Derivation and evaluation of landslide triggering thresholds by a Monte Carlo approach

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Abstract

Rainfall thresholds are the basis of early warning systems able to promptly warn about the potential triggering of landslides in an area. Following a common empirical methodology, thresholds may be derived through the analysis of historical rainfall and landslide data, by drawing an envelope curve of triggering rainfall events, represented by their intensity and duration. Nonetheless, reliability of empirical thresholds is generally affected by the historical data quality and availability. Moreover, rainfall intensity and duration alone may not be able to capture most of the uncertainty related to landslide triggering.

In this work Monte Carlo simulations are carried out to generate a synthetic rainfall series by a stochastic model and the corresponding landslide response by means of an hydrological and geotechnical model. The series are of virtually unlimited length and present no interruption in data availability, and the triggering instants can be precisely identified, overcoming some of the most important quality and availability drawbacks of using historical data. Receiver Operating Characteristic (ROC) analysis is carried out to derive and evaluate landslide triggering thresholds, considering both triggering and non-triggering rainfall. The effect of variability of both rainfall intensity within events and of initial conditions as determined by antecedent rainfall is analysed as well.

The proposed methodology is applied to the landslide-prone area of Peloritani Mountains, Northeastern Sicily, Italy.

Results show that power-law ID equations can adequately represent the triggering conditions due to transient infiltration response to temporally-variable rainfall and hence may be of good performance for a hillslope with small specific contributing area. On the other hand, as specific contributing areas become larger, past rainfall has an increasing importance, and an antecedent rainfall variable should be used in addition to ID power-laws to achieve adequate reliability. Results also indicate that for short rainfall durations uniform hyetographs may have a stronger destabilizing effect than the stochastically-variable ones, while the opposite may occur for greater durations. Thus a power-law ID
threshold may perform better than a model deterministic one that is derived considering uniform hyetographs and a prefixed initial condition.

Further analyses show that predictability of landslides decreases with soil depth and geomechanical strength.

1 Introduction

Rainfall thresholds are the basis of early warning systems able to promptly warn about the potential triggering of landslides in an area (cf., e.g., Keefer et al., 1987; Fathani et al., 2008; Takara and Apip Bagiawan, 2008; Baum and Godt, 2010; Capparelli and Versace, 2011).

Commonly, thresholds are derived by the analysis of historical rainfall and landslide data, and identified by drawing a lower-bound envelope curve of the triggering event characteristics (e.g. Campbell, 1975; Caine, 1980; Cancelli and Nova, 1985; Cannon and Ellen, 1985; Aleotti, 2004; Wieczorek et al., 2000; Guzzetti et al., 2007). A review by Guzzetti et al. (2007) indicated the prevailing use in literature of so-called power-law ID thresholds, of the form \( I = a_1D^{a_2} \), where \( D \) is rain duration to triggering and \( I \) is rain intensity \( I = W/D \), \( W \) being rainfall accumulated over duration \( D \). The \( a_1 \) and \( a_2 \) parameters have been derived for specific sites, regions or the whole globe by different researchers.

Reliability of thresholds derived by the analysis of observed data is generally limited by the quality and availability of such data. Moreover, the \((D, I)\) pair alone may not be able to capture most of the uncertainty related to landslide triggering.

More in detail, adequate historical data on landslides and simultaneous rainfall are in most cases available for a relatively short period, which may not be sufficiently significant from a statistical point of view. Even assuming completeness of landslide archives (i.e. all landslide events occurred in the historical period are known), the identification of the triggering instant is in many cases significantly uncertain. This has a direct consequence on threshold derivation, because critical duration \( D \), assumed as the time
interval from rainfall event start and the triggering instant, cannot be computed accurately. Another key factor is the criterion used for rainfall identification, and in particular how the beginning of a rainfall event is identified. Many authors either do not specify the criteria used for rainfall identification or apply qualitative criterion, and indeed only few works in literature (Aleotti, 2004; Brunetti et al., 2010; Tiranti and Rabuffetti, 2010; Berti et al., 2012) explicitly addressed this problem. This makes thresholds subjective, as in analysing the data the criterion may have been modified from one rainfall event to another; furthermore, it impairs comparisons of results obtained by different researchers.

Moreover, since in many countries automatic rain gauge networks have been installed only quite recently, commonly rainfall records are practically available for a long period only at the daily aggregation time scale (cf. Guzzetti et al., 2007, and references therein). Since many landslides, especially the most devastating shallow rapidly-moving ones, may be triggered by rainfall events of few hours (cf., e.g., Highland and Bobrowsky, 2008), consequently, daily rainfall may not be adequate for threshold derivation in these cases.

Use of characteristic variables for the representation of rainfall events, such as mean intensity and duration, introduces an intrinsic uncertainty factor, because they may not be adequately representative of rainfall triggering characteristics. In fact, rainfall events represented by the same pair \((D, I)\) may correspond to totally different hyetographs that thus may or may not result in triggering. Sirangelo and Versace (1996) proposed an empirical method based on the use of convolution between rainfall time series and a filter function, which attempts to overcome this uncertainty.

Also, use of the Duration–Intensity pair \((D, I)\) in threshold formulation implies that the effect of initial wetness on triggering rainfall is neglected. Regarding this issue, several authors have added to \(D\) and \(I\) antecedent rainfall as a control parameter, though the empirical analyses have not yet provided unequivocal indications on the role of antecedent rainfall and different researchers used diverse temporal horizons for the computation of antecedent cumulative rainfall (Guzzetti et al., 2007).
Furthermore, many thresholds have been derived by analysing triggering events only, neglecting the non-triggering ones. This may lead to an underestimation of the triggering conditions, i.e. to thresholds that may produce an unacceptable degree of false alarms, causing populations to no longer rely on early warnings. In fact, thresholds should always be provided with a measure of their reliability. To this end, Berti et al. (2012) proposed Bayesian probabilistic analysis to evaluate landslide triggering thresholds in the presence of uncertainty.

On the other hand, physically-based models that couple hydrological and slope stability models (Montgomery and Dietrich, 1994; Wu and Sidle, 1995; Baum et al., 2002, 2008; Iverson, 2000; D’Odorico et al., 2005; Rosso et al., 2006) have been proposed to analyse landslide triggering by rainfall, partly overcoming some of the above-mentioned drawbacks of empirical thresholds. From such models the triggering conditions for uniform hyetographs and given initial conditions may be derived as analytical relations, which can be referred to as model deterministic thresholds (cf., e.g., Salciarini et al., 2008). Such thresholds generally deviate from a straight line in the log(D)−log(I) plane (cf., e.g., Rosso et al., 2006; Salciarini et al., 2008), thus casting some doubts on the use of parametric power-law as a proper functional form in deriving rainfall thresholds.

In spite of their limits, ID rainfall thresholds are widely applied for landslide early warning systems. Among the reasons of their success, it may be worthwhile to mention their relative simplicity, which makes them more easily understood by stakeholders and decision makers than more complex, albeit more accurate, models.

In this work a Monte Carlo based methodology to derive and evaluate rainfall landslide-triggering thresholds is proposed, which makes use of an existing body of stochastic and physically-based models. From the Monte Carlo simulations synthetic rainfall series are generated by a stochastic model and corresponding triggering/non-triggering conditions by means of an hydrological/geotechnical model. The generated results can then be analysed in order to derive and evaluate ID thresholds that take into account the variability of both rainfall intensity within events and initial conditions determined by past rainfall, as well as triggering/non-triggering events.
In particular, Monte Carlo synthetic data generation from a combined rainfall stochastic model and hillslope hydrological and slope stability model is used to obtain a long rainfall-landslide dataset with no interruption in data availability and precise knowledge of the triggering instants. For this last point we adopt a precise rainfall identification criterion.

The developed methodology enables to evaluate rainfall thresholds, as the performances with respect to triggering and non-triggering events are taken into account, making use of a metric derived from Receiver Operating Characteristics (ROC) analysis.

Furthermore, the derived thresholds are compared with model deterministic thresholds (where uniform hyetographs and constant initial conditions are considered) in order to assess the effect of rain intensity variability and variable initial conditions. This analysis is related to the one by D’Odorico et al. (2005), that analysed the effect of rain intensity variability within events by considering beta-shaped hyetographs and the model of Iverson (2000) for derivation of hillslope response. Nevertheless, in their work, the variability of initial conditions as dependent from antecedent rainfall is not considered because the steady-state asymptotic solution of Montgomery and Dietrich (1994) is utilized for computation of initial conditions. From their study they conclude that beta-shaped non-uniform hyetographs have a stronger destabilizing effect than uniform hyetographs of the same volume. In this study we instead use hyetographs generated by a Neyman–Scott Rectangular Pulses (NSRP) stochastic model and account for variability of initial conditions using a water table recession model to derive the initial water table height from the response to rainfall events preceding the current one. The transient response to rainfall events is computed by a model based on the TRIGRS program (Baum et al., 2008).

Finally, the effect of rain intensity variability and of past-rainfall-dependent initial conditions, and their consequences on the performances of ID thresholds is investigated by measuring the variation of optimal performances with the following control-variables:
specific upslope contributing area, soil depth $d_{LZ}$ and soil mechanical properties, represented by the critical wetness ratio $\zeta_{CR}$.

The proposed simulation-optimization methodology is applied to the highly landslide-prone area of the Peloritani Mountains, Northeastern Sicily, Italy.

2 Monte Carlo synthetic data generation

The Monte Carlo simulation procedure for synthetic rainfall-landslide data generation consists of the following steps:

1. A stochastic rainfall model, calibrated on observations at a representative site, is used to generate a 1000-years long hourly rainfall time series.

2. The synthetic rainfall time series is pre-processed in order to identify rainfall events and their inter-arrival durations. In particular, when two wet spells are separated by a dry time interval less than $\Delta t_{\text{min}}$, these are considered to belong to the same rainfall event; otherwise two separate rainfall events are considered. In our analyses we assume $\Delta t_{\text{min}} = 24 \text{ h}$. A number $N_{\text{RE}}$ of rainfall events results from this step.

3. An initial value of the water table height is fixed to start simulations of the hydrological response for the whole rainfall time series. For the analysed case-study area and many similar cases, it may be assumed that at the beginning of each hydrological year the water table is at the basal boundary, because an almost totally-dry season comes before. As this is valid also for the first year, simulation for first event is conducted considering the water table at the basal boundary.

4. The following procedure is then applied to each rainfall event $i = 1, 2, ..., N_{\text{RE}}$:

(a) Response in terms of pressure head $\psi$ within rainfall events is computed using the TRIGRS model. As pressure head rise may continue after the end
of rainfall, the TRIGRS transient-response simulation-interval is prolonged \( \Delta t_a = 23 \) h after the ending time \( t_{end,i} \) of rainfall events.

(b) The instant \( t_{f,i} = \max(t_{end,i}, t_{max,i}) \) is searched, \( t_{max} \) being the time instant at which maximum transient pressure head occurs. It follows that the final response to rainfall event \( i \), in terms of water table height, is \( \psi(d_{LZ}, t_{f,i})/\beta \), where \( \beta = \cos^2 \delta \) (slope parallel flow is assumed), and \( d_{LZ} \) is the soil depth. Moreover, the time interval \( \Delta t_{i+1} = t_{(in)}_{i+1} - t_{f,i} \) is computed, with \( t_{(in)}_{i+1} \) the instant at which rainfall event \( i + 1 \) begins.

(c) The water table height at the beginning of rainfall event \( i + 1 \) is computed by a sub-horizontal drainage model which uses \( \psi(d_{LZ}, t_{f,i})/\beta \) and \( \Delta t_{i+1} \).

5. The result is a series of maximum pressure head, or minimum factor of safety, responses to the \( N_{RE} \) rainfall events. These series are together analysed to derive and evaluate landslide triggering thresholds via a ROC-based approach (see Sect. 3).

In the following subsections the rainfall stochastic model, the TRIGRS model and the sub-horizontal drainage model are described.

2.1 Rainfall stochastic model

We model rainfall at a site as a Neyman–Scott Rectangular Pulses (NSRP) process (Rodriguez-Iturbe et al., 1987a, b). This process may be represented in time \( t \) as in Fig. 1, and it is obtained by the following steps:

- First, storm origins arrive governed by a Poisson process of parameter \( \lambda t \).

- For each storm origin, rectangular pulses (rain cells) are generated. The number of pulses \( C \) associated to each storm is extracted from another Poisson distribution. In order to have realizations of \( C \) not less than one, it is assumed that
$C' = C - 1$, with $c' = 0, 1, 2, ...$ (which implies $c = 1, 2, 3, ...$), is Poisson distributed with mean $\nu - 1$.

- Each cell has origin at time $\tau_{i,j}$ with $j = 1, 2, ..., c_i$ measured from $t_i$, according to an exponential random variable of parameter $\beta$.

- A rectangular pulse of duration $d_{i,j}$ and intensity $x_{i,j}$ is associated to each rain cell. Pulses have duration exponentially distributed with parameter $\eta$ while intensities $X$ are extracted from a Weibull distribution (cf. Cowpertwait et al., 1996), which has cdf $F(x; \xi, b) = 1 - \exp(-\xi x^b)$.

- Finally, the total intensity at any point in time is given by the sum of the intensities of all active cells at that point.

We calibrate the NSRP model by the method of moments, i.e. using the properties of the aggregated NSRP process $Y(\tau)$ at different time scales of aggregation $\tau$ (cf., e.g., Rodriguez-Iturbe et al., 1987a, b; Cowpertwait et al., 1996; Calenda and Napolitano, 1999). According to this method, model parameters, i.e., $\lambda, \nu, \beta, \eta$ and $\xi$ ($b$ is typically fixed, in the range $0.6 \leq b \leq 0.9$, see Cowpertwait et al., 1996) are estimated using at least as many moments as the parameters of the model, considering different statistics (moments) at various time aggregations, and solving the related equation system, where the theoretical expressions, containing the parameters, are equated to the sample moments. Theoretical moments of $Y(\tau)$, such as the mean $\mu(\tau)$, variance $\gamma(\tau)$ and autocorrelation at lag $k$, $\rho(\tau, k)$, are given by formulas derived by Rodriguez-Iturbe et al. (1987a). Transition probabilities were derived as well, by Cowpertwait (1991), and have been included in the calibration process. The non linear equation system is solved by numerical minimization of an objective function $S(\lambda, \nu, \beta, \eta, \xi)$, that measures the global relative error between theoretical and sample moments.
2.2 Hillslope hydrological and stability model

The total pressure head response $\psi$ of an hillslope soil to a rainfall event can be subdivided into a transient part $\psi_1$ and an initial part $\psi_0$. The transient part is due to infiltration of event rainfall, while the initial part depends on rainfall time history before the current rainfall event. As pointed out by Iverson (2000), for soils that are relatively shallow, i.e. when the ratio between soil depth and the square-root of the upslope contributing area is small, $\varepsilon = d_{LZ}/\sqrt{A} \ll 1$, the prevailing process that determines $\psi_1$ is 1-D vertical infiltration, while in the dry periods in between events, the prevailing process is of sub-horizontal drainage.

Based on these considerations, we use a vertical infiltration model for computing $\psi_1$, the TRIGRS unsaturated model (Baum et al., 2008, 2010), and a linear reservoir sub-horizontal drainage model to compute the initial conditions from the water table height at the end of the rainfall event preceding the current one (which in turn depends on past rainfall time history), which is derived using the same assumptions of Rosso et al. (2006).

From pressure head response, the factor of safety FS for slope stability is computed, using a indefinite slope model.

2.2.1 Initial conditions model

The initial condition to rainfall event $i$ is computed from the response at the end of rainfall event $i - 1$, using a water table height $h$ recession model between storms based on the following mass-conservation equation (Rosso et al., 2006):

$$BhK_s \sin \delta = -A (\theta_s - \theta_r) \frac{dh}{dt},$$  \hspace{1cm} (1)

where $A$ is the contributing area draining across the contour length $B$ of the lower boundary of the hillslope, $K_s$ is the saturated hydraulic conductivity and $\theta_s - \theta_r$ is soil porosity, $\theta_s$ and $\theta_r$ being the saturated and residual soil water contents respectively.
The ratio $A/B$ is the well-known specific upslope contributing area $A/B$. For instance, it is $A/B = BN_d$, where $N_d$ is the number of cells draining into the local one, if one determines flow paths via the non-dispersive single direction (D8) method (O’Callaghan and Mark, 1984).

The solution to Eq. (1) is used to compute the water table height at the beginning of rainfall event $i$:

$$h_i = \frac{\psi(d_{LZ}, t_{i,i-1})}{\cos^2 \delta} \exp\left(-\frac{K_s \sin \delta}{BN_d(\theta_s - \theta_r)} \Delta t_i \right).$$  \hspace{1cm} (2)

where interarrival time $\Delta t_i$ has been defined in Sect. 2.

### 2.2.2 Transient infiltration model

Reference scheme for a simulated hillslope is shown in Fig. 2. Infiltration in the unsaturated zone is modeled through Richards’ (1931) vertical-infiltration equation for a sloping surface particularized for the Gardner’s (1958) exponential soil-water characteristic curve $K(\psi) = K_s \exp\{\alpha(\psi - \psi_0)\}$:

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial Z} \left[ K(\psi) \left( \frac{1}{\cos^2 \delta} \frac{\partial \psi}{\partial Z} - 1 \right) \right]$$ \hspace{1cm} (3)

where $K_s$ is the saturated hydraulic conductivity, $\alpha$ is the SWCC parameter, $\psi_0 = -1/\alpha$ is the pressure head at the top of the capillary fringe, $\theta_r$ is the residual water content, $\theta_s$ is the water content at saturation and $\alpha_1 = \alpha \cos^2 \delta$.

A closed-form solution to this equation for $\delta = 0$ has been provided by Srivastava and Yeh (1991) and extended to a sloping surface by Savage et al. (2004), and used in the TRIGRS unsaturated model (Baum et al., 2008, 2010).

The solution to Richards’ equation provides the pore pressure profile in the unsaturated zone, and a flux to the saturated zone $q(d_u, t)$. This flux is not equal to the infiltrating water because soil absorbs part of the infiltrating water and determines a lamination.
effect on $q(d_u, t)$. The TRIGRS model then computes water table rise using $q(d_u, t)$ subtracting from it a leakage flow rate given by $q_l = \min\{c_d K_s(1 - \cos^2 \delta), q(d_u, t)\}$ (vertical drainage at the basal boundary, which is not assumed perfectly impervious), where $c_d$ represents the ratio between saturated hydraulic conductivities of the basal boundary layer and of the regolith surficial layer. In the case that no specific information on $c_d$ ratio is available, a reasonable value may be $c_d = 0.1$ (cf. Baum et al., 2008), which means that hydraulic conductivity of the layer below depth $Z = d_{LZ}$ is of one order of magnitude less than the regolith surficial layer.

The resulting water table rise is computed by comparing this excess flux accumulating at the top of the capillary fringe to the available pore space directly above it.

Pressure head rise is assumed transient in the saturated zone as well, and computed by formulas adapted from analogous heat-flow problems.

### 2.2.3 Slope stability model

For analysis of hillslope stability we assume an indefinite slope scheme, and compute minimum factor of FS with the following formula (Taylor, 1948):

$$FS(d_{LZ}, t) = \tan \phi' \tan \delta + \frac{c' - \psi(d_{LZ}, t)\gamma_w \tan \phi'}{\gamma_s d_{LZ} \sin \delta \cos \delta},$$  \hspace{1cm} (4)

where $c'$ is soil cohesion for effective stress, $\phi'$ is the soil friction angle for effective stress, $\gamma_w$ is the unit weight of groundwater, $\gamma_s$ is the soil unit weight and $\delta$ is the slope angle. In this scheme the failure occurs at the basal boundary $Z = d_{LZ}$.

It is useful to consider the critical wetness ratio, derived from Eq. (4) letting $FS = 1$, which is a parameter that for a given hillslope (given slope $\delta$ and soil depth $d_{LZ}$), depends only on the geotechnical characteristics of the soil:

$$\zeta_{CR} = \frac{\gamma_s}{\gamma_w} \left[\left(\frac{c'}{\gamma_s d_{LZ} \sin \delta \cos \delta} - 1\right) \frac{\tan \delta}{\tan \phi'} + 1\right].$$  \hspace{1cm} (5)
A hillslope with high $\zeta_{CR}$ has good geotechnical characteristics, while the opposite holds for a soil with low $\zeta_{CR}$.

### 3 Threshold derivation and evaluation by ROC analysis

For a hillslope of given properties, Monte Carlo simulations lead to a series of computed failures, i.e. time instants at which the factor of safety drops below the value of 1. A triggering rainfall may be associated to each failure, in the same way as when analysing historical observed series of rainfall and slope failures.

We adopt the following rainfall identification criterion, similar to that applied by Brunetti et al. (2010). First, we isolate rainfall events when their dry interarrival is longer than 24 h. Rainfall events then have a total duration $D_{tot}$ and mean intensity $I_{tot} = W_{tot}/D_{tot}$, where $W_{tot}$ is the total event cumulative rainfall.

For a triggering event, triggering may occur before or after the end of the rainfall event. In the first case, the critical duration $D_{cr}$ is the time interval that starts at the beginning of the rainfall event and finishes at the triggering instant, and critical intensity is given by $I_{cr} = W_{cr}/D_{cr}$, where $W_{cr}$ is rainfall accumulated over duration $D_{cr}$. In the second case it is instead characterized with $D_{tot}$ and $I_{tot}$. Moreover, the $P_0$ events that have at their beginning a water table height $h_i \geq d_{LZ}\zeta_{cr}$ (corresponding to $FS \leq 1$) are removed from the analysis, as the triggering is due to the preceding events, which have been already included in the set of triggering points.

Non-triggering events are represented by $D_{tot}$ and $I_{tot}$.

In this way, analysis of the Monte Carlo simulation produces two sets: the set of positives $P$, i.e. of triggering events and the set of Negatives $N$, i.e. of non-triggering events. These sets may be represented as scatter plots in a double-logarithmic $(D, I)$ plot, and in general there is a region where both sets are present – lets say, an intersection region $P \cap N$. In our framework this is due to two separate factors:
to a given \((D, I)\) pair may correspond diverse variable NSRP-simulated hyetographs, because \(I\) is the mean intensity \(I = W/D\) (rain-intensity variability within events).

– To a given \((D, I)\) pair may correspond diverse initial conditions (variability of initial conditions, due to variability of rainfall before the current event).

When \(A/B = 0\), only the first part determines a non-null intersection set \(P \cap N\), being \(h_i = 0\) for \(i = 1, \ldots, N_{\text{RE}}\).

As a consequence of the presence of the region \(P \cap N\), when a triggering rainfall threshold is fixed, say a power-law one \(I = a_1D^{a_2}\), the four cases of True Positives, True Negatives (correct predictions), False Positives and False Negatives (wrong predictions) can occur, as illustrated in Table 1. In general, to each pair of parameters \(a_1\) and \(a_2\) corresponds a prediction performance that may be measured by indices based on the number of occurrences in the four cases, denoted respectively as TP, FN, TN, FP (or ROC-based indices).

Well-known commonly-used examples of indices are the accuracy ACC and the precision PRE:

\[
\text{ACC} = \frac{TP + TN}{P + N} \quad (6)
\]

\[
\text{PRE} = \frac{TP}{TP + FP} \quad (7)
\]

Nonetheless, the following reasoning can be made. Considering the four possible outcomes of a prediction process, one may consider the advantage of a TP (correct alarm) as an avoided FN (missed alarm), because if the threshold was higher a TP would have been a FN. The same reasoning may apply to a TN (correct non alarm) because if the threshold was lower it would have resulted into a FP. Based on these considerations one may use the following index:

\[
f_1 = \frac{TP - FN}{2} + \frac{TN - FP}{2} \quad (8)
\]
This index depends on sample size; in order to compare results obtained from samples whose sets \( P \) and \( N \) have different size, a normalized index may be preferable. The following index is obtained from normalization of Eq. (8):

\[
\Delta = \frac{1}{2} \left[ \frac{TP - FN}{TP + FN} + \frac{TN - FP}{TN + FP} \right] = TPR - FPR
\]  

where TPR and FPR are the True Positive and the False Positive Rates, given by:

\[
TPR = \frac{TP}{P} = \frac{TP}{TP + FN}
\]

\[
FPR = \frac{FP}{N} = \frac{FP}{TN + FP}
\]

It is \( \Delta = 0 \) for TPR = FPR (random guess) and \( \Delta = 1 \) for a perfect prediction (TPR = 1 and FPR = 0). In fact this index \( \Delta \) is bounded in the interval \([-1, 1]\), but negative values are fictitious as an inversion of the triggering threshold use brings \( \Delta \) to its absolute value, that is always in the interval \([0, 1]\) (i.e. saying that values below the threshold trigger landslides and vice-versa values above the threshold do not trigger). Based on these considerations, we use this \( \Delta \) index for calibration of the threshold, i.e. we estimate the best performing power-law threshold \( I = a_1D^{a_2} \) as the one that gives the maximum value of \( \Delta = \Delta(a_1, a_2) \).

At the same time the simulation-optimization methodology enables to evaluate the use of power-law ID thresholds, as the value of the objective function is a measure of the maximum performances that can be expected, and thus a measure of the validity of the adopted functional form for the threshold.

To this regard we compare the results with the model deterministic thresholds. Deterministic thresholds represent the model response to uniform hyetographs and a prefixed initial condition (cf., e.g. Rosso et al., 2006; Salciarini et al., 2008; Tarolli et al., 2011). Hence a univocal triggering threshold exists \( I = f(D) \), for given hillslope properties. Due to the complexity of the TRIGRS unsaturated model it is possible to determine
these thresholds only numerically (not in closed-form). Hence we have derived model
deterministic thresholds by simulation of infiltration and slope stability using constant-
intensity hyetographs in the \((D,I)\) domain discretized at a sufficient level, and searching
the triggering curve by interpolation of the results. In doing this we have assumed an
initial water table height of zero, \(h(0) = 0\).

4 Investigated area and data

The described methodology is applied to the Peloritani Mountains area, Northeastern
Sicily (see Fig. 3). From 2006 to now, several landslide events occured in this area, and
in particular on 15 September 2006, 25 October 2009, 1 March 2011 and 23 November
2012. Among these events, the 1 October 2009 one was the most severe; debris-
flows on this occasion provoked 37 casualties in the surroundings of the municipality
of Giampilieri.

Rainfall series measured from 21 February 2002 to 9 February 2011 (almost 9 years)
at the Fiumedinisi rain gauge installed in the area has been used for calibration of the
NSRP model. Based on a preliminary analysis of monthly statistics, six homogeneous
rainfall seasons have been identified: (i) September and October, (ii) November, (iii)
December, (iv) January–March, (v) April and (vi) May–August. Separate sets of pa-
rameters of the NSRP model have been determined for each one of the four rainy
seasons (in total \(5 \cdot 4 = 20\) parameter values), while the last two seasons have been
considered to have negligible rainfall. The Weibull shape parameter \(b\) has been fixed
to 0.6 for all seasons, based on different trials. Parameters obtained from calibration
are shown in Table 2.

For our application, we consider as representative for the case study area a hills-
lope of slope \(\delta = 40^\circ\) and depth \(d_L = 2\) m, and hydraulic and geotechnical properties
of Table 3, which can be assumed as representative of measurements from soil sam-
ple in the area. Fractured metamorphic rocks are present at the base of the regolith
strata; hence for computation of basal boundary loss it has been assumed \(c_d = 0.1\)
(see Sect. 2.2.2). The region characterized by small catchments and a representative specific catchment area may be $A/B = 10\text{ m}$.

5 Results and discussion

By applying the rainfall event identification criteria, the 1000-years generated hourly rainfall series breaks down to $N_{RE} = 19826$ rainfall events. After simulation of these events the related factors of safety have been computed and each event has been characterized in terms of duration $D$ and Intensity $I$. Thus the series of rainfall characteristics $(D,I)$ has been dichotomized as triggering and non triggering events according to the related factor of safety. In Fig. 4 the scatter plots of such events in the $\log(D) - \log(I)$ plane are shown for different values of the ratio $A/B = 0, 10, 20\text{ m}$. Related results are also shown in Table 4. In the figure, red points represent triggering rainfall events, or the set of Positives $P$, while green points represent the non-triggering ones, or the set of Negatives $N$.

Optimal thresholds have been derived by maximization of the $\Delta$ index (see Eq. 9), preliminarily by considering both the power-law coefficient $a_1$ and exponent $a_2$ as variable parameters. Inspection of the results revealed minimal changes of the exponent $a_2$ with changing ratio $A/B$, and so a second optimization has been carried out only with reference to the $a_1$ parameter, fixing the exponent $a_2$ to its mean value of $a_2 = -0.8$. Fixing the exponents forces the different thresholds corresponding to different $A/B$ ratios to be parallel and therefore to not intersect each other, which is consistent with the fact that as the $A/B$ increases, landslides are more likely to occur.

The case $A/B = 0$ (Fig. 4a) is equivalent to considering constant initial conditions, in which case variability is due only to the variation of rain intensity within events. From Fig. 4a it can be inferred that in this case the region in which triggering and non-triggering events coexist is quite narrow; moreover, a power-law relation between $I$ and $D$ represents the transition between triggering and non-triggering conditions well.
fact, the optimal power-law threshold in this case has a reliability of $\Delta = 0.991$, practically equal to the ideal value of 1.

The effect of variability of rain intensity during events may be analysed by comparing the scatter plots for $A/B = 0$ with the model deterministic threshold, represented in the plots as a dashed black line.

More specifically, Fig. 4a reveals that the deterministic threshold approximates the lower envelope curve of critical events (red dots) for short durations, say $D \leq D_s$, that in this case correspond to about 12 h. For higher durations, this is no longer true and variable-intensity hyetographs start to have a higher destabilizing effect than the constant-intensity ones of same rainfall volume. The variability of rainfall intensity within events leads to a deviation from the deterministic line of the triggering NSRP rainfall event points, making the scatter of triggering points more similar to a straight line rather than to the curved deterministic threshold. This behavior is essentially due to the presence of the leakage term $q_l = \min\{c_dK_s(1-\cos^2\delta), q(d_u, t)\}$, whose effect is stronger for uniform hyetographs than for variable ones, since in the former case there are no peaks of intensity. In particular, a uniform hyetograph produces no water table rise if intensity is below a rate slightly greater than $c_dK_s(1-\cos^2\delta)$, because all infiltrating water, after percolating through the unsaturated-zone, goes to basal loss. The same does not generally occur for a variable intensity hyetograph of same volume, because instantaneous intensity may be significantly higher than the event mean intensity $W_{tot}/D_{tot}$, and consequently a water table rise is produced. The opposite behavior for short durations is due to the fact that in this case variable hyetographs may have peaks of intensity higher than infiltration capacity, and thus not all rainfall infiltrates in the soil. Due to these reasons, the model deterministic threshold results poor performing ($\Delta = 0.642$).

For the $A/B = 10$ m case (Fig. 4b), which may represent prevalent flow convergence conditions for the Peloritani Mountains area, scattering of the red dots increases due to the introduced variability of initial conditions. Consequently, performances of predictions based only on intensity and duration of rainfall events become worse, but may still be acceptable.
Simulations for larger values of specific catchment area (e.g. $A/B = 20$ m, Fig. 4c) confirm this conclusion.

Table 4 shows the comparison of the ROC-based performances relative to the derived optimal thresholds with those of the model deterministic thresholds, derived assuming a constant rainfall intensity and fixed initial conditions. From the table, the relatively poor performances of the model deterministic threshold can be inferred. Such performances get worse as larger values of the specific catchment area are considered, which is consistent with the fact that a constant initial condition is assumed for their derivation whereas the effect of antecedent soil moisture is more significant for large $A/B$ values.

Results of Table 4, show also how the two well-known indices ACC and PRE may not behave adequately. More specifically, the accuracy index assumes high values both for the case of optimized and model deterministic thresholds. This stems from the fact that the index does not adequately account for false predictions, and the high number of True Negatives in both cases makes this index always close to 1. Precision has an opposite behavior to $\Delta$, as high values of PRE are obtained for the model deterministic threshold and low ones for the optimized threshold. This is essentially due to the fact that precision does not account for False Negatives. The results show the suitability of the $\Delta$ index, as it maximizes True Positives, limiting False Negatives at the same time.

As a practical result, the threshold:

$$I = 71.52D^{-0.8}$$

may be a reasonable choice for the Peloritani Mountains area (where $A/B = 10$ m can be assumed), and the Monte Carlo methodology enables to state that expected performances are of $\Delta = 0.862$.

We use this result to perform a global validation test to our modeling and threshold derivation methodology.

In order to validate the derived threshold, a simulation using observed rainfall and landslide data in a historical period, for which both data are available has been carried
out. In particular we refer to the period 2006–2011, in which four landslide events occurred (see Sect. 4) and Fiumedinisi rain gauge data is available. This rainfall series contains 190 events, for each of which the temporal evolution of accumulated intensities $I(D) = W(D)/D$ with duration has been compared with the derived threshold. Figure 5 presents the results of this test, and indicates positive validation of the methodology, as the events in the $I$–$D$ plane that exceed the threshold are all and only the four events that have triggered landslides in the considered period (red-line time histories).

In order to better understand the influence of soil properties on the performances of the derived thresholds, the parameter $a_1$ of the ID threshold and the related performances in terms of maximum $\Delta$ have been considered, by varying the $\zeta_{CR}$ parameter between 0.1 and 0.9 and by assuming different values of the soil depth $d_{LZ} = 1, 1.5$ and 2 m. In all cases the value of the parameter $a_2$ has been fixed to $-0.8$. The objective is to analyse how soil mechanical strength, represented by $\zeta_{CR}$, and soil depth influences the reliability of ID thresholds and the predictability of landslides. Such analysis is shown only for the two values of $A/B = 0$ and 10 m. The results are shown in Fig. 6 from which it can be inferred that the values of the optimal $a_1$ increase with both $\zeta_{CR}$ and $d_{LZ}$.

On the other hand, performances in terms of $\Delta$ decrease for decreasing $d_{LZ}$ and $\zeta_{CR}$. This happens because as soon soil depth and mechanical properties decrease, hillslope response in terms of factor of safety becomes more sensitive to rainfall input changes. Hence uncertainty increases and landslides become less predictable. Nonetheless, results indicate that performances of the $I$–$D$ thresholds may remain valid ($\Delta > 0.8$), especially for soils of relatively good mechanical properties that are not too shallow ($\zeta_{CR} > 0.5$ and $d_{LZ} > 1$ m, for the considered case).

The model deterministic threshold performs poorly in all the range of $\zeta_{CR}$. 
6 Conclusions

Traditionally for derivation of landslide triggering thresholds has been carried out by either empirical approaches, based on the analysis of historical rainfall and landslides, or deterministic approaches based on hydrological-geotechnical models and assuming constant rainfall intensity and fixed soil water initial conditions. In the present work, a Monte Carlo methodology, consisting in coupling a stochastic rainfall generator with hillslope infiltration and slope stability models, is proposed in order to derive rainfall thresholds that take into account rainfall variability, soil water initial conditions, as well as the trade-off between correct and wrong predictions by means of Receiver Operating Characteristics (ROC) analysis.

In particular, the large number of synthetic rainfall events and corresponding landslide triggering/non-triggering hillslope responses generated by the Monte Carlo simulations were analysed in order to derive Intensity-Duration thresholds by means of an optimization procedure as well as to investigate the effects of the different sources of variability into their predictive ability. Thresholds were derived by maximizing a ROC-based normalized index $\Delta$ that represents the performances of the threshold in terms of the positive trade-off between correct and wrong warnings. The effect of rainfall intensity variability within events was investigated by comparison of optimal thresholds with the model deterministic threshold, i.e. a threshold derived from the hillslope infiltration and slope stability model considering constant-intensity hyetographs and a prefixed initial condition. Furthermore, by varying the specific upslope contributing area $A/B$, the influence of antecedent soil water conditions was also investigated.

The methodology has been applied with reference to a hillslope whose characteristics are representative of catchments of the highly landslide-prone area of Peloritani Mountains, Northeastern Sicily, Italy.

Results indicate that: (1) when the variability of antecedent soil water conditions is negligible a power-law ID threshold derived by the optimization procedure is able to correctly distinguish between triggering and non-triggering events, as proven by the
fact that optimal power-law ID thresholds perform well in the $A/B = 0$ case ($\Delta \approx 1$). On the other hand, a model deterministic threshold performs adequately only for low rainfall durations while for higher durations it lies consistently above the triggering events in an double-logarithmic ID plane, therefore leading to potential missing alarms. (2) As the effect of antecedent soil water conditions increases ($A/B = 10$ and $20 \text{ m}$), so does prediction uncertainty due to the effect of variable initial conditions, and therefore performances of ID power-law thresholds worsen. (3) Threshold performances are also affected by the depth of the basal boundary and the soil geotechnical characteristics. In particular, uncertainty of landslide triggering prediction increases with decreasing soil depths as well as when soils of poorer geotechnical properties are considered, since in this case the effect of rainfall variability becomes more significant. Nonetheless, results indicate that performances of ID thresholds may remain valid ($\Delta > 0.8$), especially for soils of relatively good mechanical properties and that are not that shallow ($\zeta_{CR} > 0.5$ and $d_{LZ} > 1 \text{ m}$, for the considered case).

The main conclusion of the paper is that the widely-used power-law functional form for ID thresholds may be adequate when the effect of antecedent soil water conditions is negligible (e.g. small upslope catchment areas). However one should be aware that ID thresholds are intrinsically affected by uncertainty stemming from the fact that a great deal of information about rainfall and soil pore pressure variability is lost when rainfall is characterized only in terms of the intensity-duration pair and soil water initial conditions are not considered. Use of the optimization procedure proposed in the paper enables to take into account such uncertainty by considering the trade-off between correct alarms and false alarms. On the other hand, model deterministic thresholds do not consider such uncertainty and may lead to unacceptable missing alarms rates.

Further ongoing research is oriented to introduce additional information in the derivation of the thresholds, such as antecedent precipitation as well as indices representative of the shape of the hyetograph.
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**Table 1.** Possible success and failure cases of a warning process for a landslide triggering threshold.

<table>
<thead>
<tr>
<th>Predicted</th>
<th>Actual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Landslide: $I \geq f(D)$</td>
<td>True Positive, TP</td>
</tr>
<tr>
<td>No Landslide: $I &lt; f(D)$</td>
<td>False Negative, FN</td>
</tr>
<tr>
<td></td>
<td>False positive, FP</td>
</tr>
<tr>
<td></td>
<td>True Negative, TN</td>
</tr>
</tbody>
</table>
Table 2. Parameters of NSRP rainfall model after calibration on Fiumedinisi rainfall data, for the four homogeneous rainy seasons.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Jan, Feb, Mar</th>
<th>Sep, Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$ [h$^{-1}$]</td>
<td>0.002295</td>
<td>0.021195</td>
<td>0.001485</td>
<td>0.003185</td>
</tr>
<tr>
<td>$\nu$</td>
<td>44.28</td>
<td>1.57</td>
<td>42.41</td>
<td>42.61</td>
</tr>
<tr>
<td>$\beta$ [h$^{-1}$]</td>
<td>0.010161</td>
<td>2.1179</td>
<td>0.0059551</td>
<td>0.0098760</td>
</tr>
<tr>
<td>$\eta$ [h$^{-1}$]</td>
<td>0.72113</td>
<td>0.83999</td>
<td>0.94053</td>
<td>0.67735</td>
</tr>
<tr>
<td>$\xi$ [h$^b$mm$^{-b}$]</td>
<td>1.13441</td>
<td>0.46260</td>
<td>0.69261</td>
<td>1.03521</td>
</tr>
</tbody>
</table>
Table 3. Material strength and hydraulic properties for regolith strata in the Peloritani Mountains.

<table>
<thead>
<tr>
<th>$\phi'$</th>
<th>$c'$</th>
<th>$\gamma_s$</th>
<th>$\theta_s$</th>
<th>$K_s$</th>
<th>$\theta_r$</th>
<th>$\alpha$</th>
<th>$D_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>['']</td>
<td>[kPa]</td>
<td>[Nm$^{-3}$]</td>
<td>[-]</td>
<td>[m s$^{-1}$]</td>
<td>[-]</td>
<td>[m$^{-1}$]</td>
<td>[m$^2$ s$^{-1}$]</td>
</tr>
<tr>
<td>37</td>
<td>5.7</td>
<td>19 000</td>
<td>0.35</td>
<td>$2 \times 10^{-5}$</td>
<td>0.045</td>
<td>3.5</td>
<td>$5 \times 10^{-5}$</td>
</tr>
</tbody>
</table>
Table 4. ROC-based indices for the derived best power-law thresholds (Opt.) and comparison with model deterministic ones (Det.).

<table>
<thead>
<tr>
<th>$\frac{A}{B}$ [m]</th>
<th>$a_1$ [mm h$^{-1}$]</th>
<th>TP</th>
<th>TN</th>
<th>FN</th>
<th>FP</th>
<th>$P_0$</th>
<th>TPR</th>
<th>FPR</th>
<th>$\Delta$</th>
<th>ACC</th>
<th>PRE</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 Opt.</td>
<td>101.49</td>
<td>81</td>
<td>19558</td>
<td>0</td>
<td>187</td>
<td>0</td>
<td>1.00</td>
<td>0.009</td>
<td>0.991</td>
<td>0.991</td>
<td>0.302</td>
</tr>
<tr>
<td>0 Det.</td>
<td>–</td>
<td>52</td>
<td>19744</td>
<td>29</td>
<td>1</td>
<td>0</td>
<td>0.642</td>
<td>0.000</td>
<td>0.642</td>
<td>0.998</td>
<td>0.981</td>
</tr>
<tr>
<td>10 Opt.</td>
<td>71.52</td>
<td>104</td>
<td>19037</td>
<td>11</td>
<td>672</td>
<td>2</td>
<td>0.904</td>
<td>0.034</td>
<td>0.870</td>
<td>0.966</td>
<td>0.134</td>
</tr>
<tr>
<td>10 Det.</td>
<td>–</td>
<td>52</td>
<td>19708</td>
<td>63</td>
<td>1</td>
<td>2</td>
<td>0.452</td>
<td>0.000</td>
<td>0.452</td>
<td>0.997</td>
<td>0.981</td>
</tr>
<tr>
<td>20 Opt.</td>
<td>42.95</td>
<td>164</td>
<td>17131</td>
<td>26</td>
<td>2488</td>
<td>17</td>
<td>0.863</td>
<td>0.127</td>
<td>0.736</td>
<td>0.873</td>
<td>0.062</td>
</tr>
<tr>
<td>20 Det.</td>
<td>–</td>
<td>52</td>
<td>19618</td>
<td>138</td>
<td>1</td>
<td>17</td>
<td>0.274</td>
<td>0.000</td>
<td>0.274</td>
<td>0.993</td>
<td>0.981</td>
</tr>
</tbody>
</table>
Fig. 1. Representation of the Neyman–Scott Rectangular Pulses stochastic process for at-site rainfall modeling.
Fig. 2. Soil 1-D vertical scheme used to model infiltration and slope stability based on the TRIGRS unsaturated model (adapted from Baum et al., 2008).
Fig. 3. Location of the Peloritani Mountains, NE Sicily, Italy, of Fiumedinisi rain gauge and of the municipality of Giampilieri. On 1 October 2009, 37 people were killed by debris flows triggered by heavy rainfall in the surroundings of this municipality.
Fig. 4. Derivation of thresholds from ROC optimization of Monte Carlo simulations. Red points represent triggering simulated rainfall, while green ones represent the non-triggering. Best power-law thresholds are derived maximizing the Δ ROC-based index for these points. The model deterministic threshold is determined considering the response to uniform hyetographs and water table initially at the basal boundary. Subfigures differ for the value of the upslope specific contributing area: (a) $A/B = 0$ m, (b) $A/B = 10$ m, and (c) $A/B = 20$ m.
Fig. 5. Validation of threshold-derivation procedure with observed rainfall events in the period 2006–2010. Red lines indicate mean intensity \( I(D) = \frac{W(D)}{D} \) time histories that exceed the derived threshold. Green lines represent observed events that do not exceed the threshold.
Fig. 6. Variation of the optimal ID power-law threshold parameter $a_1$ and of relative performances $\Delta$ with critical wetness ratio $\zeta_{cr}$ and soil depth $d_{LZ} = 1$, 1.5, and 2 m. Different upslope specific catchment areas are considered, $A/B = 0$ and 10 m. The performances of the model deterministic thresholds are shown as well.