner et al. (2002). These can be attributed to (i) the different type of sediments used to derive the different approaches and (ii) the assumption that the phase shift is constant over a wide frequency range was not achieved in our data. Therefore, the prediction of $K$ was examined using the relaxation time ($\tau$). The improved Debye decomposition procedure was used to calculate ($\tau$) values from the measured spectra. It was observed that a direct relation between the $K$ and $\tau$ was not successful. In addition, the relation between the real resistivity part ($\rho'$) and $K$ was weak. Another practical application of $\rho'\tau-K$ was tested. Our results revealed that the integration of the logarithmic weighted average relaxation time ($\tau_{lw}$) and DC resistivity yielded a power law relation to predict the $K$ of unconsolidated sandy aquifer particularly if only few data points are present for correlation. However, it would be more meaningful if the above relation is tested in areas with diverse geological environments. Moreover, with the number of samples increasing, a multivariate power law relation ($K = A(\rho')^B(\tau_{lw})^C$) is predictable to improve the prediction of $K$ values and, consequently, this relation should be examined. A further, an application of this approach on 2-D SIP data is recommended.

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References


Fig. 1. (left) Location map of Schillerslage test site (Germany), available borehole, SIP field measurements and SIP profile numbers (inset map, © Google Earth map). (Right) Lithology of the Schillerslage test site.
Fig. 2. SIP sounding inversion results. Apparent resistivity amplitude (a) and phase angle (b) data and the inversion results (c and d, respectively) in comparison with well ENG_03 (see Fig. 1).
Fig. 3. (a) The resistivity amplitude ($|\rho|$), (b) conductivity phase shift ($\varphi$) and (c) coverage sections of profile P1 (0.625 Hz data).
Fig. 4. (a) The resistivity amplitude (|ρ|), (b) conductivity phase shift (ϕ) and (c) coverage sections of profile P2 (0.625 Hz data).
Fig. 5. Laboratory SIP measurements showing the sample holder using SIP Fuchs equipment.
Fig. 6. Exemplary of SIP response of three samples at different depths (2.8 m, 8.82 m and 11.8 m).
Fig. 7. Comparison of sounding, profiling and lab data at well ENG_03. Resistivity (left) and phase (right).
Fig. 8. Log-log plot of the measured resistivity versus the hydraulic conductivity ($K$) of nine core samples from the 1st aquifer (see Fig. 1). Green solid circles (left) denote real resistivity parts ($\rho'$), while the white circles (right) represent imaginary ($\rho''$) resistivity parts.
Fig. 9. Hydraulic conductivity at the location of borehole Eng_03. The black line: from own approach. The green line: from coring sleeves and grain size analysis using Kozeny equation. The Pink line: from Slater and Lesmes model. The blue line: from Börner model. Note that the models are applied on lab data.
Fig. 10. Hydraulic conductivity at the location of borehole Eng_03. The green line: from coring sleeves and grain size analysis using Kozeny equation. Red and pink lines: from Slater and Lesmes model using sounding and profiling data, respectively. Brown and blue lines: from Börner model using 1D and 2D data, respectively.
**Fig. 11.** SIP lab data of one core sample at 6.68 m depth. (a) Resistivity amplitude ($|\rho|$). (b) Measured phase ($\varphi$) (blue line) values, electromagnetic (EM) term (green line) and phase values (red line) after removing EM effect. (c) The fitting between the phase values after removing EM effect (blue line) and the fitted curve (red line). (d) The variation of spectral chargeability with relaxation time by using DD model.
Fig. 12. Log-log plot of the maximum relaxation time ($\tau_{\text{max}}$), median relaxation time ($\tau_{50}$) and weighted average relaxation time ($\tau_{lw}$) against the measured hydraulic conductivity ($K$) of nine core samples from the 1st aquifer (see Fig. 1).
Fig. 13. Log-log plot of the maximum relaxation time ($\tau_{\text{max}}$), median relaxation time ($\tau_{50}$) and weighted average relaxation time ($\tau_{\text{lw}}$) multiplied by $\rho'$ against the measured hydraulic conductivity ($K$) of nine core samples from the 1st aquifer (see Fig. 1).