Improving multi-objective reservoir operation optimization with sensitivity-informed dimension reduction

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Abstract

This study investigates the effectiveness of a sensitivity-informed method for multi-objective operation of reservoir systems, which uses global sensitivity analysis as a screening tool to reduce the computational demands. Sobol’s method is used to screen insensitive decision variables and guide the formulation of the optimization problems with a significantly reduced number of decision variables. This sensitivity-informed method dramatically reduces the computational demands required for attaining high quality approximations of optimal tradeoff relationships between conflicting design objectives. The search results obtained from the reduced complexity multi-objective reservoir operation problems are then used to pre-condition the full search of the original optimization problem. In two case studies, the Dahuofang reservoir and the inter-basin multi-reservoir system in Liaoning province, China, sensitivity analysis results show that reservoir performance is strongly controlled by a small proportion of decision variables. Sensitivity-informed dimension reduction and pre-conditioning are evaluated in their ability to improve the efficiency and effectiveness of multi-objective evolutionary optimization. Overall, this study illustrates the efficiency and effectiveness of the sensitivity-informed method and the use of global sensitivity analysis to inform dimension reduction of optimization problems when solving the complex multi-objective reservoir operation problems.

Keywords water supply; complexity reduction; multi-objective optimization; preconditioning; sensitivity analysis; reservoir operation
Introduction

Reservoirs are often operated considering a number of conflicting objectives (such as different water uses) related to environmental, economic and public services. The optimization of Reservoir Operation Systems (ROS) has attracted substantial attention over the past several decades. In China and many other countries, reservoirs are operated according to reservoir operation rule curves which are established at the planning/design stage to provide long-term operation guidelines for reservoir management to meet expected water demands. Reservoir operation rule curves usually consist of a series of storage volumes or levels at different periods (Liu et al., 2011a and 2011b).

In order to solve the ROS problem, there are different approaches, such as implicit stochastic optimization (ISO), explicit stochastic optimization (ESO), and parameter-simulation-optimization (PSO) (Celeste and Billib, 2009). ISO uses deterministic optimization, e.g., dynamic programming, to determine a set of optimal releases based on the current reservoir storage and equally likely inflow scenarios (Young, 1967; Karamouz and Houck, 1982; Castelletti et al., 2012; François et al., 2014). Instead the use of equally likely inflow scenarios, ESO incorporates inflow probability directly into the optimization process, including stochastic dynamic programming and Bayesian methods (Huang et al., 1991; Tejada-Guibert et al., 1995; Powell, 2007; Goor et al., 2010; Xu et al., 2014). However, many challenges remain in application of these two approaches due to their complexity and ability to deal with conflicting objectives (Yeh, 1985; Simonovic, 1992; Wurbs, 1993; Teegavarapu and
In a different way, PSO predefines a rule curve shape and then utilizes optimization algorithms to obtain the combination of rule curve parameters that provides the best reservoir operating performance under possible inflow scenarios or a long inflow series (Nalbantis and Koutsoyiannis, 1997; Oliveira and Loucks, 1997). In this way, most stochastic aspects of the problem, including spatial and temporal correlations of unregulated inflows, are implicitly included, and reservoir rule curves could be derived directly with genetic algorithms and other direct search methods (Koutsoyiannis and Economou, 2003; Labadie, 2004). Because PSO reduces the curse of dimensionality problem in ISO and ESO, it is widely used in reservoir operation optimization (Chen, 2003; Chang et al., 2005; Momtahen and Dariane, 2007). In this study, the PSO-based approach is used to solve the ROS problem.

In the PSO procedure to solve the ROS problem, the values of storage volumes or levels in reservoir operation rule curves are optimized to achieve one or more objectives directly. Quite often, there are multiple curves, related to different purposes of reservoir operation. The dimension of a ROS problem depends on the number of the curves and the number of time periods. For a cascaded reservoir system, the dimension can be very large, which increases the complexity and problem difficulty and poses a significant challenge for most search tools currently available (Labadie, 2004; Draper and Lund, 2004; Sadegh et al., 2010; Zhao et al., 2014).

In the context of multi-objective optimal operation of ROS, there is not one single operating policy that improves simultaneously all the objectives and a set of
non-dominating Pareto optimal solutions are normally obtained. The traditional
approach to multi-objective optimal reservoir operation is to reformulate the
multi-objective problem as a single objective problem through the use of some
cALARIZATION methods, such as the weighted sum method (Tu et al., 2003 and 2008;
Shiau, 2011). This method has been developed to repeatedly solve the single objective
problem using different sets of weights so that a set of Pareto-optimal solutions to the
original multi-objective problem could be obtained (Srinivasan and Philipose, 1998;
Shiau and Lee, 2005). Another well-known method is the ε-constraint method (Ko et
al., 1997; Mousavi and Ramamurthy, 2000; Shirangi et al., 2008): all the objectives
but one are converted into constraints and the level of satisfaction of the constraints is
optimized to obtain a set of Pareto-optimal solutions. However, with the increase in
problem complexity (i.e., the number of objectives or decision variables), both
approaches become inefficient and ineffective in deriving the Pareto-optimal
solutions.

In the last several decades, bio-inspired algorithms and tools have been developed
to directly solve multi-objective optimization problems by simultaneously handling
all the objectives (Nicklow et al., 2010). In particular, multi-objective evolutionary
algorithms (MOEA) have been increasingly applied to the optimal reservoir operation
problems, with intent of revealing tradeoff relationships between conflicting
objectives. Suen and Eheart (2006) used the non-dominated sorting genetic algorithm
(NSGAII) to find the Pareto set of operating rules that provides decision makers with
the optimal trade-off between human demands and ecological flow requirements.
Zhang et al. (2013b) used a multi-objective adaptive differential evolution combined with chaotic neural networks to provide optimal trade-offs for multi-objective long-term reservoir operation problems, balancing hydropower operation and the requirement of reservoir ecological environment. Chang et al. (2013) used an adjustable particle swarm optimization – genetic algorithm (PSO-GA) hybrid algorithm to minimize water shortages and maximize hydro-power production in management of Tao River water resources.

However, significant challenges remain for using MOEAs in large, real-world ROS applications. The high dimensionality of ROS problems makes it very difficult for MOEAs to identify ‘optimal or near optimal’ solutions with the computing resources that are typically available in practice. Thus the primary aim of this study is to investigate the effectiveness of a sensitivity-informed optimization methodology for multi-objective reservoir operation, which uses sensitivity analysis results to reduce the dimension of the optimization problems, and thus improves the search efficiency in solving these problems. This framework is based on the previous study by Fu et al. (2012), which developed a framework for dimension reduction of optimization problems that can dramatically reduce the computational demands required to obtain high quality solutions for optimal design of water distribution systems. The ROS case studies used to demonstrate this framework consider the optimal design of reservoir water supply operation policies. Storage volumes at different time periods on the operation rule curves are used as decision variables. It has been widely recognized that the determination of these decision variables requires
a balance among different ROS objectives. Sobol’s sensitivity analysis results are used to form simplified optimization problems considering a small number of sensitive decision variables, which can be solved with a dramatically reduced number of model evaluations to obtain Pareto approximate solutions. These Pareto approximate solutions are then used to pre-condition a full search by serving as starting points for the multi-objective evolutionary algorithm. The results from the Dahuofang reservoir and inter-basin multi-reservoir system case studies in Liaoning province, China, whose conflicting objectives are minimization of industry water shortage and minimization of agriculture water shortage, illustrate that sensitivity-informed dimension reduction and pre-conditioning provide clear advantages to solve large-scale multi-objective ROS problems effectively.

2 Problem formulation

Most reservoirs in China are operated according to rule curves, i.e., reservoir water supply operation rule curves. Because they are based on actual water storage volumes, they are simple to use. Fig. 1 shows an illustration of rule curves for Dahuofang reservoir based on 36 10-day periods.

As we know, water demand could be fully satisfied only when there is sufficient water in reservoir. Water supply operation rule curve, which is used to operate most reservoirs in China, represents the limited storage volume for water supply in each period of a year. In detail, water demand will be fully satisfied when the reservoir storage volume is higher than water supply operation rule curve; whereas water
demand needs to be rationed when the reservoir storage volume is lower than water supply operation rule curve. In general, a reservoir has more than one water supply target, and there is one to one correspondence between water supply rule curve and water supply target. The water supply with lower priority will be limited prior to the water supply with higher priority when the reservoir storage volume is not sufficient. To reflect the phenomenon that different water demands can have different reliability requirements and thus different levels of priority in practice, the operation rule curve for the water supply with the lower priority is located above the operation rule curve for the water supply with the higher priority.

Fig. 1 shows water supply operation rule curves for agriculture and industry where the maximum storage is smaller in the middle due to the flood control requirements in wet seasons. In Fig. 1, the red line with circle represents water supply rule curve for agriculture, the green line with triangle represents water supply rule curve for industry. The water supply rule curve for agriculture with lower priority is located above the water supply rule curve for industry with higher priority. The water storage available between the minimum and maximum storages is divided into three parts: zone 1, zone 2 and zone 3 by the water supply rule curves for agriculture and industry.

Specifically, both the agricultural demand $D_1$ and the industrial demand $D_2$ could be fully satisfied when the actual water storage is in zone 1, which is above the water supply rule curve for agriculture. When the actual water storage is in zone 2, the industrial demand could be fully satisfied, and the agricultural demand has to be rationed. Both the agricultural demand and the industrial demand have to be rationed
when the actual water storage is in zone 3. The water supply rule for a specific water
user consists of one water supply rule curve and rationing factors that indicate the
reliability and priority of the water user. The rationing factors used to determine the
amount of water supply for different water demands can be either assigned according
to the experts’ knowledge or determined by optimization (Shih and ReVelle, 1995). In
this paper, rationing factors are given at the reservoir’s design stage according to the
tolerable elastic range of each water user in which the damage caused by rationing
water supply is limited. Assuming that the specified water rationing factor $\alpha_1$ is
applied to the water supply rule curve for agriculture in Fig. 1, the agricultural
demand $D_1$ could be fully supplied without rationing when the actual water storage
is in zone 1, however, when the water storage is in zone 2 or zone 3, the agricultural
demand has to be rationed, i.e., $\alpha_1 \times D_1$. Similarly, assuming that the specified water
rationing factor $\alpha_2$ is applied to the water supply rule curve for industry in Fig. 1, the
industrial demand $D_2$ could be fully supplied without rationing when the actual
water storage is in zone 1 or zone 2, however, when the water storage is in zone 3, the
industrial demand has to be rationed, i.e., $\alpha_2 \times D_2$.

To provide long-term operation guidelines for reservoir management for meeting
expected water demands for future planning years, the projected water demands and
long-term historical inflow are used. The optimization objective for water supply
operation rule curves is to minimize water shortages during the long-term historical
period. The ROS design problem is formulated as a multi-objective optimization
problem, i.e., minimizing multiple objectives simultaneously. In this paper, the
objectives are to minimize industry and agriculture water shortages:

\[
\min f_i(x) = SI_i = \frac{100}{N} \sum_{j=1}^{N} \left( \frac{D_{i,j} - W_{i,j}(x)}{D_{i,j}} \right)^2
\]  

(1)

where \( x \) is the vector of decision variables, i.e., the water storages at different periods on a water-supply rule curve; \( SI_i \) is the shortage index for water demand \( i \) (agricultural water demand when \( i = 1 \), industrial water demand when \( i = 2 \)), which measures the average annual shortage occurred during \( N \) years, and is used as an indicator to reflect water supply efficiency; \( N \) is the total number of years simulated; \( D_{i,j} \) is the demand for water demand \( i \) during the \( j \)th year; \( W_{i,j}(x) \) is the actually delivered water for water demand \( i \) during the \( j \)th year. The term \( W_{i,j}(x) \) is calculated below using agricultural water demand \( (i = 1) \) as an example. If the actual water storage is above the water supply rule curve for agricultural water demand \( (i = 1) \) at period \( t \) in a year, the delivered water at period \( t \) is its full demand without being rationed, \( D_{1,t} \). If the actual water storage is below the water supply rule curve for agricultural water demand at period \( t \), the delivered water for agricultural water demand at period \( t \) is its rationed demands, \( \alpha_1 D_{1,t} \).

For the ROS optimization problem, the mass balance equations are:

\[
S_{t+1} - S_t = I_t - R_t - SU_t - E_t
\]  

(2)

\[
R_t = g(x), SU_t = k(x), E_t = e(x)
\]  

(3)

\[
ST^\text{min}_t \leq S_t \leq ST^\text{max}_t, ST^\text{min}_t \leq x \leq ST^\text{max}_t
\]  

(4)

where \( S_t \) is the initial water storage at the beginning of period \( t \); \( S_{t+1} \) is the ending water storage at the end of period \( t \); \( I_t, R_t, SU_t \) and \( E_t \) are inflow, delivery for water use, spill and evapotranspiration loss, respectively; and \( ST^\text{max}_t \) and \( ST^\text{min}_t \)
are the maximum and minimum storage, respectively. Additionally, because $W_{i,j}(x)$
in Equation (1) is the actually delivered water for water demand $i$ during the $j$th year, $R$ in that year is equal to the sum: $W_{1,j}(x) + W_{2,j}(x)$.

3 Methodology

Pre-conditioning is a technique that uses a set of known good solutions as starting points to improve the search process of optimization problems (Nicklow et al., 2010). It is very challenging to determine good initial solutions, and different techniques including the domain knowledge can be used. This study utilizes a sensitivity-informed dimension reduction to develop simpler search problems that consider only a small number of highly sensitive decisions. The results from these simplified search problems can be used to successively pre-condition search for larger, more complex formulations of ROS design problems. The $\varepsilon$-NSGAII, a popular multi-objective evolutionary algorithm, is chosen as it has been shown effective for many engineering optimization problems (Kollat and Reed, 2006; Tang et al., 2006; Kollat and Reed, 2007). For the two-objectives considered in this paper, their epsilon values in $\varepsilon$-NSGAII ($\varepsilon_{S1}$ and $\varepsilon_{S2}$) were chosen based on reasonable and practical requirements and were both set to 0.01. According to the study by Fu et al. (2012), the sensitivity-informed methodology, as shown in Fig. 2, has the following steps:

1. Perform a sensitivity analysis using Sobol’’s method to calculate the sensitivity indices of all decision variables regarding the ROS performance measure;

2. Define a simplified problem that considers only the most sensitive decision
variables by imposing a user specified threshold (or classification) of sensitivity;

3. Solve the simplified problem using \( \varepsilon \)-NSGAII with a small number of model simulations;

4. Solve the original problem using \( \varepsilon \)-NSGAII with the Pareto optimal solutions from the simplified problem fed into the initial population.

### 3.1 Sobol’s sensitivity analysis

Sobol’s method was chosen for sensitivity analysis because it can provide a detailed description of how individual variables and their interactions impact model performance (Tang et al., 2007b; Zhang et al., 2013a). A model could be represented in the following functional form:

\[
y = f(x) = f(x_1, \ldots, x_p)
\]  

(5)

where \( y \) is the goodness-of-fit metric of model output, and \( x = (x_1, \ldots, x_p) \) is the parameter set. Sobol’s method is a variance based method, in which the total variance of model output, \( D(y) \), is decomposed into component variances from individual variables and their interactions:

\[
D(y) = \sum_i D_i + \sum_{i<j} D_{ij} + \sum_{i<j<k} D_{ijk} + \cdots + D_{12\cdots m}
\]  

(6)

where \( D_i \) is the amount of variance due to the \( i \)th variable \( x_i \), and \( D_{ij} \) is the amount of variance from the interaction between \( x_i \) and \( x_j \). The model sensitivity resulting from each variable can be measured using the Sobol’s sensitivity indices of different orders:

**First-order index:** \( S_i = \frac{D_i}{D} \)  

(7)

**Second-order index:** \( S_{ij} = \frac{D_{ij}}{D} \)  

(8)
where $D_{-i}$ is the amount of variance from all the variables except for $x_i$, the first-order index $S_i$ measures the sensitivity from the main effect of $x_i$, the second-order index $S_{ij}$ measures the sensitivity resulting from the interactions between $x_i$ and $x_j$, and the total-order index $S_{Ti}$ represents the main effect of $x_i$ and its interactions with all the other variables.

### 3.2 Performance metrics

Since MOEA uses random-based search, performance metrics are used in this study to compare the quality of the approximation sets derived from replicate multi-objective evolutionary algorithm runs. Three indicators were selected: the generational distance (Veldhuizen and Lamont, 1998), the additive ε-indicator (Zitzler et al., 2003), and the hypervolume indicator (Zitzler and Thiele, 1998).

The generational distance measures the average Euclidean distance from solutions in an approximation set to the nearest solution in the reference set, and indicates perfect performance with zero. The additive ε-indicator measures the smallest distance that a solution set needs to be translated to completely dominate the reference set. Again, smaller values of this indicator are desirable as this indicates a closer approximation to the reference set.

The hypervolume indicator, also known as the S metric or the Lebesgue measure, measures the size of the region of objective space dominated by a set of solutions. The hypervolume not only indicates the closeness of the solutions to the optimal set, but also captures the spread of the solutions over the objective space. The indicator is
normally calculated as the volume difference between a solution set derived from an optimization algorithm and a base solution set. In this study, the worst case solution is chosen as base. For example, the worst solution is (1, 1) for two minimization objectives in the normalized objective space. Thus larger hypervolume indicator values indicate improved solution quality and imply a larger distance from the worst solution.

4 Case study

Two case studies of increasing complexity are used to demonstrate the advantages of the sensitivity-informed methodology: (1) the Dahuofang reservoir, and (2) the inter-basin multi-reservoir system in Liaoning province, China. The inter-basin multi-reservoir system test case is a more complex ROS problem with Dahuofang, Guanyinge and Shenwo reservoirs. In the two ROS problems, the reference sets were obtained from all the Pareto optimal solutions across a total of 10 random seed trials, each of which was run for a maximum number of function evaluations (NFE) of 500,000. Additionally, the industrial and agricultural water demands in the future planning year, i.e., 2030, and the historical inflow from 1956 to 2006 were used to optimize reservoir operation and meet future expected water demands in the two case studies.

4.1 Dahuofang reservoir

The Dahuofang reservoir is located in the main stream of Hun River, in Liaoning province, Northeast China. The Dahuofang reservoir basin drains an area of 5437 km²,
and within the basin the total length of Hun River is approximately 169 km. The main
purposes of the Dahuofang reservoir are industrial water supply and agricultural water
supply to central cities in Liaoning province. The reservoir characteristics and yearly
average inflow are illustrated in Table 1.

The Dahuofang ROS problem is formulated as follows: the objectives are
minimization of industrial shortage index and minimization of agricultural shortage
index as described in Equation (1); the decision variables include storage volumes on
the industrial and agricultural curves. For the industrial curve, a year is divided into
24 time periods (with ten days as the scheduling time step from April to September,
and one month as the scheduling time step in the remaining months). Thus there are
twenty-four decision variables for industrial water supply. The agricultural water
supply occurs only in the periods from the second ten-day of April to the first ten-day
of September, thus there are fifteen decision variables for agricultural water supply. In
total, there are thirty-nine decision variables.

4.2 Inter-basin multi-reservoir system

As shown in Fig. 3, Dahuofang, Guanyinge and Shenwo reservoirs compose the
inter-basin multi-reservoir system in Liaoning province, China.

Liaoning province in China covers an area of $146 \times 10^3$ km$^2$ with an extremely
uneven distribution of rainfall in space. The average amount of annual precipitation
decreases from 1100 mm in east to 600 mm in west (WMR-PRC, 2008). However, the
population, industries, and agricultural areas mainly concentrate in the western parts.
Therefore, it is critical to develop the best water supply rules for the inter-basin
multi-reservoir system to decrease the risk of water shortages caused by the mismatch of water supplies and water demands in both water deficit regions and water surplus regions. Developing inter-basin multi-reservoir water supply operation rules has been promoted as a long-term strategy for Liaoning province to meet the increasing water demands in water shortage areas. In the inter-basin multi-reservoir system of Liaoning province, the abundant water in Dahuofang, Guanyinge and Shenwo reservoirs is diverted downstream to meet the water demands in water shortage areas, especially the region between Daliaohekou and Sanhekou hydrological stations.

The main purposes of the inter-basin multi-reservoir system are industrial water supply and agricultural water supply to eight cities (Shenyang, Fushun, Anshan, Liaoyang, Panjin, Yingkou, Benxi and Dalian) of Liaoning province, and environmental water demands need to be satisfied fully. The characteristics of each reservoir in the inter-basin multi-reservoir system are illustrated in Table 2.

The flood season runs from July to September, during which the inflow takes up a large part of the annual inflow. The active storage capacities of Dahuofang and Shenwo reservoirs reduce significantly during flood season for the flood control.

The inter-basin multi-reservoir operation system problem is formulated as follows: the objectives are minimization of industrial shortage index and minimization of agricultural shortage index as described in Equation (1). Regarding Shenwo reservoir, which has the same water supply operation rule curve features as Dahuofang reservoir, the decision variables include storage volumes on the industrial and agricultural curves and there are thirty-nine decision variables. Regarding Guanyinge reservoir,
the decision variables include storage volumes on the industrial curve and water
transferring curve due to the requirement of exporting water from Guanyinge
reservoir to Shenwo reservoir in the inter-basin multi-reservoir system, which is
similar to the water supply operation rule curve for industrial water demand, and there
are forty-eight decision variables. Therefore, the inter-basin multi-reservoir system
has six rule curves and \(39 \times 2 + 48 = 126\) decision variables in total.

5 Results and discussions

5.1 Dahuofang reservoir

In the Dahuofang reservoir case study, a set of 2000 Latin Hypercube samples
were used per decision variable yielding a total number of \(2000 \times (39 + 2) = 82000\) model simulations used to compute Sobol’s indices. Following the
recommendations of Tang et al. (2007a, b) boot-strapping the Sobol’’ indices showed
that 2000 samples per decision variable were sufficient to attain stable rankings of
global sensitivity.

The first-order indices representing the individual contributions of each variable to
the variance of the objectives are shown in blue in Fig. 4. The total-order indices
representing individual and interactive impacts on the variance of the objectives are
represented by the total height of bars. Agr4_2 represents decision variable
responding to water storage volume on the agricultural curve at the second ten days of
April and ind3_3 represents decision variable responding to water storage volume on
the industrial curve at the last ten days of March, and so on. Considering the shortage
index for the industrial water demand, the water storages at time periods ind1, ind2, ind3, ind10, ind11, and ind12, i.e., the water storages at time periods 1, 2, 3, 10, 11, and 12 of water supply operation rule curves for industrial water demand are the most sensitive variables, accounting for almost 100% of the total variance. Considering the agricultural shortage index, the water storages at time periods from agr4-2 to agr5-3, i.e., the water storages at the first five time periods of water supply operation rule curves for agricultural water demand are the most sensitive variables. The explanation for the most sensitive variables in water supply operation rule curves for industrial and agricultural water demands will be provided in section 5.1.3.

5.1.1 Simplified problems

Building on the sensitivity results shown in Fig. 4, one simplified version of the Dahuofang ROS problem is formulated: only 11-periods are considered for optimization, i.e., time periods ind1, ind2, ind3, ind10, ind11, and ind12 for industrial curve and agr4-2, agr4-3, agr5-1, agr5-2, and agr5-3 for agricultural curve based on a total-order Sobol’’s index threshold of greater than 10%. The threshold is subjective and its ease-of-satisfaction decreases with increasing number of parameters or parameter interactions. In all of the results for the Sobol’’s method, parameters classified as the most sensitive contribute, on average, at least 10 percent of the overall model variance (Tang et al., 2007a, b). The full search 39-period problem serves as the performance baseline relative to the reduced complexity problem.

5.1.2 Pre-conditioned optimization

In this section, the pre-conditioning methodology is demonstrated using the
11-period simplification of the Dahuofang ROS test case from the prior section, while the insensitive decision variables are set randomly first with domain knowledge and kept constant during the solution of the simplified problem.

Using the sensitivity-informed methodology, the 11-period case was first solved using $\varepsilon$-NSGAII with a maximum NFE of 2000, and the Pareto optimal solutions combined with the constant insensitive decision variables were then used as starting points to start a complete new search with a maximum NFE of 498,000. The standard search using $\varepsilon$-NSGAII was set to a maximum NFE of 500,000 so that the two methods have the same NFE used for search. In this case, 10 random seed trials were used given the computing resources available. The search traces in Fig. 5 show for all three metrics (generational distance, additive epsilon indicator, and hypervolume) that the complexity-reduced case can reliably approximate their portions of the industrial and agricultural water shortage tradeoff given their dramatically reduced search periods. All three metrics show diminishing values at the end of the reduced search periods. The pre-conditioning results are shown in Fig. 5 in red search traces continuing from the blue reduced complexity search results.

Fig. 5 clearly highlight that the sensitivity-informed pre-condition problems dramatically enhance search efficiency in terms of the generational distance, additive epsilon indicator, and hypervolume metrics. Overall, sensitivity-informed dimension reduction and pre-conditioning yield strong efficiency gains and more reliable search (i.e., narrower band widths on search traces) for the Dahuofang ROS test case.

Fig. 6(a) shows Pareto fronts from a NFE of 3000, 5000 and 8000 in the evolution
process of one random seed trial. In the case of the pre-conditioned search, the solutions from 3000, 5000 and 8000 evaluations are much better than the corresponding solutions in the case of standard baseline search. The results show that the Pareto approximate front of the pre-conditioned search is much wider than that of the standard search, and clearly dominates that of the standard search in all the regions across the entire objective space.

Fig. 6(b) shows the best and worst Pareto fronts from a NFE of 500,000 and 8000 in the evolution process of ten seed trials. In the case of the pre-conditioned search, the best solutions from 500,000 evaluations are better than the corresponding solutions in the case of standard baseline search. Although it is obvious that there are not many differences between solutions obtained from pre-conditioned search and solutions from standard baseline search due to the complexity of the problem, the best Pareto fronts from a NFE of 8000 in the case of the pre-condition search are approximate the same as the best Pareto fronts from a NFE of 500,000 in the case of the standard baseline search.

Fig. 7 shows the computational savings for two thresholds of hypervolume values 0.80 and 0.85 in the evolution process of each seed trial. In both cases of the thresholds of hypervolume values 0.80 and 0.85, NFE of the pre-conditioned search is less than standard baseline search for each seed. In the case of the threshold of hypervolume value 0.80, the average NFES of full search and pre-conditioned full search are approximately 94,564 and 25,083 for one seed run respectively, and the computation is saved by 73.48%. Although the NFE of Sobol’s analysis is 82,000, the
average NFEs of pre-conditioned full search is approximately $25,083 + 82,000/437 = 33,283$ for each seed run, and the computational saving is 64.80%.

Similarly, in the case of the threshold of hypervolume value 0.85, which is extremely difficult to achieve, the average NFEs of full search and pre-conditioned full search are approximately 214,049 and 105,060 for each seed run respectively, and the computation is saved by 50.92%. When the computation demand by Sobol’s analysis is considered, the computational saving is still 47.09%.

5.1.3 Optimal operation rule curves

The rule curves for Dahuofang reservoir from the final Pareto fronts based on the projected water demands and long-term historical inflow are shown in Fig. 8 (S2). The effectiveness and reasonability of the rule curves for Dahuofang reservoir are analyzed as follows.

Firstly, the optimal operational rule curves in Fig. 8 (S2) have the same characteristics as they are used in practice. During the pre-flood season (from April to June), the curves gradually become lower so that they can reduce the probability of limiting water supply and empty the reservoir storage for the flood season (from July to early September). During the flood season, the curves also stay in low positions owing to the massive reservoir inflow and the requirement of flood control, so that it is beneficial to supply as much water as possible. However, during the season from mid-September to March, the curves remain high, especially from mid-September to October, in order to increase the probability of limiting water supply and retaining enough water for later periods to avoid severe water-supply shortages as drought
Secondly, Fig. 8 (S2) shows that different water demands occur at different periods, e.g., industrial water demand occurs throughout the whole year, and agricultural water demand occurs only at the periods from the second ten-day of April to the first ten-day of September. Specially, during the flood season, there are still agricultural water demands due to temporal and spatial variations of rainfall though they are significantly reduced. Also note that the water supply curves are developed based on a historical, long-term rainfall series and the projected demands are also based on historical demands, covering stochastic uncertainties in demands and rainfalls. Due to the higher priority of industrial water supply than agricultural water supply, the industrial water supply curve is more close to minimum storage throughout the year than the agricultural water supply curve. Due to the conflicting relationship between industrial and agricultural water demands, the industrial water supply curve is higher during the non-flood season, compared to the same curve in the flooding season. Thus, if the industrial water supply curve is too low during the non-flood season from January to April, which implies that the industrial water demand is satisfied sufficiently, there would not be enough water supplied for the agricultural water demand in the same year. Similarly, if the industrial water supply curve is too low during the non-flood season from September to December, there would not be enough water supplied for the agricultural water demand in the next one or more years.

Thirdly, the inflow and industrial water demands are relatively stable during the
non-flood seasons from January to March and from October to December, so one month is taken as the scheduling time step, which is in accordance with the requirement of Dahuofang reservoir operation in practice. Due to the larger amount of industrial water demand in periods 1, 2, 3, 10, 11 and 12 (January-March and October-December) than other periods, the water storages at these time periods are very important to industrial water supply, making them the most sensitive variables. Because the agricultural water demand is very high during the non-flood period from April to May, the agricultural water supply curve at this time period is higher, and the water storages at time periods from agr4-2 to agr5-3, i.e., the water storages at the first five time periods of water supply operation rule curve for agricultural water demand, are the most important variables. On the other hand, in practice, if the agricultural water demand could not be satisfied at the first few periods of water supply operation rule curve, the agricultural water supply at each period throughout the year would be limited, i.e., the interactive effects from variables are noticeable at time periods from agr4-2 to agr5-3.

Additionally, comparisons are made among the optimized solutions from the final Pareto fronts, including industry-favoring solution (S0), agriculture-favoring solution (S1) and compromised solution (S2). The comparisons of water shortage indices among different solutions are shown in Table 3, and the optimal rule curves for different solutions are shown in Fig. 8.

It could be seen from Table 3 and Fig. 8 that there are larger differences among different solutions. With industry-favoring solution (S0), the agricultural water supply
curve at the period from April to May is the highest among the three solutions. Because the agricultural water demand is very high during the non-flood period from April to May, the highest position of agricultural water supply curve at these periods could cause that the agricultural water demand would not be satisfied at the first few periods of agricultural water supply operation rule curve, and the agricultural water supply at each period throughout the year would be limited easily. Therefore, in S0, the industrial water demand could be fully satisfied through limiting agricultural water supply to a large extend, and lowering the industrial water supply curve; industrial and agricultural water shortage indices are 0.000 and 3.550, respectively. Opposite to S0, the agricultural water demand in S1 could be satisfied largely through lowering the agricultural water supply curve on the period from April to May and raising the industrial water supply curve; and industrial and agricultural water shortage indices are 0.020 and 1.380, respectively. Compared with solutions S0 and S1, two objectives are balanced in compromised solution (S2), where industrial and agricultural water shortage indices are 0.007 and 1.932, respectively.

5.2 Inter-basin multi-reservoir system

5.2.1 Sensitivity analysis

Similarly to the Dahuofang case study, a set of 2000 Latin Hypercube samples were used per decision variable yielding a total number of $2000 \times (126 + 2) = 256,000$ model simulations to compute Sobol”s indices in this case study.
Similarly to the results obtained from the Dahuofang ROS Problem in Fig. 4, the variance in the two objectives, i.e., industrial and agricultural shortage indices, are largely controlled by the water storages at time periods from agr4-2 to agr5-3 of Shenwo reservoir water supply operation rule curves for agricultural water demand, the water storages at time periods from agr4-2 to agr5-3 of Dahuofang reservoir water supply operation rule curves for agricultural water demand, the water storages at time periods ind1, ind2, ind3, ind7-1, ind10, ind11, and ind12 of Dahuofang reservoir water supply operation rule curves for industrial water demand based on a total-order Sobol’s index threshold of greater than 3%, which is subjective and its ease-of-satisfaction decreases with increasing numbers of parameters or parameter interactions. These 17 time periods are obvious candidates for reducing the dimension of the original optimization problem and formulating a pre-conditioning problem. Therefore, the simplified problem is defined from the original design problem with the 109 intensive time periods removed, while the insensitive decision variables are set randomly first with domain knowledge and kept constant during the solution of the simplified problem. It should be noted that the increased interactions across sensitive time periods in this test case. These interactions verify that this problem represents a far more challenging search problem.

5.2.2 Pre-conditioned optimization

Using the sensitivity-informed methodology, the simplified problem was first solved using e-NSGAII with a maximum NFE of 5000, and the Pareto optimal solutions combined with the constant insensitive decision variables were then used as
starting points to start a complete new search with a maximum NFE of 495,000. The standard search using ε-NSGAII was set to a maximum NFE of 500,000 so that the two methods have the same NFE used for search. In this case, 10 random seed trials are used given the computing resources available. Similarly to the results obtained from the Dahuofang ROS problem in Fig. 5, the search traces in Fig. 10 show all three metrics (generational distance, additive epsilon indicator, and hypervolume) that represent performance metrics for the inter-basin multi-reservoir water supply operation system problem. Similarly, the pre-conditioning results are shown in Fig. 10 in red search traces continuing from the blue reduced complexity search results. It is clear that the sensitivity-informed pre-condition problems enhance search efficiency in terms of the generational distance, additive epsilon indicator, and hypervolume metrics. However, with the increase in problem complexity in comparison to the first case study (i.e., the number of decision variables from 39 to 126), the search of ROS optimization problem becomes more difficult, and so the metrics obtained from pre-conditioned search are not improved greatly compared with the standard baseline search. Both Figures 5 and 10 show that sensitivity-informed dimension reduction and pre-conditioning could also yield strong efficiency gains and more reliable search (i.e., narrower band widths on search traces) for Inter-basin multi-reservoir system.

Fig. 11(a) shows Pareto fronts from a NFE of 6000, 8000 and 10,000 in the evolution process of one random seed trial. In the case of the pre-conditioned search, the solutions from the three NFE snapshots are much better than those from standard baseline search. Similar to Fig. 6(a), the results show that the Pareto approximate
front of the pre-conditioned search is much wider than that of the standard search, and
clearly dominates that of the standard search in all the regions across the entire
objective space. Additionally, in the case of the pre-conditioned search, the solutions
from 6000 evaluations are as good as those from 8000 evaluations and 10,000
evaluations. And they are much better than the solutions from the standard baseline
search. It should be noted that the slow progress in the Pareto approximate fronts from
6000 to 10,000 evaluations reveals the difficulty of the inter-basin multi-reservoir
operation system problem.

Fig. 11(b) shows the best and worst Pareto fronts from a NFE of 500,000 in the
evolution process of ten seeds trials. Although it is obvious that the best Pareto
approximate front of the pre-conditioned is as good as that of the standard search in
all the regions across the entire objective space approximately, the Pareto solutions
from 10 trials of the pre-conditioned search have significantly reduced variation,
indicating a more reliable performance of the pre-conditioned method. In other words,
the results show that the Pareto solution from one random seed trial of the
pre-conditioned search is as good as the best solution from ten random seed trials of
the standard search. That is to say, in the case of the pre-conditioned search, one
random seed trial with a NFE of 500,000 is sufficient to obtain the best set of Pareto
solutions, however, in the case of the standard search, ten seed trials with a total of
500,000 \times 10 = 5,000,000 NFE are required to obtain the Pareto solutions. Note that
the NFE of Sobol’s analysis is 256,000, which is about half of the NFE of one
random seed trial. Thus, an improvement in search reliability can significantly reduce
The computational demand for a complex search problem such as the multi-reservoir case study, even when the computation required by sensitivity analysis is included.

5.3 Discussions

The methodology tested in this study aims to reduce the number of decision variables through sensitivity-guided dimension reduction to form simplified problems. The optimization results from the two ROS problems show the reduction in decision space can make an impact on the reliability and efficiency of the search algorithm. For the Dahuofang ROS problem, recall that the original optimization problem has 39 decision variables, and the simplified problem has 11 decision variables based on Sobol’s analysis. In the case of the inter-basin multi-reservoir operation system, the original optimization problem has 126 decision variables, and the simplified problem has a significantly reduced number of decision variables, i.e., 17. Searching in such significantly reduced space formed by sensitive decision variables makes it much easier to reach good solutions.

Although Sobol’s global sensitivity analysis is computationally expensive, it captures the important sensitive information between a large number of variables for ROS models. This is critical for correctly screening insensitive decision variables and guiding the formulation of ROS optimization problems of reduced complexity (i.e., fewer decision variables). For example, in the Dahuofang ROS problem, accounting for the sensitive information, i.e., using total-order or first-order indices, result in a simplified problem for threshold of 10% as shown in Fig. 4. Compared with the
standard search, this sensitivity-informed method dramatically reduces the 
computational demands required for attaining high quality approximations of optimal 
ROS tradeoffs relationships between conflicting objectives, i.e., the best Pareto fronts 
from a NFE of 8000 in the case of the pre-condition search are approximately the 
same as the best Pareto front from a NFE of 500,000 in the case of the standard 
baseline search.

In reality for a very large and computationally intensive problem, the full search 
with all the decision variables would likely be so difficult that it may not be optimized 
sufficiently. However, as shown here, these simplified problems can be used to 
generate high quality pre-conditioning solutions and thus dramatically improve the 
computational tractability of complex problems. The framework could be used for 
solving the complex optimization problems with a large number of decision variables.

For example, Fu et al. (2012) has used the framework for reducing the complexity 
of the multi-objective optimization problems in water distribution system (WDS), and 
applied it to two case studies with different levels of complexity - the New York 
Tunnels rehabilitation problem and the Anytown rehabilitation/redesign problem. For 
the New York Tunnels network, because the original optimization problem has 21 
decision variables (pipes) and each variable has 16 options, the decision space is 
$16^{21} = 1.934 \times 10^{25}$. The simplified problem with 8 decision variables based on 
Sobol’s analysis have a decision space of $16^8 = 4.295 \times 10^9$. To obtain the same 
threshold of hypervolume value 0.78 for the New York Tunnels rehabilitation problem, 
the most pre-conditioned search need is 30 to 40% of the NFE compared to the full
search through 50 random seed trials. In the case of the Anytown network, the original
problem has a space of $2.859 \times 10^{73}$, and the simplified problem has a significantly
reduced space of $8.364 \times 10^{38}$. Through 50 random seed trials for the Anytown
rehabilitation/redesign problem, the full search requires average of 800000
evaluations to reach hypervolume value 0.77, and the pre-conditioned search exceeds
hypervolume value 0.8 in all trials in fewer than 200000 evaluations. The results also
show that searching in such significantly reduced space formed by sensitive decision
variables makes it much easier to reach good solutions, and the sensitivity-informed
reduction of problem size and pre-conditioning improve the efficiency, reliability and
effectiveness of the multi-objective evolutionary optimization.

It should be noted that the framework for sensitivity-informed dimension
reduction of optimization problems is completely independent of multi-objective
optimization algorithms, that is, any multi-objective algorithms could be embedded in
the framework. When dealing with three or more objectives, the formulation of the
optimization problems with a significantly reduced number of decision variables will
dramatically reduce the computational demands required to attain Pareto approximate
solutions in a similar way to the two-objective optimization case studies considered in
this paper.

6 Conclusions

This study investigates the effectiveness of a sensitivity-informed optimization
method for the ROS multi-objective optimization problems. The method uses a global
sensitivity analysis method to screen out insensitive decision variables and thus forms simplified problems with a significantly reduced number of decision variables. The simplified problems dramatically reduce the computational demands required to attain Pareto approximate solutions, which themselves can then be used to pre-condition and solve the original (i.e., full) optimization problem. This methodology has been tested on two case studies with different levels of complexity— the Dahuofang reservoir and the inter-basin multi-reservoir system in Liaoning province, China. The results obtained demonstrate the following:

1. The sensitivity-informed dimension reduction dramatically increases both the computational efficiency and effectiveness of the optimization process when compared to the conventional, full search approach. This is demonstrated in both case studies for both MOEA efficiency (i.e., the NFE required to attain high quality tradeoffs) and effectiveness (i.e., the quality approximations of optimal ROS tradeoffs relationships between conflicting design objectives).

2. The Sobol’s method can be used to successfully identify important sensitive information between different decision variables in the ROS optimization problem and it is important to account for interactions between variables when formulating simplified problems.

Overall, this study illustrates the efficiency and effectiveness of the sensitivity-informed method and the use of global sensitivity analysis to inform dimension reduction. This method can be used for solving the complex multi-objective optimization problems with a large number of decision variables, such
as optimal design of water distribution and urban drainage systems, distributed hydrological model calibration, multi-reservoir optimal operation and many other engineering optimization problems.

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Table 1 Reservoir characteristics and yearly average inflow ($10^6$ m$^3$)

<table>
<thead>
<tr>
<th>Reservoir name</th>
<th>Minimum capacity</th>
<th>Utilizable capacity</th>
<th>Flood control capacity</th>
<th>Yearly average inflow</th>
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<tr>
<td>Dahuofang</td>
<td>134</td>
<td>1430</td>
<td>1000</td>
<td>1570</td>
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Table 2 Characteristics of each reservoir in the inter-basin multi-reservoir system

<table>
<thead>
<tr>
<th>Reservoir</th>
<th>Active storage ($10^6$ m$^3$)</th>
<th>Role in water supply project</th>
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<tbody>
<tr>
<td></td>
<td>Flood season</td>
<td>Non-flood season</td>
</tr>
<tr>
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<td>Shenwo</td>
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<td>543</td>
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Table 3 Comparisons of water shortage indices among different solutions

<table>
<thead>
<tr>
<th>Solutions</th>
<th>Water Shortage Index ((-))</th>
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<th></th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Industrial water demand</td>
<td>Agricultural water demand</td>
<td></td>
</tr>
<tr>
<td>(S0) Industry-favoring solution</td>
<td>0.000</td>
<td>3.550</td>
<td></td>
</tr>
<tr>
<td>(S1) Agriculture-favoring solution</td>
<td>0.020</td>
<td>1.380</td>
<td></td>
</tr>
<tr>
<td>(S2) Compromised solution</td>
<td>0.007</td>
<td>1.932</td>
<td></td>
</tr>
</tbody>
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