Dear Reviewer #2,

Thank you very much for your comments and recommendations, which have been very helpful to improve the quality of our manuscript. We shall revise the paper according to your comments, but for now we would like to provide replies to the issues you raised.

**General comments:**

1. The objective is to predict river discharge from observed tidal water levels. The method of the authors is limited to upstream sections where river discharge is dominated over tidal discharge. The example of Datong represents a station at 600 km from the mouth where the tidal range is 0.1-0.2 m (Fig. 5). The measured values may be easily disturbed by ship motions and other variations. The authors should explain why this topic is so important. River discharges are very well known from upstream data. Discharge-stage relationships based on data are available for most rivers. These are easy to use. The method of the authors is fairly complicated and it will be difficult to determine for which river section it will be sufficiently accurate. Figure 11 shows that the model is not so accurate at low river discharges. The authors should comment on these outliers.

Our reply: We appreciate the comments given by the reviewer. It is true that the proposed approach is only applicable to upstream sections where river discharge is dominated over tidal discharge. For the Yangtze estuary, the model is applicable to the river section upstream from around 350 km in the dry season and upstream from 150 km in the flood season. This limitation is due to the fact that the fresh water discharge is usually small compared to the amplitude of the tidal discharge in the seaward sections of an estuary, where the cross-sectional area is generally orders of magnitude larger than the cross-section of the river. Thus the influence of river discharge on tidal dynamics in these downstream parts is usually negligible, which suggests that there is no significant correlation between observed tidal water levels and fresh water discharge. We note that this is also the case for other methods to predict fresh water discharge in estuaries, such as by Jay and Kukulka (2003) and Moftakhari et al. (2013).

We agree that the relatively small values of the tidal range used in the analytical model could affect the performance of the proposed method. To reduce the statistical uncertainties, we used the monthly averaged tidal range in Maanshan (x=430 km) and Wuhu (x=482 km) stations and estimated the fresh water discharge at the location in between (i.e., x=456 km).

In the new version of the manuscript, we shall add a paragraph in the introduction to clarify the importance of our work:

“Due to the general dominance of tidal flows in the tidal region of an estuary, it is often difficult to determine the magnitude of the fresh water discharge accurately. Thus, discharge gauging stations are usually situated at locations outside the tidal region, even
though there may be additional tributaries or drainage areas within the tidal region. Knowing the fresh water discharge within the tidal region, however, may be important for water resource assessment or flood hazard prevention (e.g., Madsen and Skltern, 2005; Erdal and Karakurt, 2013; Liu et al., 2014), or for the analyses of sediment supply (e.g., Syvitski et al., 2003; Prandle, 2004; Wang et al., 2008), or for irrigation or estimating the effect of water withdrawals on salt intrusion (e.g., MacCready, 2007; Gong and Shen, 2011; Zhang et al., 2012), and for assessing the impacts of future climate change (e.g., Kukulka and Jay, 2003a, 2003b; Moftakhari et al., 2013). Although it is possible to estimate river flow by upscaling the gauged part of a catchment, such an estimate may be inaccurate, especially in poorly gauged catchments or in high-precipitation coastal areas (Jay and Kukulka, 2003).

Meanwhile, we shall provide more explanations of the proposed analytical approach. In fact, we can see that the introduced damping equation (i.e., Eq. T4 in Table 2) is a modified ‘Discharge-stage relationship’ that accounts for the effects of residual water level slope (i.e., \( d\bar{h} / dx \), where \( \bar{h} \) is the tidally averaged depth) and tidal damping (i.e., \( d\eta / dx \), where \( \eta \) is the tidal amplitude), while the resulted predictive Eq. (25) is a modified Manning equation that is applicable to estuaries. A detailed derivation can be found in Appendix A at the end. It is worth mentioning that such a modified Manning’s equation does provide more insights into our understanding of the interaction between fresh water discharge and tide in estuaries.

In Figure 11, the deviation from observations is mainly due to the statistical uncertainties in estimating tidal damping \( \delta = \frac{1}{\eta} \frac{d\eta}{dx} \), which is rather sensitive to changes in observed tidal amplitudes. In the revised paper, we propose to use a moving average to reduce the statistical uncertainties in the observed tidal damping \( \delta \) (see Figure R1a below). It can be seen from Figure R1b that the correspondence with observations is significantly improved by using a moving average of 5 months. In the revised paper we shall extend the discussion on this aspect.
2. The model equations can only be understood by a few specialists but not by a common reader. It is to the editor to decide whether the paper is intended for the audience of HESS. It is suggested to transfer all equations to an appendix. The text and figures should be given in physical descriptions and explanations. The model should be made available as e.g. a spreadsheet or otherwise (freeware) so that an interested reader can use and check the model. If the authors are unable to do so the I would advise to reject the paper (however to be decided by the editor).

Our reply: We apologize for the confusion of the many equations. It indeed takes time and effort to understand the whole story. Generally, this study is a subsequent contribution which builds on the previous work published in HESS entitled as: “Linking the river to the estuary: influence of river discharge on tidal damping” (Cai et al., 2014). Readers can obtain more details about the analytical model by reading this publication. We agree that the model should be made available for readers so that they could attempt to use the model. Detailed Matlab scripts will be given in the new version of the manuscript, including both the forward model (determining tidal properties from fresh water discharge) and the inverse model (determining fresh water discharge from tidal properties).
3. The authors should compare their model results to one-dimensional numerical model results to show that their model is sufficiently accurate. 1D numerical models are widely available and easy to operate. A simple estuary can be modeled in a few days with such a model. The authors should made clear what are the advantages of their model compared to a 1D numerical model.

Our reply: We very much appreciate this comment, which was also raised in our previous publication in HESS, i.e., Cai et al., 2014. For detailed comparison between analytical results and 1D numerical model, readers can refer to Cai et al. (2014). And we shall give more explanations of the advantage of analytical model compared with numerical models. Generally, the most important advantage of analytical tools is that they can offer a more efficient way of assessing the impact of future changes (e.g., fresh water withdrawal). Moreover, they provide direct insights into cause-effect relations, which generally are non-linear.

Specific comments:

1. Page 7064, line 6: please indicate what the phase lag is for a progressive wave. Do the authors refer to a frictionless progressive wave in a prismatic channel? The authors should further clarify whether the wave from their model is really progressive or not. In other words: is there only one wave travelling upstream or is there a second wave propagating in downstream direction due to continuous reflection by the convergence of the estuary. A discussion on this aspect would be very helpful in understanding tidal propagation in converging estuaries.

Our reply: Thank you very much for your suggestions. In convergent estuaries, the value of the phase lag $\varepsilon$ is always between 0 and $\pi/2$ (i.e., mixed wave, see Savenije, 2005, 2012). If $\varepsilon=\pi/2$, the tidal wave is a progressive wave, which corresponds to a frictionless wave in a prismatic channel. If $\varepsilon=0$, the tidal wave is an “apparently standing” wave (the wave is not formally a standing wave generated by the superimposition of incident and reflected waves; rather it is an incident wave that mimics a standing wave with a phase difference of 90° between water level and velocity and a wave celerity tending to infinity).

In our analytical model, we focus on analytical solutions for infinite length estuaries (long coastal plain estuaries), where there is not a reflected wave (see also Jay, 1991). We shall clarify this assumption in the revised paper. Moreover, the variation of the phase lag $\varepsilon$ along the estuary axis can be interpreted by using the phase lag Eq. (T1), i.e.,

$$\tan(\varepsilon) = \lambda / (\gamma - \delta)$$

in Table 2. We can see that a standing wave ($\varepsilon=0$) is characterized by an infinite wave celerity ($\lambda \to 0$) and that friction tends to move the system far from this asymptotic condition. On the other hand, a progressive wave ($\varepsilon=\pi/2$) is obtained only when the difference between $\gamma$ and $\delta$ is vanishingly small, i.e., when both friction and convergence are negligible (i.e., frictionless prismatic channel).

2. Page 7064, line 16: It is no clear why the influence of river discharge is that of increasing friction (by comparing Eq. (19) with Eq. (14). This could be shown in more detail in the appendix.
Our reply: We shall clarify the difference between Eq. (19) and Eq. (14) by introducing an artificial friction number as $\chi_r = \kappa \chi$, where $\kappa$ is a correction coefficient of the friction term due to river discharge. In particular, the following derivation will be included as an appendix in the revised paper.

In case of negligible river discharge, the damping equation is given by (see Cai et al., 2012):

$$
\delta = \frac{\mu^2}{1 + \mu} \left[ \gamma - \mu \lambda \left( \frac{2}{3} \mu \lambda + \frac{8}{9\pi} \right) \right] = \frac{\mu^2}{\beta \mu^2 + 1} \left[ \gamma \theta - \mu \lambda \left( \frac{2}{3} \mu \lambda + \frac{8}{9\pi} \right) \chi_r \right]
$$

(1.1)

To illustrate the influence of river discharge on the friction term, we introduce an artificial friction number $\chi_r$ due to river discharge. When accounting for the effect of river discharge, the damping Eq. (1.1) is modified as (see Cai et al., 2014):

$$
\delta = \frac{\mu^2}{\beta \mu^2 + 1} \left[ \gamma \theta - \mu \lambda \left( \frac{2}{3} \mu \lambda + \frac{8}{9\pi} \right) \right] = \frac{\mu^2}{\beta \mu^2 + 1} \left[ \gamma \theta - \mu \lambda \left( \frac{2}{3} \mu \lambda + \frac{8}{9\pi} \right) \chi_r \right]
$$

(1.2)

where $\beta$ and $\theta$ are defined in Eq. (5) and the coefficients $\kappa_1$ and $\kappa_2$ are given by

$$
\kappa_i = \begin{cases} 
1 + \frac{8}{3} \xi + \frac{\phi}{\mu \lambda} \left( \frac{\phi}{\mu \lambda} \right)^2 & \text{for } \phi < \mu \lambda \\
\frac{4}{3} \xi + \frac{2 \phi}{\mu \lambda} + \frac{4}{3} \xi \left( \frac{\phi}{\mu \lambda} \right)^2 & \text{for } \phi \geq \mu \lambda 
\end{cases}
$$

(1.3)

$$
\kappa_i = \frac{3 \pi}{16} L_\lambda - \frac{\pi L_n \xi}{8 \mu \lambda}
$$

(1.4)

As can be seen from Eqs. (1.1) and (1.2), the influence of fresh water discharge is basically that of increasing friction by a factor which is a function of $\phi$. Expressing the artificial friction number as $\chi_r = \kappa \chi$ provides an estimation of the correction of the friction term

$$
\kappa = \chi_r = \frac{2}{3} \mu \lambda \kappa_i + \frac{8}{9\pi} \kappa_2
$$

(1.5)

which is needed to compensate for the lack of considering fresh water discharge. It should be noted that both $\beta$ and $\theta$ are equal to unity if $\phi=0$. For $\phi>0$, the correction factors $\theta$ and $\beta$ have values smaller than unity, but are close to unity as long as $\xi<1$. Thus the influence of river discharge introduced by these parameters are less prominent compared with that of the friction term.

3. Page 7067, line 25: During calibration of the model the river discharge should be known. This is in contradiction to the conclusion that river discharges could be deduced from tidal water level observations only.

Our reply: Actually, there are two methods to determine the parameters $r_5$ and $K$. If there are some measurements of fresh water discharge, then the parameters $r_5$ and $K$ can be determined by calibrating the analytical model (i.e., Eq. (25)) against observations. Otherwise, these two parameters can be obtained by calibrating the analytical model for
tidal wave propagation without considering the effect of river discharge (e.g., Cai et al., 2012) against the observed tidal amplitude in the seaward part of the estuary, where the influence of river discharge on tidal damping is negligible. With these two calibrated parameters, the analytical model can be used to hindcast fresh water discharge based on the tidal water level observations. We shall clarify this point in the revised paper.

4. Page 7068, Eq. (25): From Eq. (21) it follows that $\alpha_1$ is always negative for relatively small values of $\zeta$. If $\alpha_2 > 0$ (which is not trivial) then the solution given by Eq. (25) is indeed positive (thus assuming $\zeta << 1$). Can the authors prove that the 2nd (positive) root never results in a real solution for?

Our reply: Indeed, $\alpha_1$ is always negative (indicating the denominator of Eq. (25) is always negative). It should be noted that the critical value for $\zeta$ (tidal amplitude to depth ratio) is 0.75 due to the Taylor approximation of the exponent of the hydraulic radius in the friction term (see Eq. (4)). In fact, we can see from Eq. (22) that $\alpha_2$ is also always negative since all the parameters are positive except $\delta$ for given $\zeta < 0.75$. Consequently, 

$$-\alpha_2 + \sqrt{\alpha_2^2 - 4\alpha_1\alpha_3}$$

is always positive. Thus the only positive solution can only be given by Eq. (25) with numerator of $-\alpha_2 - \sqrt{\alpha_2^2 - 4\alpha_1\alpha_3}$. We shall mention this in the revised paper.

Text comments:
We agree with the suggested corrections, which will be made in the revised paper. We thank the reviewer for the detailed reading.

Figure 5: how can the water depth decrease in upstream direction if there is a net river discharge? Table 3 suggests that a constant water depth for the 2 sections is being used (10.4 and 9.2 m). Or can the model handle a non-zero bed slope? Some explanation on this is required in the text.

Our reply: In Figure 5, it presents the averaged water depth rather than the averaged water level. It can be seen from the Figure R2 below that the averaged water level would not decrease in upstream direction.

We shall clarify that the model is able to account for variable depth along the estuary axis. The averaged depths presented in Table 3 are only used to show the characterized depths over the corresponding reach. The model uses variable depth.
Figure R2. Comparison between analytically computed monthly-averaged values (left-hand vertical scale: tidal amplitude; right-hand vertical scale: residual water level) and observations in the Yangtze estuary in 2005.

Appendix A: Revisiting the Manning equation

The momentum equation when written in a Lagrangean reference frame reads (Savenije, 2005, 2012):

$$\frac{dV}{dt} + g \frac{\partial h}{\partial x} + g \frac{\partial z_b}{\partial x} + \frac{h}{2} \frac{\partial \rho}{\partial x} + \frac{\rho \frac{\partial n^2}{\partial x}}{R} = 0$$  \hspace{1cm} (R1)

where $V$ is the Lagrangean velocity for a moving particle, $g$ is the acceleration due to gravity, $h$ is the water depth, $z_b$ is bottom elevation, $\rho$ is the water density, $n$ is Manning’s coefficient, and $R$ is the hydraulic radius.

For uniform steady flow in a prismatic channel, Eq. (R1) can be simplified as the well-known Manning equation by neglecting the first, the second and the fourth terms:

$$V = \frac{1}{n} R^{2/3} S^{1/2}$$  \hspace{1cm} (R2)

where $S = -\partial z_b / \partial x$ is the slope of the channel.

Hence the expression for river discharge is given by:

$$Q = AV = \frac{1}{n} AR^{2/3} S^{1/2}$$  \hspace{1cm} (R3)

where $A$ is the cross-sectional area.
For steady flow when depth may vary along a short section of the channel (e.g., during a flood), the residual water level slope \( (\partial h / \partial x) \) should be taken into account and Eq. (R1) reduces to:

\[
\frac{\partial h}{\partial x} + \frac{\partial z}{\partial x} + n^2 \frac{V|V|}{R^{4/3}} = 0
\]  

(R4)

Consequently, the Manning’s equation (R2) is modified as:

\[
V = \frac{1}{n} R^{2/3} \left( S - \frac{\partial h}{\partial x} \right)^{1/2}
\]  

(R5)

while the river discharge becomes:

\[
Q_t = Q_0 \left(1 - \frac{\partial h}{\partial x} \right)^{1/2}
\]  

(R6)

In the Lagrangean reference frame, the continuity equation can be written as:

\[
dV \frac{dt}{d} = r_s cV \frac{dh}{dx} - cV \left( \frac{1}{b} - \frac{1}{\eta} \frac{d\eta}{dx} \right)
\]  

(R7)

where \( r_s \) is the storage width ratio, \( b \) is the convergence of width, \( c \) is the wave celerity.

In a tidal region, it is noted that both depth and discharge change along the channel axis (i.e., varied unsteady flow). Thus, Eq. (R1) when combined with (R7) becomes (see Savenije, 2005, 2012):

\[
r_s cV \frac{dh}{dx} - cV \left( \frac{1}{b} - \frac{1}{\eta} \frac{d\eta}{dx} \right) + g \frac{\partial h}{\partial x} + g \frac{\partial z}{\partial x} + g \frac{h}{2\rho} \frac{\partial p}{\partial x} + gn^2 \frac{V|V|}{R^{4/3}} = 0
\]  

(R8)

An analytical expression for the tidal damping can be obtained by subtracting high water (HW) and low water (LW) envelopes while accounting for the effect of river discharge (Cai et al., 2014):

in the downstream tide-dominated zone, where \( U, < u \sin(\varepsilon) \),

\[
\frac{1}{\eta} \frac{d\eta}{dx} \left( \theta - r - \frac{\phi}{\sin(\varepsilon)} \varepsilon + \frac{g\eta}{c u \sin(\varepsilon)} \right) = \theta - f \frac{\varepsilon}{h c \sin(\varepsilon)} + \frac{16}{9} \phi \varepsilon \sin(\varepsilon) + \frac{2}{3} \frac{\phi^2}{\sin(\varepsilon)} + \frac{L_0}{6} - \frac{L_1}{9} \frac{\varepsilon}{\sin(\varepsilon)}
\]  

(R9)

in the upstream river discharge-dominated zone, where \( U, > u \sin(\varepsilon) \),

\[
\frac{1}{\eta} \frac{d\eta}{dx} \left( \theta - r - \frac{\phi}{\sin(\varepsilon)} \varepsilon + \frac{g\eta}{c u \sin(\varepsilon)} \right) = \theta - f \frac{\varepsilon}{h c \sin(\varepsilon)} - \frac{8}{9} \phi \varepsilon \sin(\varepsilon) + \frac{4}{3} \phi + \frac{\phi^2}{9 \sin(\varepsilon)} \varepsilon + \frac{L_0}{6} \frac{\varepsilon}{9 \sin(\varepsilon)}
\]  

(R10)

where \( a \) is the convergence of cross-sectional area, \( \varepsilon \) is the phase lag between high water and high water slack (or low water and low water slack), \( \nu \) is the velocity amplitude, \( \varepsilon \) is the tidal amplitude to depth ratio \( (\varepsilon = \eta / \overline{\eta}) \), \( \phi \) is the river flow velocity to velocity amplitude ratio \( (\phi = U, / \nu) \), \( L_0 \) and \( L_1 \) are linear coefficients as a function of \( \phi \) (Dronkers, 1964, P272-275), \( \theta \) is a correction factor for wave celerity \( (\theta = 1 - \sqrt{1 + \varepsilon^2 - 1} \phi / \sin(\varepsilon)) \), and \( f \) is the dimensionless friction factor \( (f = g / \left[ K h \overline{\eta}^{1/3} (1 - 16\varepsilon^2 / 9) \right]) \).

When river discharge dominates over tide \( (\phi \geq 1) \), it is noted that

\[
L_0 = -2 - 4\phi^2, \quad L_1 = 4\phi
\]  

(R11)

Substituting Eq. (R11) into Eq. (R10) then yields a quadratic equation for the dimensionless river discharge \( \phi \):

\[
\sigma_1 \phi^2 + \sigma_2 \phi + \sigma_3 = 0
\]  

(R12)

with
\[
\sigma_1 = -\frac{4}{3} \frac{f u a \zeta}{hc \sin(\epsilon)}
\]

\[
\sigma_2 = \frac{1}{\eta} \frac{d \eta}{d x} a \zeta - 2 \frac{f u a}{hc} \left( \frac{1}{\eta} \frac{d \eta}{d x} a - 1 \right) \sqrt{1 + \zeta - 1} \sin(\epsilon)
\]

\[
\sigma_3 = -\frac{f u a}{hc} \left[ \frac{8}{9} \zeta \sin(\epsilon) + \frac{2}{9} \zeta \right] - \frac{d \eta}{\eta} a \left[ \frac{1 + \frac{g \eta}{c u \sin(\epsilon)}}{1} \right]
\]

where the unknown variables \( \epsilon, c, \eta \) can be calculated with the explicit equations (i.e., the phase lag equation, the celerity equation and the scaling equation in Table 2 in the manuscript) for given water level observations.

Eq. (R12) gives two solutions:
\[
\varphi_1 = -\sigma_2 + \sqrt{\sigma_1^2 - 4\sigma_1 \sigma_2}, \quad \varphi_2 = -\sigma_2 - \sqrt{\sigma_1^2 - 4\sigma_1 \sigma_2}
\]
in which the first root is always negative since both \( \sigma_1 \) and \( \sigma_2 \) are always negative. Hence the positive solution for \( \varphi \) can only be given by the second root, which can be rewritten as:
\[
U_r = \nu - \frac{\sigma_2 - \sqrt{\sigma_1^2 - 4\sigma_1 \sigma_2}}{2\sigma_1}
\]

We can see that Eq. (R17) is actually a modified Manning equation, accounting for friction and the effects of residual water level slope (i.e., \( \frac{d h}{d x} \)) implicitly included in the parameter of the cross-sectional area convergence \( a \), since \( \frac{1}{a} = \frac{1}{b} + \frac{1}{d} = -\frac{1}{B} \frac{d B}{d x} - \frac{1}{h} \frac{d h}{d x} \)
and tidal damping (i.e., \( \frac{d \eta}{d x} \)). It can be seen from Figure R3 that the residual water level slope indeed has substantial influence on the seasonal variation of the cross-sectional area convergence \( a \).
Figure R3. Seasonal variation of the cross-sectional area convergence $a$ due to the changes in residual water level slope $\frac{d\bar{h}}{dx}$ at $x=456$ km in the Yangtze estuary.

References:


