Notes on the estimation of resistance to flow during flood wave propagation

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Abstract. The paper discusses methods of expressing and evaluating resistance to flow in an unsteady flow. Following meaningful trends in hydrological sciences, the paper suggests abandoning, where possible, resistance coefficients in favour of physically based variables such as shear stress and friction velocity to express flow resistance. Consequently, an acknowledged method of flow resistance evaluation based on the relations derived from flow equations is examined. The paper presents both a theoretical discussion of various aspects of flow resistance evaluation and the application of the method to field data originating from artificial dam-break flood waves in a small lowland river. As the method is prone to many errors due to the scarcity and the uncertainty of measurement data, the aim of the paper is to provide suggestions on how to apply the method to enhance the correctness of the results. The main steps in applying the method include consideration of the shape of the channel, the type of wave, the method of evaluating the gradient of the flow depth, and the assessment of the uncertainty of the result. Friction velocity and the Manning coefficient are compared in terms of resistance to flow variability during flood wave propagation. It is concluded that the Manning coefficient may be a misleading indicator of the magnitude of resistance in unsteady flow, and to be inferior to physically based variables in such cases.

1 Introduction

Resistance is one of the most important factors affecting the flow in open channels. In simple terms it is the effect of water viscosity and the roughness of the channel boundary which result in friction forces that retard the flow. The largest input into the resistance is attributed to water-bed interactions.

The resistance and its impact on flow parameters is traditionally characterised by resistance coefficients such as Manning $n$, Chezy $C$ or Darcy-Weisbach $f$. However, their application has been challenged in recent years (Carrivick, 2010; Ferguson, 2010; Knight and Shino, 1996; Lane, 2005;
Strupczewski and Szymkiewicz, 1996a, b; Whatmore and Landström, 2010). The strongest critique is directed towards the most popular resistance coefficient – Manning \( n \). This was supposed to be invariant with the water stage; however, in practise \( n \) very often varies (Ferguson, 2010). It is not clear how to interpret this variability in the light of resistance definition. The flow resistance equation (Eq. 1) relating flow parameters through Manning \( n \) was originally derived for steady uniform flow conditions:

\[
n = \frac{1}{U} R^{2/3} S^{1/2},
\]

(1)

where \( R \) – hydraulic radius [m], \( S \) – friction slope [-], \( U \) – mean cross-sectional velocity [\( m s^{-1} \)]. For this reason, the resistance coefficient is meaningful only in such cases. However, its application is accepted for gradually varied flows, especially in flood wave modelling. Constant values of Manning \( n \) are usually applied in such studies; a procedure which has been questioned (Julien et al., 2002), as resistance coefficients have been shown to vary during flood wave propagation (Fread, 1985; Julien et al., 2002). However, in such cases resistance coefficients are mainly model parameters, since physical interpretation of variable Manning \( n \) is not obvious. Further aspect of possible misinterpretation of variable Manning \( n \) is the fact that trend of \( n \) versus flow rate \( Q \) may be falling or rising depending on the geometry of wetted area. Fread (1985) reported, based on computations of \( n \) from extensive data of flood waves in American rivers, that the trend is falling when inundation area is relatively small compared to inbank flow area; in reverse case the trend is rising. This inconsistency stems from the fact that additional factors affect flow resistance in unsteady flow compared to steady flow. As Yen (2002) presents after Rouse (1965), besides water flow-channel boundary interactions represented by skin friction and form drag, resistance has two more components: wave resistance from free surface distortion and resistance due to local acceleration or flow unsteadiness. The concept of resistance coefficients is not able to reflect the variability of the resistance in such cases.

Many authors argue that the description of resistance to flow is unsatisfactory (Beecham et al., 2005; Chaudhry, 2011; Knight, 2013a, b; Knight et al., 2010; Powell, 2014; Schmidt and Yen, 2008; Singh, 1996; Yen, 2002). For the above reasons, it seems more meaningful to consider friction force, rather than resistance coefficients, as a basic term expressing resistance to flow. In this respect, resistance is represented by boundary shear stress \( \tau \) which refers directly to the shearing force acting on the channel boundary, with the dimension of Pascal [Pa]. Alternatively, shear stress is expressed in velocity units [\( m s^{-1} \)] by friction (shear) velocity \( u_* \), which is related to the shear stress and friction slope by the equation:

\[
 u_* = \sqrt{\frac{\tau}{\rho}} = \sqrt{gRS},
\]

(2)

where \( g \) – gravity acceleration [\( m s^{-2} \)], \( \rho \) – density of water [\( kg m^{-3} \)]. As shear stress and friction velocity describe directly physical processes, there are no background theoretical doubts about
their soundness in unsteady flow unlike in the case of Manning $n$. Moreover, interpretation of their variability is straightforward – they rise with rising resistance to flow.

The proper definition and understanding of shear stress and friction velocity is of great importance, since shear stress is an intrinsic variable in a number of hydrological problems, such as bed load transport, rate of erosion and contaminants transport (Garcia, 2007; Julien, 2010; Kalinowska and Rowiński, 2012; Kalinowska et al., 2012; van Rijn, 1993). Boundary shear stress is expressed on a range of spatial scales from a point value to a global one (Yen, 2002). The following types of boundary shear stress are defined: local bed shear stress (Khodashenas et al., 2008), average bed shear stress; average wall shear stress (Khiadani et al., 2005); and finally average boundary shear stress, i.e. averaged over a wetted perimeter (Khiadani et al., 2005). It should be noted that the nomenclature is inconsistent, and other authors may use different terminology (Ansari et al., 2011; Khiadani et al., 2005; Khodashenas et al., 2008; Knight et al., 1994). Moreover, a number of definitions of friction velocity exist (Pokrajac et al., 2006). Hence, for clarity a reference to a definition is necessary in each study.

It is difficult to measure bed shear stress directly. The direct method, which uses a floating element balance type device, enables the measurement of the force acting tangentially on a bed, and is used in both field (Gmeiner et al., 2012) and laboratory studies (Kaczmarek and Ostrowski, 1995); however, the results are prone to high uncertainty. The majority of methods measure bed shear stress indirectly, e.g. using hot wire and hot film anemometry (Albayrak and Lemmin, 2011; Nezu et al., 1997), a Preston tube (Molinas et al., 1998; Mohajeri et al., 2012), methods that take advantage of theoretical relations between shear stress and the horizontal velocity distribution (Graf and Song, 1995; Khiadani et al., 2005; Sime et al., 2007; Yen, 2002), methods based on Reynolds shear stress (Biron et al., 2004; Campbell et al., 2005; Czernuszenko and Rowiński, 2008; Dey and Barbhuiya, 2005; Dey and Lambert, 2005; Dey et al., 2011; Graf and Song, 1995; Nezu et al., 1997; Nikora and Goring, 2000) or turbulent kinetic energy (Galperin et al., 1988; Kim et al., 2000; Pope et al., 2006), or methods that incorporate double-averaged momentum equation (Pokrajac et al., 2006). Despite the fact that there is a variety of methods, a handful of them are feasible for application in unsteady flow conditions.

In this paper we apply formulae derived from flow equations to evaluate both friction velocity and Manning $n$, because these formulae require input variables which are feasible to be monitored during passage of a flood wave – flow rate or velocity and water stage. They have been claimed to be reasonable means of friction velocity assessment in unsteady flow by a number of authors, e.g. Afzalimehr and Anctil (2000); De Sutter et al. (2001); Ghimire and Deng (2011, 2013); Graf and Song (1995); Guney et al. (2013); Rowiński et al. (2000); Shen and Diplas (2010); Tu and Graf (1993); nonetheless, in-depth analysis is still needed because this method provides uncertain results when measurement data are scarce. For this reason, the paper aims to complement the existing research studies in this field.
The objectives of this paper are: (1) to describe how to determine friction velocity for unsteady flow with critical review of existing methods based on flow equations, (2) to provide methodology to minimise uncertainty of resistance to flow evaluated by relationships derived from flow equations, (3) to illustrate inconsistency between the results of friction velocity and Manning $n$ results in the context of resistance evaluation in unsteady flow. The paper is structured as follows. Section 2 presents settings of dam-break field experiment and measurement data. Methodology of evaluation of friction velocity and Manning $n$ in unsteady flow with focus on detailed aspects of application of formulae derived from flow equations is outlined in Sect 3. In Sect 4 results of computations of friction velocity and Manning $n$ are presented for field experiment. In Sect 5 conclusions are provided. The problem presented herein has been partially considered in the unpublished Ph.D. thesis of the first author of this paper (Mrokowska, 2013).

2 Experimental data

The data originate from an experiment carried out in the Olszanka River which is a small lowland river in central Poland (see upper panel of Fig. 1). The aim of the experiment was to conduct measurements of hydraulic properties during flood wave propagation. To achieve this goal, a wooden dam was constructed across the river, then the dam was removed in order to initiate a wave. Then, measurements were carried out at downstream cross-sections. Two variables were monitored: the velocity and the water stage. Velocities were measured by propeller current meter in three verticals of a cross-section at two water depths. Water stage was measured manually by staff gage readings. Geodetic measurements of cross-sections were performed prior to the experiment. An in-depth description of the experimental settings in the Olszanka River may be found in (Szkutnicki, 1996; Kadłubowski and Szkutnicki, 1992), and a description of similar experiments in the same catchment is presented in (Rowiński and Czernuszenko, 1998; Rowiński et al., 2000).

In the study, two cross-sections, denoted in Fig. 1 as CS1 and CS2, are considered. Cross-section CS1 was located about 200 m from the dam, and cross-section CS2 about 1600 m from it. The shape of the cross-sections is presented in the bottom panel of Fig. 1. Both were of trapezoidal shape with side slopes of $m_1 = 1.52$, $m_2 = 1.26$ and $m_1 = 1.54$, $m_2 = 1.36$ for CS1 and CS2, respectively (Fig. 2). The bed slope $I$ was 0.0004 for CS1 and 0.0012 for CS2.

Four data sets are used in this study, denoted as follows: Ol-1, Ol-2, Ol-3, Ol-4. The first set was collected in cross-section CS1 and the others in cross-section CS2. Data sets Ol-1 and Ol-2 were collected during the passage of the same wave on 26 April 1990. Data set Ol-3 was collected on 27 April 1990, and Ol-4 on 9 May 1991. Measurement data used in this paper were collected at the beginning of vegetation season when banks were slightly vegetated. The bed was composed of sand and silt with no significant bed forms. Figure 3 illustrates the results of the measurements — the temporal variability of mean velocity ($U$) and flow depth ($h$). Mean velocity has been evaluated by
the velocity-area method from propeller current meter readings and flow depth has been calculated
from geodetic data and measurements of water stage. Please note the time lag between maximum
values of $U$ and $h$, which indicates the non-kinematic character of the waves. Similarly, the time lag
may be observed in the data of Shen and Diplas (2010). Consider that waves represent a gradually-
varied one-dimensional subcritical flow, with a Froude number ($Fr = U/\sqrt{gh}$) smaller than 0.33.
The loop-shaped relationship between flow rate ($Q$) and water stage ($H$) may be observed in Fig. 4.

From the figure it can be seen that the rating curves are not closed for Ol-1, Ol-2 and Ol-3, which is
probably caused by too short series of measurement data.

Data set Ol-1 was applied in (Mrokowska and Rowiński, 2012; Rowiński et al., 2000), and data
set Ol-4 in (Mrokowska et al., 2013), and to the authors knowledge, none of the data sets have been
utilised elsewhere in the context of the evaluation of friction velocity.

3 Methods

Formulae for friction velocity under unsteady flow conditions are usually derived from flow equa-
tions – the momentum conservation equation and the continuity equation in both forms: the 2D
Navier-Stokes Reynolds averaged equations (Dey and Lambert, 2005; Graf and Song, 1995; Nezu et al.,
1997) and 1D St. Venant model (Ghimire and Deng, 2011; Rowiński et al., 2000; Shen and Diplas,
2010). Despite the fact that there are many ways of deriving the formulae, when the same assump-
tions of flow conditions are made, the formulae are equivalent. Please note that the same approach
may be applied to evaluate Manning $n$ from flow equations.

The St. Venant model for a rectangular channel which comprises Eqs. (3) and (4) is the most
frequently used mathematical model to derive formulae on resistance:

\begin{align}
U \frac{\partial h}{\partial x} + h \frac{\partial U}{\partial x} + \frac{\partial h}{\partial t} &= 0, \\
\frac{\partial h}{\partial x} + \frac{h}{g} \frac{\partial U}{\partial x} + \frac{1}{g} \frac{\partial U}{\partial t} + S - I &= 0,
\end{align}

where $I$ – bed slope [-], $t$ – time [s], $x$ – longitudinal coordinate [m]. Equation (3) is the continuity
equation and Eq. (4) is the momentum balance equation which the terms represent as follows: the
gradient of flow depth (hydrostatic pressure term), advective acceleration, local acceleration, friction
slope and bed slope. Further on, derivatives will be denoted by Geek letters to stress that they are
treated as variables, namely $\zeta = \frac{\partial U}{\partial t}$ [m s$^{-2}$], $\eta = \frac{\partial h}{\partial t}$ [m s$^{-1}$], $\vartheta = \frac{\partial h}{\partial x}$ [-].

To evaluate Manning $n$ or friction velocity, friction slope $S$ is extracted from the above set of
equations and incorporated into Eq. (1) or Eq. (2), respectively. We would like to stress again that
it is questionable if this way of $S$ assessment in evaluation of Manning $n$ is meaningful. However,
relationships derived this way are applied in this paper for comparative purposes.

Scarce and uncertain measurement data very often restrict the relationships on resistance to sim-
plified forms. Among simplifications applied in literature there are simplifications of momentum
balance equation terms (i.e. type of wave) and simplifications of channel geometry that affect the number of terms in the relationships. Another simplification which is very often applied due to limited spatial data refers to the evaluation of $\frac{\partial h}{\partial x}$. Simplified methods are welcome, especially for practitioners. However, they must be justified properly, and there seems to be a gap here. It is crucial to choose the best method for a case under study. In proceeding sections a thorough review of each aspect of simplifications, description of uncertainty evaluation and finally a methodology for evaluation of resistance to flow is given. One may choose the best method to considered case from the presented herein. To the best of our knowledge such analysis is presented for the first time.

### 3.1 Simplification of momentum balance equation terms (type of wave)

Equations (3) and (4) in the full form represent a dynamic wave. If the acceleration terms of Eq. (4) are negligible, they may be eliminated, and the model for a diffusive wave is obtained. Further omission of the hydrostatic pressure term leads to the kinematic wave model, in which only the term responsible for gravitational force is kept. The simplifications of the St. Venant model have been investigated in many papers in the context of flood wave modelling (Aricó et al., 2009; Dooge and Napiórkowski, 1987; Moussa and Bocquillion, 1996; Yen and Tsai, 2001). Some authors have concluded that the diffusive approximation is satisfactory in the majority of cases (Ghimire and Deng, 2011; Moussa and Bocquillion, 1996; Yen and Tsai, 2001), especially for lowland rivers. However, according to Gosh (2014); Dooge and Napiórkowski (1987); Julien (2002), in the case of upland rivers, i.e. for average bed slopes, it could be necessary to apply the full set of St. Venant equations. Aricó et al. (2009) have pointed that this may be the case for mild and small bed slopes. Moreover, artificial flood waves, such as dam-break-like waves (Mrokowska et al., 2013), and waves due to hydro-peaking (Shen and Diplas, 2010; Spiller et al., 2014), are of a dynamic character. On the other hand, when the bed slope is large, then the gravity force dominates and the wave is kinematic (Aricó et al., 2009). Because of the vague recommendations in the literature, we suggest analysing whether simplifications are admissible separately in each studied case.

The friction velocity derived from the Eqs. (3) and (4) represents the value averaged over a wetted perimeter: the bulk variable. If the channel width is much larger than the flow depth, the mean cross-sectional velocity $U$ is equivalent to the depth-averaged velocity above any location of the bed, and the hydraulic radius $R$ may be substituted by the flow depth $h$. Consequently, the bulk friction velocity is equivalent to the bed friction velocity.

Formulae for friction velocity encountered in the literature may be classified into five groups according to the type of flow. They are the formulae on both bed $u_{b}$ and bulk $u_{a}$ friction velocity. They may be recalculated to Manning $n$ if necessary.

1. Formulae for unsteady non-uniform flow in a rectangular channel (dynamic wave):
Graf and Song (1995) derived the formula from the 2D momentum balance equation:

\[ u_{ab} = \left[ ghI + (-gh\vartheta(1-(Fr)^2)) + (\eta - h\zeta) \right]^{\frac{1}{2}}, \]  

(5)

where Fr – Froude number [-].

Rowiński et al. (2000), and next Shen and Diplas (2010) applied the formula derived from the St. Vernant set of equations:

\[ u_{ab} = \left[ gh\left(I + \left(\frac{U^2}{gh} - 1\right)\vartheta + \frac{U}{gh}\eta - \frac{1}{g}\zeta\right)\right]^{\frac{1}{2}}. \]  

(6)

Tu and Graf (1993) derived the equation from the St. Venant momentum balance equation:

\[ u_{ab} = \left[ gh\left(I + \frac{1}{C^2} \eta - \frac{1}{g}\zeta\left(1 - \frac{U}{C}\right)\right)\right]^{\frac{1}{2}}, \]  

(7)

where \(C\) – wave celerity [\(m\ s^{-1}\)].

Dey and Lambert (2005) derived the formula from the 2D Reynolds equations which incorporated data on bed roughness. To see the equation please refer to (Dey and Lambert, 2005).

2. Formulae for diffusive wave approximation:

Guney et al. (2013) applied the formula derived from the St. Venant momentum balance equation:

\[ u_{as} = \left[ gR(I - \vartheta)\right]^{\frac{1}{2}}. \]  

(8)

Ghimire and Deng (2011) combined the diffusive wave formula with the kinematic wave assumption to assess \(\vartheta\), and obtained the following formula:

\[ u_{as} = \left[ gR\left(I + \frac{1}{BC^2} \frac{\partial Q}{\partial t}\right)\right]^{\frac{1}{2}}, \]  

(9)

where \(B\) – width of rectangular channel [\(m\)], \(Q\) – flow rate [\(m^3\ s^{-1}\)].

3. Formula for steady non-uniform flow derived by Afzalimehr and Anctil (2000) from the 1D continuity and momentum balance equations:

\[ u_{ab} = \left[ gh\left(I - \vartheta \left(1 - (Fr)^2\right)\right)\right]^{\frac{1}{2}}. \]  

(10)

4. Formula for flow with negligible advective acceleration derived by Nezu et al. (1997) for \(\tau = u_z^2\) from the 2D momentum and continuity equation:

\[ \tau = gS_w R - \frac{1}{B} \frac{\partial Q}{\partial t}, \]  

(11)

where \(S_w\) – water surface slope \(S_w = I - \vartheta\) [-].
5. Formula for steady flow or kinematic wave which neglect all variables responsible for the temporal and spatial variability of flow:

\[ u_s = \left[ gRI \right]^\frac{1}{2}. \]  

(12)

### 3.2 Cross-sectional channel geometry

Besides a rectangular channel, another widely analysed channel shape is a trapezoidal one. The distribution of the shear stress in the steady flow along the boundary of a trapezoidal channel has been studied experimentally (Knight et al., 1992, 1994) and theoretically (Ansari et al., 2011). The bulk friction velocity for a dynamic wave in a trapezoidal channel may be evaluated from the relation derived from the St. Venant model (Eqs. 13 and 14) (Mrokowska et al., 2013). The cross sectional shape with symbols is depicted in Fig. 2.

\[
U(b + mh) \frac{\partial h}{\partial x} + \left( b + \frac{m}{2} h \right) h \frac{\partial U}{\partial x} + \frac{1}{g} \frac{\partial U}{\partial t} + S - I = 0, \tag{13}
\]

\[
\frac{\partial h}{\partial x} + \frac{U}{g} \frac{\partial U}{\partial x} + \frac{1}{g} \frac{\partial U}{\partial t} + S - I = 0, \tag{14}
\]

where \( b \) – width of river bed [m], \( h \) – here: the maximum flow depth in the channel section (trapezoidal height) [m], \( m = m_1 + m_2 \), \( m_1 \) and \( m_2 \) – side slopes [-] defined as \( m_1 = l_1/h \) and \( m_2 = l_2/h \). The friction velocity derived analytically from the set of equations is represented by the following formula:

\[
u_{sa} = \left[ gR \left( I + \left( \frac{U^2}{g} b + mh \frac{b h m^2}{2} - 1 \right) \vartheta + \frac{U}{g} \frac{b + mh}{bh + m b^2} \left( \frac{1}{g} \eta - \frac{1}{g} \zeta \right) \right) \right]^\frac{1}{2}. \tag{15}\]

Equation (15) is considered in this study, as Olszanka River has nearly trapezoidal cross-section.

### 3.3 Evaluation of the gradient of flow depth \( \vartheta \)

The gradient of flow depth \( \vartheta = \frac{\partial h}{\partial x} \) is a significant variable in both dynamic (Eqs. 5, 6, 15) and diffusive (Eq. 8) friction velocity formulae. Moreover, the evaluation of \( \vartheta \) is widely discussed in hydrological studies on flow modelling and rating curve assessment (Dottori et al., 2009; Perumal et al., 2004; Schmidt and Yen, 2008). The gradient of flow depth is evaluated based on flow depth measurements at one or a few gauging stations. Due to the practical problems with performing the measurements, usually only one or two cross-sections are used. This constitutes one crucial obstacle when seeking friction velocity.

#### 3.3.1 Kinematic wave concept

According to the kinematic wave concept, the gradient of flow depth is evaluated implicitly based on measurements in one cross-section by Eqs. (16) or (17) (Graf and Song, 1995; Perumal et al., 2004). This approach is encountered in friction velocity assessment studies (De Sutter et al., 2001;
Graf and Song, 1995; Ghimire and Deng, 2011, 2013; Tu and Graf, 1993). However, this method has been challenged in rating-curve studies (Dottori et al., 2009; Perumal et al., 2004; Schmidt and Yen, 2008) due to its theoretical inconsistency. As Perumal et al. (2004) presented, Jones introduced the concept in 1915 in order to overcome the problem of \( \frac{\partial h}{\partial x} \) evaluation in reference to looped rating curves, i.e. non-kinematic waves. The looped shape of non-kinematic waves results from the acceleration of flow and the gradient of flow depth (Henderson, 1963; Silvio, 1969). The kinematic wave, on the other hand, has a one-to-one relationship between the water stage and flow rate, which is equivalent to a steady flow rating curve. Both rating curves are illustrated in the upper panel of Fig. 5 after (Henderson, 1963). The kinematic wave concept results in the Jones formula which is applied in this study:

\[
\theta_{\text{kin}} = \frac{\partial h}{\partial x} = -\frac{1}{C} \frac{\partial h}{\partial t}. \tag{16}
\]

Furthermore, \( \theta \) may be expressed by the temporal variation of the flow rate instead of the flow depth (Ghimire and Deng, 2011; Julien, 2002), which leads to the following approximation:

\[
\theta = \frac{\partial h}{\partial x} = -\frac{1}{BC^2} \frac{\partial Q}{\partial t}. \tag{17}
\]

Both approximations, Eqs. (16) and (17), affect the time instant at which \( \frac{\partial h}{\partial x} = 0 \). As shown in the upper panel of Fig. 5, in the case of a non-kinematic subsiding wave, the peak of the flow rate \( \frac{\partial Q}{\partial t} = 0 \) in a considered cross-section is followed by the temporal peak of the flow depth \( \frac{\partial h}{\partial t} = 0 \), while the spatial peak of the flow depth \( \frac{\partial h}{\partial x} = 0 \) is the final one. The bottom panel of Fig. 5 presents schematically the true arrival time of \( \frac{\partial h}{\partial x} = 0 \) for the non-kinematic wave, and the arrival time approximated by the kinematic wave assumption in the form of Eq. (16) and Eq. (17). Both formulae underestimate the time instant at which \( \frac{\partial h}{\partial x} = 0 \). As a matter of fact, from the practical point of view, the evaluation of the friction velocity is exceptionally important in this region, as intensified transport processes may occur just before the wave peak (Bombar et al., 2011; De Sutter et al., 2001; Lee et al., 2004). Consequently, it seems that the admissibility of the kinematic wave assumption should be thoroughly verified for a wave under consideration.

In order to apply the kinematic wave approximation, the wave celerity must be evaluated. Celerity can be assessed by the formula for a wide rectangular channel derived from the Chezy equation (Eq. 18) (Henderson, 1963; Julien, 2002), and it is applied in this study.

\[
C = \frac{3}{2} U. \tag{18}
\]

Tu (1991); Tu and Graf (1993) proposed another method for evaluating \( C \):

\[
C = U + h \frac{\partial U}{\partial t} / \frac{\partial h}{\partial t}. \tag{19}
\]

However, we would like to highlight the fact that in Eq. (19) \( \frac{\partial h}{\partial t} \) is in the denominator, which constrains the application of the method. As a result, a discontinuity occurs for the time instant at which
\[ \frac{\partial h}{\partial t} = 0. \] When the results of Eq. (19) are applied in Eq. (16), the discontinuity of \( \vartheta \) as a function of time occurs at the time instant at which \( C = 0 \), which is between \( t(\frac{\partial h}{\partial t} = 0) \) and \( t(\frac{\partial h}{\partial t} = 0) \). This effect is illustrated in the section on field data application (Sect 4.1).

We propose another approach for evaluation of \( \vartheta \), which is compatible with the kinematic wave concept, but does not require the evaluation of temporal derivatives, and for this reason may appear to be easier to be used in some cases. Let us assume a reference cross-section \( P_0 \) and two cross-sections \( P_1 \) and \( P_2 \) located at a small distance \( \Delta s \) downstream and upstream of \( P_0 \), respectively. Knowing the \( h(t) \) relationship, let us shift this function to \( P_1 \) and to \( P_2 \) by \( \Delta t = \frac{\Delta s}{c} \) in the following way: \( h_1(t) = h_0(t - \Delta t) \), and \( h_2(t) = h_0(t + \Delta t) \). The spatial derivative \( \frac{\partial h}{\partial x} \) is next evaluated as follows:

\[ \vartheta_{m} = \frac{\partial h}{\partial x} = \frac{h_2(t) - h_1(t)}{2\Delta s}. \]

The method is denominated as wave translation method and is denoted as \( \vartheta_{m} \), and is applied in this study.

### 3.3.2 Linear approximation based on two cross-sections

Because of the drawbacks of kinematic wave approximation, it is recommended to evaluate the gradient of the flow depth based on data from two cross-sections (Aricó et al., 2008, 2009; Dottori et al., 2009; Julien, 2002; Warmink et al., 2013), which is, in fact, a two-point difference quotient (backward or forward). Nonetheless, a number of problematic aspects of this approach have been pointed out. Firstly, Koussis (2010) has stressed the fact that flow depth is highly affected by local geometry; hence, the proper location of the cross-sections is a difficult task. Moreover, Aricó et al. (2008) have pointed that lateral inflow may affect the evaluation of the gradient of flow depth, and for this reason the cross-sections should be located close enough to each other to allow the assumption of negligible lateral inflow. On the other hand, the authors have claimed that the distance between cross-sections should be large enough to perform a robust evaluation of the flow depth gradient. The impact of distance between cross-sections on the gradient of flow depth has been studied in (Mrokowska et al., 2015) with reference to dynamic waves generated in a laboratory flume. The results have shown that with a too long distance, the gradient in the region of the wave peak is misestimated due to the linear character of approximation. On the other hand, with a too short distance, the results may be affected by fluctuations of the water surface which in such case are large relative to the distance between cross-sections.

Another drawback of the method is the availability of data. Very often, data originate from measurements which have been performed for some other purpose. Consequently, the location of gauging stations and data frequency acquisition do not meet the requirements of the evaluation of the gradient of flow depth (Aricó et al., 2009). The latter problem applies to the case studied in this paper.
3.3.3 Higher order approximation

Due to the linear character of a two-point (backward and forward) difference quotient, it is not able to represent properly the peak region of a flood wave. The better approximation of the derivative requires a difference quotient of a higher order. Then, the question arises as to how many measurement cross-sections are necessary to properly reflect the realistic value of the derivative. In (Mrokowska et al., 2015) it has been stated that for better representation of $\vartheta$ the central difference quotient (Eq. (21)) should be applied:

$$\frac{\partial h}{\partial x} \approx \frac{h(x + \Delta x) - h(x - \Delta x)}{2\Delta x},$$

(21)

where $\Delta x$ – distance between cross-sections [m]. It is difficult to draw conclusions about the application of the method in natural conditions, as similar problems to those described in Sect 3.3.2 are likely to occur. The feasibility of the application of the method in the field requires further analysis. Due to not enough measurement cross-sections, this approach could not be tested for data sets from the Olszanka River.

3.3.4 Uncertainty of input data and the results

The friction velocity, as with other physical variables, should be given alongside the level of uncertainty of the results (Fornasini, 2008). The uncertainty of results depends on the evaluation method and the quality of the data. As shown in the proceeding sections, neither of these is perfect when a friction velocity assessment is performed. For this reason, an appropriate method of uncertainty evaluation must be chosen in order to obtain information about the quality of the result. Friction velocity is usually applied to further calculations, and for this reason information about the uncertainty of results is of high importance. In the case of unrepeatable experiments Mrokowska et al. (2013) have suggested applying deterministic approach – the law of propagation of uncertainty (Holman, 2001; Fornasini, 2008), which for Eq. (15) takes the form of Eq. (22):

$$\Delta u_{max} \approx \left| \frac{\partial u_s}{\partial \zeta} \right| \Delta \zeta + \left| \frac{\partial u_s}{\partial \eta} \right| \Delta \eta + \left| \frac{\partial u_s}{\partial \vartheta} \right| \Delta \vartheta + \left| \frac{\partial u_s}{\partial \Delta} \right| \Delta \Delta + \left| \frac{\partial u_s}{\partial \Delta} \right| \Delta \Delta + \left| \frac{\partial u_s}{\partial \Delta} \right| \Delta \Delta + \left| \frac{\partial u_s}{\partial \Delta} \right| \Delta \Delta + \left| \frac{\partial u_s}{\partial \Delta} \right| \Delta \Delta + \left| \frac{\partial u_s}{\partial \Delta} \right| \Delta \Delta.$$

In this method the highest possible values of uncertainty are assessed based on the knowledge of measurement techniques and experimental settings. Hence, this method provides maximum uncertainty of a result.

3.3.5 Suggestions on the application of formulae on resistance to flow – methodology

The preceding sections have demonstrated that the application of friction velocity formulae requires a thorough analysis of flow conditions and available methods. To sum up, the following issues should be considered during the evaluation of resistance to flow from flow equations:
1. What is the shape of the channel – is simplification of the channel geometry applicable?

2. What methods of evaluating input variables, especially $\partial = \frac{\partial h}{\partial x}$, are feasible in the case under study?

3. Is it admissible to apply simplified formula with regard to the type of wave?

4. What is the uncertainty of the input variables, and which of them are most significant?

Although the above considerations seem to be quite universal, their significance will be illustrated based on a set of data from an experiment carried out in natural settings. The detailed analyses shown for these practical cases may provide advise on how to proceed in similar situations.

4 Results

Methods described in Sect 3 are applied to experimental data from the Olszanka River. As in the case under study, the channel is of a trapezoidal shape with a small width to depth ratio, Eq. (15) is applied.

4.1 Evaluation of the gradient of flow depth

As presented in Sect 2 a number of measurements were performed in the Olszanka River. Nonetheless, the location and the number of cross-sections constrain the evaluation of spatial derivative $\partial$. It is feasible to use the data from only two subsequent cross-sections: for data set Ol-1, $\partial$ could be evaluated based on cross-sections CS1 and CS1a located 107 m downstream of CS1, and for the other data sets based on CS2 and CS2a located 315 m upstream of CS2 (upper panel of Fig. 1).

The following methods of evaluating $\partial$ are examined:

- Linear approximation denoted as $\partial_{\text{lin}}$

- Kinematic wave approximation in the form of the Jones formula (Eq. 16), denoted as $\partial_{\text{kin}}$ with $C$ evaluated from Eq. (18)

- Wave translation (Eq. 20) denoted as $\partial_{\text{wt}}$ proposed in this paper with $\Delta s = 10$ m, and $C$ evaluated from Eq. (18)

- Method presented by Tu (1991); Tu and Graf (1993) based on Eq. (19) which is denoted as $\partial_{\text{Tu&Graf}}$.

As can be seen from Fig. 6, $\partial_{\text{kin}}$ and $\partial_{\text{wt}}$ provide compatible results. Nonetheless, huge discrepancies in the $\partial_{\text{lin}}$ values are evident compared to $\partial_{\text{kin}}$ and $\partial_{\text{wt}}$. The reason for this is that the linear method is applied to data from two cross-sections, which are located at a considerable distance apart. Moreover, due to the linear character of this method, $\partial_{\text{lin}}$ is unsuitable to express the variability of
the flood wave shape. As a result, it overestimates the time instant at which $\theta = 0$ when the downstream cross-section is taken into account (as in Ol-1), and underestimates the time instant when the upstream cross-section is used (as in Ol-2, Ol-3, Ol-4). Next, the lateral inflows might have an effect on the flow, and thus the estimation of $\theta$ by the linear method. When it comes to $\frac{\partial h}{\partial t}$, the results are in line with $\theta_{\text{kin}}$ and $\theta_{\text{wt}}$ except for the region near the peak of the wave where discontinuity occurs. This occurs due to the form of Eq. (19), which cannot be applied if $\frac{\partial h}{\partial t} = 0$, as was theoretically analysed in Sect 3.3.1. Consequently, the method must not be applied in the region of a rising limb in the vicinity of the wave peak and in the peak of the wave itself.

4.2 Type of wave

In order to assess to which category of flood wave (dynamic, diffusive or kinematic) the case under study should be assigned, the terms of the momentum balance equation are compared. The results are shown in Fig. 7. All terms are evaluated analytically from measurement data. The results for data sets Ol-2, Ol-3, Ol-4 are similar, as they originate from the same cross-section. The bed slope is of magnitude $10^{-3}$, the maximum flow depth gradient is of magnitude $10^{-4}$, and the other terms are negligible. On the other hand, for data set Ol-1, the bed slope and the maximum flow depth gradient are of magnitude $10^{-4}$. Moreover, the acceleration terms reach the magnitude of $10^{-4}$ along the rising limb. However, the acceleration terms are of opposite signs, and the overall impact of flow acceleration on the results might not be so pronounced. The comparison between Ol-1 and Ol-2, which originate from the same experiment, shows that in cross-section CS1, which is closer to the dam, more terms of the momentum balance equation are significant. From the results for CS2 it may be concluded that the significance of the temporal variability of flow parameters decreases along the channel.

It may be concluded that the waves from cross-section CS2, i.e. Ol-2, Ol-3, and Ol-4, are of a diffusive character. Consequently, the formula for diffusive waves, Eq. (8), may be applied, and friction slope (indicated by the red line in Fig. 7) is well approximated by the water surface slope. In the case of data set Ol-1, along the rising limb local acceleration term is slightly bigger than the advective one, which may indicate dynamic character of a wave.

Another method which may be used to identify the type of wave is analysis of the sensitivity of friction velocity to input variables (Mrokowska and Rowiński, 2012; Mrokowska et al., 2013) or a kind of stability analysis in which one observes the impact of a small change in the value of the input variable on the friction velocity value (Mrokowska et al., 2013).

4.3 Evaluation of friction velocity

Figure 8 presents the results for the friction velocity evaluated using the formula for the dynamic wave (Eq. 15), using different methods to evaluate $\theta$. As can be seen from the figure, $u_{\text{kin}}$ and $u_{\text{wt}}$ agree well with each other. There is also good agreement with $u_{\theta \text{Tu&Graf}}$ along the falling limbs of
waves. In Ol-1, Ol-3, and Ol-4 it is observed that the discontinuity occurs between the time instants of maximum $U$ and maximum $h$, as is noted in the theoretical part of this paper (Sect 3.3.1). The effect of the discontinuity depends on the time step applied in the analysis, and when the step is large enough, as in the case of Ol-2, the discontinuity may be overlooked. When it comes to $u_{lin}$, it deviates to high extent from the previous results, and is considered as not reliable due to the comments on $\phi_{lin}$ presented in Sect 3.3.2.

Figure 9 presents the comparison of the results of dynamic $u_{dyn}$ (Eq. 15), diffusive $u_{dif}$ (Eq. 8) and steady flow $u_{st}$ (Eq. 12) formulae. Additionally, uncertainty bounds are presented for each result. Uncertainty bounds are represented by the maximum deterministic uncertainty evaluated by the law of propagation of uncertainty (Eq. 22). The uncertainties of the input variables are assessed based on knowledge of measurement techniques and experimental settings as follows: $\Delta h = 0.01$ m, $\Delta U = 10\% U$ (measurement performed by a propeller current meter), $\Delta R = 0.01$ m, $\Delta \zeta = 0.0001$ [m s$^{-2}$], $\Delta \eta = 0.0001$ [m s$^{-1}$], $\Delta \vartheta = 0.00001$ [-], $\Delta I = 0.0001$. As can be seen from Fig. 9, the results for friction velocity obtained by the formula for dynamic waves (Eq. 15) and the formula for diffusive waves (Eq. 8) agree well with each other. The slight difference between the results occurs in data set Ol-1. This is caused by the acceleration terms, which appear to be significant in Ol-1 along the leading edge (Fig. 7). Consequently, in this region, the application of Eq. (15) may be considered. However, the results of Eq. (8) lie within the uncertainty bounds of the results of Eq. (15); hence, the application of the simplified formula for diffusive wave is acceptable.

On the other hand, the results obtained by Eq. (15) and by formula for steady flow (Eq. 12) differ from each other. For Ol-1, Ol-2 and Ol-4 the results of Eq. (12) fall outside the uncertainty bounds of Eq. (15) along the substantial part of leading edge of the waves. In data set Ol-4, the time period could be observed in which the uncertainty bounds of Eq. (15) and Eq. (12) do not overlap. The significant discrepancies along the leading edge of a flood wave indicate that the application of Eq. (12) in this region is incorrect.

### 4.4 Analysis of the Manning coefficient

Manning $n$ is calculated from Eq. (1) with $S$ derived analytically from the St. Venant model for data sets Ol-1, Ol-2, Ol-3 and Ol-4. In fact, Manning $n$ may be also recalculated from friction velocity results. All analyses of simplifications and evaluation of $\vartheta$ presented above apply to evaluation of Manning $n$, as well. Figure 10 presents the results of $n$ for $\vartheta$ evaluated by the wave translation ($n_{wt}$) and linear approximation ($n_{lin}$) methods. In addition, $n_{st}$ is evaluated for $S = I$. Discrepancies between $n_{st}$ and $n_{wt}$ result from the difference between $I$ and $S$ depicted in Fig. 7, and the discrepancies are most pronounced for Ol-1. Moreover, it can be seen that $n_{lin}$ differs considerably from the other results in all cases. This indicates that the method of evaluating $\vartheta$ may have a significant effect on $n$. Note that Manning $n_{st}$ reaches its minimum value at the time instant of maximum $U$, hence it
decreases with increasing velocity. On the other hand, it may not be true for \( n_{\text{wt}} \) and \( n_{\text{lin}} \), because their values depend additionally on variable \( S \).

Values of \( n_{\text{wt}} \), which are reference values here, range in the following intervals: [0.015, 0.039] for Ol-1, [0.024, 0.032] for Ol-2, [0.025, 0.033] for Ol-3, and [0.053, 0.095] for Ol-4. The values of Manning \( n \) for data sets Ol-1, Ol-2, and Ol-3 correspond with the values assigned to natural minor streams in the tables presented in (Chow, 1959). The minimum values of Ol-2 and Ol-3 correspond with "clean straight, full stage, no rifts or deep pools", while the minimum value of Ol-1 does not match \( n \) for natural streams presented in the tables. The maximum values may be assigned to "same as above, but more stones and weeds". The minimum value of \( n \) for Ol-4 may be assigned to "sluggish reaches, weedy, deep pools" and the maximum value to "vary weedy reaches, deep pools". The higher values of \( n \) for data set Ol-4 compared to the other data sets result from the fact that \( U \) is smaller than in the other cases (Fig. 4). The Manning \( n \) coefficients have been evaluated in a completely different way for the measurement data from this field site by Szkutnicki (1996); Kadlubowski and Szkutnicki (1992). In that study, \( n \) was treated as a constant parameter in the St. Venant model, and its value was assessed by optimising the model performance. The authors have reported that for spring conditions, \( n \in [0.04, 0.09] \). In this analysis, the results for Ol-1, Ol-2, Ol-3 are smaller, and the results for Ol-4 fall within the mentioned bounds.

4.5 The variability of resistance to flow during flood wave propagation – comparison between friction velocity and Manning \( n \)

The variability of resistance to flow in unsteady flow is very often analysed in terms of flow rate \( Q \), and Manning \( n \) is considered as a reference variable (Fread, 1985; Julien et al., 2002). It should be emphasised that variable \( n \) is against the idea behind the derivation of the Manning resistance relation, and it is difficult to interpret the values in terms of resistance to flow.

On the other hand, friction velocity provides straightforward interpretation, as we discussed in the introduction. To illustrate the inconsistency of such analysis, the comparison between Manning \( n \) and friction velocity vs. flow rate \( Q \) is illustrated in Fig. 11. As can be seen from the figure, the Manning \( n \) decreases with increasing flow rate. This trend is characteristic of the majority of streams with inbank flow (Chow, 1959), which has been observed by Fread (1985) when the inundation area was relatively small compared to inbank flow area. This is the case considered herein, as the experiment was performed under inbank flow conditions. The reverse trend has been observed by Julien et al. (2002) for flood waves in the River Rhine. The authors discussed extensively impact of bed forms on Manning \( n \). However, we would like to emphasise another aspect – the shape of inundation area which determines the reverse trend. In (Julien, 2002) interpretation of rising \( n \) as rising resistance is qualitatively correct, while in the case of the Olszanka River false conclusions may be drawn from the analysis of Manning \( n \), that the bulk resistance decreases with flow rate. As the results for
friction velocity show, the maximum values of resistance are in the rising limb of the waves, before the maximum flow rate $Q$.

5 Concluding remarks

In the paper, two methods of expressing flow resistance in unsteady flow are considered, namely the physically based variable which is friction velocity, and the Manning coefficient. Both, friction velocity and the Manning coefficient, are derived from flow equations. The analysis proved that friction velocity is superior to the Manning coefficient when the physical interpretation of resistance is necessary. The advantage of friction velocity lies in the fact that it refers directly to the friction force; hence, the variability of friction velocity (or alternatively shear stress) may be interpreted in a straightforward way. On the other hand, when the Manning coefficient is considered, its interpretation is subjective to a great extent, as a number of factors affect the coefficient, e.g. roughness, vegetation and meandering. Discrepancy between trends of Manning $n$ versus flow rate for the Olszanka River studied herein and the River Rhine reported in (Julien et al., 2002) is most likely due to different geometry of inundation area between these two cases. Resistance cannot be compared between these two cases based on Manning $n$. Moreover, the comparison between the results for friction velocity and Manning $n$ has shown that the theoretical interpretation of $n$ in unsteady flow should be avoided. However, this remark does not apply to modelling studies, where $n$ is treated as an optimisation parameter. For the above reasons, following a large group of researchers, we suggest considering friction velocity (or shear stress) as a reference parameter for resistance to flow.

As friction velocity is recommended to express flow resistance, the method of friction velocity derived from flow equations is scrutinised. It also applies to Manning $n$. Analysis of resistance evaluation in the Olszanka River shows that friction velocity is highly uncertain and difficult to determine. However, we believe that when friction velocity relations are applied with an awareness of their constraints, and proper effort is made to minimise the uncertainty of the input data, the method of friction velocity evaluation is likely to provide reliable results. For this reason, in-depth description of how to determine friction for unsteady flow has been provided. The suggestions on the evaluation of friction velocity given in the paper should be helpful in reaching a compromise between scarcity of data and the correctness of simplifying assumptions. We have demonstrated that some simplifications such as linear approximation for $\frac{dh}{dz}$ evaluation may result in high incorrectness of results. On the other hand, as simplified methods are very often a must when data are scarce, when data from only one cross-section are available we recommend the translation method, introduced in this paper, to evaluate $\frac{dh}{dz}$ instead of Jones formula. The simplifications applied and their possible impact on the assessed value of the friction velocity should be clearly stated when the results are presented. The paper has demonstrated the application of friction velocity relations to experimental data; hence, the detailed conclusions drawn in the study apply to similar cases.
Flood wave phenomena are so complex that it is currently impossible to provide a comprehensive analysis, and the problem of resistance to flow in unsteady non-uniform conditions still poses a challenge. For this reason, more research on resistance in unsteady non-uniform conditions is necessary.

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Figure 1. The site of the experiment in Olszanka River (upper panel), and the shape of measurement cross-sections CS1 and CS2 (lower panel).

Figure 2. Trapezoidal cross-section of a channel with definitions of symbols used in the text.
Figure 3. Temporal variability of flow depth $h$ and mean velocity $U$ for experimental flood waves in the Ol-
szanka River.
Figure 4. Rating curves of experimental flood waves in the Olszanka River.
Figure 5. Comparison between rating curve for flood wave and steady flow with characteristic points, based on (Henderson, 1963) (upper panel), and impact of kinematic wave approximation (Eqs. (16) and (17)) on the assessment of time instant at which $\frac{\partial h}{\partial x} = 0$ (lower panel).
Figure 6. Temporal variability of gradient of flow depth $\vartheta = \frac{\partial h}{\partial x}$ for experimental flood waves in the Olszanka River.
Figure 7. Comparison of terms of the momentum balance equation for experimental flood waves in the Ol-
szanka River.
Figure 8. Comparison of friction velocity $u_*$ evaluated by different methods (symbols defined in the text) for experimental flood waves in the Olszanka River.
Figure 9. Comparison of friction velocity evaluated by formulae for dynamic $u_{*\text{dyn}}$, diffusive wave $u_{*\text{dif}}$ and steady uniform flow $u_{*\text{st}}$ with uncertainty bounds for experimental flood waves in the Olszanka River.
Figure 10. Temporal variability of Manning $n$ evaluated for different assumptions about friction slope $S$ for experimental flood waves in the Olszanka River.
Figure 11. Comparison of the relation of Manning $n$ vs flow rate $Q$ and friction velocity $u_*$ vs $Q$ along rising and falling limbs of waves for experimental flood waves in the Olszanka River.