Interactive comment on “Simplified stochastic soil moisture models: a look at infiltration” by J. Rigby and A. Porporato

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General comment

The manuscript entitled “Simplified stochastic soil moisture models: a look at infiltration” by Rigby and Porporato presents an interesting comparison between two simplified infiltration models, both based on a lumped description of the water balance in the rooting zone, but differing in the manner how they treat the precipitation input. My overall impression of the manuscript is very positive, since the paper is well organized and the results are novel and interesting. However, there seems to be a problem in the manner how rainfall is treated in the rectangular pulse model, and there are some other possible modifications that could improve the quality of the manuscript.
Rainfall modelling

The first problem is an analytical one: at page 1346 and 1347 the Authors assume that the distribution of the wet periods durations, \( f_W(w) \), is exponential with mean \( \delta \), and that the distribution of the rainfall depths, \( f_D(D) \), is also exponential with mean \( \alpha \). By assuming independence between \( w \) and \( D \) they obtain the marginal distribution of the rainfall average intensities, \( P = D/w \), see Eq. (8). However, assuming independence of \( w \) and \( D \), as done in the manuscript to obtain Equation (8), is not the same as assuming independence between the intensities \( P \) and the wet-period durations, \( w \), as Eagleson did in his 1978’s paper. In fact, independence between \( D \) and \( w \) does not imply independence between \( P \) and \( w \), as erroneously mentioned at page 1347, line 17-18: on the contrary, under the hypothesis that \( D \) is independent of \( w \), the average rain intensity and duration become mutually dependent variables, with a (strong) negative correlation.

This is easily demonstrated: if \( P \) and \( w \) were independent, the distribution of their product \( D \) would be \( f_D(D) = \int_0^\infty 1/w f_P(D/w) f_W(w) dw \). If one uses Equation (8) and \( f_W(w) = 1/\delta e^{-w/\delta} \) in this expression, one does not find the expected exponential distribution of the rainfall depths. The correct solution is found by considering that \( P \) and \( w \) are mutually dependent, in which case one has \( f_D(D) = \int_0^\infty 1/w f_P|W(D/w|w) f_W(w) dw \), where the conditional distribution \( f_P|W(P|w) \) has taken the place of the marginal distribution of \( P \). This conditional distribution \( f(P|w) = f(D/w|w) \) can be obtained by considering the known distribution of the rainfall depths and treating \( w \) as a parameter: one obtains \( f_P|W(P|w) = w/\alpha e^{-Pw/\alpha} \). Using this expression in \( f_D(D) = \int_0^\infty 1/w f_P|W(D/w|w) f_W(w) dw \) one obtains the desired exponential distribution for \( f_D \).

This is not only a matter of notation, since one should account for the dependence between \( P \) and \( w \) when carrying out the simulations: in fact, one should first sample a wet-period duration \( w \), and then sample a rainfall intensity \( P \) from the conditional distribution of \( P \) given \( w \), \( f(P|w) \) rather than from the marginal distribution of \( P \), as
done in the manuscript. The use in the simulations of the marginal distribution likely produces an overestimation of the number of rainfall events with long duration and large intensity, which in turn increases the probability of having Hortonian runoff. The two infiltration models considered in the paper will then probably produce even closer results when the correct $P$ distribution is adopted in the simulations.

An additional problem could be the necessity to find out a physical justification for the resulting negative correlation between $P$ and $w$. If the Authors feel uncomfortable with the presence of this correlation, they could still proceed in another manner: assume independence between $P$ and $w$ and keep the usual exponential distributions for $w$ and $D$. In this case the marginal distribution of the intensities $P$ turns out to be a Dirac-delta function $f_P(P) = \Delta(P - \alpha/\delta)$, i.e. the intensities become constant, non-random, variables ($P = \alpha/\delta$).

**Lack of generality**

Another area of possible improvement regards the possibility to make the comparison a more general one: the Authors selected a set of parameter values, which are listed in Table 1, with only a couple of these parameter values which are allowed to vary. Since the aim of the paper is to define the conditions when the two infiltration models can be considered to be equivalent, the parameter space should be explored in greater detail: for example, the saturated hydraulic conductivity $k_s$ is taken to be 200 mm/d in the manuscript, but one can guess from equation (14) that this value has a strong influence in limiting the relevance of the Hortonian runoff component (i.e., decreasing $k_s$ the differences between the two models could become much more relevant). My suggestion is then that the Authors include some more Figures and comments to better describe the influence of the climate and soil parameters (e.g., $k_s$, or the rooting depth $Z_r$) on the two infiltration models.

**Minor Corrections**

- The last sentence of the abstract is unjustified, due to the very limited space allocated
in the paper to the mentioned improvement (few lines at page 1355).

- At page 1341, line 19 “rainfall events which ignore ...”. Is “which” referred to the rainfall events? Please rephrase the sentence.
- In equation (9) there should be a 1/2 factor before $S(s_0)$ (for consistency with eq. (12)).
- The normalized rainfall intensity $\bar{P}$ in Equation (17) is not defined.
- Please change $R_t$ to $P$ in Equation (20).

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