Uncertainty in river discharge observations: a quantitative analysis

G. Di Baldassarre¹ and A. Montanari²

¹School of Geographical Sciences, University of Bristol, UK
²Faculty of Engineering, University of Bologna, Bologna, Italy

Received: 31 October 2008 – Accepted: 6 November 2008 – Published: 6 January 2009

Correspondence to: G. Di Baldassarre (g.dibaldassarre@bristol.ac.uk)

Published by Copernicus Publications on behalf of the European Geosciences Union.
Abstract

River discharge data are known to be potentially affected by a significant uncertainty. Nevertheless, the measurement error of river flow observations is often considered negligible with respect to other uncertainties that affect hydrological studies. This paper aims at analysing and quantifying the uncertainty that may be present in river discharge records. A numerical analysis was performed on a 330-km reach of the Po River (Italy). The results show that errors in river flow data are indeed far from negligible.

1 Introduction

In the recent past there has been an increasing interest in assessing uncertainty in hydrology and analysing its possible effects in hydrological modelling. Uncertainty has been recognised to be important in the communication with end users (Beven, 2006; Montanari, 2007) and to play a key role in the context of prediction in ungauged basins (PUB). Accordingly, uncertainty assessment is one of the key tasks of the PUB initiative launched in 2003 by the International Association of Hydrological Sciences (Sivapalan et al., 2003).

Indeed, hydrologists are well aware that a significant approximation affects the output of hydrological models. Uncertainty is caused by many sources of error that propagate through the model therefore affecting its output. Three main sources of uncertainty have been identified by hydrologists, namely: (a) observable uncertainty, that is, the approximation in the observed hydrological variables (typically rainfall, temperature and river flows); (b) parameter uncertainty, that is induced by imperfect calibration of hydrological models; and (c) model structural uncertainty, that is originated by the inability of the hydrological model to perfectly schematise the physical processes involved in the rainfall-runoff transformation. Among these sources of uncertainty, observable uncertainty is often believed to play a marginal role, given that it is often considered negligible with respect to (b) and (c). Accordingly, only a few attempts have been made to quan-
tify the effects of observable uncertainty in hydrological modelling (see, for instance, Clarke (1999)).

Already 20 years ago, Pelletier (1987) reviewed 140 publications dealing with uncertainty in the determination of the river discharge, thereby providing an extensive summary. He referred to the case where river discharge is measured by using the velocity-area method, that is, by applying the relationship

\[ Q'(t) = A(t) \cdot v(t), \]

where \( t \) is the sampling time, \( Q'(t) \) is the river discharge, \( A(t) \) is the cross sectional area of the river and \( v(t) \) is the velocity of the river flow averaged over the cross section. Therefore the error in measuring \( Q'(t) \) is originated by uncertainties in both \( A(t) \) and \( v(t) \), which in turn are originated by uncertainty in the current meter, variability of the river flow velocity over the cross section and uncertainty in the estimation of the cross section geometry. Pelletier (1987) highlighted that the overall uncertainty in a single determination of river discharge, at the 95% confidence level, can vary in the range [8%–20%], mainly depending on the exposure time of the current meter, the number of sampling points where the velocity is measured and the value of \( v(t) \).

Another interesting contribution was provided by the European ISO EN Rule 748 (1997), that quantified the expected errors in the determination of the river discharge with the velocity-area method. The conclusions were similar to the ones of Pelletier (1987).

However, one should consider that in many cases, including the usual practice in many countries of Europe, the river discharge is not currently estimated by using the velocity-area method, being the rating curve method instead used, which is easier to apply in practice (see, for instance, World Meteorological Organisation (1994)). Accordingly to the rating curve method, one measures the river stage that is subsequently converted to river discharge by means of a rating curve. This latter is preliminarily estimated by using observations collected with the velocity-area method (see Sect. 2 for more details). Therefore an additional error is induced by imperfect estimation of the
rating curve. In what follows, the river discharge estimated through the rating curve method at time \( t \) will be denoted with the symbol \( Q(t) \).

The purpose of the present study is assess the global uncertainty affecting \( Q(t) \). This type of uncertainty depends on: i) the methodology used to estimate river discharge (e.g., Aricò et al., 2008); and ii) the test site under study. We base our analysis on the outcomes presented by the European ISO EN Rule 748 (1997) for what concerns the uncertainty of \( Q'(t) \) (velocity-area method) and estimate the additional uncertainty in the determination of the rating curve by referring to the case study of a 330-km long reach of the Po River, in Italy.

2 Uncertainty in river discharge measurements collected with the rating curve method

A full comprehension of the uncertainty that affects the rating curve method for discharge measurement requires a detailed explanation of the procedure itself. First of all it is necessary to estimate the rating curve. To do so, field campaigns are carried out to record contemporaneous measures of river stage \( h(t) \) and river discharge \( Q'(t) \) (with the velocity-area method) in steady flow conditions at time \( t \). Such measures allow one to identify discrete points \( ((Q'(t), h(t)) \) that belong to the steady state rating curve. These points are subsequently interpolated through an analytical relationship which approximates the rating curve itself. In many cases a 3-parameter cubic relationship is used, that is,

\[
Q(t) = c_1 h(t) + c_2 h(t)^2 + c_3 h(t)^3 \tag{2}
\]

where \( c_1 \), \( c_2 \) and \( c_3 \) are calibrated parameters. Once the rating curve is defined, \( Q(t) \) at a arbitrary time \( t \) can be operationally estimated by measuring the river stage \( h(t) \) to be plugged in Eq. (2).

A critical step in the procedure above is the estimation of the rating curve. In particular, reducing the uncertainty in the estimation of \( Q'(t) \) during the field campaigns is
a compelling requirement. The European ISO EN Rule 748 (1997) provides guidelines to this end by establishing an international standard for Europe. The main requirements are the following:

- when the width, $B$, of the cross section exceeds 10 m $v(t)$ should be measured along at least 20 vertical segments laying on the cross section itself;

- the vertical segments above should be placed so that the river discharge in each subsection is less than 5% of the total. In no case it should exceed 10%;

- the number and spacing of the velocity measurements along each vertical should be selected so as the difference in readings between two adjacent points is no more than 20% of the higher value.

Once the velocity readings along each vertical are plotted against depth, the area of the obtained velocity curve gives the discharge per unit width along that vertical. The average of two subsequent area values gives the discharge per unit width in the subsection encompassed by the two verticals. By summing up the discharges in each subsection $Q(t)$ is obtained. When the river discharge is measured through the rating curve method the uncertainty is expected to be much more significant with respect to what was found for the velocity-area method, because of the uncertain definition of the rating curve.

In what follows, the uncertainty induced in the rating curve method by imperfect observation of the river stage is considered to be negligible. Moreover, let us assume that the river cross sections are not changing in time, that is, the rating curve itself is not changing in time with the exception of the seasonal variations (which are considered and analysed here below; see Sect. 2.4). This latter assumption has been introduced because the uncertainty induced by these changes is heavily dependent on the considered case study and no general rule can be suggested. However, one should note that in assuming stationarity in time of the rating curve we are ignoring one of the most relevant sources of uncertainty that usually affects the river discharge...
measurements. Therefore, this analysis is likely to underestimate the uncertainty that actually one would experience in many real world applications.

In view of the assumptions above one can identify the following main sources of uncertainty affecting the estimation of \( Q(t) \):

- inherent uncertainty of the velocity-area method;
- uncertainty induced by interpolation and extrapolation of the rating curve;
- uncertainty induced by the presence of unsteady flow conditions;
- uncertainty induced in the rating curve by seasonal changes of the river roughness.

The contribution of each of the sources of uncertainty above to the formation of the total uncertainty of \( Q(t) \) is qualitatively assessed here below.

2.1 Uncertainty in river discharge measurements collected with the velocity-area method

The uncertainty affecting the \( Q'(t) \) observations obtained with the velocity-area method is mainly due to the following causes (European ISO EN Rule 748, 1997):

- the river flow during the measurement may be unsteady;
- the presence of wind may affect the reliability of the velocity measurement;
- the velocity measurement by the current meter may be not precise even in ideal conditions;
- the measurement of the width, \( B \), of the cross section and water depth, \( h_i \), along each \( i \)-th vertical segment may be affected by errors;
– the estimation of the area of the velocity curve along the vertical segments and the mean discharge per unit width in each cross subsection may be affected by errors due to the spatial variability of the flow velocity.

In order to quantify the total uncertainty affecting \( Q'(t) \) one first needs to quantify the individual uncertainties above. A relevant contribution to this end is delivered by the European ISO EN Rule 748 (1997), where indications are provided on the magnitude of the errors above, at the 95% confidence level. In brief, the following assessment is reported:

– the uncertainty \( X_e \) affecting the measurement of the local flow velocity is about ±6%, when the velocity itself is about 0.5 m/s and the exposure time is 2 min;

– the uncertainty \( X_c \) affecting the rating of the rotating element of the current-meter is about ±1%, when the flow velocity is about 0.5 m/s;

– the uncertainty \( X_B \) affecting the measurement of \( B \) is about ±1%;

– the uncertainty \( X_d \) affecting the measurement of \( h_i \) is about ±1%;

– the uncertainty \( X_p \) in the estimation of the mean velocity along each vertical segment is about ±5% when at least 5 point measurements are collected;

– the uncertainty \( X_A \) in the estimation of the area of each subsection is about ±5% when the number of vertical segments is about 20.

It is important to remark that the uncertainties above refer to standard working conditions. In the operational application the experience of the user may suggest different estimates.

The total uncertainty affecting \( Q'(t) \) can be obtained by integrating the individual sources of uncertainty above. By following the approach proposed by the European ISO EN Rule 748 (1997), let us assume that the current meter is operated in ideal conditions, without any systematic uncertainty and in absence of significant wind and
unsteady flow. Moreover, let us assume that the errors are independent and normally distributed and that the number of vertical segments is at least 20, with an even distribution of discharge along the river cross subsections. Therefore, the global uncertainty affecting \( Q'(t) \), at the 95% confidence level, can be computed as (European ISO EN Rule 748, 1997)

\[
X_Q' = \pm \sqrt{X_e^2 + \frac{1}{m} \left( X_c^2 + X_B^2 + X_d^2 + X_p^2 \right)}
\]

\[
= 5.3\%.
\]

Thus, it can be concluded that any river discharge measurement that is used to calibrate a rating curve is affected by an uncertainty of about 5% at the 95% confidence level.

2.2 Uncertainty induced by interpolating and extrapolating the rating curve

In order to quantify the uncertainty induced by an imperfect rating curve a set of numerical experiments were performed. The numerical analysis made use of the one-dimensional (1-D) model HEC-RAS (Hydrologic Engineering Center, 2001). HEC-RAS solves the 1-D differential equations for unsteady open channel flow (De Saint Venant equations), by using the finite difference method and a four point implicit method (box scheme; Preismann, 1961).

The numerical study focused on a 330 km-reach of the Po River from Isola Sant’Antonio to Pontelagoscuro (see Fig. 1). The Po River is the longest river in Italy (the total length is about 652 km) and it drains a large part of northern Italy, with a contributing area at the closure section of about 70,000 km².

The geometry of the Po River reach from Isola Sant’Antonio to Pontelagoscuro was described by 275 cross sections surveyed in 2005. Figure 2 shows the longitudinal profile of the river bed, along with the profile of the levees between Isola Sant’Antonio (\( X = 233,976 \) m) and Pontelagoscuro (\( X = 564,229 \) m). The main geometric characteristic of the reach are summarised in Table 1.

46
In the year 2000 a major flood event occurred along the Po River, with a peak of about to 10 500 m$^3$/s at Isola Sant’Antonio, 12 000 m$^3$/s at Piacenza and 9800 m$^3$/s at Pontelagoscuro. The observations of river discharge and river stage collected during that flood were used to calibrate the 1-D model. In detail, the Manning roughness coefficient was allowed to vary in the intervals 0.01–0.06 m$^{-1/3}$ s for the main channel and 0.05–0.15 m$^{-1/3}$ s for the floodplain. Several simulations were carried out with boundary conditions given by:

- the upstream hydrograph observed during the October 2000 flood event at Isola S. Antonio (upstream boundary condition, see Fig. 3);
- the observed lateral inflow from the major tributaries;
- the stage hydrograph observed during the flood event occurred in 2000 at Pontelagoscuro (downstream boundary condition).

To check the model reliability, observed and simulated stage hydrographs were compared by referring to two internal cross sections, namely, Casalmaggiore ($X=423 940$ m, see Fig. 2) and Boretto ($X=439 446$ m, see Fig. 2). The best performance was obtained by using Manning’s values equal to 0.03 m$^{-1/3}$ s for the main channel and 0.09 m$^{-1/3}$ s for the floodplain. These values agree with what is recommended by the literature. In fact, Chow et al. (1988) suggest for this type of rivers Manning coefficients ranging in the intervals 0.03–0.04 m$^{-1/3}$ s and 0.08–0.12 m$^{-1/3}$ s for active bed and floodplain, respectively. Figure 4 shows the simulated and observed hydrographs in the internal cross sections. The results point out that the 1-D model provides a satisfactory simulation.

In order to inspect the uncertainty induced by a imperfect estimation of the rating curve the study focused on 20 cross sections placed between Casalmaggiore and Boretto (Fig. 3). For each of them the 1-D model was used for estimating the rating curve in steady flow conditions for discharges ranging from 1000 and 12 000 m$^3$/s.
Then, for each cross section, the rating curve was approximated with the analytical relationship Eq. (2) (see Sect. 2) by interpolating only 6 \((Q'(t), h(t))\) points obtained through the numerical simulation and corresponding to river discharges equally distributed in the range 1000–6000 m\(^3\)/s (see Fig. 5). The rating curve was estimated by using the least squares method, that is, the most used approach in the usual practice (see, for instance, Petersen-Øverleir, 2004). Finally the errors were computed after estimating the river discharge with the calibrated relationship Eq. (2) in the range 1000–6000 m\(^3\)/s at steps of 1000 m\(^3\) (interpolation error) and 6000–12 000 m\(^3\)/s at steps of 1000 m\(^3\) (extrapolation error). We found an average percentage error at the 95% confidence level, \(\chi''_Q\), equal to 2.32% and 17.53% for the interpolation and extrapolation error, respectively.

2.3 Uncertainty induced by the presence of unsteady flow conditions

It is well known that in unsteady flow conditions there is not a one-to-one relationship between the river stage and the river discharge. Actually, during a flood the same river stage corresponds to different river discharges in the two limbs of the hydrograph, where the higher discharge occurs in the raising limb.

In order to assess the magnitude of the error that can be induced by the presence of unsteady flow, for each cross section of the Po River reach described above the 1-D model was used for estimating the \((Q'(t), h(t))\) unsteady flow relation (see Fig. 6) by referring to the flood event occurred in the year 2000. Then, for each cross section, we compared any river discharge simulated by the model for the flood occurred in 2000 (at hourly time step) with its counterpart estimated by using the steady state rating curve. An average percentage error, \(\chi'''_Q\), equal to 15.12%, at the 95% confidence level, was found.
2.4 Uncertainty induced in the rating curve by seasonal changes of the river roughness

It is well known that the river roughness changes with season, depending on the state of the vegetation in floodplains. This causes significant changes in the rating curve that may affect the river discharge estimation (Franchini et al., 1999).

Along the course of the Po River floodplains are largely abandoned or covered by broad leaved woods. Figure 7 shows two rating curves for one cross section along the Po River calculated by the 1-D model. They refer to values of the Manning coefficient in the floodplains equal to 0.09 m$^{-1/3}$ s and 0.12 m$^{-1/3}$ s. The former is the calibrated value which refers to October, when the 2000 flood occurred. The latter is a value that might be representative of Spring conditions, according to Chow et al. (1988). The average difference between the two rating curves, $X''''$, at the 95% confidence level, amounts to 7.16%.

2.5 Computation of the total uncertainty and discussion

Let us assume that the errors estimated in the Sects. 2.1, 2.2, 2.3 and 2.4 are independent and Gaussian. Also, let us focus on the case in which the rating curve is extrapolated beyond the range of the data that were used for its calibration. Therefore the total uncertainty, in standard conditions and at the 95% confidence level, affecting river discharge measurements obtained with the rating curve method can be computed through the relationship

$$X_Q = \pm \sqrt{X'^2 + X''^2 + X'''^2 + X''''^2} = 24.80\%.$$  \hspace{1cm} (4)

In other words, one may expect an error of about 25%, at the 95% confidence levels, when measuring river discharges along the Po River by using the rating curve method in standard conditions.

It is worth recalling that the analysis considered a river discharge varying from 6000 to 12 000 m$^3$/s, which correspond to a return period approximatively varying from 5 to
100 years. The estimated total error is averaged over the above range of discharges. This means that the error is expected to be less and more significant for the lower and upper river discharges in the above range, respectively. Also, it was assumed that the rating curve is calibrated by using observed river discharge data never greater than 6000 m$^3$/s. Moreover, it is worth remarking once again that systematic errors were excluded as well as errors induced by changes in the rating curve due to variations of the river bed geometry.

The dependence of the errors estimated above, averaged over the considered reach of the Po River, on the river discharge is shown in Fig. 8.

3 Conclusions

The uncertainty affecting hydrological observations is known to be potentially significant, although in many cases its effects on hydrological modelling are neglected. The main reason for the latter assumption is that modellers are often not able to quantitatively assess the reliability of rainfall or river discharge measurements. This study is a first attempt to quantify the uncertainty that one may expect when measuring the river discharge by applying the rating curve method. This type of uncertainty strictly depends on the case study under consideration. The present analysis referred to the case of a 330-km reach of the Po River, that is the longest river in Italy and is representative of the conditions of many long rivers in Europe. Under simplifying assumptions we found that the error affecting river discharge measurement, when extrapolating the rating curve, is about 25% at the 95% confidence level. This value is averaged over river discharges ranging between 6000 m$^3$/s and 12 000 m$^3$/s. The aforementioned simplifying assumptions brought us to underestimate the total uncertainty because we are forced to neglect some sources of uncertainty which one cannot quantify. As a matter of fact, the above estimated error is significant and can heavily influence the output of hydrological studies.
Acknowledgements. The study has been partially supported by the Italian Government through its national grants to the programmes on “Advanced techniques for estimating the magnitude and forecasting extreme hydrological events, with uncertainty analysis”. The authors are extremely grateful to E. Todini and G. Schumann for providing valuable suggestions and to the Interregional Agency for the Po River (Agenzia Interregionale per il Fiume Po, AIPO, Italy) and Po River Basin Authority (Autorità di Bacino del Fiume Po, Italy) allowing access to their topographic and hydrological data.

References


Clarke, R. T.: Uncertainty in the estimation of mean annual flood due to rating curve indefiniti...
Table 1. Geometric characteristic of the Po River reach from Isola S. Antonio to Pontelagoscuro.

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Main channel width [m]</td>
<td>200–500</td>
</tr>
<tr>
<td>Main channel depth [m]</td>
<td>10–15</td>
</tr>
<tr>
<td>Floodplain width [m]</td>
<td>1000–3000</td>
</tr>
<tr>
<td>Average bed slope [–]</td>
<td>0.02</td>
</tr>
</tbody>
</table>
Fig. 1. The Po River basin and the river reach considered in the study (from Isola Sant’Antonio to Pontelagoasco). Altimetry of the basin (from white, 0 m a.s.l., to brown, 4810 m a.s.l.).
Fig. 2. Longitudinal profile of the Po River reach from Isola Sant’Antonio to Pontelagoscuro.
Fig. 3. October 2000 flood event: discharge hydrograph at Isola Sant'Antonio.
Fig. 4. Calibration of the 1-D hydraulic model along the Po River. Observed and simulated stage hydrographs in Casalmaggiore (upper panel) and Boretto (lower panel).
**Fig. 5.** True and interpolated rating curve for one cross section of the Po River.
Fig. 6. Steady and unsteady flow rating curve for one cross section of the Po River.
Fig. 7. Steady state rating curves for one cross section of the Po River for different values of the Manning Coefficient (0.09 m\(^{-1/3}\) s for the Autumn curve and 0.12 m\(^{-1/3}\) s for the Spring curve).
Fig. 8. Absolute percentage errors (at the 95% confidence level) versus river discharge averaged over the considered reach of the Po River. Interpolation error (green), extrapolation error (red), error due to roughness changes (blue) and error due to unsteady flow (black).