Comment on “A dynamic rating curve approach to indirect discharge measurement” by Dottori et al. (2009)

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1 Introduction

The estimation of transient streamflow from stage measurements is indeed important and the study of Dottori, Martina & Todini (2009) (henceforth DMT) is useful, however, DMT seem to miss its practical aspects. The goal is to infer the discharge from measurements of the stage conveniently and with accuracy adequate for practical work. Measuring at two cross-sections is not convenient; also, the two gauges would have to be so positioned that the recorded stages give a good representation of the slope of the wave profile. This is not a trivial requirement, because depth is controlled by the local stream geometry, in contrast to the flow rate that varies in space more gradually. That gauging stations will be at hand where needed is all the more doubtful, if not unlikely, as monitoring networks are shrinking worldwide and are increasingly difficult to maintain. The Jones formula was modified by Thomas (Henderson, 1966) to include the temporal derivative of the depth, instead of the spatial one, to specifically allow discharge estimation from at-a-section stage observations. In view of this approximation, the outcome of the comparison is not surprising. However, this discussion intends to show that, properly evaluated, the praxis-oriented Jones formula performs better than DMT imply. It will be also documented that the DMT methodology restates a known method for computing flood depth profiles.

2 The implications of the Jones Formula for flood routing and discharge estimation

First, there seems to be an oversight in the sign of Eq. (4) of DMT for the celerity \( c \) of the kinematic wave (KW), which should be positive: \( c = \frac{dQ}{dA}|_{x=\text{const.}} = B^{-1} \frac{dQ}{dy}|_{x=\text{const.}} \). \( Q \) is the flow rate passing through the cross-section of a channel of area \( A \), with top width \( B \) at depth \( y \), at location \( x \) along the stream axis. DTM imply that the Jones formula is evaluated explicitly. This is feasible, if the KW celerity is computed based on the rating relationship for uniform flow, \( c = \frac{dQ_o}{dA}|_{x=\text{const.}} \), and such results are good as
long as the flow conditions are quasi-kinematic. But when the flow departs markedly
from the KW status, the Jones formula should be evaluated with $c(Q)$ computed on
the looped rating curve. An indication of the correctness of computing the KW celerity
on the loop-shaped rating curve is that $c(Q_{\text{max}}) = 0$: the maximum discharge does not
propagate downstream! However, care must be exercised in the iterative calculation
of $c$, to ensure convergence (Koussis, 1975; Weinmann, 1977; Weinmann and Lau-
renson, 1979; Ferrick et al., 1984; Perkins and Koussis, 1996). We demonstrate this
point below with an example of Weinmann (1977), also reported by Weinmann and
Laurenson (1979).

Weinmann considered a rapidly rising flood wave (rate of rise of the inflow wave
$\sim 1 \text{ m/h}$) in a trapezoidal prismatic channel, with a fairly mild slope $S_o = 2 \times 10^{-4}$ and Man-
ning’s $n = 0.04$; the cross-section has a base width $b = 50 \text{ m}$ and side slopes $1 \text{ V:1.5 H}$. The slope ratio (Koussis and Chang, 1984) $\text{SR} = |\partial y/\partial x/S_o| \approx |\partial y/\partial t/cS_o|$ is $\sim 1$, well
over the limit of 0.5 suggested in DMT. The inflow hydrograph was of the form of
Eq. (36) of DMT, with $Q_b = 100 \text{ m}^3/\text{s}$, $Q_p = 1000 \text{ m}^3/\text{s}$, $t_p = 10 \text{ h}$ and $\beta = 6.67$, and was
routed for $40 \text{ km}$ using the St. Venant equations and a diffusive-wave equivalent model
developed by Koussis (1975). The latter is a nonlinear KW model corrected for wave
diffusion (by matching the routing scheme’s numerical to the physical diffusion coeffi-
cient); however, in contrast to Muskingum-Cunge and storage-type models, (i) stage-
discharge conversions are based on the flow rating formula of Jones and (ii) the KW
celerity $c(Q)$ is computed via the Jones formula (steady flow rating curves may be used
if the flow is quasi-kinematic). The Jones formula is readily modified to a form that is
appropriate when a discharge hydrograph is given, by replacing the term $\partial y/\partial t/cS_o$ by
$B\partial Q/\partial t/c^2S_o$. The modified Jones rating relationship is useful when, e.g., the chan-
nel inflow is a basin’s outflow computed by a watershed model, or a routed flow hydro-
graph. Figure 1 (top) shows the flood routing results and (bottom) the routed flow hydro-
graph. Figure 1 (top) shows the flood routing results and (bottom) the routed flow hydro-
graph. Figure 1 (top) shows the flood routing results and (bottom) the routed flow hydro-
graph.
Koussis (1975) verified the good accuracy of the KCD model (more appropriately termed, Kinematic wave Corrected for Diffusive effects), which incorporates the Jones formula, by comparing it to a complete routing solution (idealised model of the lower Mississippi) with SR ≈ 0.25; Bowen et al. (1989) showed the KCD model to be a useful tool for the design of and the simulation of flows in storm drain networks. But the KCD model’s performance is remarkable in the case of Weinmann’s wave because the slope ratio is ~1. The wide loop of the rating curve reflects the strongly transient character of the flow even after 40 km (~2 m-wide at 500 m³/s); Weinmann (1977) showed that also in this case the acceleration terms be ignored (his CI = Complete Implicit and ACI = Approximately Complete Implicit models give indistinguishable results). In contrast, when c is computed from steady/uniform-flow rating curves, as DMT apparently did in their tests, the routed outflow hydrograph differs substantially from the reference solution of the St. Venant equations. The Jones formula would perform very well in the tests of DMT, if c(Q) = dQ/dA|\text{\textit{x}}=\text{\textit{const}} were computed from it iteratively. This assertion rests on the fact that the DMT transients are milder than Weinmann’s wave (gauged by the SR values). Therefore, the application range of the Jones formula should be greater than the DMT tests suggest.

Figure 2 shows the discharge variation of the wave celerity, computed from the rating curve for transient flow by the KW formula c(Q) = dQ/dA|\text{\textit{x}}=\text{\textit{const}} = B^{-1} dQ/dy|\text{\textit{x}}=\text{\textit{const}}. The graph shows that c(Q) has two branches, one for the rising flood and one for the flood recession, so c takes on two different values for the same Q, a higher one on the rising-flood limb and a lower one on the falling-flood limb. It is also worthy of note that: (i) the wave celerity vanishes at the discharge maximum, c(Q_{\text{max}}) = 0, indicating that the peak of wave does not propagate, (ii) that c→\infty at the depth maximum, y_{\text{max}}, indicating quasi-steady flow (\partial y/\partial t = 0), and (iii) the Jones formula ignores the c-region between y_{\text{max}} and Q_{\text{max}}. It is thus clear that only for mild transients – in the sense of the slope ratio SR – is the error of computing c(Q) from the steady-flow (kinematic) rating curve small.
In computing $c(Q)$ by the Jones formula, it should be realised that the rising and falling limbs of $Q(y)$ intersect at $(Q_{\text{max}}, y_{\text{max}})$ and that $c = B^{-1} \left. \frac{dQ}{dy} \right|_{x=\text{const}}$ is numerically discontinuous there. For this reason the iteration of the looped rating curve near $(Q_{\text{max}}, y_{\text{max}})$ must be executed with care. At $(Q_{\text{max}}, y_{\text{max}})$, the KW theory gives the finite celerity $c(Q_{\text{max}}) = B^{-1} \left. \frac{d(Q_{\text{steady}}/dy)}{dy} \right|_{\text{max}}$, indicating that the maximum flow propagates downstream. In a proper routing procedure, however, the presence of wave diffusion guarantees peak attenuation. This wave diffusion originates in the free-surface slope $\partial y/\partial x$ and appears either directly, as in the actual diffusion wave model, or indirectly, as in flood routing schemes of the matched artificial diffusion type or diffusion-wave equivalent type, such as, e.g., the Muskingum-Cunge routing scheme (Koussis, 2009). Given that the routing scheme ensures wave attenuation, it is argued here that attempting to correct the Jones formula at the wave crest, by introducing higher-order derivatives (e.g., formulae of Fenton and Perumal 2) while incurring numerical oscillations, does not seem advisable, especially when considering the morphologic variability of natural streams.

It appears reasonable that simply structured models, such as the Jones formula, should be more adept in handling the complications of real streams, which often test the limits of one-dimensional hydraulics, especially flood plains yielding flow rating curves with distinct branches for in-bank and out-of-bank flows (Price, 1973; Natural Environment Research Council, 1975; Wang and Laurenson, 1983). I contend that no procedure based on hydraulic equations, no matter how mathematically elaborate, can eliminate a judgement-based “required extrapolation of the rating curve beyond the range of actual measurements used for its derivation”, as DMT state in their Abstract; and DMT’s claim (Conclusions) for the DyRaC approach that “its calibration procedure only requires the evaluation of roughness coefficient, thus eliminating the extrapolation errors” seems overly optimistic.
The calculation of flood level profiles by a standard-step method for steady flow and its relation to the DTM method

A recent review of storage routing methods (Koussis, 2009) lists among the methods for the computation of depth profiles, after the flow routing step, the option “to calculate, over a $\Delta t$, quasi-steady flood level profiles by standard-step methods (Henderson, 1966), with the mean of in- and outflow over $\Delta t$ as discharge, starting at a section with known conditions (BGS, 2000)”. It thus follows that the idea of using stage observations at two cross-sections to estimate flood flows is a reversing of the BGS procedure. This standard procedure of BGS (Darmstadt, Germany) was adopted in the modelling of flood flows in the River/Canal Kiphissos, in Athens, Attica Region, Greece (Koussis et al., 2003; Mazi and Koussis, 2006).

References


Fig. 1. Comparison of routing solutions, of the St. Venant equations and of two diffusive-wave equivalent models, for a rapidly rising wave through a prismatic channel of trapezoidal cross-section: (top) in- and outflow ($x=40$ km) hydrographs; (bottom) outflow rating curves. KCD = Kinematic wave Corrected for Dynamic effects: $y–Q$ conversions and $c$ computed with the Jones formula; GK = Generalised Kinematic: diffusive-wave equivalent, but $y–Q$ conversions and $c$ computed with the steady-flow rating curve (from Weinmann, 1977).
Fig. 2. Wave celerity from looped rating curve (Weinmann, 1977, adapted from Koussis, 1975).