Dear Editor:

Please consider our updated manuscript for acceptance. We wish to thank the two reviewers for their helpful reviews that have led us to improve the paper. Below is a point-by-point discussion of the changes we have made to address the comments from both reviewers. We have also provided a “tracked-changes” draft of the main body of the paper and the three updated figures following this discussion.

General Comment
In our paper we extended the concept of the flood-envelope curve (a common technique to estimate the maximum-probable flood for ungaged drainage basins) to include event probability or recurrence interval explicitly. The reviewer counters that we neglected many of the known controls on discharge in our model framework. We accept his/her point but we wish to note that methods for predicting peak discharges come in many different forms with many different levels of complexity. On the simplest end of the spectrum are models that relate peak discharge to drainage area alone. These methods include the flood-envelope curve and related regional fits of peak discharge data to drainage area. Such methods neglect many known controls on discharge but they are not “wrong.” Rather, they capture the first-order control on peak discharge and have the advantage of requiring very little input data (this is an advantage because more sophisticated models with more parameters are not necessarily superior, i.e. they can be overfit). Our goal was to develop a method for predicting peak discharges that retains the simplicity of the flood-envelope curve yet allows for variable recurrence intervals. As such, while the reviewer is correct that many dozens of variables control the hydrological response of watersheds, we dispute the suggestion that all hydrology studies must explicitly include all known controls. We believe that the simplifications we have made are appropriate within the context of the goal of our project, which was to generalize the flood-envelope-curve approach to variable recurrence intervals and to understand the first-order controls on the shape of frequency-magnitude-area plots of peak discharge.

Reviewer 1

Hydrology Questions/Comments
Q1) The estimation of the losses via a runoff coefficient computed elsewhere is a significant assumption that requires validation in real watersheds of the study area (see also point 3) by comparison with observed discharge. Since the authors have used real precipitation events and not synthetic ones, this could be done. As it is, I
have very little confidence in the results of the methodology (even if they may be correct).

A) We chose to use two existing studies to estimate the runoff coefficients for our model drainage basins, one of which is, in fact, based on our study area. The runoff coefficient was estimated using Vivoni et al. (2007) for smaller basins (less than $10^3$ km$^2$) and Rosenberg et al. (2013) for larger basins.

The data for small basins comes from a model applied to a basin in Oklahoma with runoff coefficients calculated for wet, medium, and dry conditions. Not only is this one of the few studies that report runoff coefficients for such small drainage basins, it is the only study that we are aware of that reports runoff coefficients for a range of antecedent moisture conditions. Antecedent moisture conditions are undoubtedly important for hydrologic response yet are non-trivial to constrain or specify in a way that does not involve a large number of poorly constrained parameters. Runoff-coefficient data from real drainage basins of the size considered by the Vivoni et al. (2007) study simply are not available within the CRB for the range of antecedent moisture conditions.

The data for large basins comes from the aggregated annual runoff coefficients calculated for basins within our study area in the CRB. These values are directly applicable to our study areas. The use of the previously published runoff coefficients is supported by the resulting relationships between basin area and runoff coefficients that show the expected pattern with wet, medium, and dry conditions causing high, medium, and low runoff coefficients.

Although we have real precipitation data from the CRB, modeling runoff coefficients for specific basins in the CRB was not within the scope of this paper (i.e. we did not use precipitation data in a detailed way, i.e. relating it to one specific basin area) and would warrant another study altogether in order to get a representative value. This second study would need to be a regional assessment type study that is similar in size and scope to those studies and reports we used in order to find channel slope data for the CRB.

Our methods in this study are simplified relative to detailed process-based drainage basin hydrologic models, but this was by design. We believe our simplified approach is warranted by the fact that the end goal of the study is to quantify and predict the effects of basin area on peak discharges within a given region in order to provide a tool for quickly estimating the recurrence intervals of extreme floods in ungaged drainage basins using area as the sole required input parameter.

Q2) It is not clear how the runoff coefficient is included in the calculations and the symbols and equations introduced never mentioned.

A) The runoff coefficients were used to remove a volume of water before the water volume was distributed and routed through the model drainage basin. This was done for the three moisture conditions separately (results shown in Fig. 7). This was described in the text, but not shown mathematically. An equation and text describing this step of the analysis has been added:

“The flow-routing algorithm we employ does not explicitly include infiltration and other losses that can further reduce $Q_{rd}$ relative to $Q_p$. In this study we modeled infiltration and evaporation losses by simply removing a volume of water per unit time
equal to one minus the runoff coefficient, i.e. the ratio of runoff to precipitation over a specified time interval, for three antecedent-moisture scenarios (wet, med, and dry). We estimated runoff coefficients for each contributing-area class and each of three antecedent-moisture scenarios using published values for annual runoff coefficients for large basins within the UCRB and LCRB (Rosenburg et al., 2013) and published values for event-based runoff coefficients for small basins modeled with a range of antecedent-moisture conditions by Vivoni et al. (2007) (Fig. 3). On average, estimated runoff coefficients are higher for smaller and/or initially wetter basins. We found the dependence of runoff coefficients on contributing area and antecedent moisture to be similar despite the large difference in time scales between event-based and annual values. Despite the difference in geographic region between our study site and that of Vivoni et al. (2007) (they studied basins in Oklahoma), the runoff coefficients they estimated are likely to be broadly applicable to the LCRB and UCRB given that basin size and antecedent moisture are the primary controls on these values (climate and soil types play a lesser role except for extreme cases).

We applied the estimated runoff coefficients for all three antecedent-moisture scenarios by simply using them to remove a portion of the $Q_p$ calculated for specific time interval and basin area

$$Q_{pm} = C*Q_p$$

where C is the runoff coefficient calculated for the specific basin area and antecedent-moisture scenario under evaluation. The newly formed $Q_{pm}$ is now the $Q_p$ value for the wet, medium, or dry antecedent-moisture scenario under analysis.”

“The assigned channel slope and width values, together with the values of $Q_{pm}$ modified for each antecedent-moisture scenario, were used to calculate the depth-average velocities, $V$ (m s$^{-1}$), in hypothetical 1D main-stem channels of idealized square drainage basins corresponding to each contributing-area and time-interval-of-measurement class. In this study, flow velocity is not modeled over space and time, but rather is set at a constant value appropriate for the peak discharge using an iterative approach that solves for the peak depth-averaged flow velocity, uses that velocity to compute the parameters of the diffusion-wave-routing algorithm, routes the flow, and then computes an updated estimate of peak depth-averaged velocity.”

**Q3)** How are the wet, medium, and dry conditions taken into account? This was not explicitly described.

A) Please see above.

**Q4)** The same problem applies for the assumption of a triangular shape of the transfer function: it requires validation.

A) We are aware that basin size, shape, and the topology of the stream network can affect flood magnitudes. However, in this study we chose to avoid such basin-specific characteristics in order to seek a more general understanding and prediction of how event discharges scale with drainage area. We chose to use a triangular basin area function on the basis of the fact that the average basin area and/or width function has been found to
be approximately triangular based on many previous studies (Marani et al., 1994; Rinaldo et al., 1995; Veneziano et al., 2000; Rodriguez-Iturbe and Rinaldo, 2001; Puente and Sivakumar, 2003; Saco and Kumar, 2008; Rigon et al., 2011). Using a triangular basin area function gives us a smooth and simplified representation of real basins without including the unique individual noise of a specific basin. A future study in this area could be the effect of the shape of the basin width function on the peak flood magnitude as well as other discharge characteristics. Text and additional references concerning this assumption have been added:

“The flow-routing algorithm routes flow along the main-stem channel of idealized square basins with sizes equal to the contributing area of each contributing-area class. The choice of a square basin is consistent with the square sample areas (see Section 3.1) and it allows for basin shape to remain the same (and therefore comparable) over the range of contributing areas used in this study. The main-stem channel, with a length of \( L \) (m), was defined as the diagonal distance from one corner to the opposite corner across the square basin (i.e. \( L \) is equal to the square root of two times the area of the square basin). This main-stem channel was used in conjunction with a normalized area function to represent the shape of the basin and the routing of runoff through the drainage basin network. By including the normalized area function, we can account for geomorphic dispersion (i.e. the attenuation of the flood peak due to the fact that precipitation that falls on the landscape will take different paths to the outlet and hence reach the outlet at different times) in our analyses. The normalized area function, \( A(x) \) (unitless), is defined as the portion of basin area, \( A_L(x) \) (m\(^2\)), that contributes flow to the main-stem channel within a given range of distances \( x \) from the outlet, normalized by the total basin area, \( A_T \) (m\(^2\); Mesa and Mifflin, 1986; Moussa, 2008). The normalized area function is assumed to be triangular in shape with a maximum value at the midpoint of the main-stem channel from the outlet. Area functions, and related width functions, from real basins used in other studies show this triangular shape in general (Marani et al., 1994; Rinaldo et al., 1995; Veneziano et al., 2000; Rodriguez-Iturbe and Rinaldo, 2001; Puente and Sivakumar, 2003; Saco and Kumar, 2008), although not all basins show this shape. The triangular area function has been shown to approximate the average area function of basins and that the peak discharge and time to peak discharge is likely more important to the shape of the flood wave (Henderson, 1963; Rodriguez-Iturbe and Valdes, 1979).”

Q5) The only validation performed is against the FEC curves published for LCRB and U.S., which are based on observed discharges, after post-processing the results via the frequency analysis. Figure 7 shows significant differences (the axis is logarithmic) between the FEC curves and those generated via the FMA method, which are based on observed precipitation. The authors have not explained the reasons of these discrepancies and it is hard to have confidence on these results, especially considering the potential use of these curves for flood-related management and design purposes.

A) It is incorrect to state that the comparison of the FMACs to the published FECs is a validation. These curves are very different and there is no reason to expect that they should match. First, the data used to create each curve are not the same (i.e. FMACs use our rainfall-runoff model-derived flood values while FECs use measured floods from the record). Second, we did not expect the FMACs to match the U.S. FEC in magnitude (and possibly even in shape) because of the variation in types of storms and flooding associated with the U.S. FEC (especially larger extreme forcings like hurricanes) that are not included within the data for the UCRB and LCRB. However, we did expect to see generally similar shapes and/or order of magnitudes between the FMACs and FECs for the LCRB because they are the same hydroclimatic region. There are discrepancies between the two curves, but exact reasons between the discrepancies are very difficult to determine when using a simplified model approach and would need to be addressed using a model that included and tested those variables. Lastly, one of the motivations for creating a new method is that we feel that the FEC curve is biased towards underestimating the size of large floods in larger drainage basins. The FEC curve is defined by the largest flood, and since there are many more small drainage basins within any hydroclimatic region than large drainage basins, it is likely that the maximum flood for the smaller drainage basins will represent a more extreme (i.e. high recurrence interval or low flow duration) event. Our method corrects for this bias.

Q6) Why have not the authors considered real basins with real stream networks? The basin shape (that affects the rainfall effectively fallen in the basin) and the stream network organization are known to have critical importance on the flood timing and magnitude.

A) We are aware that the individual basin shape and stream network are an integral part in understanding flood size, timing, and nature. Again, this paper is looking for regional trends in flood size and frequency and how they scale with drainage basin area. This approach was motivated by the history of predicting peak flood discharges from simple variables, such as basin area. Moreover, it would have been difficult or impossible to aggregate different basins together within a space-for-time substitution (in which subbasins within a given hydroclimatic regime provide replicates of each other than allow extreme floods to be estimated from a relatively short record) without subdividing each large basin into equal size smaller basins as we did. That aggregation is central to the whole idea and it would have been much more difficult if subdivided the watersheds into non-equal areas.

Q7) Given the simplified nature of the method, no contribution of snow and snowmelt was considered. This has to be stated. Regarding the snow contribution, I
have also doubts about what has been stated on p. 11759, line 28, and p. 11760, line 1: are the authors assuming that NEXRAD products provide snowfall (which they don’t)?

A) We agree that the exclusion of snow effects and the focus on rainfall-generated floods was not stated clearly. In the revised paper we have modified the discussion of the NEXRAD processing and the title of the paper to make clear that we are considering rainfall-triggered floods only (i.e. not snowmelt floods or rain-on-snow floods). On the lines pointed out by the reviewer, we explain that the NEXRAD data likely does include some snowfall measurements. These snowfall measurements, as stated in the discussion, would be identified by the NEXRAD processing as a low-intensity precipitation event. However, in this study we are only interested in the maximum precipitation intensity and therefore these values would effectively be ignored. We should also note that we choose to work on the Colorado River Basin in part because snowmelt-induced flooding is expected to be the dominant cause of flooding for only a small portion of these watersheds (e.g. Niezgoda and West, 2012, relate the predominance of snowmelt-induced flooding to the portion of drainage basins above 9000 ft in elevation in the western U.S.). We don’t think this limitation negatively impacts the importance of our work for rainfall-generated floods.

Changes to the title and the text are as follows:

“Constraining frequency-magnitude-area relationships for rainfall and flood discharges using radar-derived precipitation estimates: Example applications in the Upper and Lower Colorado River Basins, USA”

“In this study, a new method for estimating flood discharges associated with user-specified recurrence intervals is introduced that uses radar-derived precipitation estimates (in this case rainfall only), combined with the diffusion-wave flow-routing algorithm, to create frequency-magnitude-area curves (FMACs) of flood discharge. Our method (i.e. the FMAC method) retains the power of the FEC approach in that data from different drainage basins within a hydroclimatic region are aggregated by contributing area, thereby enabling large sample sizes to be obtained within each contributing-area class in order to more accurately constrain the frequencies of past extreme flood events and hence the probabilities of future extreme flood events within each class. The method improves upon the FEC approach in that the complete spatial coverage of radar-derived precipitation estimates provides for large sample sizes of most classes of contributing area (larger contributing areas have fewer samples). The radar-derived precipitation estimates include only rainfall and therefore snow and other types of precipitation are not included in the study. The precipitation estimates are then used to predict flood discharges associated with specific recurrence intervals by first accounting for water lost to infiltration and evapotranspiration using runoff coefficients appropriate for different contributing areas and antecedent-moisture conditions, and then routing the available water using a flow-routing algorithm. Predicted flood discharges are presented as FMACs on log-log plots, similar to traditional FECs, except that the method predicts a family of curves, one for each user-defined recurrence interval. These plots are then compared to FECs for the study region (Enzel et al., 1993) and the U.S. (Costa, 1987).”
“Under- and over-estimation of precipitation by NEXRAD products in relation to rain-gauge data is partly due to the difference in sampling between areal NEXRAD products and point data from rain gauges and partly due to sampling errors inherent to both methods. For example, NEXRAD products include problems such as the use of incorrect Z-R relationships for high intensity storms and different types of precipitation, such as snow and hail (Baeck and Smith, 1998). Also, because of its low reflectivity, snow in the NEXRAD products is measured as if it were light rain (David Kitzmiller, personal communication, January 10, 2012). This means the NEXRAD products likely underestimate snowfall and therefore snowfall is not fully accounted for in this study. Due to snowfall not being included in this study, associated snowpack and snowmelt effects were also not accounted for. Rain gauges can also suffer from a number of measurement errors that usually result in an underestimation of rainfall (Burton and Pitt, 2001). In addition, gridded rainfall data derived from rain gauges are not spatially complete and therefore must be interpolated between point measurements to form a spatially complete model of rainfall. It is impossible to discern which product is more correct due to the differences in measurement techniques and errors, but by taking both products and combining them into one, the Stage III NEXRAD precipitation products generate the best precipitation estimate possible for this study. Moreover, it should be noted that 100-year flood magnitude predictions based on regression equations have very large relative error bars (ranging between 37 to 120% in the western U.S.; Parrett and Johnson, 2003) and that measurements of past extreme floods can have significant errors ranging from 25% to 130% depending on the method used (Baker, 1987). As such, even a ~50% bias in NEXRAD-product-derived precipitation estimates is on par or smaller than the uncertainty associated with an analysis of extreme flood events.”

“As stated previously, the NEXRAD precipitation estimates used here do not include snowfall and other non-rainfall precipitation types. In this study we also do not include snowpack information into our flood discharge calculations. The omission of snowpack is a reasonably assumption for our low elevation, warm regions within most of the UCRB and LCRB. However, we acknowledge some of our higher elevation areas at higher latitudes may be underestimating the maximum flood discharge by only including rainfall-derived runoff. If the methodology in this paper were applied to a snowmelt-dominated region, snowpack would need to be added to accurately estimate the maximum flood discharge.”


Q8) The description of the methodology is not complete and some details not well explained. I think that more symbols and equations should be introduced to explain better each step, along with a figure that shows a schematic of the approach and an example of a basin (I found Fig. 2 not informative at all).
A) We have added to the description of the methodology and equations based on the other points brought up in this review. A schematic flow chart of the steps within the methods has been added below, please let us know if the schematic is helpful.

“Figure #. Schematic diagram of methodology used in this paper. (A) Rainfall data is sampled over spatial and temporal scales in factors of two. This sampling does not only include looking at the data within a given spatial or temporal scale, but aggregating it over that scale. These values are ranked for a given basin area and time interval to complete the frequency analysis. This results in rainfall intensities (I) for each spatial scale (basin area), temporal scale (time interval or storm duration), and frequency. (B) Intensities sampled from the rainfall data are used to calculate rainfall discharge (Q_p and Q_{pm}) values that are then put through the flow routing algorithm in order to calculate flood discharge (Q_{fd}) values. Q_{fd} values are then used to construct the frequency-magnitude-area curves (FMACs) showing the data for recurrence intervals of 10, 50, 100, and 500 years.”

**Frequency Estimation Questions/Comments**

Q9) In extreme value theory, recurrence intervals are calculated for independent events, either deriving annual maxima or through the peak over threshold approach. In both cases, a time series of a variable observed at a location or a basin is used. In the paper under review, the computation of the recurrence interval accounts for all events observed in all basins of the same drainage area. Assuming that we have N basins with the same area (e.g. 64 km2) included in the Upper and Lower CRBs, this implies that the recurrence interval is calculated by pooling together N time series of a variable. Through this method, the authors could present discharge values for the 500-year return period, using 10 years of rainfall records. However, since storms may have happened at the same time in contiguous basins, the events may not be statistically independent, as they are originated from the same weather pattern. In other words, increasing the sample size with records of contiguous basins is not a trivial operation, which requires careful evaluation. This may contradict the principle of extreme value theory. Addressing this issue is crucial to build FMA curves and the authors have not provided any justification.

A) We are aware that extreme value theory requires that values within the distribution be statistically independent of one another. The reviewer’s comments have inspired us to check our calculations and check that our methods are consistent with the peak over threshold method. We have made a few minor changes to our code that make sure we identify the peak discharge associated with rainfall events (associated with the peak intensities of individual rainfall events) of a given recurrence interval without double counting. We specify a threshold value of zero and use it to identify individual storm events in our data, i.e. storm events are identified by adjacent strings of intensity values above zero separated from other strings of intensity values by zeros.

In this method we also consider a range of possible storm durations to arrive at the peak rainfall intensities and associated discharges for a given sized watershed. However, the main purpose of specifying a threshold in the peak-over-threshold approach is to avoid “double counting,” i.e. counting multiple peaks of a single flood event as two
or more separate events. Our routing method, which uses a triangular width function and assumes constant rainfall over the duration of the storm, produces a single peak in the hydrograph. As such, there is no possibility of double counting, i.e. there is a single peak discharge associated with each rainfall event.

The minor changes to the code have changed some of our values, but only slightly. That is, those intensity, precipitation discharge, and flood discharges that have changed only changed by a very small amount, keeping the trends and conclusions in our paper the same. The largest changes were those of the errors, which in general increased slightly based on the fact that there are larger differences between the value so the specified ranks and those at the next highest rank. This is to be expected since our minor changes resulted in less duplicates and less samples overall. Changes to the text (mostly the power-law fits), tables (both tables 1 and 2), and figures (figures 6 and 7) have been incorporated. Please see marked copy of manuscript.

Q10) Additionally, in the case of precipitation, a fixed duration is utilized in extreme value theory to compute the recurrence interval (e.g. the 100 year rainfall intensity for 1-h duration). In this paper, the authors find the maximum intensity recorded for different aggregations times, chosen arbitrarily. This choice has to be supported as well.

A) The time intervals used to integrate the precipitation data were not chosen arbitrarily. We chose to use time intervals of powers of 2 to simplify the approach and to incorporate a range of time intervals from 1 hour to 64 hours. As stated in the text, this range was chosen to include short-duration precipitation events such as convective-type and/or monsoon storms (typically high intensity, short duration summer storms in the UCRB and LCRB) and long-duration precipitation events that last on the order of days such as frontal-type storms (typically lower intensity, long duration winter storms in the UCRB and LCRB). It is important to note as well that the highest maximum precipitation intensities for a given basin area (the main focus of this study) were found during smaller time intervals, so including even one larger time interval would not change the results of this study.

Reviewer 2

Q1) The authors make a strong case in the introduction about the need to incorporate recurrence intervals to the FEC methodology. However, they do not indicate that to some extent, this has already been done. The work by Castellarin et al. (2005, 2007, 2009), which is mentioned in point 5.2 should be included in the introduction to show the real state of the art. As it is now, the only papers that are mentioned in the intro are more than 10 yrs old and it looks like nobody has done anything on the subject since then. Section 5.2 should be moved to the intro as it also does not belong in the discussion (too general and without any quantitative support). This may require some rewording and a clearer statement about the novelty of the current application.

A) We have accepted this suggestion by Reviewer 1 and moved section 5.2 to the introduction. This portion of the introduction now states:
Traditional FECs also have the potential problem that the maximum flood associated with smaller drainage basins may be biased upward (or the floods of larger drainage basins biased downward) because there are typically many more records of floods in smaller drainage basins relative to larger drainage basins (because there are necessarily fewer large drainage basins in any hydroclimatic region). That is, the largest flood of record for small drainage basins within a hydroclimatic region likely corresponds to a flood of a larger recurrence interval compared with the largest flood of record for larger drainage basins. In this paper we present a method that includes recurrence-interval information and avoids any sample-size bias that might exist as a function of contributing area.

The use of FECs to quantify flood regimes is limited by the lack of recurrence-interval information (Wolman and Costa, 1984; Castellarin et al., 2005) and by the short length, incomplete nature, and sparseness of many flood-discharge records. Without recurrence-interval information, the data provided by FECs are difficult to apply to some research and planning questions related to floods. In the U.S. for example, the 100- and 500-year flood events are the standard event sizes that define flood risk for land planning and engineering applications (FEMA, 2001).

Previously published studies have looked at new approaches to approve upon the FEC method. Castellarin et al. (2005) took a probabilistic approach to estimating the exceedance probability of the FEC for synthetic flood data. The authors were able to relate the FECs of certain recurrence intervals to the correlation between sites, the number of flood observations, and the length of each observation. Later, Castellarin (2007) and Castellarin et al. (2009) applied these methods to real flood record data and extreme rainfall events for basins within north-central Italy. Castellarin et al. (2009) also created depth-duration envelope curves of precipitation to relate extreme precipitation events to mean annual precipitation. This group of studies was successful in incorporating recurrence-interval information into the traditional FEC method. However, most of the models presented in these studies were completed with synthetic data or created for design storm processes and require additional analysis. Also, most of the precipitation data used in these past studies was collected using rain gauges (point sources), while only a small subset of data in Castellarin et al. (2009) was sourced from radar-derived precipitation estimates. In contrast to these studies we formulate a simplified method (i.e. the FMAC method) that is readily applicable to any region of interest and can be directly compared to already existing FECs. Also we favor the use of spatially complete radar-derived precipitation estimates in order to apply our methods to ungauged basins.

Q2) The methodology has a number of assumptions and simplifications that are not always thoroughly justified or tested. Since the final model results are not really suitable for a validation, more emphasis should be put into the individual components of the methodology to convince the reader of the validity of the results.

Please see the above comments from Reviewer 1 in which we have responded to specific concerns about certain assumptions and variables. Please let us know if there are other locations that require additional attention.
Q3) Regarding the last point, the selection of runoff coefficients needs a lot more justification. Figure 3 does not do a good job in convincing readers of a sensible methodology. The determination of the wet, dry and intermediate antecedent conditions runoff coefficients does not agree with the data very much, and may question the assumption that such simple separation is meaningful. For example, half of the dry data of Vivoni et al. (2007) is better described by the intermediate curve, and the same goes for half of the intermediate data that falls close to the wet curve. There is also no mention of the antecedent conditions of the Rosenberg et al. 2013 data. I would also argue that the Rosenberg data does not show any dependence of the runoff coefficient with contributing area. This poor agreement with the data is reflected by the low correlation coefficient, particularly for the dry antecedent conditions (0.04). The authors should justify the validity of the runoff coefficients, and also perform a sensitivity analysis. This is particularly important since the uncertainty analysis of 3.4 does not include parameter uncertainty.

The trend lines shown in Figure 3 were found as average trends of runoff coefficients with contributing area. The data from Vivoni et al. (2007) does vary for the dry and intermediate antecedent moisture conditions, but this is due to the length and intensity of the storm used to calculate those runoff coefficients and is interpreted as showing the low and high end of possible runoff coefficients under those antecedent moisture conditions. We chose to fit a trendline to the data including both the low and high end to get an average runoff coefficient relationship to contributing area with the understanding that the trendline may have a low correlation coefficient. We feel that this is warranted based on the lack of runoff coefficient data in the literature that includes antecedent moisture data (Vivoni et al.’s study was the only study to have this type of data that the authors know of) and the uncertainty associated with the broad drainage-basin-wide conditions this study includes that affect runoff coefficients. The Rosenberg data is the only runoff coefficient data we found for our study area and no antecedent moisture conditions were given for the data. This is understandable due to the large areas and yearly time frame over which these runoff coefficients were calculated. The Rosenberg data is also just a small sample of the many basins in the CRB and may therefore not show a clear dependence with contributing area. However, we would argue that the data do show that for larger drainage basins (>10^3 km^2) the runoff coefficients are less than 0.4, which constrains our trend lines to a lower runoff coefficient for larger basins than smaller basins. This constraint leaves a trendline with a predictable relationship of higher runoff coefficients occurring in smaller drainage basins.

Overall, it is unfortunate that a national assessment of runoff coefficients for each hydroclimatic region does not exist. This sort of study would need to use rainfall data (NEXRAD data or similar), soil moisture data (possibly from the NCEP reanalysis), and discharge data (USGS gages or similar) for available basins to calculate a runoff coefficient. This would be very helpful for many hydrologic and ecological studies. In the end we felt that this type of study was beyond the scope of our study and chose to rely on previously published studies for runoff coefficient information.
Thank you,

Caitlin A. Orem, oremc@email.arizona.edu

Jon D. Pelletier, Professor, jdpellet@email.arizona.edu
Constraining frequency-magnitude-area relationships for rainfall and flood discharges using radar-derived precipitation estimates: Example applications in the Upper and Lower Colorado River Basins, U.S.A.

Caitlin A. Orem*, Jon D. Pelletier

[1] {Department of Geosciences, The University of Arizona, 1040 E. 4th Street, Tucson, AZ 85721, USA}

Correspondence to: C.A. Orem (oremc@email.arizona.edu)

Abstract

Flood-envelope curves (FEC) are useful for constraining the upper limit of possible flood discharges within drainage basins in a particular hydroclimatic region. Their usefulness, however, is limited by their lack of a well-defined recurrence interval. In this study we use radar-derived precipitation estimates to develop an alternative to the FEC method, i.e. the frequency-magnitude-area-curve (FMAC) method, that incorporates recurrence intervals. The FMAC method is demonstrated in two well-studied U.S. drainage basins, i.e. the Upper and Lower Colorado River basins (UCRB and LCRB, respectively), using Stage III Next-Generation-Radar (NEXRAD) gridded products and the diffusion-wave flow-routing algorithm. The FMAC method can be applied worldwide using any radar-derived precipitation estimates. In the FMAC method, idealized basins of similar contributing area are grouped together for frequency-magnitude analysis of precipitation intensity. These data are then routed through the idealized drainage basins of different contributing areas, using contributing-area-specific estimates for channel slope and channel width. Our results show that FMACs of precipitation discharge are power-law functions of contributing area with an average exponent of $0.82 \pm 0.06$ for recurrence intervals from 10 to 500 years. We compare our FMACs to published FECs and find that for wet antecedent-moisture conditions, the 500-year FMAC of flood discharge in the UCRB is on par with the U.S. FEC for contributing areas of $\sim 10^2$ to $10^3$ km$^2$. FMACs of flood discharge for the LCRB exceed the published FEC for the LCRB for contributing areas in the range of $\sim 10^3$ to $10^4$ km$^2$. The FMAC method retains the power of the FEC method for constraining flood hazards in basins that are ungauged or have short flood
records, yet it has the added advantage that it includes recurrence interval information necessary for estimating event probabilities.

1. **Introduction**

1.1 **Flood-Envelope Curves**

For nearly a century, the flood-envelope curves (FEC), i.e. a curve drawn slightly above the largest measured flood discharges on a plot of discharge versus contributing area for a given hydroclimatic region (Enzel et al., 1993), have been an important tool for predicting the magnitude of potential future floods, especially in regions with limited stream-gauge data. FECs assume that, within a given hydroclimatic region, maximum flood discharges for one drainage basin are similar to those of other drainage basins of the same area, despite differences in relief, soil characteristics, slope aspect, etc. (Enzel et al., 1993). This assumption enables sparse and/or short-duration flood records over a hydroclimatic region to be aggregated in order to provide more precise constraints on the magnitude of the largest possible (i.e. long-recurrence-interval) floods.

FECs reported in the literature have a broadly similar shape across regions of widely differing climate and topography. For example, FECs for the Colorado River Basin (Enzel et al., 1993), the central Appalachian Mountains (Miller, 1990; Morrison and Smith, 2002), the 17 hydrologic regions within the U.S. defined by Crippen and Bue (1977), the U.S. as a whole (Costa, 1987; Herschy, 2002), and China (Herschy, 2002) are all concave-down when plotted in log-log space, with maximum recorded flood discharges following a power-law function of contributing area for small contributing areas and increasing more slowly at larger contributing areas (i.e. the curve “flattens”).

Traditional FECs also have the potential problem that the maximum flood associated with smaller drainage basins may be biased upward (or the floods of larger drainage basins biased downward) because there are typically many more records of floods in smaller drainage basins relative to larger drainage basins (because there are necessarily fewer large drainage basins in any hydroclimatic region). That is, the largest flood of record for small drainage basins within a hydroclimatic region likely corresponds to a flood of a larger recurrence interval compared with the largest flood of record for larger drainage basins. In this paper we present a method that includes recurrence-interval
information and avoids any sample-size bias that might exist as a function of contributing
area.

The use of FECs to quantify flood regimes is limited by the lack of recurrence-interval information (Wolman and Costa, 1984; Castellarin et al., 2005) and by the short length, incomplete nature, and sparseness of many flood-discharge records. Without recurrence-interval information, the data provided by FECs are difficult to apply to some research and planning questions related to floods. In the U.S. for example, the 100- and 500-year flood events are the standard event sizes that define flood risk for land planning and engineering applications (FEMA, 2001).

Previously published studies have looked at new approaches to approve upon the FEC method. Castellarin et al. (2005) took a probabilistic approach to estimating the exceedance probability of the FEC for synthetic flood data. The authors were able to relate the FECs of certain recurrence intervals to the correlation between sites, the number of flood observations, and the length of each observation. Later, Castellarin (2007) and Castellarin et al. (2009) applied these methods to real flood record data and extreme rainfall events for basins within north-central Italy. Castellarin et al. (2009) also created depth-duration envelope curves of precipitation to relate extreme precipitation events to mean annual precipitation. This group of studies was successful in incorporating recurrence-interval information into the traditional FEC method. However, most of the models presented in these studies were completed with synthetic data or created for design storm processes and require additional analysis. Also, most of the precipitation data used in these past studies was collected using rain gauges (point sources), while only a small subset of data in Castellarin et al. (2009) was sourced from radar-derived precipitation estimates. In contrast to these studies we formulate a simplified method (i.e. the FMAC method) that is readily applicable to any region of interest and can be directly compared to already existing FECs. Also we favor the use of spatially complete radar-derived precipitation estimates in order to apply our methods to ungauged basins.

In this study, a new method for estimating flood discharges associated with user-specified recurrence intervals is introduced that uses radar-derived precipitation estimates (in this case rainfall only), combined with the diffusion-wave flow-routing algorithm, to
create frequency-magnitude-area curves (FMACs) of flood discharge. Our method (i.e. the FMAC method) retains the power of the FEC approach in that data from different drainage basins within a hydroclimatic region are aggregated by contributing area, thereby enabling large sample sizes to be obtained within each contributing-area class in order to more accurately constrain the frequencies of past extreme flood events and hence the probabilities of future extreme flood events within each class. The method improves upon the FEC approach in that the complete spatial coverage of radar-derived precipitation estimates provides for large sample sizes of most classes of contributing area (larger contributing areas have fewer samples). The radar-derived precipitation estimates include only rainfall and therefore snow and other types of precipitation are not included in the study. The precipitation estimates are then used to predict flood discharges associated with specific recurrence intervals by first accounting for water lost to infiltration and evapotranspiration using runoff coefficients appropriate for different contributing areas and antecedent-moisture conditions, and then routing the available water using a flow-routing algorithm. Predicted flood discharges are presented as FMACs on log-log plots, similar to traditional FECs, except that the method predicts a family of curves, one for each user-defined recurrence interval. These plots are then compared to FECs for the study region (Enzel et al., 1993) and the U.S. (Costa, 1987).

1.2 Study Area

This study focuses on the Upper and Lower Colorado River Basins (UCRB and LCRB, respectively; Fig. 1) as example applications of the FMAC method. Although the methods we develop are applied to the UCRB and LCRB in the western U.S. in this study, the methods are applicable to any region of interest where radar-derived precipitation estimates are available (i.e. the entire U.S. and at least 22 countries around the world; Li, 2013; RadarEU, 2014). We focus on the UCRB and LCRB because they have been a focus of flood-hazard assessment studies in the western U.S. and hence the FECs available for them are of especially high quality. In addition, the distinctly different hydroclimatic regions of the UCRB and LCRB (Sankarasubramanian and Vogel, 2003) make working in these regions an excellent opportunity to test and develop the new methods of this study on different precipitation patterns and storm types.
Precipitation and flooding in the LCRB are caused by convective-type storms, including those generated by the North American Monsoon (NAM), and frontal-type and tropical storms sourced from the Pacific Ocean and the Gulf of California (House and Hirschboeck, 1997; Ethridge et al., 2004). In the UCRB, the influence of the NAM and tropical storms is diminished and floods are generally caused by Pacific frontal-type storms (Hidalgo and Dracup, 2003). In both regions, the El Niño Southern Oscillation (ENSO) alters the frequency and intensity of the NAM, tropical storms, and the Pacific frontal systems, and can cause annual variations in precipitation and flooding (House and Hirschboeck, 1997; Hidalgo and Dracup, 2003). Winter storms in both regions are also intensified by the occurrence of atmospheric rivers (Dettinger et al., 2011), which can cause total winter precipitation to increase up to approximately 25% (Rutz and Steenburgh, 2012). The radar-derived precipitation estimates used in this study record this natural variability in precipitation in the two regions.

The methods used in this study to calculate precipitation and flood discharges of specified recurrence intervals from radar-derived precipitation estimates require a few main assumptions. The first assumption is that of climate stationarity, i.e. the parameters that define the distribution of floods do not change through time (Milly et al., 2008). Climate is changing and these changes pose a challenge to hazard predictions based on the frequencies of past events. Nevertheless, stationarity is a necessary assumption for any probabilistic analysis that uses past data to make future predictions. The results of such analyses provide useful starting points for more comprehensive analyses that include the effects of future climate changes. The second assumption is that the sample time interval is long enough to correctly represent the current hydroclimatic state (and its associated precipitation patterns and flood magnitudes and risks) of the specified study area. Our study uses data for the 1996 to 2004 water years and therefore may be limited by inadequate sampling of some types of rare weather patterns and climate fluctuations within that time interval. To address whether or not the sample time interval used in this study includes major changes in circulation and weather patterns, and therefore is a good representation of climate in the CRB, we investigated the effect of the El Niño Southern Oscillation (ENSO) on precipitation intensity within the UCRB and LCRB. ENSO is a well-known important influence on the hydroclimatology of the western U.S. (Hidalgo...
and Dracup, 2003; Cañon et al., 2007). In general, winter precipitation in the southwestern U.S. increases during El Niño events and decreases during La Niña events (Hidalgo and Dracup, 2003). The opposite effects are found in the northwestern portions of the U.S. (including the UCRB; Hidalgo and Dracup, 2003). The last assumption of the method is that all basins of similar contributing area respond similarly to input precipitation, i.e. that they have similar flood-generating and flow-routing mechanisms. Specifically, the method assumes that basins of similar contributing area have the same runoff coefficient, flow-routing parameters, basin shape, and channel length, width, and slope. This assumption is necessary in order to aggregate data into discrete contributing-area classes so that the frequency of extreme events can be estimated from relatively short-duration records. In this study, high-recurrence-interval events (i.e. low frequency events) can be considered despite the relatively short length of radar-derived-precipitation-estimate records because the number of samples in the radar-derived record is extremely large, especially for small contributing areas and small duration floods. For example, for a 1-h time-interval-of-measurement and a contributing area of 4,096 km$^2$ event in the UCRB, there are approximately 40 (number of spatial scale samples) times 55000 (number of temporal scale samples in nine years of data) samples of precipitation values (and associated modeled discharges obtained via flow routing). As contributing area and time intervals of measurement increase there are successively fewer samples, within any particular hydroclimatic region, thus increasing the uncertainty of the resulting probability assessment for larger areas and longer time periods.

2. **Next-Generation-Radar (NEXRAD) Data**

The specific radar-derived precipitation estimates we use in this study come from the Stage III Next-Generation-Radar (NEXRAD) gridded product, which is provided for the entire U.S., Guam, and Puerto Rico. NEXRAD was introduced in 1988 with the introduction of the Weather Surveillance Radar 1988 Doppler, or WSR-88D, network (Fulton et al., 1998). The WSR-88D radars use the Precipitation Processing System (PPS), a set of automated algorithms, to produce precipitation intensity estimates from reflectivity data. Reflectivity values are transformed to precipitation intensities through the empirical $Z-R$ power-law relationship,
$Z = \alpha R^\beta$  \hspace{1cm} (1)

where $Z$ is precipitation rate (mm h$^{-1}$), $\alpha$ and $\beta$ are derived empirically and can vary depending on location, season, and other conditions (Smith and Krajewski, 1993), and $R$ is reflectivity (mm$^6$ m$^{-3}$; Smith and Krajewski, 1993; Fulton et al., 1998; Johnson et al., 1999). Precipitation intensity data are filtered and processed further to create the most complete and correct product (Smith and Krajewski, 1993; Smith et al., 1996; Fulton et al., 1998; Baeck and Smith, 1998). Further information and details about PPS processing are thoroughly described by Fulton et al. (1998).

Stage III NEXRAD gridded products are Stage II precipitation products mapped onto the Hydrologic Rainfall Analysis Project (HRAP) grid (Shedd and Fulton, 1993). Stage II data are hourly precipitation intensity products that incorporate both radar reflectivity and rain-gauge data (Shedd and Fulton, 1993) in an attempt to make the most accurate precipitation estimates possible. The HRAP grid is a polar coordinate grid that covers the conterminous U.S., with an average grid size is 4 km by 4 km, although grid size varies from approximately 3.7 km (north to south) to 4.4 km (east to west) in the southern and northern U.S., respectively (Fulton et al., 1998).

3. Methods

3.1 NEXRAD Data Conversion and Sampling

NEXRAD Stage III gridded products (hereafter NEXRAD products) for an area covering the Colorado River basin from 1996 to 2005 were downloaded from the NOAA HDSG website (http://dipper.nws.noaa.gov/hdsb/data/nexrad/cbrfc_stageiii.php) for analysis. The data files were converted from archived XMRG files to ASCII format (each data file representing the mean precipitation intensity within each 1 h interval) using the xmrgrtoasc.c program provided on the NOAA HDSG website. The ASCII data files were then input into a custom program written in IDL for analysis.

We quantified hourly precipitation intensities (mm h$^{-1}$) over square idealized basins (i.e. not real basins, but square basins as shown schematically in Fig. 2) of a range of areas from 16 km$^2$ to 11,664 km$^2$ (approximately the contributing area of the Bill Williams River, AZ, for readers familiar with the geography of the western U.S.) by successively spatially averaging precipitation-intensity values at HRAP pixel-length
scales of powers of two (e.g. 4, 16 pixel², etc.) and three (e.g. 9, 81 pixel², etc.; Fig. 2).
Spatial averaging is done by both powers of 2 and 3 simply to include more points on the
FMACs than would result from using powers of 2 or 3 alone. The number of samples
within each contributing area class limited the range of contributing areas used in this
study.
UCRB and LCRB boundaries from GIS hydrologic unit layers created by the
USGS and provided online through the National Atlas site
(http://www.nationalatlas.gov/atlasftp.html#hucs00m) were projected to HRAP
coordinates using the methods of Reed and Maidment (2006). These boundaries were
used to delineate the region from which precipitation data were sampled from the
NEXRAD products, i.e. when averaging precipitation data by powers of two and three a
candidate square drainage basin was not included in the analysis if any portion of the
square fell outside of the boundaries of the UCRB or LCRB (Fig. 2). Throughout the
analysis, the HRAP pixel size was approximated by a constant 4 km by 4 km size despite
the fact that HRAP pixel sizes vary slightly as a function of latitude (Reed and Maidment,
2006). Our study basins span latitudes between approximately 31°N and 43°N resulting
in a maximum error of 15%. However, by keeping the pixel size constant, all pixels could
be treated as identical in size and shape allowing us to sample the NEXRAD products in
an efficient and automated way over many spatial scales.
For larger contributing areas, necessarily fewer samples are available within a
given hydroclimatic region, thus increasing the uncertainty associated with the analysis
for those larger contributing-area classes. For the UCRB and LCRB specifically, the
uncertainty in the analysis becomes significant for contributing-area classes equal to and
larger than \(10^3\) to \(10^4\) km² depending on the recurrence interval being analyzed. Of
course, if the hydroclimatic region is defined to be larger, more samples are available for
each contributing-area class and hence larger basins can be analyzed with confidence.
In addition to computing precipitation intensities as a function of spatial scale, we
averaged precipitation intensities as a function of the time interval of measurement
ranging from 1 to 64 hours in powers of two by averaging contiguous hourly precipitation
intensity records over the entire 9-year study period. This range in time intervals was
chosen in order to capture precipitation events that last on the order of ~1 hour
(convective-type storms) to days (frontal-type storms).

3.2 Precipitation and Flood Calculations

Two types of variables were calculated from the precipitation intensities sampled
over the contributing-area and time-interval-of-measurement classes: (1) precipitation
discharge, $Q_p$, and (2) peak flood discharge, $Q_{fa}$. The variable $Q_p$ is defined as the
average precipitation intensity over a basin and time interval of measurement multiplied
by the contributing area, resulting in units of m$^3$ s$^{-1}$. The variable $Q_{fa}$ is the peak flood
discharge (m$^3$ s$^{-1}$) calculated via the diffusion-wave flow-routing algorithm for a
hypothetical flood triggered by a precipitation discharge, $Q_p$, input uniformly over the
time interval of measurement to idealized square basins associated with each
contributing-area class.

The flow-routing algorithm we employ does not explicitly include infiltration and
other losses that can further reduce $Q_{fa}$ relative to $Q_p$. In this study we modeled
infiltration and evaporation losses by simply removing a volume of water per unit time
equal to one minus the runoff coefficient, i.e. the ratio of runoff to precipitation over a
specified time interval, for three antecedent-moisture scenarios (wet, med, and dry). We
estimated runoff coefficients for each contributing-area class and each of three
antecedent-moisture scenarios using published values for annual runoff coefficients for
large basins within the UCRB and LCRB (Rosenburg et al., 2013) and published values
for event-based runoff coefficients for small basins modeled with a range of antecedent-
moisture conditions by Vivoni et al. (2007) (Fig. 3). On average, estimated runoff
coefficients are higher for smaller and/or initially wetter basins. We found the
dependence of runoff coefficients on contributing area and antecedent moisture to be
similar despite the large difference in time scales between event-based and annual values.
Despite the difference in geographic region between our study site and that of Vivoni et
al. (2007) (they studied basins in Oklahoma), the runoff coefficients they estimated are
likely to be broadly applicable to the LCRB and UCRB given that basin size and
antecedent moisture are the primary controls on these values (climate and soil types play
a lesser role except for extreme cases).
We applied the estimated runoff coefficients for all three antecedent-moisture scenarios by simply using them to remove a portion of the $Q_p$ calculated for specific time interval and basin area.

$$Q_{pm} = C \cdot Q_p$$  \quad (2)$$

where $C$ is the runoff coefficient calculated for the specific basin area and antecedent-moisture scenario under evaluation. The newly formed $Q_{pm}$ is now the $Q_p$ value for the wet, medium, or dry antecedent-moisture scenario under analysis.

The flow-routing algorithm routes flow along the main-stem channel of idealized square basins with sizes equal to the contributing area of each contributing-area class. The choice of a square basin is consistent with the square sample areas (see Section 3.1) and it allows for basin shape to remain the same (and therefore comparable) over the range of contributing areas used in this study. The main-stem channel, with a length of $L$ (m), was defined as the diagonal distance from one corner to the opposite corner across the square basin (i.e. $L$ is equal to the square root of two times the area of the square basin). This main-stem channel was used in conjunction with a normalized area function to represent the shape of the basin and the routing of runoff through the drainage basin network. By including the normalized area function, we can account for geomorphic dispersion (i.e. the attenuation of the flood peak due to the fact that precipitation that falls on the landscape will take different paths to the outlet and hence reach the outlet at different times) in our analyses. The normalized area function, $A(x)$ (unitless), is defined as the portion of basin area, $A_T(x)$ (m$^2$), that contributes flow to the main-stem channel within a given range of distances ($x$) from the outlet, normalized by the total basin area, $A_T$ (m$^2$; Mesa and Mifflin, 1986; Moussa, 2008). The normalized area function is assumed to be triangular in shape with a maximum value at the midpoint of the main-stem channel from the outlet. Area functions, and related width functions, from real basins used in other studies show this triangular shape in general (Marani et al., 1994; Rinaldo et al., 1995; Veneziano et al., 2000; Rodriguez-Iturbe and Rinaldo, 2001; Puente and Sivakumar, 2003; Saco and Kumar, 2008), although not all basins show this shape.

The triangular area function has been shown to approximate the average area function of
basins and that the peak discharge and time to peak discharge is likely more important to
the shape of the flood wave (Henderson, 1963; Rodriguez-Iturbe and Valdes, 1979).

A 1-dimensional channel with simplified width and along-channel slope
appropriate for channels in the CRB is used to approximate the geometry of the main-
stem channel of the idealized basin in the flow-routing algorithm. In addition, values for
channel slope, $S$ (m/m), and channel width, $w$ (m), are assigned based on the contributing
area of the idealized basin and the results of a least-squares regression to channel-slope
and channel-width data from the CRB. We assume here that the assigned channel slopes
and widths represent the average value for the entire idealized basin. To find the best
approximations for channel slope and width values, we developed formulae that predict
average channel slope and channel width as a function of contributing area based on a
least-squares fit of the logarithms of slope, width, and contributing area based on
approximately 100 sites in the Colorado River Basin (CRB; Fig. 4). The data used in
these least-squares regressions included slope, width, and contributing area information
from all sites in the LCRB and southern UCRB presented in Moody et al. (2003) and
additional sites from USGS stream-gauge sites from across the CRB.

The assigned channel slope and width values, together with the values of $Q_{pm}$
modified for each antecedent-moisture scenario, were used to calculate the depth-average
velocities, $V$ (m s$^{-1}$), in hypothetical 1D main-stem channels of idealized square drainage
basins corresponding to each contributing-area and time-interval-of-measurement class.
In this study, flow velocity is not modeled over space and time, but rather is set at a
constant value appropriate for the peak discharge using an iterative approach that solves
for the peak depth-averaged flow velocity, uses that velocity to compute the parameters
of the diffusion-wave-routing algorithm, routes the flow, and then computes an updated
estimate of peak depth-averaged velocity. To calculate the depth-averaged velocity, $V$, we
used Manning’s equation, i.e.

$$V = \frac{1}{n_M} \frac{R^{\frac{2}{3}} S^{\frac{1}{2}}}{},$$  \hspace{1cm} (3)

where $n_M$ is Manning’s n (assumed to be equal to 0.035), and $R$ is the hydraulic radius
(m) calculated with the assigned channel width, and $S$ (m/m) is the assigned channel
slope. In order to calculate $R$, water depth, $h$, of the peak discharge needed to be
determined. In this study $h$ was iteratively solved for based on the peak-flow conditions (i.e. the depth-averaged velocity, $V$, associated with the peak-flood discharge, $Q_{fd}$) with $h$ set at 1 m for the first calculation of the flow-routing algorithm. At the end of each calculation, $h$ is recalculated using Manning’s equation. These iterations continue until the water depth converges on a value (i.e. the change from the last calculation of $h$ to the next calculation of $h$ is $\leq 0.1$ m) corresponding to a specific recurrence interval, contributing-area class, and time-interval-of-measurement class.

The method we used to model flow through the main-stem channel is the diffusion-wave flow-routing algorithm. This approach is based on the linearized Saint-Venant equations for shallow-water flow in one dimension. To find a simpler, linear solution to Saint-Venant equations, Brutsaert (1973) removed the acceleration term from the equations, leaving the diffusion and advection terms that often provide a reasonable approximation for watershed runoff modeling (Brutsaert, 1973). Leaving the diffusion term in the flow-routing algorithm includes hydrodynamic dispersion of the flood wave in the calculation of the flood hydrograph. In the case where the initial condition is given by a unit impulse function (Dirac function), the cell response function of the channel, $q_d$ (units of $s^{-1}$), is given by:

$$q_d = \frac{x}{(2\pi)^{1/2}bt_{r}^{3/2}} \exp \left[ -\frac{(x - at_{r})^2}{2b^{2}t_{r}} \right]$$  \hspace{1cm} (4)

where $x$ is the distance along the channel from the location where the impulse is input to the channel, $t_r$ is time since the impulse was input into the channel, and the drift velocity $a$ (m s$^{-1}$) and diffusion coefficient $b^2$ (m$^2$ s$^{-1}$) are defined as

$$a = (1 + a_0)V$$  \hspace{1cm} (5)

$$b^2 = \frac{V^3}{gSF^2} (1 - a_0^2F^2)$$  \hspace{1cm} (6)

where $F$ is the Froude number, $g$ is the acceleration due to gravity (m s$^{-2}$), and $a_0$ is a constant equal to 2/3 when using Manning’s equation (Troch et al., 1994). The large floods modeled in this study are assumed to have critical-flow conditions and therefore the Froude number is set to a constant value of 1.

The unit response discharge, $q_{fd}$ (m$^2$ s$^{-1}$), at the outlet of a drainage basin can be computed from equations (3)-(5) by integrating the product of the cell response function
$q_d(x,t)$ corresponding to a delta-function input of the normalized area function, $A(x)$, i.e. the spatial distribution of precipitation input. The integral is given by

$$q_{id}(t_r) = \int_0^{t_r} \frac{Q_p}{w} \, dt \int_0^L q_d(x,t_r - t') A(x) \, dx$$

where $t_p$ is the time interval of measurement over which the unit impulse input (i.e. $Q_p$) is applied to the idealized square drainage basin, and $t_r$ is the time after the input of the unit impulse that is long enough to capture the waxing the waning portions and the flood peak of the flood wave. The final peak discharge value, or $Q_{fd}$ (m$^3$ s$^{-1}$), was calculated by multiplying the unit discharge $q_{id}$ (m$^2$ s$^{-1}$) by the channel width found through the formula derived from CRB data in Figure 4, and then selecting the largest value from the resulting hydrograph.

### 3.3 Recurrence Interval Calculations

To determine the precipitation-intensity values and $Q_p$, associated with a user-specified recurrence interval, maximum precipitation intensities of storm events sampled from the NEXRAD data for each contributing-area and time-interval-of-measurement class was first ranked from highest to lowest. Storm events were identified as adjacent precipitation intensity values separated by instances of zero values in time for each spatial scale. The relationship between recurrence intervals and rank in the ordered list is given by the probability-of-exceedance equation:

$$RI = \frac{(n + 1)}{m}$$

where $RI$ is the recurrence interval (yr), defined as the inverse of frequency (yr$^{-1}$) or probability of exceedance, $n$ is the total number of samples in each contributing-area and time-interval-of-measurement scaled to units in years (resulting in units of yr), and $m$ is the rank of the magnitude ordered from largest to smallest (unitless). The resulting precipitation intensities associated with a user-specified recurrence interval and contributing-area and time-interval-of-measurement class was then used to calculate the $Q_p$ value.

At the end of the calculations described above we have datasets of precipitation-intensity, $Q_p$, and $Q_{fd}$ values for each combination of the eight contributing-area classes,
the seven time-interval-of-measurement classes, and the four recurrence intervals. We then find the maximum values of precipitation intensity, $Q_p$, and $Q_{fd}$ associated with a given contributing-area class and recurrence interval among all values of the time-interval-of-measurement class (i.e. the values calculated for 1 to 64 h time intervals). This step is necessary in order to find the maximum values for a given contributing area class and recurrence interval independent of the time-interval-of-measurement, i.e. independent of storm durations and associated types of storms. These maximum values are used to plot the FMAC for a given recurrence interval.

3.4 Estimation of Uncertainty

Confidence intervals (i.e. uncertainty estimates) were calculated to quantify the uncertainty in calculated precipitation intensities and associated $Q_p$ and $Q_{fd}$ values. In this study we estimated confidence intervals using a non-parametric method similar to that used to calculate quantiles for flow-duration curves (Parzen, 1979; Vogel and Fennessey, 1994). Like quantile calculations, which identify a subset of the ranked data in the vicinity of each data point to estimate expected values and associated uncertainties, we estimated confidence intervals for our predictions based on the difference in $Q_p$ values between each point and the next largest value in the ranked list. This approach quantifies the variation in the precipitation intensity value for a given contributing area and recurrence interval. In some cases the calculated uncertainties for precipitation intensities and associated $Q_p$ and $Q_{fd}$ values are infinite due to the values being past the frequency-magnitude distribution, i.e. there are not enough samples for these values to be determined and there are no finite numbers to sample. These values are not used in this study.

The resulting confidence intervals of precipitation intensity were used to calculate confidence intervals for $Q_p$ and $Q_{fd}$. Confidence intervals for $Q_p$ values were equal to the confidence intervals for precipitation intensity propagated through the calculation of $Q_p$ (i.e. multiplying by contributing area). Confidence intervals for $Q_{fd}$ values were calculated to be the same proportion of the $Q_{fd}$ value as that set by the precipitation intensity value and it’s confidence intervals. For example, if the upper confidence interval was 120% of a precipitation intensity value, the upper confidence interval for the $Q_{fd}$
value associated with the precipitation intensity value is assumed to be 120% of the \( Q_{fd} \) value. This approach to propagation of uncertainty treats all other variables in the calculations as constants and additional uncertainty related to regression analyses on variables used in the flow-routing algorithm such as slope, channel width, and runoff coefficients was not included.

3.5 Testing the Effects of Climate Variability

To quantify the robustness of our results with respect to climate variability, we separated the NEXRAD data into El Niño and La Niña months using the multivariate ENSO index (MEI). All months of data with negative MEI values (La Niña conditions) were run together to calculate the precipitation intensity and \( Q_p \) values for contributing areas of 16, 256, and 4096 km\(^2\), time intervals of 1 to 64 hours, and for 10-, 50-, 100-, and 500-year recurrence intervals. This was repeated with all months of data with positive MEI values (El Niño conditions). Figure 5 shows the distribution of negative and positive MEI values during the 1996 to 2004 water years used in this study.

4. Results

4.1 Channel Characteristics and Runoff Coefficients

Least-squares regression of channel slopes and channel widths from the CRB versus contributing area was used to estimate channel slope, channel width, and runoff coefficients for each idealized basin of a specific contributing-area class. Channel slope decreases as a power-law function of contributing area with an exponent of -0.30 \( (R^2 = 0.39) \), whereas channel width increases as a power-law function of contributing area with an exponent of 0.28 \( (R^2 = 0.65; \text{Fig. 4}) \). These results follow the expected relationships among channel slopes, widths, and contributing area, i.e. as contributing area increases the channel slope decreases and the channel width increases.

Runoff coefficients for wet, medium, and dry antecedent-moisture conditions all decrease with increasing contributing area following a logarithmic function, with the slope of the line decreasing from wet to dry conditions. The fitness of the line to the data also decreases for the wet to dry conditions, with the \( R^2 \) values for wet, medium, and dry conditions equal to 0.78, 0.45, and 0.04, respectively. Runoff coefficients decrease with
increasing contributing area due to the increased probability of water loses as basin area increases. Also, as expected, runoff coefficients are highest in basins with wet initial conditions that are primed to limit infiltration and evapotranspiration.

4.2 Trends in Precipitation Intensity

Maximum precipitation intensities (i.e. the maximum among all time-interval-of-measurement classes) for each contributing-area class and recurrence interval decrease systematically as power-law functions of increasing contributing area for all recurrence intervals with an average exponent of \(-0.18 \pm 0.06\) (error is the standard deviation of all calculated exponents found from a weighed least-squares regression; average coefficient of determination \(R^2 = 0.78\)). Note that maximum-precipitation-intensity results are not presented because they are closely related to the plots of \(Q_p\) versus contributing area in Figure 6, i.e. \(Q_p\) is simply the precipitation intensity multiplied by the contributing area.

The decrease in maximum precipitation intensity with contributing area can be seen in Table 1, where maximum precipitation intensities over contributing areas of 11,664 km\(^2\) are 45% to 8% of maximum precipitation intensity values for basin areas of 16 km\(^2\) in both the UCRB and LCRB (Table 1). The largest decrease in maximum precipitation intensity values between the smallest and largest contributing areas were found for the largest recurrence interval (e.g. 500-year) for both the UCRB and LCRB. The decrease in maximum precipitation intensity with increasing contributing area suggests that there is a spatial limitation to storms of a given precipitation intensity.

Differences among maximum precipitation intensities for the four recurrence intervals as a function of contributing area are larger in the UCRB than in the LCRB (Table 1). This larger “spread” in the maximum precipitation intensities in the UCRB relative to the LCRB is also propagated throughout the maximum precipitation and flood discharge calculations. For both the UCRB and LCRB, the difference between the 50- and 100-year recurrence interval values was the smallest (Table 1). These trends show that maximum precipitation intensities vary much more as a function of recurrence interval in the UCRB compared with the LCRB.

Maximum precipitation intensities associated with a 10-year recurrence interval are similar in the LCRB and UCRB, while intensities were higher in the UCRB than the
LCRB for recurrence intervals of 50-, 100-, and 500-years (Table 1). The results of the comparison between the two basins suggest that common (i.e. low-recurrence-interval) precipitation events will have similar maximum precipitation intensities in the UCRB and LCRB, but that rare (i.e. high-recurrence-interval) precipitation events will have higher maximum precipitation intensities in the UCRB than in the LCRB for the same recurrence interval.

4.3 Trends in $Q_p$

Maximum precipitation discharges ($Q_p$ hereafter) increase with contributing area as power-law functions with an average exponent of $0.82 \pm 0.06$ (error is the standard deviation of all calculated exponents) based on weighed least-squares regressions on the data ($R^2 = 0.98$) for all recurrence intervals and for both the UCRB and LCRB (Fig. 6). These $Q_p$ values for a given contributing-area class and recurrence interval are the largest values taken from the multiple values calculated for each of the seven time intervals of measurement as explained in Section 3.3. By taking the maximum values, the resulting $Q_p$ FMACs approximate the upper envelope of values of a given recurrence interval. In this study the FMAC follows a power-law function that shows that $Q_p$ increases predictably across the range in contributing areas. As with the maximum precipitation intensity results, differences between $Q_p$ values of different recurrence intervals for a given contributing area were larger for the UCRB than the LCRB (Fig. 6).

In general, confidence intervals for $Q_p$ values increase with increasing contributing-area class (Table 1 and Fig. 6). The large values of the highest contributing-area classes and highest recurrence intervals show the spatial limitation of the method, meaning that at these contributing-area classes and recurrence intervals the values are sampled from the largest ranked value and have infinite confidence intervals. These values include the 50-, 100-, and 500-year recurrence intervals for the UCRB and the 100- and 500-year recurrence intervals for the LCRB at the 11,664 km$^2$ contributing-area class. These values also include the 100- and 500-year recurrence intervals for the UCRB and the 500-year recurrence intervals for the LCRB at the 4,096 km$^2$ contributing-area class. Values with infinite confidence intervals are not included in Fig. 6 due to their high uncertainties.
4.4 Trends in $Q_{fd}$

Maximum $Q_{fd}$ values (hereafter $Q_{fd}$), i.e. the largest values taken for the multiple values calculated for each time interval of measurement for a given contributing-area class and recurrence interval, were used to plot FMACs for wet, medium, and dry conditions for both the UCRB and LCRB (Fig. 7). In general, FMACs for $Q_{fd}$ values follow the power-law relationship shown in the $Q_p$ FMACs until contributing areas of $\approx 1,000$ km$^2$, where the curves begin to very slightly flatten or decrease. As with the $Q_p$ values, $Q_{fd}$ values representing some of the higher recurrence intervals converge to the same value (i.e. the value corresponding to the highest precipitation intensity for the contributing-area class) at contributing areas of $\approx 10,000$ km$^2$ and the confidence intervals become infinite (Table 2). This convergence of $Q_{fd}$ values at the largest contributing areas is due to the reduction in the range of values and the number of samples from which to calculate the associated values for each recurrence interval.

In general, The UCRB $Q_{fd}$ FMACs (Fig. 7A, C, and E) are slightly higher in magnitude and span a larger range of magnitudes than the FMACs for the LCRB. For both basins, FMACs for the wet, medium, and dry conditions resulting in the highest, middle, and lowest magnitudes, respectively. This trend is expected due to the lowering of runoff coefficients and available water as conditions become drier.

FMACs of $Q_{fd}$ for the LCRB plot below published FECs for the LCRB and U.S. (Fig. 7B, D, F) at low contributing areas, but meet and/or exceed the LCRB FEC for contributing areas above $\approx 1,000$ km$^2$ and $\approx 100$ km$^2$ for dry and wet antecedent-moisture conditions, respectively. The FMACs for the LCRB do not exceed the U.S. FEC. All of the FMACs of $Q_{fd}$ for the UCRB exceed the LCRB FEC for wet conditions, with the FMACs of lower recurrence intervals exceeding the curve at higher contributing areas than the FMACs of higher recurrence intervals (Fig. 7A). The 500-year FMAC for wet conditions approximate the U.S. FEC for contributing areas between $\approx 100$ to 1,000 km$^2$. These results suggests that under certain antecedent-moisture conditions, and in basins of certain contributing areas, the LCRB produces floods that exceed the maximum recorded floods in the LCRB and the UCRB produces floods of magnitudes on par with the maximum recorded floods in the U.S.
4.5 The Effects of ENSO on Precipitation

Definitive differences in maximum precipitation intensities and $Q_p$ values were found between months with positive versus months with negative MEI values (Table 3). For very small contributing areas ($16 \text{ km}^2$) in the LCRB maximum precipitation intensities and $Q_p$ values are similar during negative and positive MEI conditions. Larger contributing areas ($256$ and $4,096 \text{ km}^2$) show higher maximum precipitation intensities during negative MEI conditions regardless of recurrence interval. Values of $Q_p$ show the same trend as the maximum precipitation intensity in the LCRB. In the UCRB, maximum precipitation intensities and $Q_p$ values during negative MEI conditions are higher than those during positive MEI conditions regardless of recurrence interval.

5. Discussion

5.1 Use and Accuracy of NEXRAD Products

NEXRAD products are widely used as precipitation inputs in rainfall-runoff modeling studies due to the spatially complete nature of the data necessary for hydrologic and atmospheric models (Ogden and Julien, 1994; Giannoni et al., 2003; Kang and Merwade, 2011). In contrast to past studies similar in scope to this study (Castellarin et al., 2005; Castellarin, 2007; Castellarin et al., 2009) we did not use rain-gauge data and only used NEXRAD products to determine the FMACs for precipitation and flood discharges. We favor NEXRAD products due to the spatial completeness of the data. Intuitively, NEXRAD products that are spatially complete and that average precipitation over a 4 km by 4 km area would not be expected to match rain-gauge data within that area precisely (due to the multi-scale variability of rainfall), although some studies have tried to address this discrepancy (Sivapalan and Bloschl, 1998; Johnson et al., 1999). Xie et al. (2006) studied a semi-arid region in central New Mexico and found that hourly NEXRAD products overestimated the mean precipitation relative to rain-gauge data in both monsoon and non-monsoon seasons by upwards of 33% and 55%, respectively. Overestimation of precipitation has also been noted due to the range and the tilt angle at which radar reflectivity data are collected (Smith et al., 1996).
Underestimation of precipitation by NEXRAD products relative to rain gauge data has also been observed (Smith et al., 1996; Johnson et al., 1999), however.

Under- and over-estimation of precipitation by NEXRAD products in relation to rain-gauge data is partly due to the difference in sampling between areal NEXRAD products and point data from rain gauges and partly due to sampling errors inherent to both methods. For example, NEXRAD products include problems such as the use of incorrect Z-R relationships for high intensity storms and different types of precipitation, such as snow and hail (Baeck and Smith, 1998). Also, because of its low reflectivity, snow in the NEXRAD products is measured as if it were light rain (David Kitzmiller, personal communication, January 10, 2012). This means the NEXRAD products likely underestimate snowfall and therefore snowfall is not fully accounted for in this study. Due to snowfall not being included in this study, associated snowpack and snowmelt effects were also not accounted for. Rain gauges can also suffer from a number of measurement errors that usually result in an underestimation of rainfall (Burton and Pitt, 2001). In addition, gridded rainfall data derived from rain gauges are not spatially complete and therefore must be interpolated between point measurements to form a spatially complete model of rainfall. It is impossible to discern which product is more correct due to the differences in measurement techniques and errors, but by taking both products and combining them into one, the Stage III NEXRAD precipitation products generate the best precipitation estimate possible for this study. Moreover, it should be noted that 100-year flood magnitude predictions based on regression equations have very large relative error bars (ranging between 37 to 120% in the western U.S.; Parrett and Johnson, 2003) and that measurements of past extreme floods can have significant errors ranging from 25% to 130% depending on the method used (Baker, 1987). As such, even a ~50% bias in NEXRAD-product-derived precipitation estimates is on par or smaller than the uncertainty associated with an analysis of extreme flood events.

As stated previously, the NEXRAD precipitation estimates used here do not include snowfall and other non-rainfall precipitation types. In this study we also do not include snowpack information into our flood discharge calculations. The omission of snowpack is a reasonably assumption for our low elevation, warm regions within most of the UCRB and LCRB. However, we acknowledge some of our higher elevation areas at
higher latitudes may be underestimating the maximum flood discharge by only including rainfall-derived runoff. If the methodology in this paper were applied to a snowmelt-dominated region, snowpack would need to be added to accurately estimate the maximum flood discharge.

### 5.2 Comparison of FMACs to Published FECs

FMACs of $Q_{fd}$ exhibit a similar shape and similar overall range in magnitudes as previously published FECs, derived from stream-gauge and paleoflood records, for the LCRB and U.S. (Fig. 7). In general, the FMACs exceed or match published FECs at larger contributing areas, and are lower than or on par with published FECs at the smallest contributing areas (Fig. 7).

All FMACs except the 500-year recurrence-interval curve for the UCRB under wet conditions are positioned well below the U.S. FEC presented by Costa (1987; Fig. 7A). The similarity between the 500-year recurrence interval $Q_{fd}$ FMAC for the UCRB under wet conditions and the U.S. FEC suggests that the U.S. FEC includes floods of larger recurrence-intervals, which are similar in magnitude to the 500-year recurrence-interval floods within the UCRB. The approximation of the U.S. FEC by the 500-year UCRB FMAC is a significant finding due to the fact that the U.S. FEC includes storms from other regions of the U.S. with extreme climatic forcings (i.e. hurricanes, extreme convection storms, etc.).

The $Q_{fd}$ FMACs for the LCRB can be directly compared to the FEC for the LCRB presented by Enzel et al. (1993). At contributing areas smaller than approximately 100 km$^2$, $Q_{fd}$ FMACs for wet conditions and all recurrence intervals are positioned below the LCRB FEC, but at larger contributing areas $Q_{fd}$ FMACs exceed or approximate the LCRB FEC. $Q_{fd}$ FMACs calculated for medium and dry antecedent conditions show the same trend, but exceed the LCRB FEC at a larger contributing areas ($\geq 1,000$ km$^2$). This comparison suggests that although the FMACs overlap the overall range of flood magnitudes of the LCRB FEC, the two methods are not capturing the same trend for extreme flood discharges and the LCRB is capable of producing floods larger than those on record.
The difference in the slope of the FMACs, and specifically the exceedance of the published LCRB FEC, suggests that the two methods are not capturing the same information. This difference may be due to the difference in how the data are sourced for each method. FECs are created as regional estimates of maximum flood discharges and are based on stream-gauging station and paleoflood data. The FECs are then used to provide flood information for the region, including ungauged and unstudied drainage basins. FECs are limited to the number of stream gauges employed by public and private parties and do not include all basins within a region. In general, FECs may underestimate maximum floods in larger basins, relative to smaller basins, because there are a larger number of smaller basins to sample than larger basins. This sample-size problem introduces bias in the record where flood estimates for smaller contributing areas may be more correct than estimates for larger basins. In this study, the regional precipitation information given by the NEXRAD network is used to form the FMAC, therefore taking advantage of the entire region and using precipitation data to calculate flood discharges, rather than directly measuring flood discharges. This sampling scheme allows for much larger sample sizes for the range of contributing areas, therefore minimizing the sample bias of the traditional FEC.

This study aimed to introduce the new method of the FMAC and therefore improve upon the traditional methods of the FEC. By calculating FMACs we provide frequency and magnitude information of possible flood events for a given region in contrast to the FECs that only provide an estimate of the largest flood on record. This information is vital for planning and infrastructure decisions and the accurate representation of precipitation and flooding in design-storm and watershed modeling. In addition, the fact that the FMACs match the FECs for large (500-year) recurrence intervals and do not exhibit the same trends suggests that the FMACs are capturing different samples than the FECs. This indicates that by using the NEXRAD products, the FMACs may provide a more inclusive flood dataset for a region (especially ungauged areas) than the traditional stream-gauge records.

5.3 Precipitation Controls on the Form of the FEC
FMACs were shown to have a strong (average $R^2=0.93$) power-law relationship between $Q_p$ and contributing area for all recurrence intervals. Figure 8 shows a conceptualized FEC where the concave-down shape is created when the observed envelope curve diverges from the constant positive power-law relationship between $Q_p$ and contributing area. This diversion creates a “gap” between the two curves and indicates that flood discharge is not a simple power-law function of contributing area.

Three mechanisms have been proposed to explain the “gap” and characteristic concave-down shape of FECs: (1) integrated precipitation (i.e. total precipitation over an area) is more limited over larger contributing areas compared to smaller contributing areas (Costa, 1987), (2) a relative decrease in maximum flood discharges in larger contributing areas due to geomorphic dispersion (Rodriguez-Iturbe and Valdes, 1979, Rinaldo et al., 1991, Saco and Kumar, 2004), and (3) a relative decrease in maximum flood discharges in larger basins due to hydrodynamic dispersion (Rinaldo et al., 1991). The first explanation, proposed by Costa (1987), suggests that there is a limitation to the size of a storm and the amount of water that a storm can precipitate. The effect of precipitation limitations may be evidenced by the decreasing maximum precipitation intensities with increasing contributing area. However, the strong power-law relationship between $Q_p$ and contributing area for all recurrence intervals indicates that $Q_p$ is, in general, increasing predictably over the range of contributing areas used in this study. Even if precipitation limitations affect the shape of the curve, this single hypothesis does not account for all of the concave-down shape of each FEC suggesting that other mechanisms are important to creating the characteristic shape. However, it is important to note that the importance of each mechanism may be different for different locations.

5.4 Climate Variability in the NEXRAD Data

The results from comparing negative and positive MEI conditions in the UCRB and LCRB are generally consistent with ideas about ENSO and how it affects precipitation in the western U.S. In the LCRB, during negative MEI conditions, small, frequent storms have similar or slightly higher maximum precipitation intensities and $Q_p$ values than during positive MEI conditions. This similarity between the two conditions may be explained by the balancing of increased winter moisture during El Niño in the
southwestern U.S. (Hidalgo and Dracup, 2003) and increased summer moisture through the strengthening of the NAM system and the convective storms it produces during La Niña conditions (Castro et al., 2001; Grantz et al., 2007). In general, the strengthening of the NAM may explain the higher maximum precipitation intensities and $Q_p$ values during negative MEI conditions in the LCRB. Strengthening of the NAM may be due in part to the large temperature difference between the cool sea surface of the eastern Pacific Ocean and the hot land surface of the southwestern U.S. and northwestern Mexico during La Niña conditions. The large temperature gradient increases winds inland, bringing the moisture associated with the NAM (Grantz et al., 2007). In the UCRB it is during negative MEI conditions, where the highest maximum precipitation intensities and $Q_p$ values for all recurrence intervals occur. This suggests that the UCRB is affected by ENSO much like the northwestern U.S., where wetter winters are affiliated with La Niña and not El Niño conditions (Cayan et al., 1999; Hidalgo and Dracup, 2003). It is important to note that this comparison is of intensity rates and not total precipitated moisture so the MEI condition resulting in wetter conditions is not known.

In addition to the ENSO analysis, by investigating previous studies we see that, along with natural yearly precipitation variability, the 1996 to 2004 water years included many atmospheric river events (Dettinger, 2004; Dettinger et al., 2011). It is important that these events were included due to their ability to greatly increase winter precipitation in the UCRB and LCRB (Rutz and Steenburgh, 2012). Atmospheric river events (sometimes known as Pineapple Express events) can also be tied to major Pacific climate modes such as the ENSO (Dettinger, 2004; Dettinger, 2011), the Pacific Decadal Oscillation (PDO; Dettinger, 2004), and the North Pacific Gyre Oscillation (NPGO; Reheis et al., 2012) in southern California. Unfortunately, correlations between atmospheric river events are unknown and/or less clear for the interior western U.S. However, all three of these Pacific climate modes shifted during the 9-year study period in ~1998 to 1999 (Reheis et al., 2012) indicating that both positive and negative conditions of the ENSO, PDO, and NPGO exist in the NEXRAD products used in this study.

The presence of distinct trends in maximum precipitation and $Q_p$ values calculated for negative and positive MEI conditions, as well as the information in the literature on
atmospheric river events, indicates the NEXRAD products used in this study incorporate circulation-scale weather patterns. In addition, the patterns in maximum precipitation and $Q_p$ values during different MEI conditions agree with common understanding of the effects of ENSO on the western U.S. and provide evidence that the data and methods used in this paper to analyze precipitation are reliable. This analysis shows that the NEXRAD products worked well in this location and that using radar-derived precipitation products may be useful for identifying precipitation and climatic trends in other locations where the FMAC method can be applied.

6. Conclusions

In this study we present the new FMAC method of calculating precipitation and flood discharges of a range of recurrence intervals using radar-derived precipitation estimates combined with a flow-routing algorithm. This method improves on the traditional FEC by assigning recurrence interval information to each value and/or curve. Also, instead of relying on stream-gauge records of discharge, this method uses up-to-date and spatially complete radar-derived precipitation estimates (in this case NEXRAD products) to calculate flood discharges using flow-routing algorithms. This study presents an alternative data source and method for flood-frequency analysis by calculating extreme (high recurrence interval) event magnitudes from a large sample set of magnitudes made possible by sampling the radar-derived precipitation estimates.

The FMACs for $Q_p$ and $Q_{fd}$ for the UCRB were similar to those produced for the LCRB. In general, all recurrence-interval curves followed the same general trend, indicating that the mechanisms of precipitation and flood discharge are similar for the two basins. However, there were some differences between the two basins. Overall, there were larger differences between curves of different recurrence intervals for the UCRB than the LCRB suggesting a larger range in maximum precipitation intensities, and therefore $Q_p$ and $Q_{fd}$, in the UCRB relative to the LCRB. For both the UCRB and LCRB the 50- and 100-year recurrence interval curves for all precipitation and discharge FMACs were the most similar. This similarity may mean that although historical discharge records are short, having a 50-year record may not underestimate the 100-year flood as much as one might expect. Also, for $Q_p$ and $Q_{fd}$, low recurrence-interval values
were slightly higher in the LCRB than in the UCRB. This relationship was opposite for high recurrence-interval values. This likely points to a general hydroclimatic difference between the two basins, with the LCRB receiving high intensity storms annually due to the NAM and the UCRB receiving more intense and rarer winter frontal storms.

Power-law relationships between maximum precipitation intensity, $Q_p$, and contributing area were also found in this study. Maximum precipitation intensities decreased as a power-law function of contributing area with an average exponent of $-0.18 \pm 0.06$ for all recurrence intervals. $Q_p$ values for all recurrence intervals increased as a power-law function of contributing area with an exponent of approximately $0.82 \pm 0.06$ on average. Based on the constant power-law relationship between $Q_p$ and contributing area, the “gap” or characteristic concave-down shape of published FEC are likely not caused by precipitation limitations.

In general, the FMACs of $Q_{fd}$ calculated in this study are lower than, and exceed, the published FECs for the LCRB at lower and higher contributing areas. All FMACs of $Q_{fd}$ were positioned well below the U.S. FEC except the UCRB 500-year FMAC, which approximated the U.S. FEC during wet antecedent-moisture conditions. All FMACs of $Q_{fd}$ for all moisture conditions in the LCRB closely approximated the same magnitudes as the published LCRB FEC, but exceeded it for larger contributing areas. The higher estimates of flood discharges at larger contributing areas may be the result of the difference of sampling methods and are likely not erroneous and may be proved true by future events.

Lastly, the approximately 9 years of NEXRAD products were found to be a good representation of climate in the CRB. This conclusion was made based on differences in precipitation between positive and negative ENSO conditions in both the UCRB and LCRB and additional data found in the literature. In general, the UCRB was found to have a hydroclimatic regime much like that of the northwestern U.S. where El Niño conditions result in lower maximum precipitation intensities and amounts and La Niña conditions result in higher maximum precipitation intensities. The LCRB showed a more complex trend with similar maximum precipitation intensities for both El Niño and La Niña conditions.
Here this method is applied to the UCRB and LCRB in the southwestern U.S., but could be applied to other regions of the U.S. and the world with variable climate and storm types where radar-derived precipitation estimates are available. In addition, this study used set values of contributing area, drainage basin shape, time intervals of measurement, and recurrence intervals that can be changed based on the focus of future studies. Other variables such as snowpack, elevation, and land use should be explored in conjunction with this method to better understand controls on precipitation and flooding.

Acknowledgments

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References


National Atlas: http://www.nationalatlas.gov/atlasftp.html#hucs00m, last access: 8 August 2014


RadarEU: http://www.radareu.cz/, last access 1 August, 2014.


Table 1. Maximum precipitation intensity and $Q_p$ for the Upper Colorado River Basin (UCRB) and Lower Colorado River Basin (LCRB). Note that data are all sampled from time intervals of measurement $\leq 2$ hours.

<table>
<thead>
<tr>
<th>RI</th>
<th>Area (km$^2$)</th>
<th>Intensity (mm h$^{-1}$)</th>
<th>$Q_p$ (m$^3$ s$^{-1}$)</th>
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<tr>
<td></td>
<td>UCRB</td>
<td>LCRB</td>
<td>UCRB</td>
</tr>
<tr>
<td>10</td>
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<td>36.6 ± 0.0</td>
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* Values with infinite confidence intervals, not used in this study.
Table 2. Maximum $Q_{fd}$ for the Upper Colorado River Basin (UCRB) and Lower Colorado River Basin (LCRB). Note that data are all sampled from time intervals of measurement ≤ 2 hours.

<table>
<thead>
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<th>RI Area (km$^2$)</th>
<th>Wet $Q_{fd}$ (m$^3$ s$^{-1}$)</th>
<th>Med $Q_{fd}$ (m$^3$ s$^{-1}$)</th>
<th>Dry $Q_{fd}$ (m$^3$ s$^{-1}$)</th>
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* Values with infinite confidence intervals, not used in this study.
Table 3. Maximum precipitation intensity and $Q_p$ values for 10, 50, 100, and 500-year recurrence intervals during negative (neg) and positive (pos) Multivariate ENSO Index (MEI) conditions within the Lower Colorado River Basin (LCRB) and Upper Colorado River Basin (UCRB). Note that data are all sampled from time intervals of measurement $\leq 2$ hours.

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<th>Basin</th>
<th>MEI</th>
<th>Area (km$^2$)</th>
<th>Intensity (mm h$^{-1}$)</th>
<th>$Q_p$ (m$^3$ s$^{-1}$)</th>
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* Values with infinite confidence intervals, not used in this study.
Figures

Figure 1. Map showing the locations of the Upper and Lower Colorado River Basins (UCRB and LCRB, respectively) outlined by the dotted line.
Figure 2. Schematic diagram of methodology used in this paper. (A) Rainfall data is sampled over spatial and temporal scales in factors of two. This sampling does not only ranked Max I for frequency analysis.

Results in Max I for each basin area, time interval, and frequency.

(B) Flow Routing for Discharge

Max I x basin area \( = \) \( Q_p \times C = Q_{pm} \) \( \rightarrow \) channel w/ slope and width defined by area function \( = V \)

idealized basin cross section w/ triangular area function

flow through main channel along diagonal axis (length of L) using diffusion-wave flow-routing algorithm

Results in \( Q_{fd} \) for each basin area, time interval, and frequency.
include looking at the data within a given spatial or temporal scale, but aggregating it over that scale. These values are ranked for a given basin area and time interval to complete the frequency analysis. This results in rainfall intensities (I) for each spatial scale (basin area), temporal scale (time interval or storm duration), and frequency. (B) Intensities sampled from the rainfall data are used to calculate rainfall discharge ($Q_p$ and $Q_{pm}$) values that are then put through the flow routing algorithm in order to calculate flood discharge ($Q_{fd}$) values. $Q_{fd}$ values are then used to construct the frequency-magnitude-area curves (FMACs) showing the data for recurrence intervals of 10, 50, 100, and 500 years.

Figure 3. Logarithmic relationships between runoff coefficients and contributing area using modeled data for wet (filled diamonds), medium (open squares), and dry (filled circles) antecedent-moisture conditions (Vivoni et al., 2007) and measured data for larger contributing areas (filled squares; Rosenberg et al., 2013). The medium (open squares) and dry (filled circles) data separate into two distinct groups relating to the precipitation event used to model them, with the lower group and higher group relating to a 12-h, 1-mm h$^{-1}$ event and 1-h, 40-mm h$^{-1}$ event, respectively. All points were used in the least-squares weighed-regression analysis.

Figure 4. Power-law relationships between channel slope and contributing area (A) and channel width and contributing area (B) for the Colorado River Basin.

Figure 5. Multivariate ENSO Index (MEI) of months included in Stage III NEXRAD gridded products. Months are numbered from September 1996 to September 2005 with years shown in gray. Dashed black line MEI equal to zero. Positive MEI indicates El Niño conditions, while negative MEI indicates La Niña conditions.
Figure 6. Frequency-magnitude-area (FMA) curves of $Q_p$ versus contributing area for recurrence intervals (RI) of 10, 50, 100, and 500 years for the Upper Colorado River Basin (UCRB; A) and the Lower Colorado River Basin (LCRB; B).
Figure 7. $Q_{fd}$ frequency-magnitude-area curves of 10, 50, 100, and 500 recurrence intervals (RI) and for wet, medium, and dry conditions for the Upper Colorado River Basin (UCRB) and the Lower Colorado River Basin (LCRB). Published FECs (black lines) for the Lower Colorado River Basin (solid black line) from Enzel et al. (1993) and the United States (dashed black line) from Costa (1987) are also shown.
Figure 8. Conceptual diagram of the characteristic concave-down shape of the FEC (observed) shown in comparison to a power-law function between $Q_p$ and contributing area. The “gap” between the observed curve and the predicted power law is caused by precipitation limitations and mechanisms occurring during the routing of water over the landscape.