Response to Reviewer #1

Dear Dr. David Jay,

We are very grateful for your constructive comments to improve the quality of our manuscript. We shall make appropriate changes to the paper to account for your comments. And we shall make appropriate references to the other inverse methods that have been described in the literature. Below we would like to provide a reply to the issues raised in your comments.

General comments:

The idea of estimating the freshwater flow through an estuarine cross-section using tidal theory and tidal analysis is not new [cf. Jay and Kulkulka, 2003], but it has only recently been presented with a careful verification and analysis of uncertainty [Moftakhari et al., 2013]. The latter authors also introduced the term “tidal discharge estimation” or TDE. As with any innovation, multiple approaches are useful, so the present contribution is a welcome addition to the field of applied tidal dynamics.

Our reply: We agree that the topic of predicting freshwater discharge through observed tidal water levels has been explored by many researchers (e.g., Jay and Kulkulka, 2003; Jay et al., 2006; Moftakhari et al., 2013). The aim of the present contribution is to propose an analytical relationship that can be used to predict freshwater discharge based on observed tidal water levels along the estuary axis. Such a relationship is derived based on the envelope theory developed by Savenije (2005, 2012) and Cai et al. (2014). It can be regarded as a modified Manning equation that includes the effects of residual water level slope (i.e., \( d\bar{h}/dx \), where \( \bar{h} \) is the tidally averaged depth) and tidal damping (i.e., \( d\eta/dx \), where \( \eta \) is the tidal amplitude). For detailed derivation, please refer to the response below. Unlike the previous studies that make use of statistical and harmonic analyses relating the tidal water levels to the freshwater discharge, the proposed analytical relationship is a closed form equation, which can be easily implemented for given observed tidal water levels.

Specific comments:

a: History of TDE

The history of the idea of using tidal theory and the fluvial modification of tidal properties to estimate river discharge needs some explanation, which is not provided here. This is most simply explained using the nomenclature “forward model” (determining tidal properties from river flow) and “inverse model” (determining river flow from tidal properties). Conceptually, the key idea is that river tides are very nonstationary, and that this non-stationarity, while complicating the prediction of tides, has many dynamical uses [Jay and Kulkulka, 2003], of which TDE is only one. There is an extensive literature on river tides dating to at least WWII, and I will not attempt to review it here. Jay and Flinchem [1997] added continuous wavelet transform (CWT) methods to the tidal analyst’s tool kit and provided a
simple forward model that related the tidal admittance (the complex ratio of tidal amplitude and phase at any point in the river to the tidal amplitude and phase at the ocean entrance) to river flow. Kukulka and Jay [2003a,b] provided a better forward model. Jay and Kukulka [2003] then used an inverse model to hindcast river flow for the December 1964 Columbia River, USA flood. Because this flood resulted primarily from tributary inflow below the most seaward river gauge, our estimate of its flow history is the only instrumental “measurement” available, though the usual flow routing approaches have also been used. We also verified that the method worked in the Fraser River, British Columbia, Canada, though this work has not been published. Jay et al. [2006] then provided a hindcast of the history of inflow to San Francisco Bay, using the long (1858 to date) San Francisco tidal record. This is a useful step, because the inflow to San Francisco Bay through its complex delta was not gauged by the US Geological Survey until 1930. This 2006 AGU presentation also provided the first instrumental estimate of the magnitude of the great flood of January 1862, the largest in the last two centuries in San Francisco Bay. The inverse models used in these two studies added an innovation, in that they were based on a single tide gauge. When only one gauge is available, then the admittance is formed in one of two ways: (a) if the variations of a major constituent like $M_2$ are used, then an admittance is formed using the astronomical tidal potential; or (b) if an overtide like $M_4$ is used, then the ratio $M_4/M_2^2$ is employed as an ersatz admittance. This complex admittance can be separated into an amplitude ratio and phase difference. Tidal theory suggest that the $M_4/M_2^4$ ratio should be useful for low flows, while the $M_2$ admittance is best for high flows. Practice confirms this, at least for the Columbia River and San Francisco Bay. To minimize the impact of time errors inherent in historical tidal records we have used amplitude ratios, though Kukulka and Jay [2003a] verified that the phase difference could also be represented by a forward model. More recently, Moftakhari et al. [2013] returned to the San Francisco Bay case to provide a revised estimate, a formal error analysis, and a discussion of long-term hydrologic change in the system. Also, if CWT methods are used to provide an estimate with a time-scale of a few days, the ratios actually involve the $D_2$ and $D_4$ tidal species, not the $M_2$ and $M_4$ constituents. If the $M_2$ and $M_4$ constituents are resolved via a properly windowed monthly harmonic analysis (as in Moftakhari et al. [2013]), then the time scale of flow estimates is ~18 days.

Our reply: Indeed, we shall provide a more detailed description of the history of “tidal discharge estimation” (TDE) in the new version of the manuscript. In particular, we will rewrite paragraph 2 in the introduction to clarify the importance of predicting fresh water discharge in estuaries (raised by the other reviewer) and the history of TDE.

Due to the general dominance of tidal flows in the tidal region of an estuary, it is often difficult to determine the magnitude of the fresh water discharge accurately. Thus, discharge gauging stations are usually situated at locations outside the tidal region, even though there may be additional tributaries or drainage areas within the tidal region. Knowing the fresh water discharge within the tidal region, however, may be important for water resource assessment or flood hazard prevention (e.g., Madsen and Sklær, 2005; Erdal and Karakurt, 2013; Liu et al., 2014), or for the analyses of sediment supply (e.g.,
Syvitski et al., 2003; Prandle, 2004; Wang et al., 2008), or for irrigation or estimating the effect of water withdrawals on salt intrusion (e.g., MacCready, 2007; Gong and Shen, 2011; Zhang et al., 2012), and for assessing the impacts of future climate change (e.g., Kukulka and Jay, 2003a, 2003b; Moftakhari et al., 2013). Although it is possible to estimate river flow by upscaling the gauged part of a catchment, such an estimate may be inaccurate, especially in poorly gauged catchments or in high-precipitation coastal areas (Jay and Kukulka, 2003).

It is noted that several forward models (determining tidal properties from fresh water discharge) have been presented to investigate the interaction between fresh water discharge and tide in estuaries (e.g., Dronkers, 1964; Leblond, 1978; Godin, 1985, 1999; Jay, 1991, 2001; Jay and Flinchem, 1997; Kukulka and Jay, 2003a, 2003b; Horrevoets et al., 2004; Buschman et al., 2009; Cai et al., 2012, 2014). Based on the tidal theory developed by Jay (1991, 2001), Jay and Flinchem (1997) and Kukulka and Jay (2003a, 2003b), Jay and Kukulka (2003) used an inverse model (determining fresh water discharge from tidal properties) to hindcast river flows for a very high-flow year (1948) and for a low-flow year (1992) in Columbia River. The model was further successfully applied to estimate the history of inflow to San Francisco Bay using the available tidal records (Jay et al., 2006). Recently, Moftakhari et al. (2013), building on the earlier work by Jay and Kukulka (2003), revisited the method of predicting fresh water discharge by including a quantification of uncertainties. However, such an approach is based on statistical and harmonic analyses without using an analytical relationship between the fresh water discharge and other controlling parameters (such as water level and tidal damping). In this paper, we aim to establish an analytical equation relating tidal wave propagation to the fresh water discharge from upstream. Besides the general interest of establishing an analytical relation between wave cerenity, phase lag, velocity amplitude, tidal damping, residual slope and river discharge, this relationship can be of practical use to estimate, in an inverse way, river discharge on the basis of observed tidal water levels along the estuary axis. Of course our method also has its disadvantages. It requires an exponential shape (as is the case in alluvial estuaries), it requires that the $M_2$ is dominant over other tidal constituents, and there should be a measurable influence of the river discharge (river discharge and tidal discharge being within the same order of magnitude). But as the reviewer indicates, the methods are complementary and help to approach the issue from different angles.

b. Theoretical foundation

The theoretical foundation of the TDE is also not explained here. It uses the tidal propagation theory for convergent channels of Jay et al. [1991]. The key assumptions are that: (a) there is no reflected wave; (b) the wave is critically convergent so that the real and imaginary part of the complex wave number are equal (i.e., the scale length for damping is the same as the inverse wave number); (c) the tidal velocity amplitude and river flow velocity are of the same order; and (d) the channel geometry does not change drastically with river flow. In practice, the last two assumptions are the most restrictive, though both can be stretched. With these assumptions it is simple to express the tidal admittance in terms of the wave
number, which can then be represented using the Dronkers [1964] cubic Tschebychev polynomial. The latter allows the admittance to be expressed in terms of the river flow and tidal amplitude at the ocean entrance. The tidal range terms recognizes that the relationship between river flow and damping of the tides varies over the neap-spring cycle. This is important for hindcasting flows on the scale of days, but not for hindcasts based on windowed monthly harmonic analyses. The relationship between admittance, river and tidal range is nonlinear and cannot be exactly inverted, but approximate inversion is simple, especially when windowed monthly harmonic analyses -- this scale of time averaging allows the tidal range term to be dropped. In practice, the coefficients in the equation for TDE are fit by regression using a calibration data set. On the whole, the analysis is just as rigorous as that proposed here. In both cases, the nonlinear bedstress term is approximated, and one or more constants must be determined from data.

Our reply: We agree that the theoretical foundation of the proposed approach should be explained in more detail. We use the envelope theory developed by Savenije (2005, 2012) for tidal wave propagation. The analytical model is further expanded by Cai et al. (2014) to account for the influence of river discharge. The basic assumptions made in the analytical model are that: (a) the longitudinal cross-sectional area can be described by an exponential function; (b) there is no reflected wave; (c) the ratio of tidal amplitude to depth ratio is less than unit. For predicting fresh water discharge, it is also required that the river discharge is at least in the same order of magnitude as the maximum tidal flow. In fact, we can see from the derivation below that the proposed analytical relationship relating the tidal wave propagation to the fresh water discharge is a modified Manning equation that accounts for the effects of residual water level slope and tidal damping (see detailed derivation below). We shall clarify the theoretical foundation in the new version of the manuscript.

The momentum equation when written in a Lagrangean reference frame reads (Savenije, 2005, 2012):

\[
\frac{dV}{dt} + g \frac{\partial h}{\partial x} + g \frac{\partial z_b}{\partial x} + g \frac{h \frac{\partial \rho}{\partial x}}{2\rho} + gn \frac{V^2}{R^{1/3}} = 0
\] (R1)

where \( V \) is the Lagrangean velocity for a moving particle, \( g \) is the acceleration due to gravity, \( h \) is the water depth, \( z_b \) is bottom elevation, \( \rho \) is the water density, \( n \) is Manning’s coefficient, and \( R \) is the hydraulic radius.

For uniform steady flow in a prismatic channel, Eq. (R1) can be simplified as the well-known Manning equation by neglecting the first, the second and the fourth terms:

\[
\frac{V}{n} = \frac{1}{R^{2/3}} S^{1/2}
\] (R2)

where \( S = \frac{-\partial z_b}{\partial x} \) is the slope of the channel.

Hence the expression for river discharge is given by:

\[
Q = AV = \frac{1}{n} AR^{2/3} S^{1/2}
\] (R3)

where \( A \) is the cross-sectional area.
For steady flow when depth may vary along a short section of the channel (e.g., during a flood), the residual water level slope \( \frac{\partial h}{\partial x} \) should be taken into account and Eq. (R1) reduces to:

\[
\frac{\partial h}{\partial x} + \frac{\partial z_r}{\partial x} + R^{1/3} \frac{V | V |}{R^{1/3}} = 0 \quad (R4)
\]

Consequently, the Manning’s equation (R2) is modified as:

\[
V = R^{1/3} \left( \frac{S - \frac{\partial h}{\partial x}}{R^{1/3}} \right)^{1/2} \quad (R5)
\]

while the river discharge becomes:

\[
Q_i = Q_0 \left( 1 - \frac{\partial h}{\partial x} \right)^{1/2} \quad (R6)
\]

In the Lagrangean reference frame, the continuity equation can be written as:

\[
d \frac{dV}{dt} = r_s \frac{cV d h}{h \, dx} - cV \left( \frac{1}{b} \frac{1}{\eta} \frac{d \eta}{dx} \right) \quad (R7)
\]

where \( r_s \) is the storage width ratio, \( b \) is the convergence of width, \( c \) is the wave celerity.

In a tidal region, it is noted that both depth and discharge change along the channel axis (i.e., varied unsteady flow). Thus, Eq. (R1) when combined with (R7) becomes (see Savenije, 2005, 2012):

\[
r_s \frac{cV d h}{h \, dx} - cV \left( \frac{1}{b} \frac{1}{\eta} \frac{d \eta}{dx} \right) + g \frac{\partial h}{\partial x} + g \frac{\partial z_r}{\partial x} + \frac{h}{2} \frac{\partial p}{\partial x} + g n^2 \frac{V | V |}{R^{1/3}} = 0 \quad (R8)
\]

An analytical expression for the tidal damping can be obtained by subtracting high water (HW) and low water (LW) envelopes while accounting for the effect of river discharge (Cai et al., 2014):

in the downstream tide-dominated zone, where \( U_r < v \sin(\epsilon) \),

\[
\frac{1}{\eta} \frac{d \eta}{dx} \left( \theta - r_s \frac{\phi}{\sin(\epsilon)} \zeta + \frac{g \eta}{c v \sin(\epsilon)} \right) = \theta - \frac{f}{2} \frac{v}{hc} \left( \frac{2}{3} \sin(\epsilon) + \frac{16}{9} \phi \zeta + \frac{2}{3} \frac{\phi^2}{\sin(\epsilon)} + \frac{L_0 - L_1}{6} \frac{\zeta}{9} \sin(\epsilon) \right) \quad (R9)
\]

in the upstream river discharge-dominated zone, where \( U_r \geq v \sin(\epsilon) \),

\[
\frac{1}{\eta} \frac{d \eta}{dx} \left( \theta - r_s \frac{\phi}{\sin(\epsilon)} \zeta + \frac{g \eta}{c v \sin(\epsilon)} \right) = \theta - \frac{f}{2} \frac{v}{hc} \left( \frac{8}{9} \phi \sin(\epsilon) + \frac{4}{3} \phi + \frac{8}{9} \frac{\phi^2}{\sin(\epsilon)} + \frac{L_2 - L_0}{6} \frac{\zeta}{9} \sin(\epsilon) \right) \quad (R10)
\]

where \( a \) is the convergence of cross-sectional area, \( \epsilon \) is the phase lag between high water and high water slack (or low water and low water slack), \( v \) is the velocity amplitude, \( \zeta \) is the tidal amplitude to depth ratio \( \left( \frac{\zeta}{\eta} = \frac{h}{\xi} \right) \), \( \phi \) is the river flow velocity to velocity amplitude ratio \( \left( \frac{\phi}{v} = U_r / v \right) \), \( L_0 \) and \( L_1 \) are linear coefficients as a function of \( \phi \) (Dronkers, 1964, P272-275), \( \theta \) is a correction factor for wave celerity \( \left( \theta = 1 - \left( \sqrt{1 + \xi^2} - 1 \right) \phi / \sin(\epsilon) \right) \), and \( f \) is the dimensionless friction factor \( \left( f = g / \left[ K^2 \eta^4 \left( 1 - 16 \xi^2 / 9 \right) \right] \right) \).

When river discharge dominates over tide \( \left( \phi \geq 1 \right) \), it is noted that

\[
L_0 = -2 - 4 \phi^2, \quad L_1 = 4 \phi \quad (R11)
\]

Substituting Eq. (R11) into Eq. (R10) then yields a quadratic equation for the dimensionless river discharge \( \phi \):

\[
\sigma_1 \phi^2 + \sigma_1 \phi + \sigma_5 = 0 \quad (R12)
\]

with
\[ \sigma_i = -\frac{4}{3} \frac{f u a \zeta}{h \sin(e)} \]  
\[ \sigma_2 = \frac{1}{\eta} \frac{d \eta}{dx} a \sin(e) - 2 \frac{f u a}{h c} + \left( \frac{1}{\eta} \frac{d \eta}{dx} a - 1 \right) \sqrt{1 + \frac{\zeta}{\sin(e)}} \]  
\[ \sigma_3 = -\frac{f u a}{h c} \left[ \frac{8}{9} \zeta \sin(e) + \frac{2}{9} \eta \frac{d \eta}{dx} \right] - \frac{1}{\eta} \frac{d \eta}{dx} a \left[ 1 + \frac{g \eta}{c \sin(e)} \right] \]

where the unknown variables \( e, c, \nu \) can be calculated with the explicit equations (i.e., the phase lag equation, the celerity equation and the scaling equation in Table 2 in the manuscript) for given water level observations.

Eq. (R12) gives two solutions:
\[ \varphi_1 = \frac{-\sigma_2 + \sqrt{\sigma_2^2 - 4 \sigma_3 \sigma_1}}{2 \sigma_1}, \quad \varphi_2 = \frac{-\sigma_2 - \sqrt{\sigma_2^2 - 4 \sigma_3 \sigma_1}}{2 \sigma_1} \]

in which the first root is always negative since both \( \sigma_i \) and \( \sigma_2 \) are always negative. Hence the positive solution for \( \varphi \) can only be given by the second root, which can be rewritten as:
\[ U_r = \nu \frac{-\sigma_2 - \sqrt{\sigma_2^2 - 4 \sigma_3 \sigma_1}}{2 \sigma_1} \]

We can see that Eq. (R17) is actually a modified Manning equation, accounting for friction and the effects of residual water level slope (i.e., \( d\bar{h}/dx \) implicitly included in the parameter of the cross-sectional area convergence \( a \), since \( \frac{1}{h} - \frac{1}{\bar{h}} = \frac{1}{\alpha} \left( 1 - \frac{1}{\bar{B}} - \frac{1}{\bar{B}} \frac{d \bar{B}}{dx} - \frac{1}{\bar{h}} \frac{d \bar{h}}{dx} \right) \)) and tidal damping (i.e., \( d\eta/dx \)). It can be seen from Figure R1 that the residual water level slope indeed has substantial influence on the seasonal variation of the cross-sectional area convergence \( a \).
c. Practical application
The approach presented here finds a closed form equation, which is an advantage for application. On the other hand, it is not obvious that the envelope tidal theory used here would work in tidal rivers where mixed tides are prominent. All three Eastern Pacific systems (the Fraser and Columbia Rivers and San Francisco Bay) we have examined have mixed tides. It is also unclear whether the present method could be used to estimate river flow variations on a scale of days, as is possible through use of the TDE method with CWT determination of tidal properties. In conclusion, any new methodology benefits from diverse approaches, and this is a useful contribution.

Our reply: It should be noted that the tidal theory used in this paper only focuses on a single dominated constituent (e.g., $M_2$). It is not applicable to tidal rivers where mixed tides are prominent. We shall explicitly mention this limitation in the text. On the other hand, the proposed analytical approach can be used to predict daily fresh water discharge for given observed tidal damping and residual water level on a scale of day. We are planning to collect more detailed tidal records and fresh water discharge (daily scale) in order to test the performance of the proposed method.

References:


