Authors reply on comments of referee #3

The authors would like to thank reviewer #3 for his/her valuable and detailed comments. We will first give a general reply, and then answer the specific comments. For clarification, the referee’s comments are repeated first and displayed in italic letters, followed by the authors’ replies.

General reply

The reviewer criticizes two main aspects of our work: (a) the selection and motivation of catchment and (b) the novelty of our work:

(a) Thur catchment

The investigated catchment Thur is certainly not of the same level of familiarity as other catchments are. However, it is a catchment of particular importance in Switzerland since it is the largest Swiss river without a natural or artificial reservoir. Together with a topography that ranges from 382 to 2502 meters above sea-level and the corresponding variability in precipitation climatology, and its long-term hydrological measurement series provide a unique opportunity to investigate it more closely. In fact, this catchment has been subject of many other studies in the scientific literature (search in HESS with key-word “Thur” yields approximately 90 hints), such as:


In the revised article we explained in more detail, why we investigated the Thur catchment (see section 2). Please also note, that our generated time-series will serve as input to two hydrological models that are calibrated over the Thur catchment. This (separate) analysis will be published in an upcoming article, that investigates the runoff-regime in current and future climate and based on input of different statistical downscaling methods:


In the revised article, we made it clearer, that our generated time-series are subject to a subsequent hydrological study (see section 2).

(b) Novelty of our method

The novelty of our precipitation generator has been criticized throughout the review-process. It is indeed true, that the model is not new but rather re-built after Wilks (1998). At several positions in the first round of revisions we have tried to make this aspect more clear. Based on your comments, we think that this must be emphasized even more, especially in the abstract. We also propose to change the title to “… Wilks-type multi-site precipitation generator …”. To make it as transparent as possible, we clarify the revised abstract and say explicitly that our precipitation generator is “re-built after the procedures described in Wilks (1998)”. So, to be clear, we did not add any novel aspect regarding the
basic method of the weather generator itself. Rather, our aim is, as stated several times in the manuscript, to have a tool ready that can be subsequently used as a downscaling technique in a climate change context, and to evaluate its performance for an Alpine catchment.

Regarding a further method-comparison to other Markovian models: this would certainly be of great value and give more details on the benefits and limitations of the different multi-site generators. On the other hand, we do not see the need of implementing a new methodology to our catchment as long as our generator produces results that fit our purposes. Based on our results of the study and clearly knowing that our generator is far from perfect, we still consider the generator as a valuable tool that will provide useful insights in local climate change, which is the focus of an upcoming study:


Without knowing the caveats and limitations of a downscaling technique, the simple application to a future climate, is of no great use. In the revised article, we added this point more clearly.

A formal quantitative method inter-comparison, although interesting, is out of scope here. The only thing possible was to give a short discussion in a qualitative way. Given the huge amount of resources for implementation and calibration, a quantitative comparison would rather be an own separate article. Also, note that the Alps are one of the focus regions of COST-VALUE that aims to inter-compare downscaling methods of different levels of complexity. We plan to contribute to this frame-work with our generator allowing to later inter-compare our results to other downscaling methods.

In summary, we addressed the reviewer’s concern by sharpening the scope of the work and explaining our target area in more details. In addition, we discussed in a qualitative way (discussion section, page 16, line 16-32 & page 17, line 1-5) why we have decided to implement a Wilks approach instead of using a hidden Markov model to simulate precipitation over the catchment.

Specific Comments

Comment 1: Title- Think about indicating explicitly that the presented precipitation generator is based on Markovian methods.

Reply 1: We fully acknowledge that the title should be as clear and transparent as possible. We agree on that and changed the title to: “Implementation and validation of a Wilks-type multi-site precipitation generator over a typical Alpine river catchment”

Comment 2: Page 3, 15-29: As the authors have mentioned many WG can be found in the literature. What is unique in the precipitation generator suggested in this paper? I am suggesting that the authors elaborate more about the existing WGs (especially the Markovian based models) and discuss them later on after describing their methods and results. I can suggest some relevant references for the introduction part (not all are Markovian based model):

Reply 2: We would like to apologize for missing some important papers about NHMMs. It is indeed worth mentioning them, as they represent important developments in the field of multi-site generators. We included the mentioned papers at the indicated position in the revised manuscript (page 3, line 28-29). As stated in the general reply, we provided a short comparison of the presented model with NHMMs in a rather qualitative way. Please note, as said several times in the manuscript, the model itself is not novel but rather rebuilt after Wilks. We implement and test this model to understand its limitations and benefits. This is key information to later apply the WG as a downscaling tool for a perturbed climate.
Comment 3: Page 4, 6-15: The main goal was to analyze the Thur catchment precipitation statistics and the second goal was to evaluate the multi-site model? Wasn’t it the other way around?

Reply 3: Yes, we agree. We re-wrote this text passage to clarify.

Comment 4: Figure 1: Please add to the figure scale-bar and coordinates. Maybe even add a background map of Switzerland? Most of the readers will probably find it useful. In the figure caption define what is a wet day. Wet day intensity – this is the average rain intensity per day I assume?

Reply 4: This is a good suggestion. We added a scale-bar to the figure and indicated the coordinates of the stations as well as the definition of a wet day in the caption of figure 1. Yes, a wet day is defined as a day with precipitation intensity ≥ 1mm day⁻¹. The wet day intensity is the average precipitation amount at wet days.

Comment 5: Section 3: This section, and especially subsections 3.1, 3.2 and 3.3.1, should be much shorter. Daily precipitation generators using Markov chains are not new and in fact can be found in many papers and textbooks. In my opinion, it is enough to shortly describe the Markovian method (the transmission and emission matrices and the bivariate exponential distribution that you have used) while citing the benchmark papers in this fields (e.g., Gabriel and Neumann, Wilks and Wilby). The subsection discussing the multi-site approach that was used in this study (subsection 3.3.2) can also be much shorter- no needs to discuss the Pearson spatial correlation in details and also the calibration procedure (iterations) can be moved to the supplementary section. On the other hand, I think that here is a good place to remind the readers again that there are daily precipitation generators using Markov models for multi-site locations (nonhomogeneous hidden Markov models, for example, see Robertson papers above). Here, or later in the discussion, you must state the differences / benefits of your model comparing to the ones already exist in the literature.

Reply 5: We fully agree with you. We have shortened the method section (3.1, 3.2, 3.3) in the revised manuscript. A large part was dislocated to the Supplementary (specifically, chapter 3.3.1, and large parts of 3.3.2). In addition, we now discuss the NHMMs as valuable alternative multi-site modelling approach. In the discussion section we argue why we have used the Wilks approach instead of a NHMM. This is done in a qualitative way. A quantitative comparison out of scope here. Yet, we think that the recently started downscaling inter-comparison project of COST-VALUE will shed light on the benefits/limitations of the two approaches in a quantitative way. For more information on COST-VALUE experiments we refer to: http://www.value-cost.eu/validation

Comment 6: Section 3.4.1: What happens when you have a sequence of wet (or dry) days that starts near the end of one month and continues in the next month? Do you refer it as a continuation of the first month or as a two separate sequences? How does your choice influence the model performance?

Reply 6: Yes, our model simulates the time-series continuously with monthly varying WG parameters. This means that precipitation occurrence of the last day of a certain month influences the occurrence of the first day of the next month (1st order Markov chain). The consideration of seamless simulations has an influence on the correct reproduction of the lag-1 autocorrelation of the simulated time-series. This issue is even of greater concern, when higher-order MC are used for modelling long-term persistence of dry or wet events. We have added a sentence in the method section of the revised manuscript to clarify (page 8, line 19-22).
Comment 7: Figure 3: Can this figure be in color? I suggest to set the same limits for both axes. It looks like the model underestimate the wet-wet transition and the dry-wet transitions, especially for the lower tail- is this the case?

Reply 7: Yes, we agree with you. We exchanged figure 3 in the revised manuscript with a coloured version of figure 3 and adjusted axes. Furthermore, we added the 1:1 line. As described in the revised manuscript (page 9, line 15-19) the estimated uncertainty of the transition probabilities largely depends on the prevailing precipitation regime. This implies that in very dry climate the estimation of the wet-wet transition probability is more uncertain due to the limited sample size of wet days. The same is true for the estimation of the dry-dry probability in very wet regimes.

Comment 8: Page 15, 24-26: I think that you have long enough time series (daily, 51 years) to fit a Gamma with GP distribution model. It is indeed more parameters to fit, but if it will improve the upper 20 percentile fitting then it will be useful in the long run (especially if you want later on to deal with climate change models and precipitation extreme).

Reply 8: Yes, we agree with you. However, improving our model in this direction goes beyond the scope of this study and will be part of future work. As stated in the revised manuscript (page 19, line 1-5) we plan to refine the presented WG by using multi-state Markov chains (e.g. dry, wet and very wet) in combination with different probability density functions for wet and very wet days in order to improve the simulation of one-day and multi-day extremes. Complementary to this future improvement of the WG, an adjustment will be further necessary to include also variations on the inter-annual scale. In our view, this is even of greater concern at the moment.

Comment 9: Page 17, 1-5: The papers you cited are not recent. There are some papers from the last decade using NHMM to fit different distributions to different locations. I think that using a multi-state approach nowadays is not really a challenge.

Reply 9: We would like to apologize for missing some more recent and important papers. In the original manuscript, we mentioned the multi-state (e.g. dry, wet, very wet) Markov models in terms of further model developments of the existing model, rather than as a new modelling approach with NHMMs. We complemented now the revised manuscript with more recent studies. As stated in reply 8, the extension of the existing model towards a multi-state model will be part of future work and goes beyond the scope of this study.

Comment 10: Discussion: At this section I would expect the authors to convince the reader why the models they have suggested is better than applying one of the common daily precipitation model (for example, why not applying a NHMM for the Thur catchment).

Reply 10: We mentioned in the discussion section several qualitative reasons why we have decided against the use of NHMMs to simulate precipitation over the investigated area (page 16, line 16-32, & page 17, line 1-5).
<table>
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<tr>
<th>Page</th>
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<td>Title has been changed to <em>Implementation and validation of a Wilks-type multi-site daily precipitation generator over a typical Alpine river catchment</em></td>
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<td>As suggested by the reviewer, additional citations about NHMMs and other multi-site precipitation generators were included into introduction</td>
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<td>Explanation, why we have decided to implement approach of Wilks (1998)</td>
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<td>Explanation, why we have selected the catchment of the Alpine river <em>Thur</em> for this study</td>
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<td>6</td>
<td>18-27</td>
<td>As suggested by the reviewer, the method section was shortened. The former section 3.3.1, large parts of the former section 3.3.2 as well as the former Figure 2 were moved to the Supplementary.</td>
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<td>7-14</td>
<td>1-13</td>
<td>As suggested by the reviewer, the method section was shortened. The former section 3.3.1, large parts of the former section 3.3.2 as well as the former Figure 2 were moved to the Supplementary.</td>
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<tr>
<td>23</td>
<td>11-32</td>
<td>We provided a short comparison of the presented model with NHMMs in a rather qualitative way</td>
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</tbody>
</table>
| 37-39|      | Figure 1: a scale-bar was added  
Figure caption: includes the coordinates of the stations and the definition of a wet day. |
| 39-41|      | Former Figure 2 moved to the Supplementary  
New Figure 2 (former Figure 3): now in colour and the x- and y- axes were set to the same limits. |
Implementation and validation of a **Wilks-type** multi-site daily precipitation generator over a **Swisstypical Alpine** river catchment

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**ABSTRACT**

Many climate impact assessments require high-resolution precipitation time-series that have a spatio-temporal correlation structure consistent with observations, for simulating either current or future climate conditions. In this respect, weather generators (WGs) designed and calibrated for multiple sites are an appealing statistical downscaling technique to stochastically simulate multiple realizations of possible future time-series consistent with the local precipitation characteristics and its expected future changes. In this study, we present the implementation and validation of a multi-site daily precipitation generator following ideas of re-built after the methodology described in Wilks (1998). The generator consists of several Richardson-type WGs run with spatially correlated random number streams. We investigate the applicability/capabilities, the added value and the limitations of the precipitation generator for the current climate by analysing systematic biases and stochastically generated variability and assessing the added value of a multi-site generator compared to multiple single-site WGs. Results are presented for the Swiss...
hydrological typical Alpine river catchment Thur in the Swiss Alpine region for current climate condition.

The calibrated multi-site WG is skilful at individual sites in representing the annual cycle of precipitation statistics, such as mean wet day frequency and intensity as well as monthly precipitation sums. It reproduces realistically the multi-day statistics such as the frequencies of dry and wet spell lengths and precipitation sums over consecutive wet days. Substantial added value is demonstrated in simulating daily areal precipitation sums in comparison to multiple WGs that lack the spatial dependency in the stochastic process. Limitations are seen in reproducing daily and multi-day extreme precipitation sums, observed variability from year to year and in reproducing long dry spell lengths. Given the performance of the presented generator, we conclude that it is a useful tool to generate precipitation series consistent with the mean climatic aspects of the current and future climate likely helpful to be used as downscaling technique for climate change scenarios.

1 Introduction

In Switzerland, precipitation is a key weather variable with high relevance for sectors such as energy production, infrastructure, tourism, agriculture and ecosystems. Owing to a complex topography, daily precipitation varies strongly in space and time (Frei and Schär, 1998; Isotta et al., 2013). The spatial distribution of daily precipitation frequency and intensity depends on the topography, with higher frequencies and intensities along the North-Alpine ridge during summer, and a strong north-south gradient with heavier intensities in southern Switzerland from spring to autumn. The most prominent weather situations causing these precipitation patterns are shallow pressure systems favouring convective precipitation, orographically induced precipitation (e.g. Föhn situations), and frontal passages. Precipitation amounts and frequencies are typically largest in summer, mainly due to convective processes (Frei and Schär, 1998).

Given the expected changes in the hydrological cycle over the 21st century (Allen and Ingram, 2002; Held and Soden, 2006), the need for reliable and quantitative future local precipitation projections in Switzerland is continuously growing. To effectively assess the impacts related to changes in precipitation, often highly localized daily data are needed that are ideally both consistent in time and in space (e.g. Köplin et al., 2010). Currently, in Switzerland various impact assessment reports rely on the statistically downscaled precipitation change data derived from regional climate models by the well-known and simple
delta change approach, which shifts an observed time series by a model-derived change in the
mean climate (BAFU, 2012; Bosshard et al., 2011; CH2014-Impacts, 2014). The delta change
approach accounts for changes in the mean annual cycle, but potential changes in inter-annual
variability, changes in wet-day frequency and intensity or of spell lengths are not taken into
account. Hence, the data are also not suitable for the analysis of future changes in extreme
events (Bosshard et al., 2011). It is our aim here to develop a statistical downscaling method
for Switzerland that overcomes some of these limitations and that subsequently can be easy
applied to climate model output.

Over recent years a vast number of statistical downscaling methods have been developed that
go far beyond a simple delta change approach (Maraun et al., 2010). These include bias-correction methods (e.g. Themeßl et al., 2011), regression-based methods (e.g. Hertig and Jacobeit, 2013) or weather generator (WG) approaches (e.g. Chandler and Wheater, 2002; Mezghani and Hingray, 2009). For our purposes, the latter method is
especially appealing, since it includes a stochastic component. This is a major improvement
compared to a (deterministic) delta change approach, allowing to investigate multiple time-
series and uncertainty at the local scale that are consistent with a given (current or future)
mean climate. Moreover, it allows the incorporation of changes in the temporal correlation
structure and consequently alterations of the dry-wet sequences. From an agricultural impact’s
perspective this is a key aspect of future precipitation change (e.g. Calanca, 2007) or water resource management’s (e.g. Samuels et al., 2009) perspective this is
a key aspect of future precipitation change.

A serious limitation of many WGs is that they are often calibrated to observations at single
sites only, therefore lacking the spatial correlation structure that is required for many
applications, particularly in the context of hydrological impact modelling in a topographically
complex terrain such as the Alps. A number of sophisticated approaches in time-space
precipitation simulation have been put forward in the literature to address this issue, such as
K-nearest neighbor resampling approaches (e.g. Buishand and Brandsma, 2001) or Poisson cluster models (e.g. Cowpertwait, 1995; Paterson et al., 2011) or more sophisticated field generators (e.g. Paschalis et al., 2013). K-
nearest neighbor resampling approaches represent a further possibility to ensure the spatial
coherence (e.g. Buishand and Brandsma, 2001) (e.g. Paschalis et al., 2013; Peleg and Morin,
Of increasing popularity are Markovian multi-site models (e.g. Baigorria and Jones, 2010; Wilks, 1998) and in particular non-homogeneous hidden Markov model (NHMMs) (e.g. Bellone et al., 2000; Hughes et al., 1999; Kioutsioukis et al., 2008; Robertson et al., 2004, 2009). The latter approach models transitions between pre-defined precipitation state patterns conditional on the synoptic-scale circulation. Each of these time-space WGs come with method-specific benefits and limitations for the reproduction of the daily precipitation statistics and consequently its use in impact models. For instance, some of them do better in simulating more realistically longer-term variability (e.g. generalized linear model (GLM) based multi-site WGs, Chandler, 2014), while some are explicitly adapted to deal with extreme precipitation (e.g. Huser and Davison, 2014)

The main purpose of our precipitation generator is its use as a downscaling tool in a climate change context. It should be easily transferable to different climatological regions and time-periods and its generated time-series should serve several impact applications that have different needs in terms of time-space consistency. For these reasons we opt for a precipitation generator whose degree of complexity and associated calibration requirements are still sufficiently easy to handle. This is accomplished with Mehrotra et al. (2006) inter-compared three stochastic multi-site precipitation occurrence generators over a region over Australia and found that the generator by Wilks (1998) outperforms Hidden Markov models and K-nearest neighbour resampling techniques in terms of overall performance, time required for model running and simplicity of the model structure. Hence the multi-site precipitation generator proposed by Wilks (1998) that is therefore serves our purposes. It is a relatively simple tool based on a Richardson-type WG (Richardson, 1981) run with spatially correlated random number streams.

In this study, we implement and validate to investigate the capabilities, the added value and the limitations of this multi-site generator for the Swiss catchment Thur in the Swiss Alpine region under current climate conditions to document the specific challenges encountered during the setup. The Thur catchment serves as an ideal testbed with different precipitation characteristics mainly due to the complex topography. Understanding its capabilities and systematic biases in current climate is key to later better interpret the climatic changes in the simulated time-series for a future climate. Of which is part of an upcoming study. In particular-relevance is the actual amount of stochastically generated variability—A second goal of the study is to assess will be assessed as well as the added value of a multi-site
model against multiple single-site models. To accurately quantify these aspects, we choose a rather long calibration period that minimizes the effect of sampling uncertainties.

The structure of this paper is as follows: Sect. 2 introduces the hydrological catchment. The analysis is done for the Swiss catchment Thur. While being not of the river Thur together with the used station data, same level of familiarity as other catchments, the Thur catchment serves as an ideal testbed for our purposes, as will be detailed in Sect. 1. In Sect. 3 we first describe the statistical models for simulating precipitation occurrence and amount and show how these models are combined. We recapitulate the basic procedures to multi-site precipitation simulation. The validity of our generated multi-site precipitation series and the comparison to after Wilks (1998) and detail how the generator was calibrated over the catchment. Results of the validation against observations and against single-site generators will be presented in Sect. 4. Sect. 5 includes. We end the article with a discussion and finally Sect. 6 provides (Sect. 5) and a summary and an outlook (Sect. 6).

2 Data

2.1 Selection of catchment area

This study focuses on the hydrological catchment of the river Thur, which is located in the north-eastern part of Switzerland (Figure 1, Figure 1a). The river Thur is a feeder river of the Rhine with a length of about 135 km and a catchment area of approximately 1696 km². It represents the largest Swiss river without a natural or artificial reservoir and therefore exhibits discharge fluctuations similar to unregulated Alpine rivers. Its flow regime is nivo-pluvial that is heavily influenced by snowmelt (BAFU, 2007). This particular catchment was selected for mainly two reasons:

(a) In an upcoming study our generated synthetic time-series over the Thur catchment will serve as input to two hydrological models to assess the runoff regime under current and future climate (similar as in Jasper et al., 2004). The Thur is a well-studied and well-observed river catchment in Switzerland (e.g. Fundel et al., 2013; Kunstmann et al., 2006) providing high-quality hydrological measurement series for a robust calibration of hydrological runoff models. It further represents the largest Swiss river without a natural or artificial reservoir and therefore exhibits discharge fluctuations similar to unregulated Alpine rivers.
Owing to the complex topography over this catchment area, (ranging from less than 400 meters a.s.l. to more than 2500 meters a.s.l.), precipitation exhibits a large variability both in space and in time. This is illustrated in (see Figure 1) based on gridded observational data from Frei and Schär (1998). Over 1961-2011 and for a winter and summer month, the data clearly show larger precipitation frequencies and intensities over higher-elevated regions compared to the lowlands. Additionally, this catchment lies in one of the Swiss regions featuring well above-average precipitation. A large portion of these precipitation characteristics can be explained by a north-east to south-west lying mountain range (Alpstein) extracting precipitation from westerly flows and triggering convective storms. These spatio-temporal variations serve hence as an ideal observation basis to validate and analyse the capabilities and limitations of the WG.

For the purpose of this study, we selected eight evenly distributed measurement stations (Figure 1) of MeteoSwiss that all provide homogenized time-series covering a 51-year period from 1961-2011 (Begert et al., 2003), and that sufficiently cover the elevation profile of the catchment area from Andelfingen lying at 382 meters a.s.l. to Saentis lying at 2502 meters a.s.l.

3 Method

3.1 Precipitation occurrence and amount model

The core of our multi-site WG is a Richardson-type precipitation generator (Richardson, 1981), that relies on the concept of modelling two processes at one single station: an occurrence and an amount process. Following Wilks (1998), this single-site WG is then extended in order to simultaneously generate precipitation at several sites taking into account the complex spatio-temporal correlation structure.

In the following, we explain the setup of our multi-site generator step by step: Sect. 3.1 and 3.2 present the concepts of statistically characterizing occurrence and amount at single sites. The simulation procedure of new synthetic time-series is detailed in Sect. 3.3. In Sect. 3.4 we give a description of how we implemented the multi-site WG over the Thur catchment.
3.1 Precipitation occurrence process

(Richardson, 1981) consisting of an occurrence and amount model. To model occurrence at a single station we rely on a first-order two-state Markov chain (Gabriel and Neumann, 1962; Richardson, 1981). The first order two-state Markov chain is a statistical model describing the probability to stay in the same state or switch to the other state. In this context, first order implies the state at a given day depends only on the state at the previous day (Gabriel and Neumann, 1962; Richardson, 1981; Wilks and Wilby, 1999). The use of a first-order model in our WG was justified by inspecting the Akaike information criterion (AIC) (Akaike, 1974) and the Bayesian information criterion (BIC) (Schwarz, 1978). Both the AIC and the BIC criterions revealed a substantial improvement when going from a zero-order to a first-order model, but the additional gain at a second- or higher-order model was negligible (not shown). We used a specific wet-day threshold of 1 mm day$^{-1}$ to discretize a given daily precipitation time-series $X(t)$ at a given site into the two states ‘dry’ ($X(t) < 1$ mm day$^{-1}$, $J_t = 0$) and ‘wet’ ($X(t) \geq 1$ mm day$^{-1}$) and to subsequently generate a binary series (i.e. $J_t$ with $J_t = 0$ for a dry state and $J_t = 1$ for a). The wet state). Four transitions are possible: a dry day following a dry day ($00$), a wet day following a dry day ($01$), a dry day following a wet day ($10$) and a wet day following a wet day ($11$). Transition probabilities suffice to fully specify the first-order two-state Markov chain model:

The first-order two-state Markov chain model can be specified by formulating the probabilities (p) of these state transitions:

\[
\begin{align*}
 p_{11} &= P[J_t = 1 | J_{t-1} = 1] \\
 p_{01} &= P[J_t = 1 | J_{t-1} = 0]
\end{align*}
\]

(1)

The corresponding counterparts: For an estimate of the transition probabilities ($p_{00}$ and $p_{10}$) can then easily be derived, since the sum of two probabilities conditioned on the same state at the previous day equals one:

\[
\begin{align*}
 p_{11} + p_{10} &= 1 \\
 p_{00} + p_{01} &= 1
\end{align*}
\]

(2)

The two transition probabilities of Eq. (1) suffice to fully specify the first-order two-state Markov chain model. For the remaining part of this study, we therefore concentrate on these two parameters when addressing state transitions. For an estimate we rely on their conditional relative frequencies (Wilks, 2011):
where \( n_{01} \) and \( n_{11} \) are the number of transitions from dry to wet and wet to wet in the binary series and \( n_0 \) and \( n_1 \) are the total number of zero’s and one’s in the series followed by any of the two states. From the transition probabilities of Eq. (3) other important precipitation indices can be inferred. The wet day frequency (wdf, \( \pi \)) is defined as the ratio of the number of wet days to the total number of days over a given time period. It can be expressed in terms of the two transition probabilities (Wilks, 2011):

\[
\pi = \frac{p_{01}}{1 + p_{01} - p_{11}}
\]

(4)

Similarly, the lag-1 autocorrelation \( r_2 \) is defined as the difference between the transition probabilities (Wilks, 2011):

\[
r_1 = p_{11} - p_{01}
\]

(5)

Since day-to-day precipitation generally exhibits positive serial correlation (i.e. \( r_1 \) greater than 0), \( p_{11} \) is usually larger than \( p_{01} \) and the wdf is between the two. Note, that a first-order two-state Markov chain does not imply independence for lags greater than one. The autocorrelation \( r_L \) (6) decays exponentially with larger lags \( L \):

\[
r_L = \left( p_{11} - p_{01} \right)^L
\]

(6)

### 3.2 Precipitation amount process

As will be detailed in Sect. 3.3, precipitation amounts at wet days are drawn from probability density functions (PDFs) fitted at single stations. Many studies use either an exponential (Richardson, 1981) or a gamma distribution (Buishand, 1978; Katz, 1977) to model non-zero precipitation amounts \( X(t) \geq 1 \text{ mm day}^{-1} \). Both distribution types, however, do not appropriately characterize the frequency of the heavily right skewed precipitation amounts: they underestimate either light precipitation (exponential distribution) and/or heavy precipitation (exponential and gamma distribution). As an alternative, a mixture model of two exponential distributions has been proposed to provide better overall fits and to better represent precipitation extremes (Wilks, 1999a). The PDF can be formulated as:
\begin{equation}
\begin{aligned}
f(x) &= \frac{w}{\beta_1} \exp \left( -\frac{x}{\beta_1} \right) + \frac{1-w}{\beta_2} \exp \left( -\frac{x}{\beta_2} \right)
\end{aligned}
\end{equation}

\(f(x)\) is a weighted average (weight \(w\)) of two exponential distributions with means \(\beta_1\) and \(\beta_2\).

The quantile function exists in a closed form. Consequently, random samples from this distribution can easily be obtained by inversion (Wilks, 2011). Other important precipitation indices can be inferred. The wet day frequency (wdf, \(\pi\)) is defined as the ratio of the number of wet days to the total number of days over a given time period:

\begin{equation}
\pi = \frac{P_{01}}{1 + P_{01} - P_{11}}
\end{equation}

Similarly, the lag-\(k\) autocorrelation \(r_k\) is defined as:

\begin{equation}
r_k = (P_{11} - P_{01})^k
\end{equation}

Since day-to-day precipitation generally exhibits positive serial correlation (i.e. \(r_1 > 0\)), \(P_{11}\) is usually larger than \(P_{01}\) and the wdf is between the two. Note that a first-order Markov chain does not imply independence for lags greater than one. The autocorrelation \(r_k\) decays exponentially with larger lags \(k\).

Given a simulated wet day from the occurrence model, precipitation amounts are set. This is done by sampling from a mixture model of two exponential distributions (Wilks, 1999a):

\begin{equation}
\begin{aligned}
f(x) &= \frac{w}{\beta_1} \exp \left( -\frac{x}{\beta_1} \right) + \frac{1-w}{\beta_2} \exp \left( -\frac{x}{\beta_2} \right)
\end{aligned}
\end{equation}

\(f(x)\) is a weighted average (weight \(w\)) of two exponential distributions with means \(\beta_1\) and \(\beta_2\). The parameters \(w\), \(\beta_1\) and \(\beta_2\) are estimated by using the concept of maximum-likelihood (Tallis and Light, 1968). Note that the estimation of PDF parameters is subject to sampling uncertainty from the available number of wet days in a given calendar month.

### 3.3.2 Stochastic modelling of multi-site daily precipitation

#### 3.3.1 Single-site

In this section, we demonstrate how the occurrence (Sect. 3.1) and amount model (Sect. 3.2) are applied to stochastically simulate daily precipitation at a single site. The simulation process is based on Richardson (1981) with the five above-introduced parameters serving as
input in Figure 2: i.e. the transition probabilities $p_{11}$ and $p_{01}$ as well as $w_1$, $\beta_1$ and $\beta_2$. The simulation of precipitation at a given day and a given station (say $A$) is accomplished in four main steps (see yellow circles in Figure 2):

1. A random number $u_{t,A}$ is drawn from a standard normal distribution.
2. The conditional wet day probability $p_{t,A}$ is determined depending on the state of the previous day. It is set to $p_{11,A}$ or $p_{01,A}$ depending on whether the previous simulated day was wet or dry, respectively.
3. The random number $u_{t,A}$ is compared to the standard normal quantile function $Q$, evaluated at $p_{t,A}$: if $u_{t,A}$ is larger than $Q(p_{t,A})$, a dry day ($J_t,A'=0$) is simulated and else a wet day ($J_t,A'=1$) is set.
4.1 In case of a dry day, the simulated amount $X_{t,A}'$ is set to zero.
4.2.1 In case of a wet day, a second random number $v_{t,A}$ (independent from $u_{t,A}$) is drawn from a standard normal distribution.
4.2.2 The corresponding quantile of the random number $v_{t,A}$ is then inserted into the quantile function of the mixture model yielding the corresponding precipitation amount ($x_{t,A}$) at a given day.

Note that this simulation procedure could be simplified by taking random uniform [0,1] numbers instead of Gaussian random numbers. We use the latter here in order to be consistent with the multi-site extension introduced later (Sect. 3.3.2).

Steps 1-4 are repeated over all remaining days within a certain simulation period. Based on this procedure time series of arbitrary length can be generated that resemble observed climatological precipitation statistics, both in terms of frequency and intensity. For a more realistic reproduction of the annual cycle of precipitation the WG is calibrated on a monthly basis (see Sect. 3.4).

3.3.2 Multi-site

So far, the procedure to generate precipitation consists of multiple single-site WGs only. Specifically, no spatial dependence in the simultaneous simulation of precipitation at several sites was taken into account. To close this gap several single-site WGs are driven simultaneously with spatially correlated but serially independent random numbers. The simulation process is based on Richardson (1981) at single stations with the five above-
introduced parameters: i.e. the transition probabilities $p_{ij}$ and $p_{0j}$ as well as $w$, $\beta_1$ and $\beta_2$. That is, a uniform random number between 0 and 1 is compared to either $p_{ij}$ or $p_{0j}$ depending on the state of the previous day and correspondingly set as either dry or wet. In case of a wet day, a second uniform random number is drawn to assign the precipitation amount based on the quantile function. For further details on the simulation of precipitation at a single location we refer to Wilks and Wilby (1999). The simulation allows time-series of arbitrary length resembling observed climatological precipitation statistics, both in terms of frequency and intensity.

The main extension to a multi-site model after Wilks (1998) is to drive several single-site WGs simultaneously with spatially correlated but serially independent random numbers. To generate correlated random number streams, we rely on a Cholesky decomposition (e.g. Higham, 2009). The latter requires matrices that are positive definite, which is not always granted. In absence, a fall-back solution based on the nearest positive correlation matrix is chosen (e.g. Higham, 1989). This problem, however, occurs only a few times in our study. One of the main hurdles in simultaneously generating precipitation at multiple sites is to ensure that the spatial dependence is also preserved in the final generated time-series (Wilks and Wilby, 1999; Wilks, 1998). This difficulty mainly arises from the stochastic process that partly destroys the initially imposed correlation structure again (Wilks, 1998). For simplicity, the concept is illustrated in Figure 2 for the example of two fictitious sites (A and B) only. The extension to several sites is straightforward. One of the main hurdles in simultaneously generating precipitation at several sites is the prescription of the spatial correlation matrices such that the dependence is also preserved in the final generated time-series (Wilks and Wilby, 1999; Wilks, 1998). This difficulty mainly arises from the stochastic process that partly destroys the initially imposed correlation structure again (Wilks, 1998). We will come back to this problem later. For the moment, let us assume that the optimal correlation matrices for both, occurrence and amount (i.e. $\phi_{AB,\text{opt}}$ and $r_{AB,\text{opt}}$), are known. In this case, the main extensions to single-site WGs are two spatially correlated but serially independent random number streams (dashed boxes in Figure 2): one for the occurrence ($u$) and the other for the amount ($v$) process. They are determined prior to the simulation process (see below) and contain the same number of days as the simulation period. Once these correlated random number streams are generated, the simulation proceeds as in Sect. 3.3.1 for all stations.
simultaneously. In practice, the multi-site WG implies the handling of three main methodological hurdles that are the following:

1) Calculating spatial correlation coefficients $\phi_{AB}$ and $r_{AB}$

Spatial dependence in binary series at site A and B is inferred by the phi-coefficient ($\phi_{AB}$).
Similarly as the Pearson correlation coefficient, the phi-coefficient $\phi_{AB}$ is bounded by $-1$ and $1$. For the precipitation amounts, the spatial correlation coefficient ($r_{AB}$) is determined by the conventional Pearson product-moment correlation coefficient. The correlation is calculated over the whole precipitation series that also include time steps with zero amounts. From a statistical point of view, this is not an optimal procedure, since the correlation coefficients could be strongly affected by the number of zeros in the time-series. However, the purpose here is to use this spatial similarity measure rather as a tool to compare the observed spatial dependencies with those in artificial data. It is assumed that the statistical limitations in the calculation apply similarly to observations and generated data. The spatial correlations between different sites are determined pair-wise. Note that the pair-wise estimation of the inter-station correlation can result in matrices that are not positive definite, especially when the number of station number is large or when there are incomplete station records.

2) Finding optimal spatial correlation coefficients $\phi_{AB, \text{sim}}$ and $r_{AB, \text{sim}}$

As mentioned above, imposing observed inter-site correlations as input to our WG does not guarantee its reproduction in the generated series. This is due to a randomization process through transition probabilities calibrated at each site separately. In general, the imposed correlation is reduced by the stochastic process, both in terms of occurrence and amount process. This characteristic is illustrated at an artificial example of two fictitious sites A and B in Supplementary Fig. 1. While the random number streams ($u_A$ and $u_B$) perfectly incorporate the observed spatial correlation in occurrence between A and B, it is essentially the two distinct transition probabilities at the two sites that lead to a final correlation in the binary series that is much reduced ($\phi_{AB, \text{sim}} = 0.6$ compared to $\phi_{AB, \text{obs}} = 0.8$). In case of precipitation amounts the mismatch in correlation magnitude is also present ($r_{AB, \text{sim}} = 0.38$ compared to $r_{AB, \text{obs}} = 0.5$) and can be mainly explained by two factors. First, precipitation amount is only simulated at wet days (i.e. where $J_t = 1$), while the correlated random number streams ($v_{A,B}(t)$)
and \( v(t) \) are representative for the full time series. Hence, the number of zeros introduced by distinct transition probabilities impact on the generated correlation coefficient. Second, if the two fitted PDFs at the two sites are markedly different, the correlation of the observed and simulated precipitation time series will deviate, even in absence of any zeros.

To overcome this inherent problem of a multi-site WG after Wilks (1998), an optimization procedure was proposed to find an input spatial correlation that ultimately yield the target correlation of the observations. This has to be done first for the occurrence process \( (\phi_{AB,\text{optim}}) \) and then in a subsequent step for the amount process \( (\tau_{AB,\text{optim}}) \). The optimization procedure iterates over an interval of input correlations, thereby running at each iteration the full occurrence and amount model of the multi-site WG (see Supplementary Fig. 2). After each iteration, the resulting correlation is compared to the target correlation of observations. To find an optimal correlation, we use a bisection method (Burden and Faires, 2010) as non-linear root finding algorithm. The iteration is repeated until the generated correlation equals the one of observations with a precision of 0.005 (see Supplementary Fig. 2). Note that this estimation procedure is done prior to the simulation and has to be repeated for each station pair and month.

3) Generation of correlated random number streams

There are several approaches to generate spatially correlated random numbers streams (e.g. Monahan 2011). For the study at hand we applied a Cholesky decomposition (e.g. Higham 2009):

1. Sample for each station a random number stream from a standard Gaussian distribution.

2. Apply a Cholesky decomposition to the optimized correlation matrix to get a lower triangular matrix and its transposed.

3. Multiply the resulting lower triangular matrix with the matrix of random number streams.

Cholesky decomposition requires matrices that are positive definite, i.e. that contain no negative eigenvalues. However, in case of the applied pairwise optimization process (see section (2) above) this is not always fulfilled. In absence of positive definite matrices, a fall-
back solution based on the nearest positive correlation matrix was chosen. The nearest positive definite matrix was found by using the algorithm proposed by Higham (1989), which uses a weighted version of the Frobenius norm. This problem occurred in our study only a few times. Note, that the temporal correlation structure of the precipitation time series at one specific site is not altered by the imposed spatial correlation, since the spatially correlated random number streams exhibit no serial correlation.

To circumvent this problem, Wilks (1998) suggested an optimization procedure based on a bisection method (Burden and Faires, 2010) that minimizes the difference between the generated spatial correlation and the target correlation of observations. In our case, the iteration is repeated until a precision of 0.005 is reached. This estimation procedure is done prior to the actual simulation and has to be done for each station pair and each month. For further details regarding the setup of stochastic simulation and in particular the implementation of multi-site simulation we refer to the Supplementary Material.

3.4.3 Implementation

3.4.3.1 Implementation of the multi-site WG over the Thur catchment

Our developed precipitation generator is calibrated on a monthly basis. First, all the single-site input parameters ($p_{11}$, $p_{01}$, $\beta_1$, $\beta_2$ and $w$) were estimated for each of the 8 stations within the catchment and for each month separately using a time-window of 51 years (1961-2011). In this study we chose a relatively long calibration period in order to minimize the effect of sampling uncertainties. This allows us to accurately assess the added value of a multi-site model against multiple single-site models and to better quantify systematic biases of the WG. For the two transition probabilities in a given month, the climatological mean over the 51 yearly values of $p_{11}$ and $p_{01}$ was taken. In the case of fitting a PDF to non-zero precipitation amounts and the estimation of $\beta_1$, $\beta_2$ and $w$, we used the daily data over all 51 years together. In addition, a three-month window centred at the month of interest was chosen, in order to increase sample size and the robustness. The distributional parameters were derived based on maximum-likelihood (Tallis and Light, 1968). Despite our three-month time-window, cases occurred when the maximum-likelihood algorithm did not converge. For such cases, a fall back solution was applied where the parameter estimates from the previous month were adopted. With the monthly parameters from all the calibrated single-site WGs and the monthly observed inter-station correlations (symmetric correlation matrices), the
optimized correlation matrices had to be found for each month based on the procedure described in Sect. 3.3.2. Optimization procedure for spatial correlation had to be applied (see Supplementary). In terms of a correct temporal correlation in the generated time-series, it was ensured that the transitions between adjacent months is continuous (i.e., the first day of a given month is conditioned on the last day of the previous month). Note, that by calibrating the multi-site WG on a monthly instead of a seasonal basis, additional sampling uncertainty is introduced due to the rather small time-window to estimate our parameters. This is the downside of prescribing an improved annual cycle in the WG parameters.

Once the multi-site WG was calibrated, we generated 100 ensembles of daily time-series, of 51-year length. All the results presented in Sect. 4 are calculated over the time-period 1961-2011.

### Reproduction and uncertainty of WG model parameters

To test whether our developed WG is properly implemented, we evaluated the reproduction of WG input parameters extracted from the generated time-series. A correct reproduction in parameters such as wet day intensity, frequency and transition probabilities is a prerequisite for all the subsequent analyses presented in Sect. 4.4. The evaluation was performed for four subjectively-defined climatic regimes: a very dry, a dry, a wet and a very wet climate. The corresponding model parameters are indicated in Figure 2 with dashed vertical lines. For each of these precipitation regimes, 100 synthetic daily time-series were generated. To test the effect of sample-size, different sizes of time-windows were used: (a) 10'000 days, (b) 1000 days, (c) 100 days and (d) 30 days. The latter corresponds to the same sample-size as used to simulate monthly precipitation occurrence over the Thur catchment. For each of the generated time-series the WG parameters were re-estimated and the 95% interquantile range was computed across the set of 100 realizations (Figure 2).

Three main results can be inferred: (a) our precipitation generator is able to correctly reproduce the key WG parameters implying that the chances for substantial coding errors are small; (b) as expected the estimate of the input parameters becomes more uncertain for smaller sample sizes; in fact, the uncertainty range increases by a factor of 18.3 when the sample size is reduced from 10000 to 30. At a sample size of 1000 the uncertainty range stays at around ± 0.03, that only marginally lowers when going to a sample of 10000. (c) the different pre-defined climate regimes affect the uncertainty, particularly in the estimated transition probabilities. In a very dry or wet climate, the wet-wet or dry-wet transition
probability, respectively, exhibits large uncertainties in the estimate. This again is mainly related to a sample size problem due to very few wet-wet or dry-wet pairs. Thus, we expect that the weather generator does not work optimally in extremely wet or arid climates.

4 Results

An in-depth evaluation of the generated time-series with our calibrated multi-site WG is now undertaken with real observations. First, the reproduction of the daily and longer-term precipitation statistics at individual sites is analysed (Sect. 4.1). In a second step, the performance of the multi-site model is investigated regarding spatially aggregated precipitation indices in comparison to WGs without incorporating spatial dependencies (Sect. 4.2).

4.1 Validation of the precipitation generator at individual sites

Based on our ensemble of synthetic time-series, each containing 51 years, we analyse the reproduction of key precipitation characteristics. This validation goes beyond the reproduction of pure model parameters used to calibrate the WG (Sect. 3.4.2) as it includes precipitation statistics that are not directly used in the specification and calibration of the model. Note, that we present this analysis for the same time-period as used for calibrating our WG. This is justified for the study here, as long as we treat and use our WG to simulate long-term monthly precipitation statistics. In such a setup the stationarity of the model is given by definition. However, in a climate prediction or projection context, this stationarity assumption would have to be tested and hence separate calibration and validation periods are needed.

4.1.1 Long-term mean and inter-annual variance of monthly precipitation sums

In a first step of validating our WG, we focus on the reproduction of the long-term mean in monthly precipitation sums. Figure 3 shows both the modelled (blue) and observed (black) long-term monthly precipitation sum for each of the eight investigated stations. In general, the annual cycle of precipitation sums is well reproduced. Consistently, this is also true for the long-term seasonal as well as for the annual precipitation sums (not shown). But the WG tends to slightly underestimate precipitation sums in June and August, and overestimate them in October. In addition, the two stations Bischofszell (BIZ) and Herisau (HES) show rather large positive deviations from the observed record during the winter.
months. In order to explain part of these deviations, we decomposed the long-term mean of
monthly (T=30 days) precipitation sums (E[S(T)]) into the product of the mean monthly wet
day frequency (wdf) and intensity (wdi) (Figure 4):

\[ E[S(T)] = T \cdot wdf \cdot wdi \]  
(5)

Since these two climatological quantities are indirectly forced (Sect. 3.3.2), we expect from
the results in Figure 2 a good match on average. As shown in Figure 3, this is true for
the wet day frequency, where the deviations between generated (red) and observed (black)
values are relatively small. The differences, however, are more pronounced in case of mean
wet day intensities. In fact, it is the wet day intensities that explain the mismatches in
precipitation sums. In case of the winter performance over Bischofszell and Herisau the
deviations can be attributed to the failure of converging in case of fitting the non-zero
precipitation amount. For those instances, the fallback solution had to be used (see 3.3.1).

Next we focus on the inter-annual variability of monthly precipitation sums, which is often
more difficult to realistically model than the long-term mean (Wilks and Wilby, 1999). The
shaded areas in Figure 4 represent the inter-quartile range of the observed (grey) and
modelled (blue) monthly precipitation sums. From Figure 4 it is obvious that the
variability of the WG is smaller than in observations for all of the analysed stations. This
implies that the stochastic model only explains part of the observed total variability. This
reduced variability is expected, as observations are subject to additional sources of variability,
which our comparable simple WG is not trained for. The WG is forced with mean observed
values, varying between months but not between different years. The annual cycle is assumed
to be stationary, and hence interannual variability, e.g. related to the North Atlantic
Oscillation (Hurrell et al., 2003) is missing. Consequently, the ratio of simulated over
observed variance accounts for approximately 33% on average. The magnitude of this result
is consistent with other studies (e.g. Gregory et al. 1993). Further insights can be gained from
a decomposition of the variance of monthly (T=30 days) precipitation sums (Var[S(T)]) into
the variance of non-zero amount (Var[X≥1 mm day⁻¹]) and the variance of the number of wet
days (Var[N(T)]) as proposed by Wilks and Wilby (Wilks and Wilby, 1999):

\[
\text{Var}[S(T)] = T \cdot wdf \cdot \text{Var}[X \geq 1 \text{ mm day}^{-1}] + \text{Var}[N(T)] \cdot wdi^2
\]

\[
\text{Var}[S(T)] = T \cdot wdf \cdot \text{Var}[X \geq 1 \text{ mm day}^{-1}] + \text{Var}[N(T)] \cdot wdi^2
\]  
(6)
Since the mean wet day frequency (wdf) and intensity (wdi) are reasonably reproduced, we expect that the reduced variability of monthly precipitation sums originate from deficiencies in correctly reproducing the inter-annual variability of the number of wet days and/or of the non-zero amount. One likely reason is the neglect of low-frequency variability in the WG parameters. It has been shown that physically based models that include large-scale circulation as a predictor could alleviate this problem (Chandler and Wheater, 2002; Furrer and Katz, 2007; Wheater et al., 2005; Yang et al., 2005).

4.1.2 Reproduction of PDF of daily non-zero amount

The adequate reproduction of the mean wet day intensity and frequency is a necessary but not sufficient precondition of a WG to be used for subsequent (impact) studies. Due to a large variability of precipitation amounts, it strongly matters how its frequency distribution is reproduced. For this, we compared simulated and observed quantiles of the daily non-zero precipitation distribution at each station (Supplementary Fig. 3). Generally, the mixture model of two exponential distributions captures the frequencies of the intensities reasonably well, even at the high-Alpine station Saentis (SAE). This is at least the case up to the 80th percentile, above which intensities are systematically underestimated at all stations. This issue could be overcome by more sophisticated amount models combining e.g. a Gamma with a Generalized Pareto distribution (Vrac and Naveau, 2007). However, this comes at the expense of fitting many parameters with a limited sample size.

4.1.3 Reproduction of multi-day statistics

While the frequencies of precipitation amounts and the frequencies of wet and dry days are realistically simulated, it remains unclear how the WG performs for multi-day spells. For many application studies, this is an essential information that requires a specific analysis. Figure 6 displays observed and modelled cumulative frequencies of dry and wet spells lengths at the example of two months and two stations. The two stations Saentis and Andelfingen are selected for display since they represent the stations with the highest and lowest elevation in the catchment. For both stations a clear seasonal difference in the probability of dry spells toward more short and less long dry spells during summer compared to winter is found. A plausible explanation are the more intermittent (convective) precipitation systems during summer. In contrast to dry spells, no seasonal differences in wet
spell length probabilities can be inferred. This is likely related to the fact that the dry-dry
transition probability $p_{00}$ exhibits a more distinct annual cycle than the wet-wet transition
probability $p_{11}$. Figure 6 also shows that the frequency at shorter spell lengths (up to 3
days) is more realistically reproduced by the model than the frequency at longer spell lengths.
Generally, a better reproduction of wet spell probabilities is seen compared to the dry spell
counterpart. Long dry spell lengths are more frequently underestimated by the model than
longer wet spell lengths. The underestimation of long wet and dry spells is a common
shortcoming of the Richardson-type weather generator and has been reported by many studies
before (e.g. Racsko et al. 1991). This deficiency mainly arises due to the
fast exponential decay of the autocorrelation function with larger lags (see Eq. (63)). Similar
to the underestimation of variability in precipitation sums, higher-order Markov chains
(Wilks, 1999b) or GLMs with additional predictors might improve this aspect, which is out of
scope in this study here.

Given that the frequency of wet spell lengths is realistically simulated, the question arises
whether this also holds for multi-day precipitation sums. Multi-day periods of rain is a
common phenomenon over Switzerland, especially during prevailing weather situations that
favour orographic uplift. We compared observed and simulated cumulative distribution
functions (CDFs) of precipitation sums over multiple consecutive wet days (Figure 7). Overall, we found that the differences between generated and observed time-series are
largest for the higher quantiles and for long lasting wet spells (5-day wet spells) where the
WG tends to underestimate large multi-day sums. This reduced skill in simulating longer wet
spell sums can be explained by the fact that our WG is only prescribed with the temporal
structure of precipitation occurrence but not in amount. In other words, the WG has memory
to realistically reproduce multi-day wet spell lengths (Figure 6), while the combined
analysis of multi-day occurrence and accumulated amount loses somewhat this memory
again. Two further noticeable features in Figure 7 are that intense one-day
precipitation sums are often overestimated by the model compared to the observations, while
a relatively good match is obtained for three-day sums. Although the deficiency in correctly
simulating multi-day sums of consecutive wet days is to be expected by construction of the
WG, it could be improved by more sophisticated precipitation models, such as multi-state
Markov-chains with different probability density distributions at each state (Buishand, 1978;
Katz, 1977). This, however, comes at the expense of fitting many additional parameters with a
limited sample size-state Markov-chains with different probability density distributions.
conditioned on pre-defined states as for instance ‘dry’, ‘wet’, ‘very-wet’ (Boughton, 1999; Gregory et al., 1993).

4.2 Performance of spatial precipitation indices

Up to this point we evaluated the generator at individual sites only. One of the key issue of this study though is the potential added value of incorporating inter-station dependencies. Similarly as in the previous section, we analyse the performance first in terms of occurrence-related statistics and second in terms of the combined occurrence and amount statistics.

4.2.1 Dry and wet spell statistics for the whole catchment

Based on the eight stations in our catchment with each being either in a wet or dry state at a given day, theoretically $2^8 = 256$ different dry-wet patterns in space are possible. In observations, though, it turns out that 70% of the investigated days over 1961-2011 are in fact either completely dry (45%) or completely wet (25%) and the remaining 254 dry-wet-patterns are subject to far smaller frequencies (around $10^{-5}$ - $10^{-3}$%). The pre-dominance of a dry or a wet catchment makes sense given that the catchment is relatively small and given that precipitation is to a large degree circulation-triggered. Analysing the synthetic time-series from our multi-site WG reveals an almost perfect match with observations (Table 1), a consequence of prescribing the spatial dependency structure in the occurrence process. Indeed, when re-doing the same experiments with multiple single-site WGs without inter-site dependencies, only about 2% of all days are completely dry in the catchment and none of the days are simulated as completely wet (Table 1). In a single-site WG setup, the chances for all stations being dry or wet ultimately depend on the calibrated wet day frequencies at the eight stations that remain below 0.5 in almost all months (see Figure 5, Figure 4). This implies that the likelihood for dry conditions over the catchment is higher than for wet conditions. Those days with complete dry or wet catchment conditions were further investigated in terms of the temporal structure. Table 1 presents observed and multi-site simulated spell length statistics for the catchment. In general, remarkably good agreement between observations and the multi-site model is found. This is also true for longer spell lengths, where the spatio-temporal correlation structure is only indirectly given as input to the WG. All of these results imply that the calibrated multi-site WG not only captures the frequencies of spatially
aggregated binary series very well, it also does a surprisingly good job in reproducing multi-
day dry/wet spells of the Thur catchment.

4.2.2 Daily non-zero precipitation sums over the catchment

The above findings on the spatio-temporal correlation structure in the occurrence process also
give confidence that daily precipitation sums aggregated over the catchment are reasonably
simulated. To answer this user-relevant question, we first analyse seasonal distributions of
single-day precipitation area sums over the time-period 1961-2011 (Figure 8, Figure 7). Area
sums are defined as the precipitation sum over the eight stations. Note, that days with an area
sum of zero were excluded from this analysis and are not shown. The observations (grey
boxplots) show in the median only a weak inter-seasonal variability with somewhat higher
sums during summer. The spread in daily precipitation is smallest for winter and spring and
largest for summer owing to the higher extreme precipitation values observed. Common to all
seasons is a distribution that is heavily right-skewed ranging from nearly dry conditions up to
about 220 mm day⁻¹. Note, that the spread shown here includes variability from year-to-year
but also within the season of the same year.

Compared to observations, the multi-site generator reproduces well the median of the
observed daily areal sums. The relative deviations remain rather small, ranging from -8.5% in
summer to +1.6% in autumn. Moreover, the multi-site model is able to capture about 95% of
the observed variability in the daily sums, while the single-site WG only explains about 13%.
Even for extreme areal precipitation, the deficiencies are rather small. Contrary to a multi-site
model, the areal sum derived from several single-site WGs over the catchment (red)
systematically underestimates median, variability and consequently the magnitude of extreme
precipitation amounts (Figure 8, Figure 7). The relative deviations from observations in the
median range from -28% in autumn to -18% in spring. The underestimation may be explained
by the fact that the single-site model rarely simulates days where all stations are wet (Sect.
4.2.1). Also, the spatial structure of the precipitation amount is not accounted for.

4.2.3 Annual maximum precipitation sums of consecutive days over the catchment

The previous analysis has revealed a pronounced added value when incorporating spatial
dependencies in the stochastic simulation of daily areal precipitation sums over the Thur.
Similarly to Sect. 4.2.1, we want to go a step beyond and additionally include the temporal structure. Note that by investigating spatial precipitation sums over multi-days, we explore the limits of our WG. We analyse in Figure 8 annual maxima of observed (grey), and modelled (blue and red for multi-site and single-site, respectively) precipitation sums over several consecutive days (2, 5, and 10 days). This means that out of the aggregated catchment-time-series we compute temporal sums over consecutive days and take the maximum in each year.

Regarding the performance of the calibrated WG in multi-site and single-site mode, Figure 9 shows that both are clearly underestimating the observed sums. Yet, the multi-site model exhibits much smaller deviations from the observed distribution than the single-site model, and hence the added value of the multi-site WG is clearly evident. In fact, the sums simulated with the multi-site WG are larger by a factor of around 1.8 than those generated with the single-site WG. Overall, deviations from observations are reduced from about -53% (single-site WG) to about -17% (multi-site WG). The added value of the multi-site model is not constant for different consecutive sums. Differences are larger at shorter multi-day sums and decrease toward longer time-windows. This is related to the fact that the spatio-temporal correlation structure at longer lags is not prescribed in the model as already seen in Sect. 4.2.1 and Table 1. The benefit of a multi-site WG in terms of maximum daily areal precipitation sums is therefore restricted to consecutive sums over a few days only. And as a consequence for time-windows of 30 days (or monthly sums), a single-site WG performs equally good as a multi-site WG (not shown), as both models are calibrated for monthly sums at the eight stations and consequently at the catchment.

5 Discussion

The incorporation of inter-station dependencies in the stochastic model brings substantial added value over multiple single-site models regarding daily and multi-day areal precipitation sums over the Thur catchment. Similar benefits from the multi-site WG would be expected for other Alpine catchments and regions with complex topography, where correlations between sites are significant but well below unity. For very homogeneous regimes (inter-station correlation near unity) one single-site WG would be sufficient for the catchment-area, whereas for low spatial correlations several independent single-site WGs can be used.
A stochastic simulation with multi-site correlation structure comes with additional uncertainty from parameter estimations, additional implementation complexity and additional computational costs. The decision for incorporating spatial dependencies must therefore be balanced with the benefit. A careful inspection of the observed precipitation regime and its spatial structure over the catchment prior to the simulation is necessary to decide in favour or against multi-site simulation. This is also important in terms of validation: for a large catchment area that is frequently affected by frontal passages, the validation of the precipitation generator should include more complex space-time dependency analyses. An example is the probability of a certain precipitation amount at a particular station given precipitation at a neighbouring station some days earlier.

In the following, we want to elaborate more on the question, why we have implemented the rather simple multi-site precipitation model of Wilks (1998) and not a more sophisticated one. As already mentioned in the introduction, one premise of our work was to implement a stochastic tool that can be subsequently applied in a climate change context. This means that the number of model parameters needs to be kept limited for practical purposes such as calibration handling and evaluation of parameter changes from multi-models. An approach, such as NHMM is conditioned on atmospheric circulation. Changes of which would need to be constrained when used as a downscaling technique. However, from model evaluation studies it is well-known that climate models are prone to substantial circulation errors (e.g. van Haren et al., 2012; van Ulden and van Oldenborgh, 2006) with effects on the local precipitation. Furthermore, the overall performance of a NHMM is highly dependent on the predictive power of atmospheric circulation patterns and the number of synoptic weather states, respectively (Schiemann and Frei, 2010). In winter, we would expect a NHMM to perform better than in summer, when precipitation process is mainly dominated by local-scale convective processes triggered by orography. However, we need a downscaling technique that equally applies to all seasons. Also, for a small catchment scale such as the Thur here, the variability of the local precipitation pattern is pre-dominantly caused by physiographic factors, such as height differences, or shielding effects, rather than by large-scale atmospheric patterns. As was shown in Table 1, at around 70% of all days over 1961-2011 all stations in the catchment are simultaneously dry or wet. For all these reasons, the precipitation generator by Wilks (1998) is in our view the more direct approach to guarantee the spatial consistency for the stations in our catchment.
For many impact applications gridded precipitation data instead of multiple scattered stations would be beneficial. This demand could be achieved by interpolating the spatially consistent synthetic station data over the area of interest. A more sophisticated and elegant method, however, is to build a field generator, for instance by high-dimensional random Gaussian fields (e.g. Pegram and Clothier, 2001), random cascade models (e.g. Over and Gupta, 1996) or Poisson cluster models (e.g. Burton et al., 2008). An alternative would be to rely on geostatistical methods, for instance by prescribing a spatial correlation function at gauged and ungauged locations, that additionally requires specifying also parameters of the WG between the sites (e.g. Wilks, 2009). In regions with complex topography this additional interpolation is not straightforward. It could be alleviated by explicitly including information of topographic aspects (e.g. altitude, aspect and slope) in a GLM- (McCullagh and Nelder, 1989) or Bayesian Hierarchical modelling-approach (Gelman and Hill, 2006). These are appealing frameworks that allow the modelling of physiographic dependencies in the precipitation amount and occurrence model. However, this alone is not sufficient for a space-time weather generator as the spatial dependence of daily precipitation is also determined by spatial autocorrelation and not just the physiographic conditioning of parameters. Clearly, the development of a gridded space-time weather generator dealing with spatial autocorrelation, physiographic conditioning, intermittence and temporal autocorrelation is highly challenging and needs fundamental methodological development. This is beyond the scope in the present study, where our main focus was to develop an easy-to-use statistical downscaling tool for current and future climate.

### 6 Summary and Outlook

The multi-site precipitation generator of Wilks (1998) has been successfully developed, implemented and tested over the Swiss alpine river catchment Thur. The precipitation generator treats precipitation occurrence as a Markov chain and simulates non-zero daily precipitation amounts from a mixture model of two exponential distributions. The spatial dependency is ensured by running the WG with spatially correlated random numbers. The model was calibrated on a monthly basis by using daily station data over a 51-year long time-period from 1961-2011, and extensively compared to the observed record and to simulations based on multiple independent single-site WGs.

Our main findings of this study are:
• The multi-site precipitation generator realistically reproduces key precipitation statistics at single stations, including the annual cycle, quantiles of non-zero precipitation amounts, multi-day spells and multi-day amount statistics.

• The precipitation generator is able to generate relatively large stochastic variability. Nevertheless, it is rather low compared to observed inter-annual variability where it underestimates inter-annual variability by a factor of 3.

• The incorporation of inter-station dependencies in the stochastic process brings substantial added value over multiple single-site WGs. The median of daily area sums are higher by about a factor of 1.3 than those from independent single-site models. In addition, the multi-site WG is able to capture about 95% of the observed variability, while the single-site WG only explains about 13%. Annual maxima of multi-day sums over the catchment increase by about a factor of 1.8 by incorporating the inter-site dependence in the stochastic simulations.

• The added value is largest when the precipitation regime is subject to a large spatial and temporal heterogeneity as it is the case over the Thur catchment.

These results provide confidence that the developed precipitation generator is a helpful tool to realistically simulate mean aspects of the current climate. We therefore conclude that this generator can subsequently be used as a statistical downscaling tool to generate synthetic time-series consistent with mean aspects of the future climate. Although there is substantial improvement compared to a simple delta-change approach, from an end-user perspective some relevant limitations need to be kept in mind: The synthetically generated time-series (for current or future climate) do not fully capture the day-to-day and multi-day variability of precipitation. Extreme values and longer spell lengths are hence underestimated. The generator further underestimates the year-to-year variability in monthly precipitation sums. Therefore, care should be taken when using the precipitation generator as a tool for a broad risk assessment, in particular with respect to extreme events.

These inherent limitations point to potential future refinements of the presented model: (a) To better reproduce extreme precipitation, we intend to implement a three-state Markov chain model with the states dry, wet, and very wet and with state-dependent PDFs. From this, we expect a substantial improvement of one-day and multi-day extremes as well as a better reproduction of multi-day precipitation sums. (b) To alleviate the underestimation of inter-
annual variability, we will introduce a non-stationary model. This could be accomplished by
sampling from a distribution of observed WG parameters (instead of taking the mean) or by
formulating a regression model using large-scale atmospheric variables as predictors (see e.g.,
Furrer and Katz, 2007). Beside these methodological improvements the precipitation generator will be subject to two
extensions: (a) the coupling of daily minimum and maximum temperature as additional
atmospheric variables and (b) the adjustment of the WG parameters to represent a future mean
climate. Finally, the time-series over the Thur catchment will serve as input for a hydrological
model to assess the added value of multi- versus single-site WGs in terms of runoff and to
assess the implications of the systematic biases of the WG for hydrological quantities.
Acknowledgements

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References


Table 1. Frequencies (given in percent) of a completely wet or dry catchment together with the frequencies of its spell lengths. The observed (OBS) frequencies are calculated over 1961-2011. The multi-site simulated frequencies are given by the mean of 100 runs over 51 years (1961-2011).

<table>
<thead>
<tr>
<th>Overall frequency</th>
<th>Wet catchment</th>
<th>Dry catchment</th>
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<tbody>
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<td></td>
<td>OBS</td>
<td>multi-site</td>
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<tr>
<td>Overall frequency</td>
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<td>25</td>
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</table>

<table>
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<th>Frequencies of spell lengths</th>
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<th>Dry catchment</th>
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</thead>
<tbody>
<tr>
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<td>multi-site</td>
</tr>
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</tr>
<tr>
<td>2</td>
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<td>29.4</td>
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<tr>
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Figure 1. a) The catchment of the river Thur, located in north-eastern Switzerland, together with the underlying topography (in m.a.s.l.). The dots indicate the locations of the
investigated stations. 1: Andelfingen (AFI, 47.60°N / 8.69°E), 2: Frauenfeld (FRF, 47.57°N / 8.89°E), 3: Bischofszell (BIZ, 47.50°N / 9.23°E), 4: Eschikon (EKO, 47.45°N / 8.97°E), 5: Ebnat-Kappel (EBK, 47.27°N / 9.11°E), 6: Herisau (HES, 47.39°N / 9.26°E), 7: Appenzell (APP, 47.34°N / 9.40°E), 8: Saentis (SAE, 47.25°N / 9.34°E).

b) Observed precipitation climatology of the wet day frequency (1961-2011) derived from a 2.2km x 2.2km gridded daily precipitation dataset (Frei and Schär, 1998) for December and June. c) The same as in b), but for wet day intensity (in mm day⁻¹). A wet day is defined as a day with precipitation amount equal or higher than 1 mm day⁻¹. The filled circle symbols point to the station locations (as in a) together with the observed station measurements.
Figure 2—Technical workflow of a multi-site precipitation generator after Wilks (1998) at the example of two fictitious sites A and B. In general, it is a combination of multiple single-site precipitation generators that are calibrated at each site individually (see input parameters) and run simultaneously with spatially correlated random number streams (dashed boxes). The correlated random number streams (of similar length as the simulation period) are determined beforehand (see Section 3.3.2). The orange labelled numbers in indicate the steps for single-site precipitation simulation (see Section 3.3.1).
Figure 3. Reproduction of average wet day frequency (wdf), mean wet day intensity (wdi), wet-wet transition probability ($p_{11}$) and dry-wet transition probability ($p_{01}$) for the four idealized climate regime ranging from very dry (left) to very wet (right) as indicated by dashed lines. The shaded areas correspond to the range between the 2.5% and the 97.5% empirical quantiles of 100 realizations. Results are shown for sample sizes of 10000, 1000, 100 and 30 (grey shading).
Figure 3. Long-term mean and variability of monthly precipitation sums during the period 1961-2011 for eight stations in the Thur catchment. The black (blue) lines refer to the mean annual cycle of observed (modelled) precipitation sums. The grey (blue) shaded areas represent the inter-quartile ranges of observed (simulated) monthly precipitation sums. The simulation comprises 100 realizations covering each 51 years. The numbers at the bottom indicate for each month the percentage of variance explained by the precipitation generator. Note that the scale of the y-axis differ between different stations.
Figure 4. Observed and modelled monthly mean wet day intensity (blue) and frequency (red) at eight stations during 1961-2011. The black (coloured) lines indicate the observed (modelled) values. The blue (red) shaded areas correspond to the inter-quartile range across the set of synthetic daily time-series. They comprise 100 runs covering each 51 years.
Figure 5. Cumulative distribution of the observed and simulated dry (left) and wet (right) spell length frequencies for the lowland station Andelfingen (top) and the mountain station Saentis (bottom). Results are for January and June during the time period of 1961-2011. The coloured area (line) represents the inter-quartile range (median) of the 100 realizations covering each 51 year-long daily time-series.
Figure 6. Cumulative distribution functions (CDFs) of multi-day precipitation sums for the three stations Andelfingen (AFI), Appenzell (APP) and Saentis (SAE). The lines represent the CDFs of non-zero precipitation amounts over one day (red), over three consecutive wet days (green) and over five consecutive wet days (blue). Darker and lighter colours refer to observations and simulations, respectively. The observed CDFs have been derived from a 51-year long daily time-series between 1961 and 2011, those of the weather generator from 100 realizations of 51-year long daily simulations. Note that the scaling of the horizontal axis differs between different stations.
Figure 7. Daily non-zero precipitation sums over the catchment for the four seasons during 1961-2011. Daily Precipitation intensity of the eight stations are summed and days with an area sum of zero are excluded. Boxplots of observed daily sums (grey), of multi-site simulated time-series (blue) and of single-site simulated time-series (red) are shown. The WG models were run 100 times over a 51 year time-period. The numbers (in percentage) indicated above the corresponding model represent the relative deviation of the simulated median from the observed.
Figure 8. Annual maximum precipitation summed over all eight stations and over consecutive days. The analysis is done for all days of year. The bars (horizontal line) indicate the range between the 2.5% and the 97.5% empirical quantiles of the yearly maximum area sums during 1961-2011. The observations are plotted in grey, the multi-site simulations in blue and the single-site simulations in red. The observations comprise 51 years, the models were run 100 times over a 51 year time-period.