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A model of landslide triggering by transient pressure waves

G. W. Waswa\textsuperscript{1,2} and S. A. Lorentz\textsuperscript{1}

\textsuperscript{1}School of Engineering and Centre for Water Resources Research, University of KwaZulu-Natal, Durban, South Africa
\textsuperscript{2}Department of Disaster Preparedness and Engineering Management, and Department of Civil Engineering, Masinde Muliro University of Science and Technology, Kakamega, Kenya

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Correspondence to: G. W. Waswa (waswageorge@gmail.com, 208529656@stu.ukzn.ac.za, gwaswa@mmust.ac.ke)

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Abstract

Previous studies indicate that most rainfall-triggered shallow landslides are initiated by a spike in rainfall intensity, which does not usually occur at the beginning of a critical storm, within which the slide is triggered, but after several minutes (or hours) of the storm. The critical storm is also usually not positioned at the beginning of a rainfall season, but after several days of antecedent period. Rainfall triggers landslides via rapid increase in pore water pressure, commonly associated with the change in water content. Consequently, many hydrologic pressure wave models assume that rapid pore water pressure responses are as a result of rapid infiltration of rainwater. On the contrary, this paper argues that, based on the above timings of landslide occurrences and the knowledge that infiltration rate decays with the soil wetness, the rapid increase in pore water pressure that triggers shallow landslides is as a result of rapid introduction of additional energy into the tension saturated (or nearly saturated) zone by the intense rainfall at the ground surface, without infiltration. Antecedent and critical precipitations are significant in creating a tension saturated zone, necessary for rapid transmission of the introduced energy from the ground surface to the lower soil horizons during the critical storm. These arguments are supported by a newly proposed one-dimensional diffusion mathematical model, which, when solved for the appropriate boundary conditions, can yield pore water pressure at any time and depth of a tension-saturated soil profile, without infiltration. The newly proposed diffusion model is mathematically similar to Iverson’s model (Iverson, 2000), except that the hydraulic diffusivity parameter in the latter is substituted with a newly proposed energy diffusivity coefficient in the former. A combination of the new diffusion model and the infinite slope model can predict the stability of a shallow slope as a result of transient pore water pressure.
1 Introduction

1.1 Antecedent and critical precipitation

Rainfall is one of the hydrologic factors that trigger shallow landslides via rapid increase in pore water pressure (Lan et al., 2003; Waswa et al., 2003; Thiebes, 2012). Consequently, rainfall thresholds and characteristics are frequently used to develop statistically based empirical models and physically based deterministic models for prediction of rainfall-triggered shallow landslides. Empirical models are usually based on general rainfall parameters (e.g. intensity, duration, magnitude, cumulative rainfall and antecedent rainfall) and the number of induced landslides. From these general rainfall parameters, more specific parameters (Fig. 1), or terms, have been suggested, in order to effectively describe rainfall thresholds (Aleotti, 2004). A critical storm is the one within which a landslide is initiated. Critical duration is the time (usually in hours) from the beginning of a critical storm to the time that a landslide is triggered. Critical precipitation is the corresponding amount of rainfall collected during the critical duration. Antecedent rainfall is the accumulated amount of rainfall over a specified number of days preceding the day on which landslide occurred. Rainfall intensity-duration and rainfall depth-duration empirical models were first proposed by Caine (1980) and have since been applied by many researchers (e.g. Wilson and Wieczorek, 1995; Matsushi and Matsukura, 2007; Crosta and Frattini, 2003; Guzzetti et al., 2008). A weakness of these models is that they treat the subsurface as a black-box, yet the stability of hillslopes is affected by elevated pressure heads that reduce the soil shear strength.

Antecedent rainfall and critical rainfall duration models (e.g. Terlien, 1998; Crosta, 1998; Crozier, 1999; Glade et al., 2000) implicitly consider soil and hillslope characteristics and may be classified as grey-box models. However, it has been reported that antecedent rainfall is not an important factor in tropical areas, as well as in slopes covered with coarse colluvium having large inter-particle voids. For instance, Rahardjo et al. (2007) found that the antecedent rainfall plays a more critical role in soils with lower permeability compared with soils with higher permeability. Aleotti (2004) states that in low permeability soils, antecedent rainfall can be an important factor because it reduces suction and increases pore-water pressures in soils. A study in Boso Peninsula, Japan, by Matsushi and Matsukura (2007) indicated that slopes underlain by permeable sandstone bedrock had a greater critical rainfall than the slopes underlain by impermeable mudstone bedrock. Guzzetti et al. (2007) found that the minimum intensity likely to trigger slope failures decreased linearly with an increase in the duration of a critical rainfall. Later, Tsai (2008) reported that for the rainstorm with greater than the minimum landslide-triggering rainfall amount, the occurrence of landslide significantly depended not only on the duration but also on the pattern of the critical rainfall. D’Odorico et al. (2005) found that hyetographs with a peak at the end of a rainfall event have a stronger destabilizing effect than hyetographs with a constant rainfall or with a peak at the beginning of a critical storm. Studies by Avanzi et al. (2004), Guzzetti et al. (2004), Chen (2006), and Matsushi and Matsukura (2007) also indicate that shallow landslides are triggered by a peak or spike in rainfall intensity that does not usually occur at the beginning, but at the middle or at the end, of a critical storm.

1.2 Zone of tension saturation

It appears, therefore, from the above studies that the rapid pore water pressure responses that trigger shallow landslides are as a result of the action of intense rainfall on a pre-wetted soil profile. The role of the antecedent rainfall, as well as the critical rainfall, is to wet the soil to a suitable condition for the spike-rainfall-intensity, which occurs in the latter part of the critical storm, to rapidly elevate the pore water pressure and trigger the slide. This is evident in the report of Iverson (2000, p. 1904), that: “...the soil was pre-wetted by application of low-intensity rainfall to raise moisture contents to near saturation levels without producing positive pressure heads ... Higher-intensity rainfall is then used to elevate groundwater pressures and trigger slope failure.” This
report indicates that, for the spike-rainfall-intensity to trigger landslide, the soil profile should be in a state of tension saturated (or nearly saturated). Certainly, Waswa et al. (2013) found that when an intense rainfall acts at the surface of a tension saturated soil profile, an additional energy, proportional to the rainfall intensity, is rapidly introduced into the profile, elevating the pressure potential at every depth.

1.3 Objectives

From the above review, it can be stated that, because the higher-intensity rainfall falls on an already saturated (tension saturated) soil profile, it may not infiltrate the soil to account for the rapidly elevated groundwater pressures that trigger slope failure. This is contrary to many existing physical models of transmission of pressure potential (Crosta, 1998; Iverson, 2000; Guzzetti et al., 2007), which assume that rapid pore water pressure responses that trigger shallow landslides are as a result of rapid infiltration of rainwater. The objectives of this paper are (1) to review some of the existing physical models of transmission of pressure potential in a pre-wetted soil profile; (2) to propose and evaluate a new transient pressure wave model that appears to represent more accurately the physical processes involved in the rapid transmission of pressure potential in a pre-wetted soil profile; and (3) apply the newly proposed model in the evaluation of the stability of a shallow slope as a result of transient pressure waves.

2 A review of some models of transmission of pressure potential in a pre-wetted soil profile

2.1 Richard’s equation

Since change in pressure potential in soil pore water is commonly associated with the change in soil water content, many existing models on the transmission of pressure potential in saturated soils (e.g. Iverson, 2000; Capparelli and Versace, 2013) and nearly saturated soils (e.g. Rasmussen et al., 2000; Cloke et al., 2006) are based on the Richards’ equation (Richards, 1931):

\[
\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial y} \left[ K(\theta) \left( \frac{\partial \psi}{\partial y} - 1 \right) \right] = \frac{\partial}{\partial y} \left[ D(\theta) \frac{\partial \theta}{\partial y} - K(\theta) \right].
\] (1)

In Eq. (1),

\[
D(\theta) = K(\theta) \frac{\partial \psi}{\partial \theta} = \frac{K(\theta)}{C(\psi)}
\] (2)

is the hydraulic diffusivity, \(C(\psi)\) is the specific water capacity, \(\theta\) is the soil water content, \(K(\theta)\) is the unsaturated hydraulic conductivity (depends on soil water content), \(\psi\) is the fluid pressure (capillary suction for unsaturated porous media), and \(y\) and \(t\) are the space and time coordinates, respectively. \(D(\theta)\) is variably called moisture diffusivity and pore water diffusivity (Barari et al., 2009).

It should be emphasized that the Richards’ equation was developed for the movement of water in the unsaturated soils, where a change in pressure potential can be expressed as a function of change in soil water content, a relationship that is defined by \(C(\psi)\). This implies that it may not be possible to define \(C(\psi)\), where the water content is constant, e.g., in a saturated zone.

From the Richards’ equation, therefore, it may be stated that, in an unsaturated porous media of specified physical properties, a given soil water content is associated with a distinct pore water pressure that is unique from other soil water contents. In other words, pore water pressure is dependent only on water content. Therefore, at any point within the porous media, pore water pressure may not change without change in water content. Furthermore, \(K(\theta)\), \(K_{sat}\), as well as \(D(\theta)\), can only be applied to describe the transmission of water (mass) through a porous media, but may be physically inconsistent if applied to the transmission of energy through pore water.
2.2 Iverson’s equation

From Eq. (1), Iverson (2000) developed a one dimensional diffusion equation,
\[
\frac{\partial \psi}{\partial t} = D_0 \cos^2 \alpha \frac{\partial^2 \psi}{\partial y^2}, \tag{3}
\]
that can be used to predict pressure heads in a saturated soil profile as a result of rainfall infiltration at the ground surface. In Eq. (3),
\[
D_0 = \frac{K_{sat}}{C_0} \tag{4}
\]
is the maximum hydraulic diffusivity, \(K_{sat}\) is the saturated hydraulic conductivity, \(C_0\) is the minimum value of \(C(\psi)\) and \(\alpha\) is the slope angle.

Iverson (2000) derived a fundamental solution to Eq. (3) under the following initial and boundary conditions:
\[
\psi(y, 0) = (y - d_y)\beta, \tag{5a}
\]
\[
\frac{\partial \psi}{\partial y}(\infty, t) = \beta, \tag{5b}
\]
\[
\frac{\partial \psi}{\partial y}(0, t) = \begin{cases} -I_y/K_y + \beta & \text{for } t \leq T \\ \beta & \text{for } t > T \end{cases} \tag{5c}
\]
in which, \(\beta\) is a constant that expresses the initial steady-state pressure head distribution, \(d_y\) is the water table depth (in vertical direction, \(y\)), \(T\) is rainfall duration, \(I_y\) is the average infiltration rate in the \(y\) direction at the ground surface, \(K_y\) is the hydraulic conductivity of the soil profile in the \(y\) direction.

Iverson has described \(D_0\) as maximum characteristic diffusivity governing transmission of pressure head, and that it thereby provides convenient reference diffusivity. This implies that this diffusivity describes the transmission of pressure head through water. The problem is that this implication and use of \(D_0\) in Eq. (3) contradict the quantitative definition of \(D_0\) in Eq. (4), which is a hydraulic diffusivity for transmission of water through porous media.

Secondly, Iverson (2000) explained the boundary condition, Eq. (5b), that, “at great depths below the water table, transient vertical groundwater flux decays to zero and steady-state pressures persist” (Iverson, 2000; p. 1902). This explanation indicates that pressure head in deep soil profile can only change as a result of (the arrival of) groundwater water fluxes (perhaps from infiltration). Two questions arise from this explanation. First, do the groundwater fluxes result in change in pressure head? To answer this, imagine monitoring pressure heads inside a soil sample on which a constant-head saturated hydraulic conductivity test is being performed. At any given value of constant-supply-head, there will be water fluxes through the sample, but the pressure head inside the soil sample will remain constant. Second, are the changes in pressure head in a deep soil profile a result of the arrival of new water? To answer this, consider a tensiometer ceramic-cup installed in a saturated soil profile, below a phreatic surface. When the ground water table is recharged, the tensiometer will indicate a positive change in pressure potential. The change in pressure potential at the depth of the ceramic cup is neither as a result of a change in water content, nor arrival of new water, but as a result of change in energy content at that depth. While the water content at that depth remains constant (at full saturation) the energy content varies. The transmission of energy from recharge water is more rapid than the transmission of new water itself.

Thirdly, the use of \(D_0\) in Eq. (4) was based on the fact that the soils were at full saturation. According to texts in soil physics and groundwater, “... the rate of infiltration decreases as the soil becomes wetter and less able to take up water”, (Shaw, 1994; p. 132). Therefore, it may be physically contradicting to use infiltration in defining the pressure head boundary condition (Eq. 5c) at the surface of a saturated soil profile.
Iverson used the hydraulic diffusivity parameter to describe the transmission of pressure head based on the assumption that transmission of pressure head (energy) is synonymous to the transmission of water (mass). This assumption contradicts what appears to be a more logical argument, on the distinction between water content-dependent and water-content-independent pressure potential transmission, that: “...the distinction between pore pressure transmission and water flux is relevant. . . ., but pore pressure change in a porous medium is largely a diffusive process that can occur with or without much water fluxes” (Iverson, 2000; p. 1899).

Fourthly, Iverson arrived at Eq. (3) from Eq. (1) by arguing that when the soils are sufficiently wet, the gravity component in Eq. (1) can be neglected, and if the soils are sufficiently dry the pressure term in Eq. (1) can be neglected. These arguments seem to be inconsistent with the physics of infiltration according to the Richards’ equation: when the soils are dry, water “diffuses” through the soils under the influence of pressure head (adhesion forces and capillary pressure), and gravity term can be neglected. But when the soils become wet, gravity forces start dominating the flow of water and the pressure term can be neglected.

Therefore, while Iverson’s model may be mathematically sound and has been widely adopted (e.g. Crosta and Frattini, 2003; Chen et al., 2005; D’Odorico et al., 2005; Tsai, 2008; Tsai et al., 2008; Berti and Simoni, 2012), it appears to be inconsistent with the physical processes involved in the diffusive transmission of pressure potential through pore water without, or with very little, water fluxes.

2.3 Rasmussen et al.’s (2000) equation

From the Richards’ equation and the kinematic wave theory, Rasmussen et al. (2000) derived an advection-diffusion equation:

$$\frac{\partial \psi}{\partial t} = D \frac{\partial^2 \psi}{\partial y^2} - \nu \frac{\partial \psi}{\partial y},$$  \hspace{1cm} (6)

for prediction of transient pressure wave velocities in an unsaturated soil profile, due to perturbations at the surface. The coefficient \(\nu\) in Eq. (6) is the wave velocity, expressed as \(dK/d\theta\). The proposed solution to Eq. (6) for spike input was given as:

$$\psi(y, t) = \frac{\nu C_0}{\sqrt{4\pi Dt^3}} \exp \left[ - \frac{(y - \nu t)^2}{4Dt} \right],$$  \hspace{1cm} (7)

in which \(C_0\) was defined as the hydraulic condition at the upper boundary of a semi-infinite column of soil and \(D\) is the hydraulic diffusivity. Equation (7) was successfully used to reproduce the shape of pressure response curves observed from a laboratory soil-column experiments, performed with the application of short-duration fluid irrigation pulses at the soil surface. Observations from these laboratory experiments indicated transmission of pressure potential through the unsaturated soil profile without detectable change in water content, a phenomenon that is inconsistent with the Richards’ equation.

Although Rasmussen et al. (2000) derived Eq. (6) from the Richards’ equation (1), they did not use Eq. (2) in estimating \(D\). Instead, based on their experimental data and Eq. (7), they estimated \(D\) using an analytical parameter estimation technique (Farlow, 1992). Since the data they used to estimate \(D\) was the pressure potential data only, and the derived parameter could effectively reproduce the observed transmission of pressure potential, which occurred without change in water content, this derived parameter could not be used practically as a water-content-dependent hydraulic diffusivity coefficient. This was apparent in the much lower specific water capacity values obtained from the observed experimental data, than the anticipated value calculated from the water retention curve of their soil sample.

Moreover, in order for Eq. (7) to fit on the observed data, Rasmussen et al. (2000) had to neglect the kinematic velocity component (i.e. \(\nu = 0\)), because it was found to be insignificant. This step, i.e. equating \(\nu\) to zero, implied that the second term on the right hand side of Eq. (6) was not necessary, effectively reducing Eq. (6) to a diffusion equation, similar to the one derived by Iverson (2000).
Lastly, when implementing Eq. (7), Rasmussen et al. (2000) did not specify the \( C_0 \). Nevertheless, to satisfy dimensional homogeneity condition, the coefficient of the exponential term in Eq. (7) should be of the same units as those on the left-hand side of the equation. A dimensionless analysis of Eq. (7) indicates that, for the units of \( \psi \) to be in length [L], as reported by Rasmussen et al. (2000), \( C_0 \) has to take on the units of Length-Time [LT], e.g. cm-day.

In summary, while the Richards’ equation, and more specifically, the embedded hydraulic parameters, e.g. hydraulic diffusivity coefficient and hydraulic conductivity, sufficiently and adequately applies where change in pressure potential is a function of soil water content, it violates the law of conservation of energy, when applied where the change in pressure potential occurs without the change in water content (e.g. Rasmussen et al., 2000; Gillham, 1984).

A new theory of transmission of pressure potential through a pre-wetted soil profile without water fluxes and change in water content is presented below.

3 A differential equation for transmission of pressure potential through a pre-wetted soil profile without water fluxes

3.1 A differential equation

We consider a tension saturated (or nearly saturated) soil profile, as idealized in Fig. 2. The zone of tension saturation, which may be as a result of antecedent precipitation and critical precipitation (Aleotti, 2004), extends to the ground surface. When a spiked-intensity rainfall, which occurs during the critical rainfall, falls on the ground surface, its kinetic energy is imparted into the tension saturated (or nearly saturated) zone as potential energy. The newly introduced potential energy is rapidly transmitted down the tension saturated soil profile, elevating the pressure potential (energy) at every depth below the ground surface. This introduced energy may trigger landslide.

From the law of conservation of energy, the diffusive transmission of pressure potential from the ground surface through the zone of tension saturation and without water fluxes can be described by the equation (Waswa, 2013):

\[
\frac{\partial h_w}{\partial t} = d_e \frac{\partial^2 h_w}{\partial y^2},
\]

in which, \( h_w \) [L] is the pressure potential (potential energy per unit weight) and \( d_e \) [L²T⁻¹] is energy diffusivity coefficient and can be expressed as:

\[
d_e = \frac{\kappa}{\rho_w g}.
\]

In Eq. (9), \( \kappa \) [MT⁻³] is the energy conductivity of the pore water and \( \rho_w \) [ML⁻³] is the density of water. \( \kappa \) and \( d_e \) are analogous to the hydraulic conductivity and hydraulic diffusivity for transmission/flow of water through porous media, respectively. They are also analogous to the thermal conductivity and thermal diffusivity for conduction of heat through solid, respectively. An expanded derivation of Eq. (8) can be found in Appendix A and a more detailed derivation can be found in Waswa (2013).

3.2 Initial and boundary conditions

3.2.1 Initial conditions

The initial condition is the pressure potential at every depth in the soil profile, just before the occurrence of the spike-intensity that rapidly elevates the pressure potential. Since we are interested in predicting the deviation behaviour of the pressure potential from the initial conditions and in one dimensional vertical direction, to simplify the mathematics and analysis, we can reduce the initial conditions everywhere to zero, i.e.,

\[
h_w(y, 0) = 0 \quad \text{for} \quad y \geq 0
\]

The actual initial conditions can always be added to the solution.
3.2.2 Boundary conditions

The conditions at the ground surface can be defined by:

\[ h_w(0,t) = h_w(t), \quad \text{for } t \geq 0, \tag{11} \]

where, \( h_w(t) \) is the pressure potential at the upper boundary of the soil profile and may be expressed as a function of time.

It is also necessary to pose the semi-infinite boundary condition, where the change in pressure potential with depth, and at any time, becomes negligible, as the depth becomes large. This can be written as:

\[ \frac{h_w(y,t)}{dy} \to 0, \quad \text{as } \ y \to \infty. \tag{12} \]

It is worth noting that the change in pressure potential at any point within a homogeneous and incompressible fluid depends on the depth of that point relative to some reference plane, and is not influenced by the size and shape of the container (Janna, 1983). In other words, the position and the shape of the lower and the side boundaries to the soil profile do not have any influence on the change in pressure potential at a point within the soil profile. More directly, the conditions of the lower and the side boundaries are redundant and would not influence or be involved in the analytical solution or observed results.

3.3 Solution to the differential equation

The solution to Eq. (8), which can also satisfy the initial and boundary conditions Eqs. (10)–(12), may take the form,

\[ h_w(y,t) = h_w(0,t) \text{ erfc} \left( \frac{y}{\sqrt{4d\varepsilon t}} \right), \tag{13} \]

which is analogous to the heat conduction solution (Carslaw and Jaeger, 1959; p. 63). The actual initial conditions can be added to Eq. (13).

3.4 Estimation of energy diffusivity coefficient

In applying Eq. (13), the value of \( d\varepsilon \) should be known. Here, this parameter is determined by fitting Eq. (13) to the experimental data, in a methodology similar to that employed by Rasmussen et al. (2000) and Flury and Gimmi (2002), which requires the determination of time to peak, \( t_p \), of response. Here, \( t_p \) is determined as the time when the second time derivative of Eq. (13) is zero, i.e. \( \frac{\partial^2 h_w(y,t)}{\partial t^2} = 0 \), which is:

\[ t_p = \frac{y^2}{6d\varepsilon}. \tag{14} \]

The theoretical results from Eq. (13) are compared with results from the field observations.

4 Evaluation and application of the new transient pressure wave model

4.1 Evaluation of the model

Equation (13) was evaluated using field observations from the Weatherley research catchment in South Africa and for the rainfall events of the summer season of 2000/01. Detailed description of the catchment can be found in Lorentz et al. (2004) and Waswa et al. (2013). The data of pore water pressure responses used to evaluate the model were obtained from an observation site that was located in the upper part of the catchment. The physical parameters of the soil at the observation site were as shown in Table 1. The instrumentation at the observation site consisted of two ceramic-cup tensiometers, installed at 20 and 100 cm vertically below the ground surface, as shown in Fig. 3.
Based on certain criteria (Waswa et al., 2013), the 2000/01 summer season had 96 rainfall events, of which 11 caused rapid pore water pressure responses at the observation site. These eleven rainfall events occurred in the middle of the rainfall season when the water table at the observation site was shallow and the zone of tension saturation extended to the ground surface. For the present purpose of appraising the model, the tensiometric responses at the observation site in two events, namely Event 47 and Event 70 (Fig. 4 and Table 1), are used. From the responses of pressure potential at the deeper tensiometer (100 cm below ground surface), the average value of the energy diffusivity coefficient of between 27–33 cm$^2$ min$^{-1}$ was calculated using Eq. (14). The observed values of pressure potential at the shallower tensiometer (20 cm below ground surface) were used as the boundary condition (Eq.11) in Eq. (13) to predict the pressure potential at the deeper tensiometer. The predicted and the observed values at the deeper tensiometer (100 cm below ground surface) agreed fairly well (Fig. 5).

### 4.2 Application of the new model to landslides prediction

The model (Eq. 13) can be combined with an infinite slope model to predict the stability of a shallow slope. The infinite slope model is commonly used to predict the stability of a shallow slope based on a dimensionless factor of safety (FS) that is generated by carrying out a limited equilibrium analysis of the ratio of driving forces to the resisting forces acting on the slope material along a potential failure plane. FS greater than 1 indicates stable conditions and FS less than 1 indicates unstable conditions. The infinite slope model can be expressed as (Iverson, 2000):

$$FS = \frac{\tan \phi}{\tan \alpha} + \frac{c}{\gamma_s \sin \alpha \cos \alpha} - \frac{h_w(y,t) \tan \phi}{\gamma_s \sin \alpha \cos \alpha}$$  \hfill (15)

where, $\phi$ is the friction angle of the soil in degrees, $\alpha$ is the slope of bedrock or potential failure plane in degrees, $c$ is the cohesion of the soil in N m$^{-2}$, and $\gamma_s$ is the saturated unit weight of the soil in N m$^{-3}$. All the terms in Eq. (15) depend on soil water content.

However, when the soil profile is in a tension saturated (or nearly saturated) state, as it usually is during the latter period of the critical storm but before landslide initiation, soil water content may not vary significantly to account for the rapid increase in pore water pressures that trigger landslide. Therefore, Eq. (15) can be reduced to be dependent only on depth and time as follows:

$$FS = A + F_c(y) + F_w(y,t) = A + \frac{B}{y} - \frac{C \cdot h_w(y,t)}{y}.$$  \hfill (16)

where, $F_c(y)$ is the depth-dependent cohesion factor and $F_w(y,t)$ is the depth- and time-dependent pore water pressure potential factor; and $A$, $B$, and $C$ are constants: $A = \frac{\tan \phi}{\gamma_s \sin \alpha \cos \alpha}$, $B = \frac{c}{\gamma_s \sin \alpha \cos \alpha}$, and $C = \frac{\tan \phi}{\gamma_s \sin \alpha \cos \alpha}$.

Using Eq. (13) and the measured soil physical properties in Table 1, the time- and space-dependent pore water pressure variable in Eq. (16) can be predicted as shown in Fig. 6. With the predicted pressure potential, together with the assumed mechanical properties of the soil (Table 1), Eq. (16) was used to generate the behavior of the factor of safety down the soil profile, at any given time (Fig. 7), as well as the history of safety factor at any given depth (Fig. 8).

### 5 Discussions

#### 5.1 The role of tension saturation

Our results from field observations indicated that the presence of a tension saturated (or nearly saturated) zone, and its extension to the ground surface, played a significant role in the transmission of pressure potential from the ground surface to the lower soil horizons when an intense rainfall occurred at the ground surface. These results support Marui et al.’s (1993) statement that “the rapid response of pore water pressure in the deep soil profile is assisted by a relatively continuous water phase” (p. 469).
Antecedent and critical precipitations are significant in the formation of a continuous tension saturated (or nearly saturated) zone from the ground surface to the deep horizons. In environments with less steep slopes, or less permeable soil, or in cold climates, the rainwater absorbed by the soils from the previous events is lost or drained at a relatively low rate. In such environments, therefore, saturation (or near saturation) conditions are more likely to be maintained for a long period, and the antecedent precipitation becomes more significant than the critical precipitation in rainfall-triggered shallow landslides. On the other hand, in environments with steep slopes, or more permeable soils, or hot climates, the drainage of rainwater absorbed by the soils from the previous events is relatively rapid and saturation (or near saturation) conditions cannot be sustained for a long period of time. In such environment, critical precipitation is significant for the formation of the tension saturated soil profile, necessary for transmission of pressure potential.

5.2 Rapid pore pressure responses and slope stability

Immediately the peak-intensity rainfall occurred (Table 1, 18:10 LT for Event 47 and 13:39 LT for Event 70), the pressure potentials at the shallower horizons were rapidly elevated (Fig. 6), but started recovering as the potential diffused to the lower horizons. From the durations of the rainfall events (Table 1) and the behavior of the pressure curves in Fig. 6, it can also be seen that the recovery of the pressure potential in the shallower horizons started before the end of the rainfall events. This can be seen when one compares the pressure curves at 12 and 24 min in Fig. 6. This is contrary to the results reported by Iverson (2000), which showed that maximum pressure potentials in the shallower horizon are attained only at the end of the rainfall event. This difference arises because Iverson's pressure potential is a function of infiltration, which depends on the duration of the event, and based on the assumption that all the rainfall is absorbed by the soil. In the present case, the pressure potential is a function of the energy conditions at the ground surface (governed by \( h_w(0,t) \) in Eq. 13), which are dependent on the intensity, and not the duration, of the rainfall event.

5.3 Transient pressure wave models

The newly introduced pressure potential caused a transient perched water table condition in the upper soil profile, above the deeper water table, e.g. at 12 min (Fig. 6). Here, it might be worth mentioning that a water table is a locus of points where the pressure potential in the soil pore water is equivalent to the atmospheric pressure (Holzer, 2010). It is realistic, therefore, for a water table to be perched above another one, as indicated by the pressure curve at 12 min in Fig. 6. It is also realistic for a water table to appear above the ground surface (similar to a potentiometric surface in confined aquifers), i.e., if a great amount of energy is introduced into the tension saturated soil profile. This is again contrary to Iverson's comment that 'pressure heads above \( \beta \) line are physically unrealistic' (Iverson, 2000; Fig. 7). Iverson's \( \beta \) line is a unit-gradient pressure line passing through zero pressure head at the ground surface, as illustrated in Fig. 6. Iverson's \( \beta \) line limits a water table to the ground surface.

The spatial and temporal behavior of the transient pressure potential is also reflected/mirrored in the factor of safety. It can be noted from Fig. 7 that in the early period of the pressure potential responses, the factor of safety, in both events, decreases rapidly in the upper horizons and starts increasing as the lower parts of the soil profile become unstable. For instance, it is shown that at 24 min, there are two horizons within which the slope can fail: 0–14 cm below the ground surface and the horizon below 38 cm (Fig. 7). The upper horizon is due to the downward transmission of pressure potential, which resulted in perched water table conditions. The instability in the lower horizon is due to the rising water table.

The present results indicate that the spike intensity rainfall may not trigger landslide via rapid infiltration, but by rapid introduction of additional pressure potential into the tension saturated (or nearly saturated) soil profile. The introduced pressure potential (potential energy) diffuses through the pore fluid from the ground surface to the lower horizons without, or with very little, water fluxes and change in water content.
Consequently, Richards’ equation, which was developed for transmission of water in the unsaturated soils and can only predict pressure potential as a function of water content, may not be extended to represent the diffusive transmission of pressure potential through a pre-wetted soil profile without a change in water content. The hydraulic parameters, e.g. the hydraulic conductivity and the hydraulic diffusivity coefficient, embedded in Richards’ equation, are only meant to describe the transmission and diffusion of water in unsaturated porous media and may not be extended to describe the diffusive transmission of pressure potential through pore water without change in water content. This was evident in the report of Rasmussen et al. (2000), in which analytically derived diffusivities (from pressure potential data only) were significantly different from the hydraulic diffusivity obtained from the water retention curve for the same soil.

Iverson’s (2000) diffusion model may be mathematically sound for transmission of pressure potential and is widely applied in the analysis of shallow landslides. Nevertheless, the hydraulic diffusivity, in the Iverson’s model, may not be an appropriate physical parameter for transmission of pressure potential. This misplacement of the hydraulic diffusivity appears to originate from the physically inconsistent assumption made during the derivation of the Iverson’s equation that: when the soils are sufficiently wet, the gravity term in the Richard’s equation can be neglected, and that if the soils are sufficiently dry, the pressure term can be neglected. Furthermore, the use of infiltration in defining the pressure potential at the ground surface in the Iverson’s model may not represent the actual field conditions in most rainfall-triggered shallow landslides.

A new diffusion model presented in this paper can predict the transmission of potential energy through a uniformly wetted soil profile and without the movement of water or change in water content. The key parameter is the energy diffusivity coefficient, which indicates the diffusive transmission behaviour of potential energy through pore fluid.

6 Conclusion

It can be concluded that the spike in rainfall intensity, which rapidly elevates the pressure potential that triggers shallow landslides, usually occurs on a tension saturated (or nearly saturated) soil profile and when infiltration is nearly zero. Infiltration, therefore, may not account for the rapid elevation of pressure potentials that trigger shallow landslides.

Consequently, the existing transient pressure wave models, which are derived from the Richards’ equation, and predict pressure potential as a function of infiltration, may not sufficiently represent the exact field conditions in most rainfall-triggered shallow landslides.

The newly proposed transient pressure wave model is mathematically similar to Iverson’s (2000) model, except that the energy diffusivity coefficient in the former substitutes the hydraulic diffusivity in the latter. Additionally, the new proposed model is solved for the ground surface energy conditions, associated with rainfall intensity (and the kinetic energy), contrary to Iverson’s model, which is solved for infiltration and rainfall duration.

The newly proposed transient pressure wave model and its solution, therefore, seems to represent more accurately the physical conditions in most intense rainfall-triggered shallow landslides, in which the soil profile is usually in tension saturated (or nearly saturated), to an extent that infiltration rate may be quite insignificant to account for the rapid increase in pressure potential that triggers shallow landslides. This model, when combined with the infinite slope model, can predict the stability of a shallow slope as a result of transient pore water pressure.
Appendix A

Derivation of Eq. (8)

A1 Assumptions

Equation (8) was derived based on the following assumptions: (1) the porous media is homogeneous and isotropic on the averaging scale; (2) the porous media is rigid and non-deformable in space and time (the physical properties of the porous medium are spatially and temporally constant).

A2 Derivation of the equation

The soil profile, as shown in Fig. 2, is considered. The profile is of uniform unit cross-sectional area, $U \, [L^2]$, and uniform saturation. The vertical space coordinate is oriented positive downwards, with the origin at the upper boundary of the soil profile. The energy coordinate is also oriented positive downwards.

It should be noted that the fundamental property here is energy, which cannot be described by a velocity field of water. This is because energy is transmitted through water without the movement of water particles. According to the basic law of conservation of energy, and for the entire height of the soil profile, $0 \leq y \leq y_{SP}$, the rate of change of energy within the soil profile is equal to the net energy transmitted through the two boundaries, $\beta_u$ and $\beta_l$.

We consider an infinitesimal section of height $dy$ in the interval $0 \leq y \leq y_{SP}$. The energy content, $dE_c = [ML^2T^{-2}]$, in this infinitesimal section is proportional to the weight of water and the fluid pressure (energy per unit weight of water), $h_w \,[L]$:

$$dE_c = U \rho_w g n h_w \, dy , \quad (A1)$$

where, $U \,[L^2]$ is the unit-cross-sectional area, $\rho_w \,[ML^{-3}]$ is the density of water, $n$ is the porosity of the porous media, $g \,[LT^{-2}]$ is the gravitational constant. Therefore, in the absence of work done, the total energy content in the interval $0 \leq y \leq y_{SP}$ is:

$$E_c = \int_{0}^{y_{SP}} U \rho_w g n h_w(y,t) \, dy . \quad (A2)$$

Based on Fourier’s law of heat conduction, the rate of energy transmitted into a body (the soil profile) through a small surface element on its boundary is proportional to the area of that element and the outward normal derivative of the pressure potential at that location. In other words, the diffusion rate of energy through a surface/boundary is proportional to the negative pressure energy gradient across the surface/boundary. Therefore, the net energy through the boundaries $\beta_u$ and $\beta_l$ is:

$$E_r(t) = \kappa U n \frac{dh_w}{dy}(y_{SP},t) - \kappa U n \frac{dh_w}{dy}(0,t) , \quad (A3)$$

where, $\kappa \,[MT^{-3}]$ is energy conductivity of the pore fluid. Energy conductivity, analogous to thermal conductivity in thermodynamics (Yunus, 1990), is the time-rate of energy transfer through a unit thickness of the material (the zone of tension saturation) per unit area per unit pressure difference.

From the law of conservation of energy, Eqs. (A2) and (A3) may be combined as:

$$\frac{dE_c}{dt} = E_r , \quad \text{or} \quad \frac{d}{dt} \int_{0}^{y_{SP}} \rho_w g h_w(y,t) \, dy = \kappa \frac{dh_w}{dy}(y_{SP},t) - \kappa \frac{dh_w}{dy}(0,t) . \quad (A4)$$

For smooth material properties, i.e., if $\rho_w$ and $\kappa$ are continuous with continuous first derivatives, the solution $h_w(y,t)$ is also continuous with continuous first partial derivatives $\partial h_w/\partial y$ and $\partial h_w/\partial t$ (Kevorkian, 1989). Hence, Eq. (A4) can also be written
in the following form, after expressing the right-hand side as the integral of a derivative:

\[
\int_0^{y_p} \left\{ \rho_w g \frac{\partial h_w}{\partial t}(y, t) - \frac{\partial}{\partial y} \left[ \kappa \frac{\partial h_w}{\partial y}(y, t) \right] \right\} \, dy = 0. \tag{A5}
\]

Since Eq. (A5) can apply to any limits \( y_1 \) and \( y_2 \), it follows that the integrand must vanish:

\[
\rho_w g \frac{\partial h_w}{\partial t} - \frac{\partial}{\partial y} \left[ \kappa \frac{\partial h_w}{\partial y} \right] = 0. \tag{A6}
\]

Equation (A6) can be written simply as:

\[
\frac{\partial h_w}{\partial t} = d_e \frac{\partial^2 h_w}{\partial y^2}. \tag{A7}
\]

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References


Thiebes, B.: Landslide Analysis and Early Warning Systems, Local and Regional Case Study in the Swabian Alb, Germany, Local and Regional Case Study in the Swabian Alb, Germany, Springer Theses, doi:10.1007/978-3-642-27526-5_2, 2012.


Table 1. Physical parameters and characteristics of the soil at the observation site in the Weatherley research catchment in South Africa and the characteristics of the rainfall Events 47 and 70 of the summer season 2000/01.

<table>
<thead>
<tr>
<th>Parameter and units</th>
<th>Symbol</th>
<th>Value</th>
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<tr>
<td>Hydraulic and energy parameters</td>
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<tr>
<td>Saturated hydraulic conductivity (cm h(^{-1}))</td>
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<td>Pore air entry pressure head (cm)</td>
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<td>Energy diffusivity coefficient (cm(^2) min(^{-1}))</td>
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<td>Soil physical parameters</td>
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<tr>
<td>Medium sand (%)</td>
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<tr>
<td>Fine sand (%)</td>
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<tr>
<td>Silt and clay (%)</td>
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<td>Mechanical properties*</td>
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<tr>
<td>Friction angle (degrees)</td>
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<tr>
<td>Cohesion (N m(^{-2}))</td>
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<td>Rainfall characteristics</td>
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<tr>
<td>Event 47</td>
<td>Event 70</td>
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</tr>
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<td>10 Mar 2001</td>
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<tr>
<td>Time of the event</td>
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<td>13:15 LT–14:24 LT</td>
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<td>36; 92</td>
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<td>Time of peak intensity (hh:mm)</td>
<td>18:10 LT; 18:17 LT</td>
<td>13:39 LT; 14:09 LT</td>
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* Assumed values.

Table A1. Nomenclature.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>(c)</td>
<td>Cohesion of the soil [ML(^{-1})T(^{-2})]</td>
</tr>
<tr>
<td>(d_e)</td>
<td>Energy diffusivity (diffusion constant) in soil water [L T(^{-1})]</td>
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<tr>
<td>(E_i)</td>
<td>Complementary error function</td>
</tr>
<tr>
<td>(E_{i1})</td>
<td>Time-rate change in energy content in the zone of tension saturation [ML(^2)T(^{-3})]</td>
</tr>
<tr>
<td>(E_{i2})</td>
<td>Energy content in the zone of tension saturation [ML(^4)T(^{-3})]</td>
</tr>
<tr>
<td>(g)</td>
<td>Gravitational acceleration [LT(^{-2})]</td>
</tr>
<tr>
<td>(h_p)</td>
<td>Pore air entry pressure of the soil [L]</td>
</tr>
<tr>
<td>(h_w)</td>
<td>Pore water pressure [L]</td>
</tr>
<tr>
<td>(K_{sat})</td>
<td>Saturated hydraulic conductivity [LT(^{-1})]</td>
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<tr>
<td>(n)</td>
<td>Porosity of the porous medium [L(^2)L(^{-3})]</td>
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<tr>
<td>(t)</td>
<td>Time coordinate [T]</td>
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<tr>
<td>(t_p)</td>
<td>Time to peak response of the potential pressure [T]</td>
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<tr>
<td>(U)</td>
<td>Unit horizontal cross-section area of the zone of tension saturation [L(^2)]</td>
</tr>
<tr>
<td>(y)</td>
<td>Space coordinate [L]</td>
</tr>
<tr>
<td>(y_{01})</td>
<td>Depth of the zone of tension saturation [L]</td>
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<td>(\alpha)</td>
<td>Slope angle of bedrock [-]</td>
</tr>
<tr>
<td>(\phi)</td>
<td>Friction angle of the soil [-]</td>
</tr>
<tr>
<td>(\infty)</td>
<td>Infinity</td>
</tr>
<tr>
<td>(cm)</td>
<td>Centimeter</td>
</tr>
<tr>
<td>(CS)</td>
<td>Coarse sand (0.50–2.00 mm)</td>
</tr>
<tr>
<td>(FS)</td>
<td>Fine sand (0.053–0.25 mm)</td>
</tr>
<tr>
<td>(H_2O)</td>
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<tr>
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<tr>
<td>(kg)</td>
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<tr>
<td>(m)</td>
<td>Meter</td>
</tr>
<tr>
<td>(mm)</td>
<td>Millimeter</td>
</tr>
<tr>
<td>(m)</td>
<td>Mass</td>
</tr>
<tr>
<td>(N)</td>
<td>Newton</td>
</tr>
<tr>
<td>(V)</td>
<td>Volume</td>
</tr>
<tr>
<td>(\beta_l)</td>
<td>Lower boundary of the zone of tension saturation</td>
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<tr>
<td>(\beta_u)</td>
<td>Upper boundary of the zone of tension saturation</td>
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</table>
Fig. 1. Rainfall parameters used in the formulation of rainfall thresholds (modified from Aleotti, 2004).

Fig. 2. Idealized soil profile with a tension saturated (or nearly saturated) zone extending to the ground surface. Key: Arrow 1 is intense rainfall (spike) that rapidly introduces additional pressure potential into the zone of tension saturation. Arrow 2 is the downward transmission of introduced potential energy (pressure potential), which rapidly elevates pore water pressure that may trigger landslides. $\beta_u$ and $\beta_l$ are the upper boundary and lower boundary, respectively, of the soil profile under consideration. The upper boundary coincides with the origin of the space coordinate ($y = 0$). Since the depth of the soil profile under consideration is $y_{SP}$, the lower boundary coincides with the space coordinate $y = y_{SP}$. $W_t$ is the pore water in tension.
Fig. 3. Instrumentation and the position of the ceramic-cup tensiometer in the soil profile at the observation site in the Weatherley research catchment in South Africa.

Fig. 4. Rapid responses of pressure potential in a tension saturated soil profile at the observation site in the Weatherley research catchment in South Africa in (a) Event 47 and (b) Event 70, of the 2000/01 summer season.
Fig. 5. Theoretical and observed changes (responses) in pressure potential at 20 and 100 cm below ground surface at the observation site in the Weatherley research catchment in South Africa in (a) Event 47 and (b) Event 70 of the 2000/01 summer season.

Fig. 6. Pressure potential in a soil profile at the observation site in the Weatherley research catchment in South Africa during (a) Event 47 and (b) Event 70 of the 2000/01 summer season. 0 min corresponds to 18:12 and 13:37 LT for (a) Event 47 and (b) Event 70, respectively (see Fig. 5).
Fig. 7. Behaviour of factor of safety as a result of change in pore water pressure in a soil profile at the observation site in the Weatherley research catchment, South Africa, during (a) Event 47 and (b) Event 70 of the summer season of 2000/01. (FS were calculated based on assumed soil mechanical properties indicated in Table 1). 0 min corresponds to 18:12 and 13:37 LT for (a) Event 47 and (b) Event 70, respectively (see Fig. 5).

Fig. 8. (a) The history of factors of safety (FS) at selected depths in a soil profile at the observation site in the Weatherley research catchment in South Africa during (a) Event 47 and (b) Event 70 of the 2000/01 summer season (FS were calculated based on assumed soil mechanical properties indicated in Table 1).