Supplement of

Hydrological model parameter dimensionality is a weak measure of prediction uncertainty

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Supplementary Material: SIXPAR code and equations

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1 Mex and C code

```c
#include "mex.h"
#include <stdio.h>
#include <stdlib.h>
#include <math.h>

/* Function declaration */
static void sixpar2(int ndata, double *xpar, double *data, double *t1);

/* The gateway routine */
void mexFunction(int nlhs, mxArray *plhs[], int nrhs, const mxArray *prhs[])
{
    double *outpt, *par;
    double *dat;
    int m, n, size;

    /* Get size of input array */
    m = mxGetM(prhs[1]);
    n = mxGetN(prhs[1]);
    size = m*n;

    /* Create matrix for the return argument */
    plhs[0] = mxCreateDoubleMatrix(size,1,mxREAL);

    /* Assign pointers to the various parameters */
    par = mxGetPr(prhs[0]);
    dat = mxGetPr(prhs[1]);
    outpt = mxGetPr(plhs[0]);
```
/* Calling the C routine*/
sixpar2(size, par, dat, outpt);
return;
/
*/ Subroutines */
static void sixpar2(int ndata, double *xpar, double *data, double *t1)
{
    int i, j;
    double um, uk, bm, z, x, iperc, a;
    double bk, us, bs, perc, r, b, s, s0;
    double rlzdr, yy, zz, d;

    /* Assign par to model parameters */
    um=xpar[0];
    uk=xpar[1];
    bm=xpar[2];
    bk=xpar[3];
    z=xpar[4];
    x=xpar[5];
    iperc=0.0;
    a=z;

    /* Initialize the states */
i=0;
    us=0.5*um;
    bs=0.5*bm;

    for (i=0;i<ndata;i++) {
        r=0.0;
        s=0.0;
        b=0.0;

        us=us+data[i];
        rlzdr= (bm-bs)/bm;
        if (rlzdr <= a) {
            goto L900;
        }
        else{
            perc=us;
            us=0.0;
            goto L910;
        }
    }

L900:
L910:

2
\[
L900:
\begin{align*}
yy &= \frac{bm \times bk}{um}; \\
zz &= \frac{bm - bs}{bm \times a}; \\
\text{if} \ (zz <= 0.0) \\
& \quad \text{zz = 0;}
\end{align*}
\]

\[
\begin{align*}
\text{else} \{ \\
& \quad zz = \text{pow}(zz, x); \\
& \text{perc} = yy + zz \times (us - yy); \\
& \text{if}(\text{iperce} == 1) \\
& \quad \text{if}(\text{perc} > us) \\
& \quad \quad \text{perc} = us; \\
& \}
\end{align*}
\]

\[
\begin{align*}
us &= us - \text{perc};
\end{align*}
\]

\[
L910:
\begin{align*}
bs &= bs + \text{perc}; \\
\text{if} \ (bs > bm) \\
& \quad \text{goto L120;}
\end{align*}
\]

\[
\begin{align*}
\text{else} \{ \\
& \quad b = bs \times bk; \\
& \quad bs = bs - b; \\
& \quad \text{goto L130;}
\end{align*}
\]

\[
L120:
\begin{align*}
d &= bs - bm; \\
b &= bm \times bk; \\
bs &= (bm - b) + d; \\
\text{if}(bs > bm) \\
& \quad \text{goto L140;}
\end{align*}
\]

\[
\begin{align*}
\text{else} \{ \\
& \quad \text{goto L150;}
\end{align*}
\]

\[
L140:
\begin{align*}
d &= bs - bm; \\
bs &= bm; \\
us &= us + d;
\end{align*}
\]
if (us > um){
  goto L160;
}
else{
  r = 0.0;
  goto L170;
}

r = us - um;
us = um;

s = us * uk;
us = us - s;
t1[i] = r + s + b;

return;

2 Equations and Explanation

The SIXPAR equations are based on modified percolation equation that appeared in Gupta and Soороoshian (1983). The SIXPAR model structure contains one lower and one upper reservoir with capacities $S_{u_{max}}$ and $S_{b_{max}}$ respectively. The following table provides a mapping between the specification of variables and parameters in the code presented above and equations to be presented below.

Runoff is the sum of outflows generated by upper, $r^u(t) = k^uS^u(t)$, and lower zones, $r^b(t) = k^bS^b(t)$, and excess that may be generated when upper zone storage exceeds its capacity, i.e. $r^e(t) = \max(S_{u_{max}} - S^u, 0)$.

Percolation from upper to lower zone depends on relative lower zone deficit, $\Delta(t) = \frac{s^b_{max} - s^b(t)}{S_{b_{max}}}$ and $z$. If $\Delta(t) \leq z$ then the following specifies the percolation flux.

$$\Delta_z(t) = \frac{\Delta(t)}{z}$$
$$\Delta_x(t) = (\Delta_z(t))^x$$
Table 1: Definitions of variables and parameters used in the code and the presented model equations. Only those variable related to main model equations are presented.

<table>
<thead>
<tr>
<th>C code</th>
<th>Equations</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameters</td>
<td></td>
<td></td>
</tr>
<tr>
<td>UM [mm]</td>
<td>$S_u^{\text{max}}$</td>
<td>upper zone capacity</td>
</tr>
<tr>
<td>UK [day$^{-1}$]</td>
<td>$k_u$</td>
<td>upper zone recession</td>
</tr>
<tr>
<td>BM [mm]</td>
<td>$S_b^{\text{max}}$</td>
<td>lower zone capacity</td>
</tr>
<tr>
<td>BK [day$^{-1}$]</td>
<td>$k_b$</td>
<td>lower zone recession</td>
</tr>
<tr>
<td>Z[-]</td>
<td>$z$</td>
<td>percolation scaling parameter</td>
</tr>
<tr>
<td>X[-]</td>
<td>$x$</td>
<td>percolation power parameter</td>
</tr>
</tbody>
</table>

| Variables |
| US[mm] | $S_u(t)$ | upper zone content                   |
| BS[mm] | $S_b(t)$ | lower zone content                   |
| b[mm/day] | $r_b(t)$ | upper zone outflow                   |
| s[mm/day] | $r_u(t)$ | lower zone outflow                   |
| r[mm/day] | $r_e(t)$ | upper zone saturation excess         |
| perc[mm/day] | $q(t)$ | upper zone to lower zone percolation |
| data[mm/day] | $p(t)$ | effective precipitation              |

$$ q(t) = \gamma + \Delta_z(t)(S_u(t) - \gamma(t)) $$

Where $\gamma(t) = \frac{S_u(t)S_b^{\text{max}}k_b}{S_u^{\text{max}}}$. However if $\Delta(t) > z$ then the relative deficit in the lower reservoir is sufficiently high to empty the entire upper zone in the form of percolation.

The mass balance equation for the two reservoirs update the contents given the above flux equations and effective precipitation, $p(t)$ that enters the upper zone. For additional details, readers are referred to Gupta and Sorooshian (1983).

References