Response to editor comments

Editor comments are in italics, followed by author response/ manuscript modifications. Line/ page numbers refer to the markup version of the manuscript (below).

Dear Authors,

The manuscript addresses an important new method of measurements of water surface elevations using SWOT, with a virtual-reality numerical experiment on two reaches of the Amazon River. However, the main issues raised by both referees is that the authors should make a further effort to highlight the originality of the work and discuss the cases where the method could be applied:

i) discuss the domains and limit of application of the new method in comparison to classical ones;

In order to include the use and limitations of classical methods of discharge estimation (i.e. via rating curves at gauging stations), a new paragraph has been included in the introduction (lines 71-97):

In order to obtain estimates of discharge from gauge measurements of river stage, a rating curve is usually constructed for each station. This relates observed water level to discharge estimated from flow measurements and river cross-sectional area collected previously across the channel for a range of different stages. Rating curves are widely acknowledged to be a limited method to estimate discharge (e.g. Clarke et al., 2000; Domeneghetti et al., 2012) since they include errors in measurements of river flow and stage used in rating curve construction, errors resulting from the necessary interpolation or extrapolation of the rating curve to the measured stage, and errors from any unsteady flow conditions or seasonal variations in roughness through changes to vegetation or other conditions (Di Baldassarre and Montanari, 2009). Improvements to discharge measurements in rating curve construction are now possible using Acoustic Doppler Current Profiler (ADCP) (for example, see Oberg and Mueller, 2007), however, the primary challenge remains: multiple measurements are required throughout the hydrograph in order to obtain accurate estimates, which may be expensive, time-consuming or impractical, particularly for remote sites. High discharge during flood events, in particular, may be poorly estimated, due to errors in extrapolation (Di Baldassarre and Claps, 2010) resulting from limited opportunities or the increased difficulty and hazard of obtaining measurements during high flows.

This then leads into the discussion of the use and development of remote sensing methods.

ii) clarify all the steps involved in matching the hydraulic model and the remotely sensed data;

A flow diagram of the processing steps performed has been produced – Figure 2. This was suggested by review 1 and helps to clarify all steps. Where specific clarifications were requested by reviewers, these have been incorporated in the text (e.g. regarding the cross-sections for the model)

iii) justify and discuss the strong assumptions of additivity of a noise related to SWOT measurements and a perfect knowledge of channel friction and bed erosion;

Included in the response to reviewer 1: the assumption of perfect knowledge of channel friction and bed elevation is a necessary element to allow the quantification of error which is contributed by SWOT observations of the water surface – which is the main focus of the paper. In reality, we acknowledge that this perfect knowledge would not be the case, and state in the conclusions (final sentences) that
further work is required to assess the relative importance of each of these. We feel that it is beyond the scope of the current work to test these factors as that would take the focus away from the direct assessment of potential SWOT observations and their contribution to error (rather than other potential unknowns). However, we now include additional discussion (lines 714-746) to make clear that these are other important factors such as channel bathymetry or depth required for the estimation of discharge, and point to other work which has focused on their assessment.

iv) how to deal with the complexity of the flood plain, eventual changes or uncertainties of topography, influence of the vegetation, etc.?

Unfortunately this is beyond the scope of this paper – but we have already stated that this research is limited to the main channel, not the floodplain where vegetation may be important. On Lines 270-274 we state: “Note that, presently, the performance of the SWOT instrument in the case of flooded vegetation is unknown, thus throughout this paper the words “floodplain” and “wetland” reference those conditions of a clear view of the sky without any flooded vegetation.”

In addition, we include the following statement in the discussion (lines 747-751):

As with other studies, the error analysis presented here excluded layover and vegetation effects, as may be found in wetlands and floodplains, or along the edges of rivers. These effects are likely to be greatest for narrower rivers with bank vegetation.

Currently we don’t have a good error characterization for flooded vegetation as may be observed by SWOT – this is an area for further research.

v) discuss the representativeness of the two studied reaches on the Amazon River;

The results are most applicable to large, lowland rivers with low temporal variability. As stated in the paper, however, further research is required to assess smaller rivers. The comparison between the Solimões and Purus Rivers is useful in this context – the Purus had markedly lower accuracy due in part to the narrower width of the river. It is likely that other, narrower, rivers would have further reductions in accuracy, but this would need to be assessed. However, we have tried to illustrate the transferability to other rivers - please see next point.

vi) give a list of rivers where SWOT could be successfully applied, and what wide the possible audience of this paper might be?

A new figure has been included to illustrate this, rather than a list, which may be quite limited – Figure 15 (page 14). This presents as contours the percentage error in discharge which may be expected using SWOT estimates of surface water slope (excluding other errors) for a range of river widths/ surface water slopes. Superimposed on this is a variety of international rivers for which published width and surface water slope were available. It can be seen that, due to very low water surface slopes, error in surface water slope estimation has a proportionately greater impact on the calculation of discharge. A discussion of Figure 15 is provided from lines 689 to 704.
vii) discuss how conclusions from the two study cases can be extended to other cases.

This has been incorporated using Figure 15 and the associated discussion of it. The transferability of the conclusions should now be clear, at least for large low land rivers without additional complications such as high spatial or temporal variability – again, this is included in the conclusions. For a particular river of interest, readers will be able to identify expected error in discharge estimates using width and water surface slope (with provisos included in the figure caption, such as exclusion of other sources of error, use of 10 km reach lengths).

The paper will be reconsidered after revisions of the points raised by both referees are being taken and included into the manuscript. Please highlight clearly what you changed in the revised manuscript, so the reviewers are able to assess your changes.

Kind regards,

Roger Moussa

The authors thank you for your consideration.

**Response to reviewer comments**

The authors would like to thank both reviewers for their constructive comments and feedback, which have allowed the manuscript to be substantially improved.

Referee comments are in italics, followed by author response/ manuscript modifications. Line/ page numbers refer to the markup version of the manuscript (below).

**Referee 1: R. Romanowicz**

_The manuscript addresses an important new method of measurements that has the potential to improve step-wise the information content of hydrological observations. The Surface Water and Ocean Topography (SWOT) mission is planned to be launched in 2020. It will provide the first routine measurements of water surface elevations in two-dimensional space. It is still in the future, but there is no doubt that it will come. Therefore it is important to develop a background of new methods, and to understand their possibilities and limitations. The authors present a virtual-reality experiment simulating the spatio-temporal sampling scheme of SWOT for the River Amazon. The experiment employs the numerical flow routing model LISFLOOD-FP. The water surface elevation errors derived from the SWOT characteristics were added to water levels simulated by the hydraulic model. The scientific approach and applied methods are suitable for a problem. The authors refer to the latest work related to the subject of_
their research. I have some concerns regarding the presentation of the research. Sentences are too long and the description is not always clear (see the specific comments). Otherwise, the structure of the paper is good.

- Thank you for your comments – SWOT indeed has the potential to provide improved information for hydrology, and it is important that algorithms are developed and tested ready for use with data when acquired (the main rational for the research presented in this paper). Where appropriate, we have restructured some of the longer sentences, used tables for values to increase readability and improved descriptions as suggested.

There are some aspects of the paper that need improvement. The question arises if there were any real observational data used in the experiment, or was it only a model to-model comparison? Another question is on the applicability of the approach. The authors test it on the River Amazon. It would be useful for the reader if a list of rivers where SWOT could be successfully applied were to be given. The answer to this question would also specify how wide the possible audience of this paper might be. The other points concern the assumptions of additivity of a noise related to SWOT measurements and a perfect knowledge of channel friction and bed elevation. These are very strong assumptions. The authors are asked to expand on those issues and provide some estimates of outcomes resulting from a situation where those assumptions are not met.

- Observational data were used in the sense that the LISFLOOD-FP model was developed and calibrated for a past flood event (1995-1997) by Trigg et al. 2009 (RMSE now stated in section 4.1 – line 478). However, these observational data are very limited with respect to surface water elevation and lack both spatial and temporal details – that is, of course, the reason for the development of SWOT in the first place. The model allows a highly detailed representation of the water surface and so is good as starting point for the assessment of details which may be resolvable by SWOT.

- Regarding the applicability to other rivers – we have tried to incorporate this issue in the discussion, and have produced a new figure (15, page 14 and associated discussion on lines 689 - 704) to illustrate a variety of global rivers and the reach-averaged discharge error which may be expected to be contributed by SWOT observation error (we make clear that other errors which may be significant that are not accounted for here). The results are most applicable to large, lowland rivers with low temporal variability. As stated in the paper, however, further research is required to assess smaller rivers. The comparison between the Solimões and Purus Rivers is useful in this context – the Purus had markedly lower accuracy due in part to the narrower width of the river. It is likely that other, narrower, rivers would have further reductions in accuracy, but this would need to be assessed.

- The assumption of perfect knowledge of channel friction and bed elevation is a necessary element to allow the quantification of error which is contributed by SWOT observations of the water surface – which is the main focus of the paper (this is now clarified in the objectives as stated lines 188-194). In reality, we acknowledge that this perfect knowledge would not be the
case, and state in the conclusions (final sentences) that further work is required to assess the relative importance of each of these. We feel that it is beyond the scope of the current work to test these factors as that would take the focus away from the assessment of potential SWOT observations (rather than other potential unknowns). However, we include additional discussion (lines 714-746) to make clear that these are other important factors such as channel bathymetry or depth required for the estimation of discharge, and point to other work which has focused on their assessment.

I found the description of SWOT observations very difficult to understand. The sentence (page 9408, lines 18-19) saying: “500m SWOT errors were downscaled to 100m resolution” is an example of a lack of precision in the description. Downsampling is an operation that can produce a serious error that has not been taken into account in the further discussion. It would be useful if the authors could provide a scheme of their virtual experiment that would include all the steps involved in matching the hydraulic model to the remotely-sensed data.

- We have revisited this section and tried to improve clarity. “Resampling” more closely reflects the procedure used rather than “downscaling” and the sentence has been modified accordingly (line 382). Readers are referred to Rodríguez 2014 for details of the Fourier transform for error generation. A schematic is a good idea and has been included as a separate figure (Figure 2, page 4).

Specific comments: page 9408, lines 1-3: It is not clear how the LISFLOOD_FP was validated. Page 9412, lines 24-27: It is not clear to me how the errors are reduced by averaging. That reasoning assumes that there is no bias.

- The LISFLOOD-FP model as applied was developed by Trigg et al. 2009 and readers are referred to this paper for a full validation. The overall RMSE accuracy for model validation was 1.26 m and 1.42 m for the Solimões and Purus rivers, respectively. This is now stated in section 4.1 (line 478). It is true that errors will only reduce through averaging if there is no bias – however, this is a valid assumption in this case as we have not introduced a bias component to the error modelling, since it is not part of the design requirement for SWOT. However, for clarity, we have stated “assuming no bias” at the end of section 4.1 (line 543).

Referee 2: Anonymous

The difficulty to estimate the river discharge of Amazon River is well known and mainly due to the influence of ocean elevation and to the width of the river. This width (several kilometres) permits to use
measurements from satellites and the future arrival of SWOT mission will provide more accurate data. The method used in the paper to assess river discharge from satellites data is (classically) based on the bed topography and the water surface slope. The data of water surface elevations from SWOT mission are replaced by water elevations coming from the calculation using hydrodynamic model and distorted to have errors inside the requirements (density and accuracy of points) proposed for SWOT objectives. The input discharge is compared to the results of the method either using the results of the hydrodynamic model or the distorted data (either using one point per cross section of the river or an average of all the points of a cross section).

The comparison is limited to two reaches of Amazon River and the only aim seems to optimize the discharge estimate for these two reaches. These two reaches are not representative of a set of American or world rivers; then, only the method can be transferred to other cases; the results of the optimization in term of length of reach or accuracy of discharge (or water surface slope) cannot be transferred. Even, the conclusion of the advantage of using SWOT cannot be transferred to other rivers very different from Amazon River. For Amazon River itself, the real applicability of the method from actual SWOT mission is not so sure because of the complexity of the flood plain, the eventual changes or uncertainties of topography and the influence of the vegetation that can bias the actual measurements.

- The comparison was necessarily restricted to two connected reaches and an optimum discharge estimate using generated SWOT data was obtained – with the express aim of characterizing the contribution of SWOT data to the estimation of discharge. While the main Solimões River is, arguably, not representative of other rivers globally due to its size, the estimation of discharge here is important, nonetheless. The very low water surface slope makes the estimation of discharge for this reach from satellite data a considerable challenge and it was for this reason that the reach was selected – along with the solid baseline available from previously published work for the river in both estimation of discharge (from satellite altimetry and SRTM) and hydraulic modelling. The Purus River tributary was included to increase the transferability of the work to other rivers – at around 1 km in width it may represent a “large” world river. The water surface slope remained a challenge for this reach, but it is clear from the work presented that narrower rivers are likely to be more challenging to estimate accurately due to the limited possibility for cross-sectional averaging. The applicability of the methods to other rivers is illustrated in a new figure with a range of rivers of different widths/ slopes and the expected contribution of SWOT observation errors to discharge errors (Figure 15, page 14).

- Further work is required to assess a range of additional rivers – this was stated in the conclusions but is beyond the scope of this paper and is now discussed in more detail in section 4.4. However, the observation and accuracy assessment of water surface slope for different reach lengths may be directly transferred (albeit in the absence of cross-channel averaging for smaller rivers). The transferability of the discharge estimation is more difficult and will depend on the characteristics of the river in question, but this is acknowledged in the text in Section 4.4 – lines 704-712 (‘... we can infer that discharge estimates may be more accurate for rivers with: (i) greater channel widths which permit a greater level of cross-section averaging and the use of shorter reach lengths; and (ii) higher water surface slopes, since, from Eq. 6, the relative error in
discharge decreases as slope increases. Conversely, discharge estimation accuracy is likely to be lowest for narrow rivers with low slopes, although further research is required to quantify errors for rivers at this scale.'). We have also provided relative discharge estimate error as percentage and using the Nash-Sutcliffe efficiency coefficient to help with transferability. Figure 15 will also aid readers relate the work to other rivers – but we are clear that there will be other errors which are excluded from this assessment (lines 714-721, Figure 15 caption and the end of the conclusions – lines 808-813)

• Further work is also required to assess vegetation effects and the research was necessarily limited to the main channel – this was stated in Section 2.1 (‘Note that, presently, the performance of the SWOT instrument in the case of flooded vegetation is unknown, thus throughout this paper the words “floodplain” and “wetland” reference those conditions of a clear view of the sky without any flooded vegetation.’), in Section 4.4 (‘Note that the error analysis presented here excluded layover and vegetation effects, as may be found in wetlands and floodplains, or along the edges of rivers. These effects are likely to be greatest for narrower rivers with bank vegetation.’), and is again referred to in the conclusions.

The presentation of the results includes both a text with a lot of numerical values that the reader cannot compare easily (one or several tables would have been much better) and graphs that contain the essential information.

• The use of tables to summarize numerical values rather than placing all of them in the text is a good idea and this has been incorporated. Table 1 summarizes the modelled slope and discharge (text removed from section 4.1, lines 486-502). Table 2 summarizes errors in slope and discharge derived from SWOT observations (text in sections 4.2 and 4.3 reduced)

From the abstract but also from the whole paper, it is difficult to understand the scientific interest of this paper. A well-known method is applied to virtual data and provides some results for two reaches of Amazon River. As not all the steps (particularly the way virtual data are obtained) are detailed, the readers will not be able to use the same method for other rivers.

• The scientific interest is related to the assessment of the potential for SWOT data to assess water surface slope and river discharge for the Amazon, a river of international significance. There is also clear scientific interest since methods used (i.e. reach and cross-channel averaging and estimation of discharge) are transferrable and easily applicable elsewhere. While “virtual missions” are needed for other rivers and for floodplain areas, once SWOT is launched in 2020, the prior development and assessment of these algorithms will be valuable. Where appropriate, some additional details are provided in the text. However, space does not permit a fully detailed description of all methods (e.g. the Fourier transform), but readers are directed to reference papers where appropriate. A schematic of the methods used (suggested by Referee 1) has been included to clarify the procedures used – Figure 2, page 4).
Other remarks:

P. 9402: the values “0.31 m” and “2 mm” are put together without clear explanation of the calculation method for linking them

- This has now been clarified in the text – it is double the RMS error (the worst-case scenario from two independent tracks), divided by the track spacing (315 km) to obtain the water-surface slope error. The text now reads: “For rivers in the Amazon basin, the OSTM altimeter has been found by Seyler et al. (2013) to have a mean RMS error of ±0.31 m for rivers over 400 m wide. Using two parallel tracks to calculate water-surface slope, as is needed for the estimation of instantaneous discharge in the absence of in-situ rating curves, this RMS error would lead to a maximum water-surface slope error of around 2 mm per kilometer (calculated using 2 x 0.31 m / 315 km).” line 142

P. 9403: “32% rivers”, “1% rivers”: the authors do not explain what are the rivers concerned by these values: which length? which width? Which water depth? So the values are meaningless.

- The percentages are referring to observations of rivers within a global database, rather than particular widths/depths, based on satellite sampling schemes. The sentence has been clarified in this respect to: “Profiling altimetry was shown by Alsdorf et al. (2007b) to miss entirely 32% of rivers in a global database, compared to only 1% of rivers being missed by an imager (based on the Terra 16-day repeat cycle, 120 km swath, 98° inclination and sun-synchronous orbit).” Lines 157-163

P. 9406: I understand that the model used includes a series of cross sections and perhaps a 2D flood plain representation; no information is provided about number and spacing of cross sections and cells, topology of the system (1D network, organization of 2D cells, etc).

- This information is now provided and the model structure as implemented is clarified: “The formulation of LISFLOOD-FP used here was the one-dimensional diffusive wave formulation of Trigg et al. (2009) for channel flow (floodplain flow was excluded), allowing complex channel bathymetry and back propagation of flow. A detailed series of channel cross-sections were used (124 for the Solimões and 48 for the Purus), with an average along-channel spacing of 2.4 km and each representing the average bed-elevation for that location.” Lines 296-306

P. 9407: what is the time spacing of data for calibration (minimization of RMSE)?
• This detail is provided in Trigg et al. 2009, but has now been included here as well – “Friction parameters for the model were obtained through a calibration based on the minimization of RMS error calculated from river levels from four gauging stations internal to the model domain and model water surface elevation obtained at a temporal resolution of 12 hours” lines 347-359 (RMS error now provided in section 4.1 as part of results)

P. 9407: does the choice of 100 m influence the results?

• This resolution was selected to be representative of the approximate SWOT image resolution – this is fixed by instrument design and an assessment of the influence of variations grid size used is, therefore, beyond the scope of the paper. A clarification regarding this selection has been added: “... 1D channel water elevations were mapped onto channel cross-sections then interpolated onto a 2D regular grid at a spatial resolution of 100 m (selected to approximately match the design requirements of SWOT as specified by Rodriguez, 2014, although this resolution will vary across the swath).” Lines 323-326

P. 9408 §3.2 l.16-20: the authors do not explain why they pass by “500 m” instead of going straight to “100 m”

The explanation is already included – “resolution limited by computational power” – line 376

P. 9409: equation 4 should be explained; for instance what is the origin of the x distances?

• This is a standard 1D polynomial slope equation and is straightforward to apply, but was included here for clarity. X is the distance (chainage) along the channel of each observation – the origin of this is not significant, but in this case is the downstream section of the channel; h is the elevation of that observation.

P. 9409: equation 5 assumes rectangular cross sections. Is it the case for all the cross sections of the model? How are averaged the width and the water depth along a channel reach (or how is selected the representative cross section)?

• Yes, this is the case – and given the large width relative to depth in the channel, this is a reasonable assumption and is common practice. The average bed elevation for the cross-section was used, so that the cross-sectional area of flow was maintained. This information has been added to Section 3.1 (lines 299-303)
Mots part of the values inserted in the text could appear in tables comparing the various methods, the various parameter values and the various locations.

- Much of these values have now been moved to the two tables already described, representing the model output and the errors in estimates of slope and discharge.

“added to the according”: one word missing

- Corrected
Swath altimetry measurements of the mainstem Amazon River: measurement errors and hydraulic implications

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Abstract. The Surface Water and Ocean Topography (SWOT) mission, scheduled for launch in 2020, will provide a step-change improvement in the measurement of terrestrial surface water storage and dynamics. In particular, it will provide the first, routine two-dimensional measurements of water surface elevations. In this paper, we aimed to (i) characterize and illustrate in two-dimensions the errors which may be found in SWOT swath measurements of terrestrial surface water, (ii) simulate the spatio-temporal sampling scheme of SWOT for the Amazon, and (iii) assess the impact of each of these on estimates of water surface slope and river discharge which may be obtained from SWOT imagery. We based our analysis on a “virtual mission” for a 200–260 km reach of the central Amazon river (Solimões River), using a hydraulic model to provide water surface elevations according to SWOT spatio-temporal sampling to which errors were added based on a two-dimension height error spectrum derived from the SWOT design requirements. We thereby obtained water surface elevation measurements for the Amazon mainstem as may be observed by SWOT. Using these measurements, we derived estimates of river slope and discharge and compared them to those obtained directly from the hydraulic model. We found that cross-channel and along-reach averaging of SWOT measurements using reach lengths of greater than 4 km for the Solimões and 7.5 km for Purus reduced the effect of systematic height errors, enabling discharge to be reproduced accurately from the water height, assuming known bathymetry and friction. Using cross-section averaging and 20 km reach lengths, results show Nash-Sutcliffe model efficiency values of 0.99 for the Solimões and 0.88 for the Purus, with 2.6% and 19.1% average overall error in discharge, respectively. We extend the results to other

rivers worldwide and infer that SWOT-derived discharge estimates may be more accurate for rivers with larger channel widths (permitting a greater level of cross-section averaging and the use of shorter reach lengths) and higher water surface slopes (reducing the proportional impact of slope errors on discharge calculation).

1 Introduction

The hydrological cycle is of fundamental importance to life and society and river gauges have long formed a basis for hydrological understanding, often providing real-time measurement capabilities of river stage or discharge and information for water management and flood warning. Yet existing in-situ gauge networks are unevenly distributed globally, with a distinct lack of measurements obtained in developing countries, particularly for areas with low population (Vörös and the others, 2001; Shiklomanov et al., 2002). In addition, gauging stations are highly variable in their accuracy and are under threat. The United States has around 7,000 stream gauges (requiring an average of 30 or more years of gauging to be considered “adequate” (USGS, 1998), contributing to an insufficient knowledge of available national water resources (NESC, 2004).

Over the latter half of the 20th century, increasing numbers of gauging stations in the United States with 30 or more years of record were discontinued each year; in the mid-1990s, this represented about 4% of the long-record stations being discontinued (USGS, 1998). The situation globally is substantially worse than in the United States, with much of the globally significant discharge occurring in sparsely gauged catchments (Alsdorf et al., 2003). The gauge density in the
Amazon, expressed as number of gauges per unit discharge, is around 4 orders of magnitude less than what is typical in the eastern United States (Alsdorf et al., 2007b), worldwide, Fekete and Vörösmarty (2007) indicate that the amount of data available through the Global Runoff Data Centre (GRDC) is in sharp decline, and now stands at less than 600 discharge monitoring stations, down from a peak of around 5,000 in 1980. Remote sensing has a potentially useful role to play to fill the gaps in river gauge data.

In order to obtain estimates of discharge from gauge measurements of river stage, a rating curve is usually constructed for each station. This relates observed water level to discharge estimated from flow measurements and river cross-sectional area collected previously across the channel for a range of different stages. Rating curves are widely acknowledged to be a limited method to estimate discharge (e.g. Clarke et al., 2000; Domeneghetti et al., 2012) since they include errors in measurements of river flow and stage used in rating curve construction, errors resulting from the necessary interpolation or extrapolation of the rating curve to the measured stage, and errors from any unsteady flow conditions or seasonal variations in roughness through changes to vegetation or other conditions (Di Baldassarre and Montanari, 2009). Improvements to discharge measurements in rating curve construction are now possible using Acoustic Doppler Current Profiler (ADCP) (for example, see Oberg and Mueller, 2007). However, the primary challenge remains: multiple measurements are required throughout the hydrograph in order to obtain accurate estimates, which may be expensive, time-consuming or impractical, particularly for remote sites. High discharge during flood events, in particular, may be poorly estimated, due to errors in extrapolation (Di Baldassarre and Claps, 2010) resulting from limited opportunities or the increased difficulty and hazard of obtaining measurements during high flows.

Remote sensing has been shown to be a valuable addition to ground-based gauges, with the added benefit of being able to reduce data access issues in international river basins, which contribute to greater than 50% of global surface flows (Wolf et al., 1999) and where obtaining information about upstream flows can be politically challenging (e.g. Hossain et al., 2007).

Remote sensing has also been shown to be a valuable addition to ground-based gauges with satellite. Satellite altimetry, in particular, has been used extensively to obtain water elevations of inland river and lake systems, including data from ERS, TOPEX/POSEIDON, Envisat and Jason 1 and 2 (e.g. Berry et al., 2005; Birkett, 1998). For example, Birkett et al. (2002) used TOPEX/POSEIDON altimetry data to analyze surface water dynamics along the Amazon River and characterized the spatially and temporally variable surface-water gradient as between 1.5 cm/km downstream to 4.0 cm/km upstream. Satellite altimetry has also been used to estimate river discharge. Birkinshaw et al. (2012) estimated discharge for the Mekong and Ob Rivers using ENVISAT altimetry over 50 km river reaches, based on the Manning’s resistance formulation of Bjerklie et al. (2003), and were able to obtain Nash-Sutcliffe efficiency values of 0.86 to 0.90. Papa et al. (2012) used Jason-2 altimetry data to estimate flux from the Ganga-Brahmaputra Rivers, based on in-situ rating curves relating water-elevation to discharge, and obtained errors of 6.5% and 13% for the Brahmaputra and Ganga rivers, respectively.

A limitation of profiling satellite altimetry for the analysis of river hydrology is that the nadir viewing geometry and narrow field of view leads to an incomplete coverage and a long revisit time. Currently operational satellite altimeters include the Ocean Surface Topography Mission (OSTM) on the Jason-2 platform (Lambin et al., 2010) which, as with its predecessors Jason-1 and Topex/Poseidon, has an orbital repeat-time of around 10 days and a ground track spacing of 315 km at the equator (Seyler et al., 2013). For rivers in the Amazon basin, the OSTM altimeter has been found by Seyler et al. (2013) to have a mean RMS-Root Mean Square (RMS) error of ±0.31 m for rivers over 400 m wide. Using two parallel tracks to calculate water-surface slope, as is needed for the estimation of instantaneous discharge in the absence of in-situ rating curves, this RMS error would lead to a maximum water-surface slope error of around 2 mm per kilometer (calculated using $2 \times 0.31$ m / 315 km). However, this represents an average slope over a large river distance and does not reflect the likely spatial variability or curvature in the water-surface due to a coarse spatial resolution. Although ascending and descending tracks may be combined to represent better this variability, errors in the estimate of water-surface slope and, hence, discharge would increase. In addition, to calculate water-surface slope, temporal interpolation of data in different tracks is needed, increasing errors particularly for smaller rivers with higher temporal variability or during periods of highly variable flow, such as flood events.

These limitations mean that, for the majority of rivers, satellite altimetry does not provide sufficient detail to capture the full spatial or temporal complexity of river hydrology. In addition, profiling altimetry has been shown Profiling altimetry was shown by Alsdorf et al. (2007b) to miss entirely 32% of rivers globally if rivers in a global database, compared to only 1% of rivers being missed by an imager (Alsdorf et al., 2007b) (based on the Terra 16-day repeat cycle, 120 km swath, ~98° inclination and sun-synchronous orbit).

In common with river gauges, measurements obtained by profiling altimetry are spatially one-dimensional, meaning that no information on water surface area or two-dimensional patterns in water surface slope are provided. However, Synthetic Aperture Radar (SAR) interferometry work by Alsdorf et al. (2007a) has shown that water flow is both spatially and temporally complex, requiring two-dimensional, multi-temporal measurements to capture sufficiently. This
means that our current, operational remote sensing has a limited capability for an important component of the water surface (Alsdorf et al., 2007b). Remote sensing has been used with some success to characterize hydraulic variables including surface water area and elevation, water slope and temporal changes, but. However, none of the existing technologies are able to provide each commensurately, as needed to model accurately the water cycle (Alsdorf et al., 2007b).

The forthcoming Surface Water and Ocean Topography (SWOT) mission (Durand et al., 2010) aims to overcome existing limitations in remote sensing by using a swath-altimetry approach to measure surface water elevation in two-dimensions, providing both surface water area and elevation simultaneously. Such measurements may allow water surface slopes to be derived instantaneously and, therefore, potentially could provide estimates of river and floodplain discharge. The objective of this work is to investigate the hydraulic implications of potential measurement errors in SWOT imagery and their hydraulic implications (independently to other potential errors) for a reach of the mainstem Amazon River and one of its tributaries.

2 The Surface Water and Ocean Topography mission

Recommended for launch by the National Research Council Decadal Survey (NRC, 2007), SWOT will provide a substantial improvement in the availability of data on terrestrial surface water storage and dynamics, achieving near-global water elevation measurements in large rivers and their large floodplains. The SWOT sensor is a Ka-band radar interferometer which will allow mapping of surface water extent and elevation at a spatial resolution of around 250 m, at centimetric vertical precision when averaged over targets of interest, every 2-11 days depending on the latitude (Durand et al., 2010; Rodríguez, 2014).

Thus, SWOT will provide the first, routine two-dimensional measurements of water surface elevation, allowing the analysis of floodplain hydrodynamics and the estimation of river discharge. While SWOT will not replace a ground-based river gauge network, it will allow large ungauged rivers to be sampled and increase the level of detail and availability in river flow estimates. In addition, the two-dimensional measurements of surface water provided by SWOT will allow the detailed observation of floodplain and wetland hydrodynamics (Durand et al., 2010).

The approach used by SWOT is similar to that of LeFavour and Alsdorf (2005) and Kiel et al. (2006), who used Shuttle Radar Topography Mission (SRTM) elevation data of the water surface to obtain slopes of the Amazon and Ohio rivers and, subsequently, to estimate channel discharge. However, for the Amazon, LeFavour and Alsdorf (2005) found vertical errors 5.51 m in water surface elevations from C-band SRTM data, meaning that a long reach length of 733 km was required to reduce errors in derived water surface slopes to 1.5 cm/km for the accurate estimation of channel discharge (6.2% error at Manacapuru; 7.6% at Itapeua). For SWOT, the science requirements are for a vertical precision of 10 cm in measurements of water surface elevation and derived water surface slopes with errors of no more than 1 cm/km when averaged over a 10 km reach length (Rodríguez, 2014), substantially more accurate than measurements obtained from SRTM. For comparison, using the simple method of LeFavour and Alsdorf (2005) to determine an appropriate reach length (2σ/Smin, where σ denotes the vertical precision of the measurements and Smin denotes the minimum slope required), indicates that, using the SWOT vertical precision of 10 cm, to achieve water surface slope errors of no more than 1 cm/km, reach lengths of 20 km may be required; for 1.5 cm/km, reach lengths of 13.3 km. However, this simple method may be overly conservative and does not take into account the potential for averaging over channel cross-sections. In this paper, we explore the implications of the SWOT science-requirements on the derivation of water surface slope and subsequent estimation of channel discharge.

2.1 Virtual mission

We used a “virtual mission” study of two-dimensional observations of water surface elevation as may be obtained by SWOT, for the estimation of discharge on a ~260 km reach of the central Amazon River (Solimões) and one of its tributaries (Purus) in Brazil (Fig. 1a). The Amazon is a globally significant river, carrying around 20% of total global continental runoff (Richey et al., 1989) with a monomodal flood pulse passing annually down the river. The middle reaches of the Amazon are characterized by very low water surface slopes of between 1 and 3 cm/km and significant backwater effects (Meade et al., 1991), with peak channel flow in the study site around ~120,000 m³/s. This combination of low water surface slope combined with high discharge makes the estimation of discharge from SWOT challenging since surface water slope errors may have a proportionately large impact. Here, we assessed the likely accuracy which may be possible. Specifically, we aimed to: (i) characterize and illustrate in two-dimensions the errors which may be found in SWOT swath altimetry measurements of terrestrial surface water, (ii) simulate the spatio-temporal sampling scheme of SWOT for the Amazon and (iii) assess the impact of each on estimates of water surface slope and river discharge which may be obtained from SWOT imagery. Note that, presently, the performance of the SWOT instrument in the case of flooded vegetation is unknown, thus throughout this paper the words “floodplain” and “wetland” reference those conditions of a clear view of the sky without any flooded vegetation.
We utilized the hydrodynamic model of Wilson et al. (2007) and Trigg et al. (2009) for the same reach of the Amazon. We used this model to generate water surface elevation “truth” images for a 22-month period comprising more than a full flood cycle (Fig. 1b-c). These “truth” images were then temporally sampled to match the orbital characteristics of SWOT, and 2D errors as defined by the SWOT design requirements were added. Thus, we obtained estimates of surface water heights as may be observed by SWOT. From both the “truth” images and the simulated SWOT observations, estimates of river slope and discharge were then derived. A schematic summary of the virtual mission and methods used is shown in Fig. 2, with details provided in the following section.

3 Methods

3.1 Generation of water surface “truth” images from hydrodynamic modeling

In order to generate water elevation “truth” images, the hydrodynamic model code LISFLOOD-FP (Bates and De Roo, 2000) was used. LISFLOOD-FP consists of a 1D representation of the river channel which comprises of a series of channel cross-sections and a 2D floodplain representation. The formulation of LISFLOOD-FP used here was based on that of Wilson et al. (2007) and included the one-dimensional diffusive wave formulation of Trigg et al. (2009) for chan-
nel flow (floodplain flow was excluded), allowing complex channel bathymetry and back propagation of flow in the main channel. In LISFLOOD-FP, this is . A detailed series of channel cross-sections were used (124 for the Solimões and 48 for the Purus), with an average along-channel spacing of 2.4 km and each representing the average bed-elevation for that location. Channel flow was implemented in the form:

\[
\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = q
\]

\[
S_0 - \frac{n^2 P^{4/3} Q^2}{A^{10/3}} - \left[\frac{\partial y}{\partial x}\right] = 0
\]

where \( Q \) is the volumetric flow rate in the channel, \( A \) the cross-sectional area of the flow, \( P \) the wetted perimeter (approximated by channel width), \( n \) is the Manning friction coefficient, \( S_0 \) is the channel bed slope, \( q \) is the lateral flow into and out of the channel, \( y \) is the channel depth, \( x \) is the distance along the river and \( t \) is time (Trigg et al., 2009). Note that \( S_0 \) is written here so as to be greater than zero in the usual case where the bed elevation decreases in the downstream direction. The diffusion term, \( \partial y/\partial x \), allows channel flow to respond to both the channel bed slope and the water surface slope. This diffusive wave approximation of the full 1D Saint Venant equations is solved using an implicit Newton-Raphson scheme.

In order to create “truth” images of water surface elevation (h[TRUE]), 1D channel water elevations were mapped onto channel cross-sections then interpolated onto a 2D regular grid at a spatial resolution of 100 m. This was selected to approximately match the design requirements of SWOT as specified by Rodríguez (2014), although resolution will vary across the swath. While this method excluded potential minor cross-channel variation in water surface elevation, variation along-channel was incorporated fully, including any backwater effects.

Upstream boundary conditions (channel discharge) for the Solimões (Fig. 1b) and Purus (Fig. 1c) were derived from rating curves and river stage measurements at in-situ gauges at Itapeua and Aruma (Fig. 1a), respectively, using data provided by the Agência Nacional de Águas (ANA), Brazil for the period 1 June 1995 to 31 March 1997. River stage measured at Manacapuru was used as the downstream boundary condition. The model developed allowed the inclusion of a detailed river bathymetry (Fig. 1d), obtained in a field survey by Wilson et al. (2007) and described in detail by Trigg et al. (2009). In the study reach, the Solimões varies in width from around 1.6 km to 5.6 km, with minimum bed elevation between -26.5 and 8.0 m (vertical datum: EGM96); the width of the Purus varies from 0.6 to 1.7 km, with minimum bed elevation between -9.8 and 9.5 m. Friction parameters for the model were obtained through a calibration based on the minimization of root mean square error (RMSE). RMS error calculated from river levels from four gauging stations internal to the model domain and model water surface elevation (Trigg et al., 2009). Manning’s \( n \) values of 0.032 for the Solimões and 0.034 for the Purus were obtained with RMSE ranging from 0.1 to 0.9 m (Trigg et al., 2009). Model validation consisted of a comparison of model water levels with an independent set of satellite altimetry data, with over RMSE found to be 1.26 m and 1.42 m for the Solimões and Purus rivers, respectively, obtained at a temporal resolution of 12 hours (Trigg et al., 2009).

### 3.2 Obtaining SWOT observations

Water surface elevations obtained from LISFLOOD-FP were used as “truth” onto which SWOT sampling and errors could be added, thereby allowing us to assess their hydraulic implications. Water surfaces were obtained from the model according to the SWOT spatio-temporal sampling scheme from an orbit with 78° inclination, 22 day repeat, 97 km altitude, and 140 km swath width. The reach length was sufficient to be covered by 6 swaths in total in each 22 day cycle (3 ascending, 3 descending), with each ground location being observed 2 or 3 times (Fig. 3a). Since the site is close to the equator, this represents the minimum frequency in sampling which may be obtained by SWOT.

Onto the water surface images, errors were added based on a two-dimension height error spectrum derived from the SWOT design requirements (Fig. 3b). 2D SWOT errors were generated by inverse Fourier transform of the design requirements error spectrum (Rodríguez, 2014). A separate error field was generated for each overlap in order to include long-wavelength errors. The 500 m SWOT errors were down scaled Error fields were then resampled to model resolution (100 m), adding random noise in order to ensure that the total error variance (spectral, integral of the design requirements error spectrum) was correct.

We thereby obtained water surface elevation measurements for the Amazon mainstem as may be observed by SWOT. Using these measurements, we derived estimates of river slope and discharge and compared them to those obtained directly from the hydraulic model. For completeness, we also compared discharge computed directly from the model output, i.e. the water surface slope prior to adding slope errors. This allowed us to characterize the error in water surface slope and discharge estimates from both the SWOT spatio-temporal sampling scheme and from the instrument measurement error.

### 3.3 Calculation of slope and discharge from water surface elevations

Initially, single-pixel SWOT water surface elevation measurements (\( h_{\text{SWOT OBS}} \)) were extracted along the chan-
channel centerline and used to calculate water surface slope \((S_{\text{SWOT OBS}})\). Note that the water surface slope is mathematically equal to the sum of the bed slope \((S_0)\) and downstream changes in water depth \([\partial y/\partial x]\):

\[
S = S_0 \cdot \frac{\partial y}{\partial x}
\]

(3)

\(S\) was derived by along-reach averaging through the fitting of 1D polynomials using least square estimation to moving windows placed on the surface water heights:

\[
S = \frac{\sum xh - \bar{x}\bar{h}}{\sum x^2 - \bar{x}^2}
\]

(4)

where \(k\) is the number of data points included in the moving window and \(x\) is the distance of the water elevation observation, \(h\), along the channel; the negative sign constrains the slopes to be greater than zero in the usual case when \(h\) is decreasing in the downstream direction. The size of the moving windows used ranged from 0.5 km up to 20 km, with larger windows leading to greater along-channel smoothing of the data. This process was then repeated using cross-section averages of SWOT water elevation measurements \((h_{\text{SWOT XS}})\), extracted by taking the arithmetic mean of pixels across-channel in a direction perpendicular to the channel centerline. \(S_{\text{SWOT XS}}\) was then calculated in the same way as \(S_{\text{SWOT OBS}}\). For comparison and to assess accuracy of derived estimates of \(Q\), true slope \((S_{\text{TRUE}})\) was also calculated using water surface elevation “truth” images \((h_{\text{TRUE}})\) using Eq. (4).

For each water surface slope \((S_{\text{SWOT OBS}}, S_{\text{SWOT XS}}, S_{\text{TRUE}})\) at each reach-length, discharge along the length of the channel was derived, following the method of LeFavour and Alsdorf (2005):

\[
Q = \frac{1}{n} w y^{5/3} S^{1/2}
\]

(5)

where \(w\) is the channel width, \(z\) is the bed elevation, \(y\) is the river depth and \(S\) is the water surface slope. In this paper, we assume that channel friction, width and bed elevation are known. Thus, the focus was here on the impact of errors in observations of water surface elevation and the derived estimates of water surface slope on the estimation of discharge. Errors in \(Q\) were approximated using first-order error propagation, via a Taylor series expansion:

\[
\sigma_Q \approx \frac{\partial Q}{\partial S} \sigma_S = \frac{1}{2} Q \frac{\sigma_S}{S}
\]

(6)

Note that we have here isolated the uncertainty in \(Q\) that derives from \(S\). Hydrographs of discharge over time for given points on the channel were then extracted, with the temporal frequency of these determined by the SWOT sampling scheme. Thus, for most locations on the channel, two values of \(Q\) were available in each 22-day cycle.

### 3.4 Accuracy assessment of SWOT derived discharge

In addition to the discharge error approximation \((\sigma_Q)\) calculated in Eq. (6), hydrographs of channel discharge obtained using along-reach averaging \((Q_{\text{SWOT OBS}})\) and with added cross-section averaging \((Q_{\text{SWOT XS}})\) were compared to hydrographs obtained using the “true” water surface elevation \((Q_{\text{TRUE}})\) using a percentage error calculation and the Nash-Sutcliffe model efficiency coefficient (Nash and Sutcliffe, 1970):

\[
E = 1 - \frac{\sum_{t=1}^{T} \left( Q_{t}^{\text{[TRUE]}} - Q_{t}^{\text{[PRED]}} \right)^2}{\sum_{t=1}^{T} \left( Q_{t}^{\text{[TRUE]}} - \bar{Q}^{\text{[TRUE]}} \right)^2}
\]

(7)

where \(Q_{t}^{\text{[TRUE]}}\) is the “observed” channel discharge derived from “true” water surface elevations at time \(t\) and \(Q_{t}^{\text{[PRED]}}\) is channel discharge derived from SWOT observations \((Q_{\text{SWOT OBS}}\) or \(Q_{\text{SWOT XS}}\)). Values of \(E\) range between \(-\infty\) and 1.0, with 1.0 indicating a perfect match between \(Q_{t}^{\text{[TRUE]}}\) and \(Q_{t}^{\text{[PRED]}}\) and values less than zero indicating that
4 Results and discussion

4.1 Model output and generation of SWOT images

The LISFLOOD-FP model was run for the full 22-month period between 1 June 1995 and 31 March 1997, taking around 82 hours to complete on a dual-processor compute server. The Manning’s friction coefficient, n, used was 0.032 for the Solimões and 0.034 for the Purus, obtained from model calibration by (Trigg et al., 2009). The overall root mean square error of the model ranged between 0.1 and 0.9 m (please see Trigg et al., 2009, for details). Model validation consisted of a comparison of model water levels with an independent set of satellite altimeter data, with RMS error found to be 1.26 m and 1.42 m for the Solimões and Purus rivers, respectively (Trigg et al., 2009).

1D channel profiles outputs from the LISFLOOD-FP model are shown in Fig. 3 for low water (September 15, 1995) and high water (June 21, 1996), including the water surface elevation, water surface slope and channel discharge. Along-channel, and are summarised in Table 1. There was substantial along-channel variation in water surface slope for the Solimões ranged from 0.15 to 9.57 cm/km at low water (mean: 1.37 cm/km; standard deviation: 1.53) and from 0.69 to 7.43 cm/km at high water (mean: 2.19 cm/km; standard deviation: 0.95). For the Purus, water surface slope ranged from 0.12 to 1.99 cm/km at low water (mean: 0.50 cm/km; standard deviation: 1.02) and from 0.17 to 3.01 cm/km at high water (mean: 0.52 cm/km; standard deviation: 0.35). Along-channel variation in discharge was also significant: for the channel discharge for the both the Solimões, this ranged from 19.765 to 32.068 m³/s at low water (mean: 26.316 ± 3.72 m³/s; standard deviation: 2.137); and 69.918 to 116.030 m³/s at high water (mean: 99.783 ± 9.59 m³/s; standard deviation: 9.172); for the Purus, discharge ranged from 2.649 to 5.314 m³/s at low water (mean: 4.58 ± 1.73 m³/s; standard deviation: 1.276); and from 6.665 to 19.276 m³/s at high water (mean: 13.466 ± 3.05 m³/s; standard deviation: 2.958). Along the channel, this along-channel variability may make the accurate estimation of discharge using reach-averaged estimates of slope a considerably greater challenge.

Fig. 5 indicates water elevation at the upstream and downstream ends of the Solimões and Purus reaches and average water surface slopes throughout the 22-month simulation period. Generally, water surface slope is lowest during the falling limb of the hydrograph and highest during the rising limb. Average water surface slope for the Solimões rose quickly to its maximum level of 2.9 cm/km during the low water period (September to November, 1995), immediately after the river level at the upstream end of the channel started to rise. The maximum water surface slope in the Purus of 1.29 cm/km occurred during the low water period (October, 1995), when backwater effects from the main Solimões channel were less important.

As detailed in Section 3.2, “truth” images of water surface elevation, \( h_{\text{TRUE}} \), were generated from LISFLOOD-FP according to the SWOT spatio-temporal sampling scheme and 2D errors were then added to these according to the 2D SWOT science requirements height error spectrum, providing SWOT images of water surface height observations, \( h_{\text{SWOT OBS}} \). An example set of six overpasses from a SWOT orbit cycle at high water (cycle 18) is shown in Fig. 6, illustrating the extent of channel which may be observed. Note that here we are focused on the main channels and have not attempted to map water elevations in the forest floodplain. A detailed inset image of the Purus/ Solimões confluence for cycle 18, overpass 6 is shown in Fig. 7, illustrating the image of \( h_{\text{SWOT OBS}} \) alongside the corresponding image of \( h_{\text{TRUE}} \) and 2D SWOT height errors.

Values of SWOT water surface height observations were extracted from images of \( h_{\text{SWOT OBS}} \) along the channel centerline and, in addition, averages of channel cross-sections taken perpendicular to the channel centerline were calculated \( h_{\text{SWOT XS}} \), plotted against distance downstream for high water (cycle 18) in Fig. 8. In these profiles, the tighter clustering of the cross-section averages to the true channel water elevation profile indicates clearly that by taking a cross-section average, errors in water surface height observations were reduced (assuming no bias).

4.2 Water surface slopes

Fig. 9 illustrates along-channel water surface slope as calculated using \( h_{\text{SWOT XS}} \) for high water (cycle 18, overpass 6), using reach-lengths between 5 and 20 km. As the length of averaging increased, errors in \( h_{\text{SWOT XS}} \) reduced substantially when compared to \( h_{\text{TRUE}} \). Overall error in the estimation of water surface slope decreased quickly with increasing reach-lengths (Fig. 10): for the Solimões, without averaging the mean of \( q_{\text{TRUE}} \) is a better predictor of true channel discharge than \( q_{\text{PRED}} \) (Legates and McCabe, 1999). Generally, values of \( E \) between 0.0 and 1.0 are considered as acceptable levels of performance (Moriasi et al., 2007).

### Table 1. Summary of along-channel variability in modelled water surface slope and channel discharge at low and high water.

<table>
<thead>
<tr>
<th>Water level</th>
<th>Slope (cm/km)</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Mean</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solimões</td>
<td>Low</td>
<td>0.15</td>
<td>9.57</td>
<td>1.37</td>
<td>1.53</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>0.69</td>
<td>7.43</td>
<td>2.19</td>
<td>0.95</td>
</tr>
<tr>
<td>Discharge</td>
<td>Low</td>
<td>19,765</td>
<td>32,068</td>
<td>26,346</td>
<td>2,137.9</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>69,918</td>
<td>116,030</td>
<td>99,783</td>
<td>9,372.3</td>
</tr>
<tr>
<td>Purus</td>
<td>Low</td>
<td>-0.12</td>
<td>4.99</td>
<td>0.5</td>
<td>1.02</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>0.17</td>
<td>3.01</td>
<td>0.52</td>
<td>0.35</td>
</tr>
<tr>
<td>Discharge</td>
<td>Low</td>
<td>-2,649</td>
<td>5,314</td>
<td>958</td>
<td>1,276.4</td>
</tr>
<tr>
<td></td>
<td>High</td>
<td>6,665</td>
<td>19,276</td>
<td>13,466</td>
<td>2,959.9</td>
</tr>
</tbody>
</table>
across channel ($S_{\text{SWOT OBS}}$) and with a short reach lengths of 0.5 km, errors in slope were high at 86.4 cm/km. These errors dropped quickly as more data were included in the estimation of slope, reducing to 0.33 cm/km at 20 km. Averaging across channel in addition to along reach lengths ($S_{\text{SWOT XS}}$) led to a further drop in errors, with 0.09 cm/km error at a reach length of 0.5 km, reducing to 0.09 cm/km at 20 km reach lengths. Slope errors were similar for the Purus without cross-section averaging (91.0 cm/km at 0.5 km; 0.31 at 20 km), and were moderately higher than the Solimões with cross-section averaging (36.0 at 0.5 km; 0.13 cm/km at 20 km) due to the narrower channel width (Table 2). The science-
requirement for the SWOT sensor is that river slopes are measured with errors less than 1 cm per km when averaged for 10 km reach length (Rodríguez, 2014). For both the Solimões and Purus, without cross-section averaging ($S_{SWOT\,OBS}$), reach-lengths of ~10 km were required to achieve this level of accuracy; with cross-section averaging ($S_{SWOT\,XS}$) accuracies better than 1 cm/km were achieved using shorter reach lengths of ~4 km and ~5 km for the Solimões and Purus, respectively. For 10 km reach lengths, incorporating cross-section averaging, water slope errors of 0.26 and 0.37 cm per km, respectively, were achieved.

### 4.3 Channel discharge

In Fig. 11, along-channel discharge estimates for high water (cycle 18, overpass 6) are shown for $Q_{SWOT\,XS}$ using reach lengths between 5 and 20 km. As with errors in slope, as reach lengths increased, the errors in estimated discharge decreased. The LISFLOOD-FP modeled discharge ($Q_{MODEL}$)
Figure 8. The 2D heights (Fig. 6) were transferred to 1D for both the Solimões and Purus by extracting values of $h_{\text{SWOT OBS}}$ along the channel centerline; to reduce errors, averages of cross-sections taken perpendicular to the channel centerline were also calculated ($h_{\text{SWOT XS}}$).

(a)  (b)

Figure 9. Slope errors: the effect of averaging along channel using reach lengths between 5 and 20 km for the (a) Solimões and (b) Purus rivers. Plots show cycle 18 (high water), overpass 6. Note that $Q_{\text{TRUE}}$ is different to $Q_{\text{MODEL}}$ since it does not take into account the full diffusive wave approximation of the Saint Venant equations (Section 3.1) and is a reach length average rather than an instantaneous discharge for a particular location.

Using reach lengths of 20 km, full discharge hydrographs were constructed for $Q_{\text{SWOT XS}}$ for several locations along the Solimões and Purus channels, and are compared to hydrographs for $Q_{\text{TRUE}}$ and $Q_{\text{MODEL}}$ in Fig. 12. $Q_{\text{SWOT XS}}$ matched well $Q_{\text{TRUE}}$ throughout the 22-month hydrograph, including both rising and falling flood wave. As with slope errors, the error in estimated discharge dropped quickly as the length of reach length averaging increased (Fig. 13). Without averaging water surface elevations across channel
At 20 km. Averaging across 5 km reach lengths, reducing to 9.7% at 20 km. Averaging across channel in addition to along reach lengths (Eq. 5), led to a further drop in errors, with 34,180 m$^3$/s (22.2%) error at a reach lengths of 5 km; reducing to 1,960 m$^3$/s (reducing to 2.6%) at 20 km. Discharge errors for the Purus without cross-section averaging were 9,682 m$^3$/s (130.9% of the mean Purus discharge) at 5 km, reducing to 2,795 m$^3$/s (35.1%) at 20 km; with cross-section averaging errors were 5,764 m$^3$/s (76.0%) at 5 km, reducing to 1,493 m$^3$/s (19.1%) at 20 km. Discharge errors are summarised in Table 2.

Nash-Sutcliffe efficiency coefficient ($E$) values for with increasing reach length averaging are shown in Fig. 13c. On the Solimões, for $Q_{[SWOT OBS]}$, $E$ was -1.92 at reach lengths of 5 km, 0.23 at 10 km and increasing to 0.89 at 20 km; for $Q_{[SWOT XS]}$, $E$ was 0.46 at 5 km, 0.93 at 10 km and increasing to 0.99 at 20 km. For the Purus, values of $E$ were lower: for $Q_{[SWOT OBS]}$, $E$ was -8.17 at reach lengths of 5 km, -0.92 at 10 km and increasing to 0.57 at 20 km; for $Q_{[SWOT XS]}$, $E$ was -1.34 at 5 km, 0.41 at 10 km and increasing to 0.88 at 20 km.

Negative values of the Nash-Sutcliffe efficiency coefficient indicate that the prediction of discharge is no better than the mean value of the observations: consequently, using cross-section averaging, reach lengths of ~4 km were required to achieve positive values of $E$ (indicating “acceptable” levels of accuracy) for the Solimões; for the Purus, ~7.5 km reach lengths were required. High values of $E$ (>0.8) were achieved with reach lengths greater than ~7.5 km for the Solimões and ~17.5 km for the Purus, indicating high accuracy in the estimation of discharge.

The above accuracy assessment of SWOT-derived discharge compares estimates obtained using SWOT observations of water elevation to those obtained using “true” water surface elevations, based on the channel discharge approximation in Fig. 14). Thus, $Q_{[TRUE]}$ and $Q_{[SWOT]}$ were also compared using $E$ to channel discharge obtained directly from LISFLOOD-FP, using $Q_{[MODEL]}$ in place of $Q_{[TRUE]}$ in Eq. (7) (Fig. 14). Thus, we were able to characterize errors in estimates of channel discharge introduced directly by errors in SWOT observations, as well as errors introduced by the calculation of $Q$ using reach length averaging of the water surface in the calculation of water surface slope. Errors in $Q_{[TRUE]}$ were low with a minimum error of 2.418 m$^3$/s (3.5%, $E = 0.99$) for the Solimões at a reach length of 0.75 km, and 486 m$^3$/s (6.6%,

**Table 2. Summary of errors in slope (S) and discharge (Q) for the Solimões and Purus channels, obtained using reach length averaging of direct SWOT observations along the channel centerline (OBS) and with additional cross-section averaging (XS).**

<table>
<thead>
<tr>
<th>Error</th>
<th>Reach length (km)</th>
<th>5</th>
<th>10</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{[SWOT OBS]}$</td>
<td>cm/km</td>
<td>2.55</td>
<td>0.91</td>
<td>0.33</td>
</tr>
<tr>
<td>$S_{[SWOT XS]}$</td>
<td>cm/km</td>
<td>0.72</td>
<td>0.26</td>
<td>0.09</td>
</tr>
<tr>
<td>$Q_{[SWOT OBS]}$</td>
<td>m$^3$/s</td>
<td>34.180</td>
<td>18.900</td>
<td>7.190</td>
</tr>
<tr>
<td>$Q_{[SWOT XS]}$</td>
<td>m$^3$/s</td>
<td>15.670</td>
<td>5.950</td>
<td>1.960</td>
</tr>
<tr>
<td>$E$</td>
<td></td>
<td>-1.92</td>
<td>0.23</td>
<td>0.89</td>
</tr>
<tr>
<td>$Q_{[SWOT OBS]}$</td>
<td>m$^3$/s</td>
<td>9.682</td>
<td>5.211</td>
<td>2.795</td>
</tr>
<tr>
<td>$Q_{[SWOT XS]}$</td>
<td>m$^3$/s</td>
<td>5.764</td>
<td>3.189</td>
<td>1.493</td>
</tr>
<tr>
<td>$E$</td>
<td></td>
<td>-8.17</td>
<td>-0.92</td>
<td>0.57</td>
</tr>
</tbody>
</table>

Figure 10. The effect of reach-length averaging on errors in the water surface slope estimation for (a) the Solimões and (b) the Purus rivers.
Figure 11. Discharge estimates accounting for slope errors but neglecting width, depth, and friction errors for reach lengths between 5 and 20 km for the (a) Solimões and (b) Purus rivers. Plots show cycle 18 (high water), overpass 6.

Figure 12. Reconstruction of channel discharge hydrographs from cross-section averaged SWOT observations ($Q_{\text{SWOT XS}}$) for the Solimões and Purus channels using 20 km reach lengths, compared to discharge obtained using water elevation “truth” images ($Q_{\text{TRUE}}$) and the original modeled channel discharge ($Q_{\text{MODEL}}$). 

As the reach length used increased, the errors in $Q_{\text{TRUE}}$ also increased. At reach lengths of 20 km, errors for the Solimões were 5,690 m$^3$/s (8.3%, $E = 0.87$) and 1,238 m$^3$/s (18.1%, $E = 0.89$) for the Purus. This increase in error with reach length is a result of the reach length averaging used for the calculation of water surface slope in Eq. (4), as compared to the instantaneous discharge obtained at a single cross-section from the LISFLOOD-FP model output. These results illustrate that there may be an optimal reach length for the es-
4.4 Implications for SWOT

These results indicate that discharge may be obtained accurately from SWOT measurements on larger, lowland rivers, assuming sufficient knowledge of channel bathymetry and frictional properties. The error in discharge of 2.6% for the Solimões using cross-channel averaging and 20 km reach lengths compares favorably with the error of ~6-8% obtained by LeFavour and Alsdorf (2005) for the same section of river using SRTM data and 733 km reach lengths. When comparing against instantaneous discharge obtained directly from model output, errors were moderately higher with accuracies of 9.1% obtained at reach lengths of 12.5 km. This suggests that SWOT data will provide both an improvement in accuracy of discharge estimates and a substantial increase in the level of along-channel detail. Since SWOT will provide 2D measurements of surface water, we were able to use cross-channel averaging to substantially improve accuracy due to the improved representation of channel water surface elevations and subsequent reductions in water surface slope errors. For the Purus, accuracy in discharge estimates was lower, which is likely to have been in large part due to the narrower width of the river leading to a reduction in averaging of height errors and consequently higher slope errors, combined with the very low water surface slopes on the river leading to a proportionately higher impact of slope errors when calculating discharge.

**Figure 13.** Errors in discharge ($Q$) as related to reach-length averaging, calculated against slope and discharge obtained using water elevation “truth” images ($Q_{\text{TRUE}}$): (a) absolute discharge error; (b) error expressed as a percentage of mean discharge; and (c) Nash-Sutcliffe efficiency coefficient. The horizontal line in (c) represents the level of “acceptable” error in modeled discharge estimates. Top row: Solimões; bottom row: Purus.

Examples of other rivers which may be observable by SWOT are shown in Fig. 15. Here, rivers are plotted according to their approximate width and water surface slopes obtained from published sources. The percentage error in calculated discharge, $Q$, resulting from errors in SWOT derived water surface slope are indicated. These errors were derived from Eq. (6) using 10 km reach lengths to estimate water surface slope and incorporating the effects of cross-channel averaging of water surface elevation. Note that as channel width increases, error in discharge decreases since greater averaging of water surface elevation is possible (water surface elevation errors will decrease by $1/\sqrt{n}$, where $n$ is the number of pixels being averaged (Rodriguez, 2014)); as water surface slope decreases, error in discharge increases since water surface slope errors become proportionately more important according to Eq. (6). From this, we can infer that discharge estimates may be more accurate for rivers with: (i) greater channel widths
which permit a greater level of cross-section averaging and the use of shorter reach lengths; and (ii) higher water surface slopes, since, from Eq. (6), the relative error in discharge decreases as slope increases. Conversely, discharge estimation accuracy is likely to be lowest for narrow rivers with low slopes, although further research is required to quantify errors for rivers at this scale.

Note that the error.

It is important to note that the errors presented here represent only the contribution to overall error in reach-averaged discharge which may be added by SWOT observations of water surface elevation. Other errors, such as those contributed by friction or bathymetry errors, or resulting from along-channel variability in discharge, are excluded but may be significant and further research is required to characterize their contribution. Other than surface water slope and elevation, parameters required in the estimation of discharge (i.e. channel width, roughness and bed elevation or channel depth) are the subject of other recent studies. For example, Durand et al. (2008) used data assimilation of synthetic SWOT measurements into a hydraulic model to estimate river bathymetric slope and depth for the same river reach as presented in this paper, obtaining RMS errors of 0.3 cm/km and 0.56 m, respectively. Similarly, Yoon et al. (2012) estimated river bathymetry for the Ohio River, United States, obtaining an RMS error of 0.52 m and obtained effective reach-averaged river roughness to within 1% of the true value. Finally, Durand et al. (2014) illustrate the use of a Bayesian algorithm to estimate river bathymetry and roughness based on observations of river $h$ and $S$ with high accuracy for the River Severn, United Kingdom, and the subsequent estimation of channel discharge. When compared to gauge estimates of discharge, Durand et al. (2014) obtained an accuracy of 10% in discharge estimation for in-bank flows, assuming known lateral inflows, decreasing to 36% without this assumption. The work presented in this paper builds on these studies in that it is the first to directly assess the implications of errors in surface water slope derived from SWOT observations of water elevation on the estimation of discharge, independent of other factors.

As with other studies, the error analysis presented here excluded layover and vegetation effects, as may be found in wetlands and floodplains, or along the edges of rivers. These effects are likely to be greatest for narrower rivers with bank vegetation. In addition, research presented here did not incorporate effects of the temporal sampling scheme on the accuracy of hydrograph estimation. For large rivers with discharge which changes relatively slowly, such as the Amazon and its sub-basins, errors introduced by SWOT temporal sampling are likely to be minimal. However, for smaller rivers with higher discharge variability, this sampling may be significant. Further research is required in this area, although it is likely that there will be an optimum level of width, slope and discharge variability for discharge estimation.

5 Conclusions

In this paper, we used a “virtual mission” study of two-dimensional water surface elevations which may be obtained
by SWOT for a reach of the central Amazon River in Brazil and investigated the implications of errors in such measurements on the estimation of water surface slope and channel discharge. The following remarks can be made following our work:

1. Using 1D polynomials with least squares estimation fitted to water elevations obtained from channel centerlines, the SWOT design requirement of slope errors less than 1 cm per km when averaged for 10 km (Rodríguez, 2014) was achieved for both the Solimões and Purus Rivers.

2. Shorter reach lengths (~4 km and ~5 km for the Solimões and Purus, respectively) were required to achieve the design level of accuracy when additionally averaging SWOT water surface height estimates across-channel; for 10 km reach lengths, higher accuracies were achieved (water slope errors of 0.26 and 0.37 cm per km for the Solimões and Purus, respectively). This indicates that the accuracy of water surface slopes estimates will be higher for rivers with wider channels, particularly those several times wider than the ~250–70–250 m nominal spatial resolution (Durand et al., 2010; Durand et al., 2010; Rodríguez, 2014).

3. SWOT data are promising for the estimation of Amazonian river discharge, with low errors in estimates (9.1% for instantaneous estimates, or 2.6% for reach-averaged discharge estimates). Discharge hydrographs could be re-constructed accurately from SWOT imagery based on the specified temporal sampling scheme (Figure 3; Rodríguez, 2014) although, for rivers with a higher discharge variability, temporal sampling is likely to be a significant source of error for hydrograph estimation.

4. A high proportion of the errors found in the instantaneous estimates derived from the method used to calculate discharge from water surface slopes, rather than from SWOT errors, suggesting that improvements to the estimation of discharge may be possible.

Overall, these findings indicate that forthcoming SWOT imagery shows considerable promise for the hydraulic characterization of large rivers such as the Amazon, although further work is required for a range of additional rivers with a variety of characteristics, particularly those with a high spatial and temporal variability in surface water slope and channel discharge. However, for large, lowland rivers, the results are directly transferable.

It should also be noted that, in this paper, we assumed knowledge of channel friction, width and bed elevation in the calculation of discharge, and excluded potential effects of vegetation on errors in SWOT surface water heights. Further work is needed to assess the relative importance of each of these factors on the estimation of channel discharge.
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References


