Ecohydrology in Mediterranean areas: a numerical model to describe growing seasons out of phase with precipitations

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Abstract

The probabilistic description of soil moisture dynamics is a relatively new topic in hydrology. The most common ecohydrological models start from the soil water balance, a stochastic differential equation where the unknown quantity is the function of the soil moisture, depending both on spaces and time. Most of existing solutions in literature are obtained in a probabilistic framework and under steady-state condition; even if this last condition allows the analytical handling of the problem, it has considerably simplified the problem by subtracting generalities from it.

The steady-state hypothesis, used in many ecohydrological works, appears perfectly applicable in arid and semiarid climatic areas like those of African’s or middle American’s savannas, but it seems to be no more valid in areas with Mediterranean climate, where, notoriously, the wet season foregoes the growing season, thus recharging the soil moisture. This initial condition, especially for deep rooted vegetation, has a great importance by enabling survival in absence of rainfalls during the growing season and, however, keeping the water stress low during its first period.

The aim of this paper is to investigate the soil moisture dynamics using a simple non-steady numerical ecohydrological model. The numerical model is able to reproduce soil moisture probability density function, obtained analytically in previous studies for different climate and soil conditions in steady state conditions.

The proposed model gives both the soil moisture time-profile and the vegetation static water stress time-profile. From the former it is possible to extract the probability density function of soil-moisture during the whole growing season, while the latter allows the estimation of the vegetation response to the water stress. Here the differences between the analytical and the numerical probability density functions are presented, showing how the numerical model is able to capture the effects of winter recharge on the soil moisture. The dynamic water stress is numerically evaluated, implicitly taking into account the soil moisture condition at the beginning of the growing season. The model proposed here is applied in the forested river basin of the Eleuterio in Sicily.
1 Introduction

The ecohydrology may be defined as that branch of hydrology that seeks to describe the hydrologic mechanisms underling ecologic pattern and processes. During last years several ecohydrological models have been developed and improved, different each other and characterised by different goals. All the ecohydrological models are based on a soil water balance, and one of the purposes of previous work has been to seek solving it in the simplest and most accurate manner.

Rodriguez-Iturbe et al. (1999a) proposed an analytical model for the study of the soil moisture temporal dynamics in water controlled ecosystems. The model provides a simplified realistic description of the interactions between climate, vegetation and soil. This kind of approach is the starting point for a quantitative valuation of the soil moisture effects on ecosystems dynamics and of the vegetation response to water stress. It is also useful in the study of the hydrological control on the nutrient cycles into the soil and the study on the competition dynamics for water among the various species (Rodriguez-Iturbe and Porporato, 2004). Laio et al. (2001b) obtained the analytical expression for the soil moisture probably density function (pdf) in steady state conditions. This model has been applied in regions where the growing season is in phase with the wet one (Laio et al., 2001a). These climatic conditions make the steady-state hypothesis reasonably satisfied, because the effects of the transient condition, due to an initial condition, are limited to a short time-period.

There are only few ecohydrological studies focusing on areas where temperature and rainfall are seasonally out of phase. Kiang (2002) and Baldocchi et al. (2004) analysed the stochastic soil moisture dynamics and the related water stress for a Californian savanna, where the climate is semi-arid and similar to Mediterranean. Using soil moisture data recorded in situ, Kiang (2002) compared these data with the predictions of the stochastic model proposed by Laio et al. (2001b), finding a general good
agreement but with some differences due to the role of the initial soil moisture transient, not included by analytic model.

For Mediterranean climate the steady-state hypothesis is not valid because the transient effects of the high initial conditions are not negligible, especially for deep rooted vegetation. For this climate the precipitations are mainly concentrated in the October–March period, when the vegetation is almost inactive. Thus the wet season increases the soil moisture, which will be available for vegetation at the beginning of the subsequently growing season (from April to September). The vegetation, adapting itself to those soil moisture dynamics, has developed an extensive water uptake strategy, by delving the roots into the soil in order to utilize the water stored in the deeper layers.

As regards the link between soil moisture and plants response, low soil moisture level implies a reduction in physiological capacities, and if it is severe or prolonged may cause permanent damages on the vegetation. Porporato et al. (2001) suggest a method for water stress quantification, defining the static water stress and the dynamic water stress. The static water stress \( \zeta \) gives a static description of physiological effects inducted by water stress. In the above definition, the incipient stomatal closure is related to the soil moisture value \( s^* \), below which it is assumed that the water stress increases. Transpiration and root water uptake continue at a reduced rate until soil moisture reaches the wilting point \( s_w \), below which the plant suffers a permanent damage which quickly leads plant to the death. The resistance mechanism to dryness adopted by plants is closely related to water stress duration. If the water stress has a short duration the vegetation does not suffer any damages, while when it remains for longer, the wilting phase takes over, with permanent damages for the plants that become unable to re-establish their own vital functions. For those reasons it is necessary to consider also the length of the time intervals in which vegetation is under stress and the number of such intervals during the growing season in order to characterize properly the vegetation water stress. Porporato et al. (2001) proposed another measure of water stress (dynamic water stress \( \theta \)) that explicitly considers the mean duration and frequency of water stress periods.
Both the previously described stress indexes are defined in steady state conditions, which are not verified in Mediterranean climate, as mentioned above.

Here a numerical model, which reproduces the soil moisture dynamics in Mediterranean climate during the whole hydrologic year, is proposed. The model works using an opportune time-scale (lower than daily), through a finite differences method, and it computes the soil moisture temporal evolution. The model implicitly takes into account the transient effects of the initial soil moisture condition at the beginning of the growing season, which is crucial for the Mediterranean ecosystems.

The soil moisture dynamics that are reproduced through the proposed numerical model, summarize the interrelationships among climate, soil and vegetation and furthermore are strongly correlated with vegetation stress, defined by Porporato et al. (2001). In particular, here the static water stress is numerically computed from the soil moisture traces. Starting from the same traces it is also possible to calculate the mean duration and frequency of water stress periods and hence a dynamic water stress index. The proposed model is here applied at the forested Eleuterio river basin in Sicily (Italy).

2 Description of existing models and of the proposed one

2.1 Soil water balance at a point

Rodriguez-Iturbe et al. (1999a) considered the water balance vertically averaged over the root zone, under the simplifying assumption that the lateral water contributions, mainly due to topographic effects, can be neglected. With the above conditions, the soil moisture balance can be expressed as:

$$n \cdot Z_r \cdot \frac{ds}{dt} = I(s, t) - E(s, t) - L(s, t)$$  \hspace{1cm} (1)

where $n$ is the porosity, $Z_r$ is the rooting depth, $s$ is the relative soil water content or soil moisture, $t$ is the time, while the following terms, all dependent on both $s$ and $t$,
represent respectively the infiltration from rainfall \( I(s, t) \), the rate of evapotranspiration \( E(s, t) \) and the rate of leakage \( L(s, t) \).

The soil water balance equation is a stochastic ordinary differential equation describing at each point the behaviour of soil moisture on time, by linking climatic, pedological and vegetational features. It is constituted by a deterministic part given from the distribution of water fluxes within the soil (infiltration, evapotranspiration and leakage), and by a stochastic part given from the uncertain nature of climatic variables (especially rainfall and temperature regimes).

The solution of this equation consists in the determination of the soil moisture pdf that in general depends on time. The equation does not have any particular time-scale. However here it will be initially consider a daily time scale.

Working in a first basic level of analysis, namely at the spatial scale of a few meters and at the temporal scale of the growing season, the rainfall input may be considered as an external forcing, independent of soil moisture state. Therefore, at daily scale, the depth of rainfall events is assumed to be an independent random variable exponentially distributed with mean value \( \alpha \), while the occurrence of rainfall is assumed to be a marked stationary Poisson process with rate \( \lambda \). The aerial apparatus of vegetation intercepts part of the rainfall; therefore it never arrives to soil surface but is lost directly through evaporation. Interception \( I(t) \) is incorporated in the stochastic model by fixing a threshold for rainfall depth \( \Delta \) (dependent on vegetation type), below which no water reaches the ground, while for rainfall depth higher than \( \Delta \), the water arriving to soil surface is equal to theirs difference \( h' = h - \Delta \).

It is also common in literature to find another alternative form of the soil water balance (1) that is

\[
\begin{align*}
 n \cdot Z_r \cdot \frac{ds}{dt} &= \varphi(s, t) - \chi(s, t) \\
(2)
\end{align*}
\]

where \( \varphi(s,t) \) is the rate of infiltration from rainfall (taking into account the amount of water lost through canopy interception \( \Delta t \), and \( \chi(s) \) is the water losses from the soil, given by the sum of the rates of evapotranspiration and leakage; namely \( \varphi(s, t) \) and...
\( \chi(s) \) represent water fluxes, incoming into and outgoing from the soil column respectively.

\[
\varphi(s, t) = R(t) - \Delta(t) - D(s, t) \tag{3}
\]

\[
\chi(s, t) = E(s, t) + L(s, t) \tag{4}
\]

where \( R(t) \) is the rainfall rate, while \( D(s, t) \) is the rate of runoff (which can be consider as the surplus of water in respect of storage capacity of the soil column, assuming a mechanism of soil saturation from below). It is important to point out that all the above mechanisms consider no interaction between active soil layer and water table.

In the term evapotranspiration are considered two different phenomenons together, which are plant transpiration and evaporation from the soil. Both take place contemporaneously and depend strongly on interactions between plant and soil; it is hence very difficult to distinguish in a quantitative way the amount of water lost through transpiration and evaporation. Therefore, as well as most of the consolidate evapotranspiration model existing in literature, they are estimated together (such as Penman-Monteith method, suggested by FAO, or those of Blaney and Criddle or Thorntwhaite).

As it is well-known, there are different type of factors influencing evapotranspiration: 1) climatic factors, such as solar radiation, temperature, air humidity, wind speed; 2) physical factors related to water and soil, such as area and form of evaporative surface, water availability and soil moisture, water height, soil colour; 3) vegetational factors, such as vegetation type, plant growth rate, roots depth and density, plant physiological activity.

Evapotranspiration valuation is related to the quantitative valuation of each factor and of theirs interaction effects on climate-soil-vegetation system.

From a mathematical viewpoint, the dependence of evapotranspiration losses on soil moisture can be summarized as (Rodriguez-Iturbe and Porporato, 2004)

\[
E = \begin{cases} 
E_w \cdot \frac{s-s_h}{s_w-s_h} & \text{for } s_h < s \leq s_w \\
E_w + (ET_{\text{max}} - E_w) \cdot \frac{s-s_w}{s^*-s_w} & \text{for } s_w < s \leq s^* \\
ET_{\text{max}} & \text{for } s^* < s \leq 1 
\end{cases} \tag{5}
\]
At daily time scale, the maximum value of evapotranspiration $ET_{\text{max}}$ can be interpreted as the average daily evapotranspiration of a unitary surface uniformly covered with vegetation under well-watered conditions during the growing season, while $E_w$, minimum evapotranspiration rate, corresponds to the direct evaporation from bare soil. In particular, for soil moisture levels below the wilting point $s_w$, soil water losses are solely due to direct evaporation from the soil, and soil moisture can decrease until hygroscopic point $s_h$ is reached. At this soil moisture level the soil water pressure is in equilibrium with atmospheric vapour pressure. Thanks to the relatively small losses involved, with $s$ varying from $s_w$ to $s_h$, it is possible to assume a linear decadence law from $E_w$ to zero.

When the soil water content is higher than field capacity $s_{fc}$, the active soil depth tends to lose water excess by gravity at the lowest boundary of the soil layer. This is the so called leakage losses phenomenon, in which the loss rate is assumed to be at the maximum (saturated hydraulic conductivity) when soil is saturated and then rapidly decays as the soil dries following the decrease of hydraulic conductivity $K(s)$. For soil moisture equal to field capacity condition, hydraulic conductivity can be assumed equal to zero (the filed capacity can be seen as the value of soil moisture at which hydraulic conductivity assumes a negligible value; e.g. less than 10% of $ET_{\text{max}}$). The decay of the hydraulic conductivity is usually modelled using empirical relationships, such as exponential law or power law. The exponential form is frequently preferable and is expressed as (Rodriguez-Iturbe and Porporato, 2004)

$$L(s) = \begin{cases} 
\frac{k_s \left[ e^{\beta(s-s_{fc})-1} \right]}{e^{\beta(s_{fc})-1}} & s_{fc} < s \leq 1 \\
0 & s \leq s_{fc}
\end{cases}$$

with $\beta = 2b + 4$, where $b$ is an index related to the type of soil and pore size (varying from $\approx 4$ for sand to $\approx 12$ for clay). This index is the same of the empirical expression of Clapp and Hornberger (1978) for retention curves, and can be experimentally determined.
Figure 1 shows the behaviour of soil water losses $\chi(s)$ as a function of relative soil water content for typical climate, soil and vegetation in semi-arid ecosystems.

Until $s$ is higher than a certain threshold value $s^*$, dependent on both vegetation and soil characteristic, plant does not have any limitations in water uptake and therefore evapotranspiration is ever at the maximum rate $ET_{\text{max}}$. When $s$ reaches $s^*$ (point of incipient stomatal closure), plant begins to reduce transpiration by closing stomata and so reducing water consumers. Namely vegetation is going to water stress phase. Another plant critical value of soil moisture $s_w$, usually varying from $-1.5$ to $-2.5$ MPa in terms of soil matrix potential, is the wilting point, below which transpiration process is finished, the stomatal closure is completed and evapotranspiration is at minimum rate $E_w$.

2.2 Analytical solution of the soil water balance

During interstorm periods the Eqs. (1) or (2) describe decay process for relative soil water content from a certain initial condition, related to the previous history of the entire process. The following ratio is called normalized loss function

$$p(s) = \frac{E(s) + L(s)}{nZ_r} = \frac{\chi(s)}{nZ_r}$$

(7)

where $E(s)$ and $L(s)$ are still the rates of evapotranspiration and leakage respectively, $\chi(s)$ is theirs sum, while $nZ_r$ is the active soil depth. Moreover, it will be assumed:

$$\eta_w = \frac{ET_w}{n \cdot Z_r}$$

(8)

$$\eta = \frac{ET_{\text{max}}}{n \cdot Z_r}$$

(9)

$$m = \frac{K_s}{n \cdot Z_r \cdot (e^{\beta(1-s_{fc})} - 1)}$$

(10)
where symbols on the right-hand sides have the same meaning as before. Assuming that the soil moisture pdf is not time dependent, it is possible to obtain an approximate analytical solution of water balance equation (Eqs. 1 or 2) that is given in Laio et al. (2001b) as follows

\[
p(s) = \begin{cases} 
\frac{C}{\eta_w} \cdot \left( \frac{s-s_h}{s_w-s_h} \right)^{\frac{\lambda'(s_w-s_h)}{\eta_w}} \cdot e^{-\gamma s}, & \text{if } s_h < s \leq s_w \\
\frac{C}{\eta} \cdot \left[ 1 + \left( \frac{\eta}{\eta_w} - 1 \right) \cdot \left( \frac{s-s_w}{s^*-s_w} \right)^{\frac{\lambda'(s^*-s_w)}{\eta-n_w}} \right]^{-1} \cdot e^{-\gamma s}, & \text{if } s_w < s \leq s^* \\
\frac{C}{\eta} \cdot e^{-(\beta+\gamma) s + \beta s_{fc}} \left( \frac{\eta e^{\beta s}}{(\eta-m)e^{\beta s_{fc}}+m e^{\beta s}} \right)^{\frac{\lambda'(s^*-s_{fc})}{\eta-n_w}} e^{\frac{\lambda'(s_{fc}-s)^*}{\eta-n_w}} \cdot e^{\frac{\lambda'(s-s_{fc})}{\eta-n_w}} \cdot e^{-\gamma s}, & \text{if } s_{fc} < s \leq 1 
\end{cases}
\]

where \( C \) is the normalization constant such that

\[
\int_{s_h}^{1} p(s) ds = 1 \tag{13}
\]

The soil water balance represents the core of several different hydrological problems and it is crucial in control of a wide range of mechanisms, such as those controlling climatic changes or desertification, or in management of water resource.
2.3 Numerical solution of the soil water balance

In the above section, the analytical model in steady state condition has been described. As above mentioned, this condition is very difficult to apply in Mediterranean areas where the soil moisture at the beginning of the growing season generates a transient dynamic. In order to overcome the limitation imposed by the analytical formulation, the soil water balance Eq. (2) can be numerically solved through a finite differences method. Once \( s \) at the generic time \( t_i \) is known, it is possible to estimate \( s \) at the time \( t_{i+1} \), rewriting the balance equation as

\[
\Delta s = s_{i+1} - s_i = \left( \frac{\varphi_i}{n \cdot Z_r} - \frac{\chi_i}{n \cdot Z_r} \right) \cdot \Delta t = \left( \frac{\varphi_i}{n \cdot Z_r} - \rho_i \right) \cdot \Delta t
\]  
(14)

where \( s_i \) and \( s_{i+1} \) are the soil moisture contents at time \( t_i \) and \( t_{i+1} \), while \( \Delta t \) is the temporal step chosen and \( \rho_i \) is expressed by the Eq. (7).

Again, any interaction between the active soil layer and the water table are considered. Moreover it is assumed that the dynamic effects driven by pouring rain are neglected, and that the porosity \( n \) depends only on texture, while the rooting-deep \( Z_r \) depends only on vegetation type, and both are time invariant.

The first step of the model consists in establish the rainfall series, that may be synthetic or it is also possible to use the historical precipitations series. In the case of synthetic series use, the occurrence of rainfall is still idealized as a series of point events in continuous time, arising according to a Poisson process of rate \( \lambda \) and each carrying a random amount of rainfall extracted from a given distribution with mean \( \alpha \) (see Sect. 2.1). It is possible to estimate \( \alpha \) and \( \lambda \) starting from the historical precipitation series.

In order to take into account the canopy interception, the rainfall process is still transformed into a new marked Poisson process, following the indications in Sect. 2.1.

The specific water volume infiltrated at time \( t_i \) is the lower between \( h_i \) (depth of rainfall event \( i \)) and the maximum volume available for water storage at time \( t_i \) given by

\[
W_{\text{max,}i} = (1 - s_{i-1}) n Z_r
\]

(15)
where $s_{i-1}$ is the soil moisture at time $t_{i-1}$. When $h_i$ is greater than $W_{\text{max},i}$, then the excess of water is lost as runoff (mechanism of saturation from below).

The water losses from the soil are given by the sum of the evapotranspiration (Eq. 5) and leakage losses (Eq. 6).

The model works at a spatial scale of few meters and considers a river basin as a collection of elementary cells, where each cell is considered homogenous with regard to soil and vegetation.

Resuming all the input to the model, it is necessary to consider for every combination soil-vegetation:

- $k_s$ = saturated hydraulic conductivity;
- $\beta$ = coefficient of power law for hydraulic conductivity;
- $s_h, s_w, s^*, s_{fc}$ = characteristic values of relative soil moisture;
- $E_w$ = evapotranspiration at the minimum rate;
- $E_{\text{max}}$ = evapotranspiration at the maximum rate.
- $\alpha$ and $\lambda$ = rainfall parameters.

According to the conveniently chosen annual discretization (division of the year in several time-invariant periods each one characterized by the own regimes of precipitation and evapotranspiration), from rainfall series viewpoint it is needed to estimate the values of $\alpha$ and $\lambda$ in relation to it. If a monthly annual discretization with the mean monthly values of precipitation in the synthetic series corresponding to historical mean monthly precipitations is needed, then the estimate twelve values for both $\alpha$ and $\lambda$ has to be carried out. Otherwise if the main target is only to distinguish between dry and wet seasons, then the estimate of only two sets of parameters $\alpha$ and $\lambda$ is needed. Analogous reasoning must be done for the estimation of evapotranspiration parameters.

In general most of input data, such as rainfall series, temperature series or others data useful for the determination of evapotranspiration are easily available at daily time.
scale. In truth is helpful to consider smaller time steps in order to obtain a better accuracy in generation of rainfall series and principally for what concerning the valuation of leakage losses. Evaluating the importance of temporal discretization $\Delta t$ in the numerical estimate of soil moisture dynamics it is possible to note that estimation of leakage losses is very sensitive to it, in particular long intervals lead to a heavy overestimation of those contributions.

A sub-daily temporal discretization is very important for an enough accurate solution of soil water balance, working with a finite differences method. It is possible to have a satisfactory reproduction of steady state analytical solution in dry climate (less than 400 mm of rainfall in the growing season) with 2–4 steps for day. The temporal step chosen must be lower as precipitations during the growing season increases.

Consequently it is necessary to scale down at the same chosen time-scale (step-scale), also all the input, such as the values of evapotranspiration (minimum and maximum) and of saturated hydraulic conductivity.

Once the synthetic rainfall series has been generated and the soil water balance equation has been solved by finite differences method for each possible soil-vegetation combination, the soil moisture values at each instant (step) of simulation (with duration equal to the synthetic series number of years) are known and then it is possible to obtain the soil moisture time-profile.

2.4 Indexes of plant water stress

Moisture reduction into the active soil layer leads to a decrease of plant water potential and consequently of transpiration, potentially dangerous for plant physiological functions.

Under steady state hypothesis, it is possible to calculate the static water stress $\zeta$ (Porporato et al., 2001). The static stress is equal to zero (minimum value) when the relative soil water content is equal or above $s^*$ (incipient stomatal closure), while when $s$ is equal or below $s_w$, the stress is equal to one (maximum value). The static water
stress can be expressed as
\[
\zeta(s) = \left[\frac{s^* - s(t)}{s^* - s_w}\right]^q \quad \text{for } s_w \leq s \leq s^* \tag{16}
\]
where the exponent \( q \) accounts for the non-linearity of the relationship between water stress and soil moisture, and its value depends on vegetation species and soil type (in linear case \( q=1 \), while in strongly non-linear case \( q=3 \)).

The pdf of static stress \( p(\zeta) \) can be easily analytically obtained from that of soil moisture \( p(s) \) described by (8). The Eq. (16) can be inverted for the entire domain, and so for each \( \zeta \) in the interval \( 0 < \zeta < 1 \), it is possible to associate a corresponded value for \( s \). The pdf of static water stress has two atom of probability, at \( \zeta = 0 \) and at \( \zeta = 1 \). Naming with \( P(s^*) \) the value of cumulative distribution of soil moisture calculated in \( s = s^* \) and with \( P(s_w) \) that calculated in \( s = s_w \), the probability of having no stress, correspondent to the probability of soil moisture above \( s^* \), represents the first atom of probability and is equal to
\[
P(\zeta = 0) = P(s > s^*) = 1 - P(s \leq s^*) = 1 - P(s^*) \tag{17}
\]
While the probability of having maximum stress \( (\zeta = 1) \), represents the second atom of probability and is equal to
\[
P(\zeta = 1) = P(s \leq s_w) = P(s_w) \tag{18}
\]
From the soil moisture pdf and for \( s_w < s \leq s^* \), it is possible to obtain the continuous part of the static stress pdf \( p(\zeta) \), by inverting the stress function \( \zeta(t) \)
\[
p(\zeta) = \frac{C_{\zeta}}{n_{w}} \cdot \left[\left(1 - \frac{n}{n_{w}}\right) \cdot \zeta^\frac{1}{q} + \frac{n}{n_{w}}\right] \cdot e^{\gamma \left[\left(s^* - s_w\right)\zeta^\frac{1}{q} - s^*\right]} \tag{19}
\]
The constant of integration \( C_{\zeta} \) can be deduced by imposing the condition
\[
\int_{0}^{1} p(\zeta) d\zeta = P(s^*) - P(s_w) \tag{20}
\]
The mean value of the static water stress \(<\zeta>\) can be obtained by integration, as

\[
\langle \zeta \rangle = \int_{0}^{1} \zeta \cdot p(\zeta) d\zeta + P(\zeta = 1)
\] (21)

It is more helpful to estimate the mean static water stress on the periods of growing season in which the water stress is really present, neglecting the periods in which \(\zeta=0\) (i.e. the static water stress modified \(<\zeta'\>)). In order to analytically obtain the latter, the following expression can be used

\[
\langle \zeta' \rangle = \frac{\langle \zeta \rangle}{P(s*)}
\] (22)

Another index for the evaluation of plant water stress under steady state condition is the dynamic water stress, which also takes into account the crossing properties. According to Porporato et al. (2001) the dynamic water stress is a measure of water stress able to combine, through the later shown variables \(<T_{s*}\>) and \(<n_{s*}\>\), the above defined mean static stress modified \(<\zeta'\)> with the mean duration and frequency of water stress.

The static water stress modified \(<\zeta'\>) takes into account the mean intensity of water deficit, but it does not contain information on its duration and frequency.

\(<T_{s*}\>) is defined as the mean duration of stress periods during the growing season and assuming a linear dependence between this duration and the intensity of dynamic water stress, the mean value of plant water stress during a stress period is a function of the product between \(< \zeta'\>) and \(<T_{s*}\>). The actual plant water stress, however, cannot increase indefinitely with this product, since there must be a point from where on the stress is at its maximum level, corresponding to the onset of permanent damages (Rodriguez-Iturbe and Porporato, 2004). A parameter \(k\) (index of plant resistance to water stress) is used to fix this upper threshold of water stress; in this way permanent damages appear when the product \(< \zeta'\><T_{s*}\>) is greater than the product \(kT_{\text{seas}}\),
where $T_{\text{seas}}$ is the growing season duration. When no specific information on the resistance of different species is available, the same value for all the species can be assumed (Rodriguez-Iturbe and Porporato, 2004 suggest to use $k=0.5$). Moreover, the parameter $k$ can be considered as the mean static stress modified $<\zeta'>$ that a plant can be experience without suffering permanent stress, when the duration of the stress period is equal to $T_{\text{seas}}$.

The number of stress periods during a growing season ($<n_{s^*}>=\text{mean number of downcrossing or mean number of the soil moisture excursions below } s^*$) and the effect that multiple periods of stress may have on the plant status are also very important.

According to Porporato et al. (2001), the dynamic water stress can be expressed by a function in which the number $<n_{s^*}>$ appears as an exponent and moreover it is in turn raised to the power of $-r$, where $r$ is a constant (according to the authors $r$ is equal to 0.5).

The dynamic water stress or mean total dynamic stress during the growing season $<\theta>$, can be hence defined according to

$$
\langle \theta \rangle = \begin{cases} 
\left( \frac{\langle \zeta' \rangle \langle T_{s^*} \rangle}{kT_{\text{seas}}} \right)^{<n_{s^*}>^{-r}} & \text{if } \langle \zeta' \rangle \langle T_{s^*} \rangle < k \cdot T_{\text{seas}} \\
1 & \text{if } \langle \zeta' \rangle \langle T_{s^*} \rangle \geq k \cdot T_{\text{seas}}
\end{cases}
$$

(23)

When $<\zeta'><T_{s^*}$ is close to $kT_{\text{seas}}$, the influence of $<n_{s^*}>$ is reduced. If the first product is equal or greater than the second one, then the dynamic water stress is equal to one, regardless of $<n_{s^*}>$ value.

The plant response in terms of dynamic water stress, is different depending on the active soil depth $Z_r$. Deep-rooted species (e.g. trees) rely on a dependable winter recharge, as opposed to shallow-rooted species (e.g. grasses) that quickly respond to the intermittent and uncertain rainfall during the growing season (Rodriguez-Iturbe et al., 2001).

The dynamic water stress is defined in steady-state conditions. Thus, whenever a
not negligible transient period is present the above definition is not valid. It is the case of Mediterranean climate where the presence of a winter recharge creates a transient in the soil moisture dynamics (whose duration depends mainly on the active soil depth $Z_r$), and where an analytical evaluation of the dynamic water stress would lead to overestimation.

Moreover, the crossing properties of the soil moisture process are also valuated during the growing season in analytical manner by Porporato et al. (2001). Therefore some of these terms involved in the dynamic water stress formulation may assume physically unrealistic values, e.g. the value of $<T_s>$ is not bounded to $T_{seas}$ and it is hence possible to have a mean duration of stress periods during the growing season higher than the duration of the growing season.

In order to overcome those limitations, a numerically estimation of the dynamic water stress is proposed and it is explained in the next section.

2.5 Numerical evaluation of vegetation water stress

Once the soil water balance equation has been solved by finite differences method obtaining the soil moisture time-profile, the next step of the proposed numerical model consists in evaluating of vegetation response in terms of water stress. Using the static water stress index (Eq. 16) it is possible to obtain static water stress time-profile, related and analogous to that of soil moisture. It is also possible to calculate provisionally to the growing season, the value of mean static water stress modified, namely the mean value of static water stress on the periods in which there is really water stress for each growing season of the rainfall series. The seasonal values of the number of periods with stress and theirs mean duration can be assessed year-by-year. Averaging the seasonal data on the whole number of simulated years, it is possible to evaluate the mean values for soil moisture $<s>$, for static water stress $<ζ>$ and static water stress modified $<ζ'>$, and finally for the variables $n_s$ and $T_s$, described in the previous section, allowing this way the final estimation of the mean dynamic water stress according to Eq. (17). It is important to point out that the main differences of the proposed approach
from the analytical one are that here the variables $n_s^*$ and $T_s^*$ are numerically computed and for this reason they are bounded. Particularly it is impossible have a value of $T_s^*$ higher than $T_{\text{seas}}$. Also the evaluation of the mean static water stress is quite different, since it is not obtained from the static stress pdf (through Eq. 15) but it is evaluated starting from the soil moisture time-profile, step-by-step valuating the static water stress by the Eq. (16) and averaging all the results. Similarly the mean static water stress modified is not obtained by the Eq. (16) but through a simple average operation on the periods of static water stress time profile in which $<\zeta>$ is different from zero.

Even if the two indexes given by Porporato et al. (2001) are defined for steady state conditions, their numerical estimation provide a complete description of the plants water stress also in presence of a not negligible transient period. The methodology described above has been applied to a small watershed located in Sicily (Italy).

3 Application

3.1 The Eleuterio river basin

The watershed studied in the present paper is located in the province of Palermo (Sicily, Italy) within the Eleuterio River Valley. This area, near the town of Ficuzza, in ancient time was a Borbonic hunting reserve, and also for this reason, is nowadays well conserved, with few anthropic actions. The watershed has an extent of almost 9.5 km$^2$ and it is within the “Bosco della Ficuzza” wood. Far from Palermo about 40 km, the natural reserve of “Bosco della Ficuzza” with an extent of 5333 ha represents one of the widest natural reserves in Sicily. The present vegetation is mainly of woody type, constituted by Quercus pubescens, Acer campestre and Fraxinus ornus. The forest presents oneself dense and covers the whole surfaces by both a superficial woody layer and a bottom layer of shrub and grass plants.

The zone under exam is constituted by several river channels merging to the Lake of Scanzano, where is located the outlet of the river basin (Fig. 2).
watershed is located at latitude 37.53°, its elevation ranges from 517.5 m a.s.l. to 1635.5 m a.s.l. with a mean elevation of 792.2 m a.s.l. and standard deviation of 194.6 m.

In a previous study (Liguori et al., 1983), three maps of the under exam zone have been product (Map of Cultures, Hydrogeological Map, Geological Map). They are taken into account in determining spatial patterns of soil texture and vegetation.

The Map of Cultures shows that there is an overriding presence of woody vegetation, even if vineyard, olive tree grove and pasture land with shrub vegetation are present too; with a low percentage a dry seminative land and bare soils are also present.

For sake of simplicity the basin is assumed covered by the following three types of vegetation:

– **Trees**, including the areas classified as woody, degraded woody, reforestation zone;

– **Shrubs**, including the areas classified as vineyard, olive tree grove and pasture land;

– **Grasses**, including the areas classified as seminative and sterile, where the presence of small grassland is possible.

This classification set aside the classical definition of tree, shrub or grass species, and it is based on deepness of vegetational root-apparatus. The vegetational parameters related to each type of vegetation are shown in Table 1, using data coming from Laio et al. (2005).

The Hydrogeological Map and the Geological Map show that the zone of interest is mainly constituted by lithological-technical complexes classifiable as incoherent soil materials in the southern part, pseudo-coherent soil materials in the middle part of the basin and coherent soil materials with pseudo-coherent levels in the northern part and in the eastern one.
The basin in exam can be considered as constituted by three soil types (according to USDA classification), whose features are summarized in the Table 2: sandy loam; loamy sand; clay.

The Fig. 3 shows as the vegetational and pedological information for the Eleuterio river basin have been used, to obtain the spatial patterns of vegetation, soil and finally the spatial overlay between these.

For the purpose of this work, only the trees, that are the main component of vegetation, are taken into account. The basin has been divided using a raster schematisation with 23,814 elementary cells (20 m × 20 m) homogeneous with regard to soil and vegetation. In this way three possible different combinations of soil-vegetation are present.

Moreover, no interaction between soil and water table is taken into account, since of the deep groundwater.

3.2 Evapotranspiration estimation

With regard to growing season, it starts on 1 April and ends on 31 October, with a duration of 214 days.

The evapotranspiration at minimum rate $E_w$, in correspondence to the wilting point $s_w$, can be fixed at value 0.1 mm/day (Rodriguez-Iturbe et al., 1999b).

The difficulty to find daily historical data series of air relative humidity, heliophany and wind speed, caused the choice to fix a monthly scale, starting from mean monthly data coming from multi-year observation time series, in order to estimate potential evaporation.

The spatial distribution of climatic variables has not been taken into account because of low variability of these within the watershed.

Monthly time series of temperature, air relative humidity, heliophany and wind speed have been extracted from records of the Ficuzza gauge station (Atlante climatografico della Sicilia, Regione Siciliana; Augi, 2003). Table 3 shows some meteo-climatic features within the basin. The higher values of mean monthly temperatures and air humidity have been observed during the growing season.
Potential evapotranspiration is estimated by Penman-Monteith method, with the following equation (Caylor et al., 2005)

\[
\lambda ET_{\text{max}} = \frac{\Delta R_n + \rho C_p g_a \cdot \delta_e}{\Delta + \gamma (1 + g_a / g_c)}
\]  

(24)

where \( ET_{\text{max}} \) is in kg/m\(^2\)s, \( \lambda \) is the latent heat of vaporization (J/kg), \( \Delta \) is the slope of the curve relating saturation vapor pressure to temperature (Pa/\(^\circ\)C), \( R_n \) is the net radiation of the plant canopy (J/m\(^2\)s), \( \rho \) is the density of air (J/\(^\circ\)C), \( C_p \) is the specific heat capacity of air (J/\(^\circ\)C), \( g_a \) is the aerodynamic conductance of the vegetation canopy (m/s), \( \delta_e \) is the vapor pressure deficit (Pa), \( \gamma \) is the psychrometric constant (Pa/\(^\circ\)C), and finally \( g_c \) is the vegetation canopy conductance (m/s).

The net radiation of the plant canopy \( R_n \) is calculated following Jones (1983) by assuming that the temperature of the vegetation canopy is equal to that in atmosphere \( (T_{\text{leaf}} = T_a) \), so that

\[
R_n = \alpha_s SW_{\text{inc}} + \sigma T_s^4 - \sigma (T_a + 273.15)^4
\]  

(25)

where \( \alpha_s \) is the albedo, \( SW_{\text{inc}} \) is the incident shortwave radiation, \( \sigma \) is the Stefan Boltzmann constant \((5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4})\) and \( T_s \) is the apparent radiative temperature of the atmosphere (K) determined using the empirical relationship given by Friend (1995) as

\[
T_s = T_a + 273.15 - 0.825 e^{3.54 \times 10^{-3} SW_{\text{inc}}}
\]  

(26)

The aerodynamic conductance \( (g_a) \) term can be calculated by using the following equation (Caylor et al., 2005)

\[
g_a = \frac{k^2 u_z}{(\ln [(h - d) / z_0]^2}
\]  

(27)
where \( k = 0.41 \) is the dimensionless von Karman constant, \( u_z \) is the average daily wind speed at the reference height \( h \) (taken to be 2m above the canopy height \( H \)), \( d = 0.64H \) is the displacement height, and \( z_0 = 0.13H \) is the roughness length (see Table 1).

The total canopy conductance \( g_c \) is the product of the vegetation maximum stomatal conductance \( g_{s,\text{max}} \) and the leaf area index \( LAI \) (Table 1)

\[
g_c = g_{s,\text{max}} \cdot LAI
\]  

(28)

3.3 Generation of the synthetic rainfall series

Fig. 4a and Fig. 4b represent the recorded precipitation in the Eleuterio river basin. The former shows the annual precipitations during the observation period, divided into precipitation during growing season (green) and that during dormant season (yellow), while the latter shows the mean monthly precipitations. The mean annual precipitation observed is 773 mm, where on average 526 mm rain during the dormant season. A strong seasonality is evident in monthly precipitation, with higher and more frequent rainfall events during the winter months (in December, January and February over 100 mm of monthly precipitation) and only 5 mm of mean monthly rainfall in July.

In order to analyse the effect of a different temporal schematisation of the year, two different schemes are considered: SCHEME A and SCHEME B.

In the SCHEME A, the year is divided into two season, the growing season (GS) and the dormant season (DS), each one with its own values of \( \alpha \) and \( \lambda \) for the precipitations and \( ET_{\text{max}} \) for the evapotranspiration, time-invariant, year-by-year, quantities representative of own season.

In the SCHEME B the year is still divided into two parts (GS and DS), but now the sets of parameters \( \alpha, \lambda \) and \( ET_{\text{max}} \) are assumed to be time-invariant quantities at monthly time-scale, so twelve sets of these parameters are present.

Starting from the historical data series, the seasonal values of \( \alpha \) and \( \lambda \) (useful for SCHEME A) and the monthly (useful for SCHEME B) have been derived (reported in Table 4). In the SCHEME A, \( \alpha \) and \( \lambda \) are constant during each season and are equal
to 5.95 mm and 0.195 1/day respectively during the growing season, and 7.01 mm and 0.493 1/day during the dormant season. In the SCHEME B $\alpha$ ranges from 2.97 mm (June) to 7.80 mm (October), while $\lambda$ ranges from 0.051 1/day (July) to 0.128 1/day (February).

Similarly, using the Eq. (18) and from the data shown in Table 1 and Table 3, the seasonal and monthly values of $ET_{\text{max}}$ for the woody component of vegetation have been obtained and reported in Table 5. Here it is possible to point out a maximum value of evapotranspiration in July (5.84 mm/day), maintaining high values for all the summer periods.

The different annual fluctuations of the three parameters $\alpha$, $\lambda$ and $ET_{\text{max}}$ used in the two schemes are emphasized in Fig. 5a and Fig. 5b.

3.4 Model application to Eleuterio

For the Eleuterio river basin, a time step, $\Delta t$, equal to four hours has been chosen. This value allows to accurately reproduce the soil moisture dynamics during the growing season because of the low values of leakage in that period, resulting also as a good compromise between results accuracy and computational effort for the wet season. All the daily input data to model ($\alpha$, $\lambda$, $ET_{\text{max}}$, $E_w$, $k_s$) have been hence scaled using the above mentioned time step.

The numerical model requires as input a synthetic rainfall series, long enough to allow long-term evaluations for the response of vegetation in a river basin. For this purpose, two synthetic series of 100 years are generated, one for each considered scheme (A and B), following the procedure described in Sect. 2.3 and using $\alpha$ and $\lambda$ parameters in Table 4.

Starting from an initial condition of soil moisture equal to the field capacity on the 1 January of the first year, and applying the Eq. (10), the soil moisture time-profile can be calculated for all the three possible soil-types.

It is subsequently possible to obtain the static water stress time-profile for both the two simulations (each one of 100 years) and the three soil-types, starting from the
knowledge of the soil moisture step per step (in Eq. 16 it is used the exponent $q$ equal to 3).

The results of simulations carried out using the two schemes are summarised in Fig. 6a and Fig. 6b. These figures show the results for a single representative year (randomly chosen among all the 100 years simulated) in order to emphasize the soil moisture annual variability and the consequent vegetation response. Both this plots show the soil moisture time-profile (middle panel) and the static water stress (bottom panel), in response to the synthetic rainfall series (top panel). The blue lines refer to loamy sand, the red to sandy loam, and the green to clay soil.

Analysing these plots is possible to note that the SCHEME A, despite of its simplicity, is able to capture interannual variability and the strong seasonality of the soil moisture dynamics. In the SCHEME B, because of its discretization, the rainfall input, the soil moisture traces and the water stress are more regular, with less abrupt variations in the temporal evolution. This scheme, for the same soil type gives higher, but similar, values for the mean soil moisture than SCHEME A (Table 6). That is due to the fact that SCHEME B simulates lower losses at the beginning of the growing season (evapotranspiration lower than SCHEME A, see Fig. 5a) and at the same time the rainfall events, more frequent, keep the soil moisture higher than the SCHEME A. Although during the driest periods (from June to August), according to SCHEME B, the evapotranspiration losses are much higher than the ones calculated with the SCHEME A, they are bounded in a shorter period, thus the mean soil moisture during the whole growing season remain higher for the SCHEME B.

Afterwards, the analysis will be only focused on the growing season for both the considered schemes, assuming a correspondence between the seasonal average values on the 100 simulated years and the long-term seasonal values obtainable with the analytical solution.

In order to give a comparison between the results obtainable from the analytical solution in stationary condition and from the two solutions of the proposed numerical model, it is possible to compare the soil moisture $pdf$ during the growing season, shown
in Fig. 7 for different soil characteristics. The analytical pdf is unimodal and symmetric, with low variance. The numerical pdf’s show a greater dispersion of the values around the mode, since the model takes into account the transient effects of soil moisture at the beginning of the growing season. The SCHEME B leads to a bimodal pdf, with the right-mode representing the effect of high soil moisture initial condition.

Using the numerical model, the mean number of periods with water stress during the growing season, and theirs mean duration can be assessed for each simulated growing season. Averaging on the whole considered period (100 years), it is possible to obtain the long-term seasonal values of the mean number of downcrossing $n_{s^*}$ and theirs mean duration $T_{s^*}$ which are necessary for the dynamic water stress determination.

Finally, using the Eq. (17) with $r=0.5$, $T_{\text{seas}}=214$ days and $k=0.7$ for the woody vegetation, the dynamic water stress representative of plant condition within the Eleuterio basin, during the growing season, for the three soil-type can be obtained for both the proposed schemes.

Although in SCHEME B the mean soil moisture during the growing season is higher, in SCHEME A the mean static water stress is lower (see Table 6). This is mainly due to the short stress periods with very high values of plant stress in SCHEME B, and it is also barely due to the non linearity ($q=3$) for the relationship between $s$ and $\zeta$.

The above considerations are also confirmed by observing the same behaviour assumed by mean static water stress modified, more sensitive to the prolonged and intense stress periods. As direct consequence, also the dynamic water stress indexes for SCHEME B are slightly higher than these for SCHEME A. Comparing the results for the three different soils it is possible to point out that the woody vegetation on a clay soil suffers less water stress than that lying in the other soils, because this soil has higher water storage capacity and moreover retains for a longer period the initial moisture during the growing season.

The differences between the results obtained with the two proposed schemes are minimal. Both the seasonal soil moisture pdf’s and the evaluations of the dynamic water stress are similar. For those reasons in the Elueterio river basin the SCHEME
A, being simpler, seems to be preferable. Anyway in watersheds characterized by a higher variability of climatic parameters during the year, the SCHEME B might be more appropriate.

4 Concluding remarks

The soil moisture dynamics and the vegetation water stress in Mediterranean climate, where the wet and the growing season are out of face, have been investigated proposing a numerical ecohydrological model which takes into account the seasonality of the rainfall and of the evapotranspiration demand. Working on the whole years, the model is able to reproduce the winter recharging processes, which gives the soil moisture condition at the beginning of each growing season, and it also describes the transient effect during the growing season. The proposed model solves the soil water balance, through a finite difference method, working with a temporal step short enough to give a satisfactory approximation of the water losses. Sampling the soil moisture values in the growing season it is possible to estimate the soil moisture pdf, which implicitly considers the transient effects. The numerical pdf's have been compared with those analytical obtained, showing important differences. The numerical pdf's are not symmetric and spread over a wide range, from the field capacity, which is a likely value at the beginning of the growing season, to the stomata closure point, which is the most likely value during the growing season.

The influence of the description of the seasonal climate variability on the soil moisture pdf has been analyzed, finding that the higher is the interannual discretization considered for rainfall and evapotranspiration parameters the more accurate is the resulting pdf. Furthermore, if the climate variability is described at monthly scale, the soil moisture pdf results as bimodal.

After solving the soil water balance equation and obtaining the soil moisture time-profile, the static and the dynamic water stress indexes introduced by (Porporato et al., 2001) have been numerically computed in order to evaluate the plant response.
numerical evaluation of the dynamic water stress is a new definition of water stress and leads to different results from its analytical estimate. It allows to consider a non steady condition for the soil moisture dynamics, and thus to calculate the vegetation water stress in Mediterranean climate where the presence of a transient period is crucial. Finally an investigation on how the water stress evaluation is influenced by the description of the seasonal climate variability has been carried out. The results obtained from the two adopted schemes adopted are quite similar, suggesting to consider a two-season division of the years for generic applications and a higher discretization for more accurate applications.

References

Augi, S.: Deduzione di serie temporali di contenuto e di stress idrico da un modello stocastico di bilancio idrologico, Università degli Studi di Palermo (Dottorato di Ricerca in Idronomia Ambientale, XV ciclo, Tesi per il conseguimento del titolo), 2003.


Table 1. Parameters describing the vegetation characteristics used in the model application (Eleuterio).

<table>
<thead>
<tr>
<th>Vegetation type</th>
<th>$Z_r$ (cm)</th>
<th>$h_\Delta$ (cm)</th>
<th>$H$ (m)</th>
<th>$\alpha_s$</th>
<th>$LAI$ (m$^2$/m$^2$)</th>
<th>$g_{s,\text{max}}$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tree</td>
<td>150</td>
<td>0.2</td>
<td>4</td>
<td>0.1</td>
<td>1.5</td>
<td>0.01</td>
</tr>
<tr>
<td>Shrub</td>
<td>40</td>
<td>0.15</td>
<td>1</td>
<td>0.15</td>
<td>0.5</td>
<td>0.0125</td>
</tr>
<tr>
<td>Grass</td>
<td>30</td>
<td>0.1</td>
<td>0.5</td>
<td>0.12</td>
<td>0.25</td>
<td>0.0167</td>
</tr>
</tbody>
</table>

Caylor et al. (2005) – “On the coupled geomorphological and ecohydrological organization of river basins” – Advances in Water Resources
Table 2. Parameters describing the soil characteristics used in the model application (Eleuterio).

<table>
<thead>
<tr>
<th>Soil type</th>
<th>$\beta$</th>
<th>$k_s$ cm/d</th>
<th>$n$</th>
<th>$s_h$</th>
<th>$s_w$</th>
<th>$s^*$</th>
<th>$s_{fc}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loamy sand</td>
<td>12.7</td>
<td>100</td>
<td>0.42</td>
<td>0.08</td>
<td>0.11</td>
<td>0.31</td>
<td>0.52</td>
</tr>
<tr>
<td>Sandy loam</td>
<td>13.8</td>
<td>80</td>
<td>0.43</td>
<td>0.14</td>
<td>0.18</td>
<td>0.46</td>
<td>0.56</td>
</tr>
<tr>
<td>Clay</td>
<td>26.8</td>
<td>2.5</td>
<td>0.5</td>
<td>0.47</td>
<td>0.52</td>
<td>0.78</td>
<td>0.91</td>
</tr>
</tbody>
</table>

Laio et al. (2001) – “Plants in water-controlled ecosystems: active role in hydrological processes and response to water stress” – Advances in Water Resources
**Table 3.** Meteo-climatic data for the Eleuterio river basin. \( \rho \) and \( C_p \) are the density and the specific heat capacity of air, respectively; \( T_{\text{max}} \) and \( T_{\text{min}} \) are the mean daily values of maximum and minimum temperatures respectively; \( \text{RH}_{\text{mean}} \) is the average daily air humidity.

<table>
<thead>
<tr>
<th>Month</th>
<th>( T_{\text{max}} )</th>
<th>( T_{\text{min}} )</th>
<th>( \text{RH}_{\text{mean}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>11.7</td>
<td>3.8</td>
<td>69</td>
</tr>
<tr>
<td>February</td>
<td>12.3</td>
<td>3.9</td>
<td>68</td>
</tr>
<tr>
<td>March</td>
<td>15.0</td>
<td>5.2</td>
<td>73</td>
</tr>
<tr>
<td>April</td>
<td>18.3</td>
<td>7.1</td>
<td>74</td>
</tr>
<tr>
<td>May</td>
<td>22.9</td>
<td>10.6</td>
<td>73</td>
</tr>
<tr>
<td>June</td>
<td>28.5</td>
<td>13.7</td>
<td>72</td>
</tr>
<tr>
<td>July</td>
<td>31.8</td>
<td>16.6</td>
<td>73</td>
</tr>
<tr>
<td>August</td>
<td>32.6</td>
<td>17.3</td>
<td>74</td>
</tr>
<tr>
<td>September</td>
<td>28.2</td>
<td>14.9</td>
<td>76</td>
</tr>
<tr>
<td>October</td>
<td>22.6</td>
<td>12.0</td>
<td>75</td>
</tr>
<tr>
<td>November</td>
<td>17.2</td>
<td>8.2</td>
<td>74</td>
</tr>
<tr>
<td>December</td>
<td>13.3</td>
<td>5.1</td>
<td>71</td>
</tr>
</tbody>
</table>
Table 4. In the upper part seasonal and annual values of $\alpha$, $\lambda$ and the total amount of rainfall $\Theta$ for the Eteterio river basin. At bottom, mean monthly values and standard deviation (S.D.) (GS = growing season; DS = dormant season).

<table>
<thead>
<tr>
<th></th>
<th>$T$</th>
<th>$\alpha$</th>
<th>$\lambda$</th>
<th>$\Theta = \alpha\lambda T$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>days</td>
<td>mm</td>
<td>1/day</td>
<td>mm</td>
</tr>
<tr>
<td>GS</td>
<td>214</td>
<td>5.95</td>
<td>0.195</td>
<td>248</td>
</tr>
<tr>
<td>DS</td>
<td>154</td>
<td>7.01</td>
<td>0.493</td>
<td>526</td>
</tr>
<tr>
<td>Annual</td>
<td>365</td>
<td>6.63</td>
<td>0.319</td>
<td>773</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Month</th>
<th>Mean</th>
<th>S.D.</th>
<th>$\alpha$ mean rainfall depth</th>
<th>$\lambda$ = mean time between two events</th>
<th>Monthly Precipitation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean</td>
<td>S.D.</td>
<td>1/day</td>
<td>1/day</td>
<td>mm</td>
</tr>
<tr>
<td>January</td>
<td>7.52</td>
<td>2.25</td>
<td>0.526</td>
<td>0.109</td>
<td>130</td>
</tr>
<tr>
<td>February</td>
<td>7.2</td>
<td>1.9</td>
<td>0.502</td>
<td>0.128</td>
<td>105</td>
</tr>
<tr>
<td>March</td>
<td>6.45</td>
<td>2.34</td>
<td>0.45</td>
<td>0.123</td>
<td>91</td>
</tr>
<tr>
<td>April</td>
<td>6.32</td>
<td>3.17</td>
<td>0.354</td>
<td>0.1</td>
<td>72</td>
</tr>
<tr>
<td>May</td>
<td>4.71</td>
<td>2.75</td>
<td>0.191</td>
<td>0.076</td>
<td>31</td>
</tr>
<tr>
<td>June</td>
<td>2.97</td>
<td>2.37</td>
<td>0.12</td>
<td>0.056</td>
<td>11</td>
</tr>
<tr>
<td>July</td>
<td>3.35</td>
<td>2.4</td>
<td>0.051</td>
<td>0.053</td>
<td>5</td>
</tr>
<tr>
<td>August</td>
<td>6.07</td>
<td>4.52</td>
<td>0.086</td>
<td>0.053</td>
<td>17</td>
</tr>
<tr>
<td>September</td>
<td>6.14</td>
<td>2.94</td>
<td>0.213</td>
<td>0.081</td>
<td>38</td>
</tr>
<tr>
<td>October</td>
<td>7.8</td>
<td>2.49</td>
<td>0.313</td>
<td>0.101</td>
<td>74</td>
</tr>
<tr>
<td>November</td>
<td>6.74</td>
<td>2.53</td>
<td>0.413</td>
<td>0.107</td>
<td>86</td>
</tr>
<tr>
<td>December</td>
<td>7</td>
<td>2.32</td>
<td>0.512</td>
<td>0.108</td>
<td>113</td>
</tr>
</tbody>
</table>
Table 5. Monthly and seasonal values of ET$_{\text{max}}$ for woody vegetation within the Eleuterio river basin (GS = growing season; DS = dormant season).

<table>
<thead>
<tr>
<th>Month</th>
<th>ET$_{\text{max}}$ mm/day</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>1.67</td>
</tr>
<tr>
<td>February</td>
<td>1.91</td>
</tr>
<tr>
<td>March</td>
<td>2.23</td>
</tr>
<tr>
<td>April</td>
<td>3.06</td>
</tr>
<tr>
<td>May</td>
<td>4.13</td>
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<tr>
<td>June</td>
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<tr>
<td>July</td>
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</tr>
<tr>
<td>August</td>
<td>5.79</td>
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<tr>
<td>September</td>
<td>4.82</td>
</tr>
<tr>
<td>October</td>
<td>3.59</td>
</tr>
<tr>
<td>November</td>
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</tr>
<tr>
<td>December</td>
<td>1.86</td>
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<table>
<thead>
<tr>
<th>Season</th>
<th>ET$_{\text{max}}$ mm/day</th>
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</thead>
<tbody>
<tr>
<td>GS</td>
<td>4.64</td>
</tr>
<tr>
<td>DS</td>
<td>2.03</td>
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</table>
Table 6. Eleuterio basin. Mean values during the growing season of soil moisture $<s>$, seasonal number of stress periods $n_{s^*}$ and its duration $T_{s^*}$, static water stress $<\zeta>$ and static water stress modified $<\zeta^*>$, and dynamic water stress $<\theta>$. Woody vegetation. $s^*$ is the soil moisture relative to the incipient stomatal clousure (for each possible soil type).

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>Scheme A</th>
<th>Scheme B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loamy sand ($s^*=0.31$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$&lt;s&gt;$ (%)</td>
<td>0.22</td>
<td>0.24</td>
</tr>
<tr>
<td>$n_{s^*}$</td>
<td>1.31</td>
<td>1.19</td>
</tr>
<tr>
<td>$T_{s^*}$ [days]</td>
<td>136.2</td>
<td>130.8</td>
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<tr>
<td>$&lt;\zeta&gt;$</td>
<td>0.29</td>
<td>0.35</td>
</tr>
<tr>
<td>$&lt;\zeta^*&gt;$</td>
<td>0.35</td>
<td>0.48</td>
</tr>
<tr>
<td>$&lt;\theta&gt;$</td>
<td>0.37</td>
<td>0.45</td>
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<tr>
<td>Sandy loam ($s^*=0.46$)</td>
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<tr>
<td>$&lt;s&gt;$ (%)</td>
<td>0.31</td>
<td>0.32</td>
</tr>
<tr>
<td>$n_{s^*}$</td>
<td>1.29</td>
<td>1.39</td>
</tr>
<tr>
<td>$T_{s^*}$ [days]</td>
<td>149.6</td>
<td>124.2</td>
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<tr>
<td>$&lt;\zeta&gt;$</td>
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<td>0.34</td>
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<tr>
<td>$&lt;\zeta^*&gt;$</td>
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<td>0.42</td>
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<tr>
<td>$&lt;\theta&gt;$</td>
<td>0.35</td>
<td>0.41</td>
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<tr>
<td>Clay ($s^*=0.78$)</td>
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<tr>
<td>$&lt;s&gt;$ (%)</td>
<td>0.65</td>
<td>0.66</td>
</tr>
<tr>
<td>$n_{s^*}$</td>
<td>1.36</td>
<td>1.3</td>
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<tr>
<td>$T_{s^*}$ [days]</td>
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<td>129.1</td>
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<tr>
<td>$&lt;\zeta&gt;$</td>
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<td>0.31</td>
</tr>
<tr>
<td>$&lt;\zeta^*&gt;$</td>
<td>0.29</td>
<td>0.4</td>
</tr>
<tr>
<td>$&lt;\theta&gt;$</td>
<td>0.32</td>
<td>0.39</td>
</tr>
</tbody>
</table>
Fig. 1. Typical behaviour of the soil water losses, $\chi(s)$, as a function of relative soil moisture (after Laio et al., 2001b).
Fig. 2. Location of the Eleuterio basin (Sicily-Italy). In blu are highlighted the main river channels.
Fig. 3. Spatial pattern of soil texture (top left) and vegetation (top right) for the Eleuterio river basin and theirs spatial overlay (at bottom).
Fig. 4. Eleuterio river basin. (a) Historical rainfall series (from Ficuzza raingauge, 1960-1988). In yellow the precipitation during the dormant season (DS) while in green that during the growing season (GS). (b) Mean monthly precipitations.
Fig. 5. Eleuterio river basin. (a): Annual fluctuation of the mean seasonal (SCHEME A) and monthly (SCHEME B) values of $\alpha$ (cm) and $\lambda$ (1/day). (b) Annual fluctuations of the mean monthly $ET_{max}$ (mm/day) used in SCHEME A and SCHEME B.
Fig. 6. Eleuterio river basin. Time-profiles for SCHEME A (a) and for SCHEME B (b). A generic representative year extracted from the 100 simulated years. Vegetation type: tree; Soil-type: loamy sand (blue), sandy loam (red) and clay (green). On the top the precipitation series, in the middle the soil moisture time-profile and at bottom the static water stress time-profile. For the SCHEME A the grid denotes borderlines among each season, while for the SCHEME B it denotes borderlines among each month.
Fig. 7. Eleuterio river basin. Probability density functions of soil moisture during the growing season relative to the analytical solution (green), and to the numerical solutions for the SCHEME A (red) and for the SCHEME B (blue). Vegetation type: tree; Soil-type: loamy sand (top left), sandy loam (top right) and clay (at bottom). In the box of each plot there are the mean values of soil moisture during the growing season.