Flexural behaviour of selected plants under static load

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Abstract

One of the principal purposes of soil bioengineering is the application of vegetation layers from a civil engineering point of view. Living plants are used to reinforce slopes and to control erosion. For a standardised implementation, it is essential to quantify the effectiveness and to assess technical parameters for such bioengineering systems. The objective of this study is to investigate the flexibility of stems and branches of different riparian species of the area of Southern Brazil suitable for soil bioengineering (*Phyllanthus sellowianus* Müll. Arg., *Sebastiania schottiana* (Müll. Arg.) Müll. Arg., *Salix humboldtiana* Willd., and *Salix × rubens* Schrank). Fifty specimens (green stem samples) were collected in the surroundings of Santa Maria, state of Rio Grande do Sul, Brazil, and subjected to static bending tests. Their overall deformation behaviour (elastic and plastic) is of crucial importance for bioengineering systems. Thus, additional to the state of the art of material parameters, a new parameter is introduced: the “angle of flexibility”. This parameter describes the elastic and plastic deformation behaviour of a plant under load in a more engineering practival experience. The results show that the species of *Phyllanthus sellowianus* is the most flexible species, followed by *Sebastiania schottiana*, *Salix humboldtiana* and *Salix × rubens*.

1 Introduction

In the centre of the federal state of Rio Grande do Sul (Brazil) many farmers have problems from the erosion of river banks which in turn causes erosion of agriculturally useful land. Consequently farmers are interested in sustainably stabilised river banks to protect their farmland which provides a more stable economic foundation. Stabilised riverbanks also guarantee that artificially constructed irrigation channels work properly. These channels can be managed efficiently when riparian areas show more or less stable morphological conditions. Additional public awareness has been gained for erosion control for riverbanks and agricultural land when natural hazards cause damage.
to infrastructure facilities. The techniques of soil bioengineering may provide suitable
instruments to counter these problems (Schiechtl et al., 1994).

Drawing on the results from a previous study which focused on the vegetative re-
production potential of suitable plants for soil bioengineering (Sutili et al., 2007), the
next step involved quantifying the biomechanical behaviour of plants under specific
load. The quantification of these processes is a crucial point establishing standards
and function related dimensioning of soil bioengineering techniques from an engineer-
point of view. The application of plants in river engineering project is based on a
proven knowledge about the interaction of plants and flow.

The hydraulic interaction of flow with plants depends not only on geometrical prop-
erties (e.g. stem and branch diameter, length and leaf density), but also on the dynamic
response of plants under flood conditions (stem/branch/leaf bending and reduction of
plant height; e.g. Fathi-Moghadam and Kouwen, 1997; Oplatka, 1998; Gerstgraser,
2000; Meixner et al., 2004; Rauch, 2005, Rhigetti and Armanini, 2002; Musley and
Cruise, 2006; McBride et al., 2007). Mechanical properties such as flexural stiffness,
modulus of elasticity and plastic deformation are indicators to assess the impact of
plants on hydraulic conditions. The modulus of elasticity, the proportional limit, as
well as the deformation and stress up to the point of rupture are shown in a typical
stress×deformation diagram (Fig. 1). The first part of the diagram is a straight line that
determines the modulus of elasticity and the limit of elastic behaviour. From this point
the deformation becomes plastic and continues up to the point of rupture (B in Fig. 1)
in a non-proportional way.

The objective of this study is to investigate the flexibility of stems and branches of
different riparian species suitable for soil bioengineering (Phyllanthus sellowianus Müll.
Arg., Sebastania schottiana Müll. Arg., Salix humboldtiana Willd., and Salix×rubens
Schrank) in the specific project area of Southern Brazil.
2 Plant material and methods

Although riparian vegetation is exposed to dynamic stress during flood events, static bending tests are useful to identify the bending behaviour of different plant species. The results have to be considered as a comparison between different species.

The specimens were collected along rivers in the surroundings of the municipality of Santa Maria. *Phyllantus sellowianus* is part of the spurge family (Euphorbiaceae) and grows up to a height of 2–3 m. It is a widely ramified bush with slender and bendable branches. *Sebastiana schottiana* is also a shrub with a maximum height of 3–4 m. It is provided with strong branches which are highly bendable. *Salix × rubens* originally comes from Europe and is a hybrid between *Salix alba* and *Salix fragilis*. The plant grows very fast and reaches heights up to 16 m. Finally, *Salix humboldtiana* is a tree which reaches a height up to 20 m and a stem diameter of about 90 cm. All of the tested species are known to be well adapted to the riverine environment conditions along rivers.

Fifty static bending tests of each species were carried out, using samples of different diameters. The minimum diameter was defined at 10 mm and the bending device limited the maximum diameter. For *Phyllantus sellowianus* and *Sebastiana schottiana*, no samples that exceeded a diameter of 50 mm for the former and 60 mm for the latter were found in the area under study. The specimens were tested with their bark immediately after harvesting. The setup of the bending tests is based on the DIN standard (DIN 52186) for 3 point loading tests. The specimens must have a minimum length of 14 times the diameter at point loading. The bending device of the laboratory is designed for samples with a maximum length of one meter, which means the maximum diameter is limited to 70 mm. The lengths of the samples were adapted according to the length-diameter ratio proposed in the DIN 52186.

The testing equipment automatically recorded the parameters load $F$ [N], displacement $f$ [mm] and time $t$ [s]. Based on the collected data sets ($F$ [N], $f$ [mm] and $t$ [s]) as well as on the measured diameters $d$ [mm] of the specimens at the point of load and on the distance between the points of support $l$ [mm], it was possible to determine
the following parameters:

- proportional limit load, $F_{\text{elast}}$ [N]
- breaking load (maximum), $F_B$ [N]
- modulus of elasticity (MOE), $E$ [N/mm$^2$]
- proportional limit stress, $\sigma_{\text{elast}}$ [N/mm$^2$]
- breaking stress (modulus of rupture, MOR), $\sigma_B$ [N/mm$^2$]
- elastic deformation, $\epsilon_{\text{elast}}$ [-]
- plastic deformation, $\epsilon_{\text{plast}}$ [-]
- breaking deformation (maximum), $\epsilon_B$ [-]
- inertial moment, $I$ [mm$^4$]
- maximum moment $M$ [N/mm]
- maximum resistance $W$ [mm$^3$]

After each bending test, a 100 mm long sample was taken to determine the following parameters:

- moisture content in wood, $u$ [%]
- basic apparent specific weight of the wood, $\rho$ [g/cm$^3$]
- thickness of bark, $t_c$ [mm]
- percentage of bark, $%c$ [%]
- age of specimen, $Y$ [years]
2.1 Calculating the parameters

The breaking load \( F_B \) and proportional limit load \( F_{\text{elast}} \) were directly taken from the load\times displacement diagram. The modulus of elasticity \( E \) (also called Young’s modulus) is used to characterise the elastic behaviour of the stems and branches. This mechanical parameter expresses the ratio between the stress and the deformation under load, i.e., how much force is required for a given unit of reversible deformation. Accordingly, the modulus of elasticity can be taken as an indicator for rigidity rather than flexibility. The modulus of elasticity \( E \) [N/mm\(^2\)] for specimens with a circular cross-section, supported on two points and exposed to a load at a central point, is calculated as:

\[
E = \frac{F_{\text{elast}} \cdot l^3}{48 \cdot F_{\text{elast}} \cdot l},
\]

where:
- \( F_{\text{elast}} \) proportional limit load [N],
- \( l \) distance between the support points [mm],
- \( f_{\text{elast}} \) displacement up to proportional limit [mm],
- \( I \) inertial moment [mm\(^4\)].

The inertial moment (\( I \)) [mm\(^4\)] for a circular section was calculated as:

\[
I = \frac{\pi \cdot d^4}{64},
\]

where \( d \) [mm] is the diameter of the specimen measured at the point of load application.

Replacing \( l \) in the formula above, the equation changes to:

\[
E = \frac{F_{\text{elast}} \cdot l^3}{3 \cdot f_{\text{elast}} \cdot \frac{\pi \cdot d^4}{4}}.
\]
The deformation $\epsilon$ is a dimensionless variable that can be calculated at the proportional limit $\epsilon_{\text{elast}}$ and at the rupture limit $\epsilon_B$, thus also including the range of plastic deformation:

$$\epsilon = f \cdot \frac{12 \cdot d}{l^2},$$

where:
- $f$ displacement (at the elastic or at the rupture limit) [mm],
- $d$ diameter of the specimen at the point of load application [mm],
- $l$ distance between the support points [mm].

The formulae used for the analysis of $E$ and $\epsilon$ are based on the assumption that the segment has a constant diameter and shear stress is neglected. Tree stems are generally tapered, in our case the diameter of the specimens were approximately constant, therefore the tapering effect is negligible. For wood shear stress is conventionally neglected if $l/d > 14$. Usually bending resistance of wood is determined with a 4-point bending test. The results of the used 3-point test are influenced by shear strains which are neglected. However, the used formulae are to be considered as an approximation but results are highly appropriate to compare the species between each other.

The breaking stress $\sigma_B$ [N/mm$^2$] (modulus of rupture, MOR) for specimens with a circular cross-section was obtained by:

$$\sigma_B = \frac{M}{W} = \frac{F_B \cdot l / 2}{\frac{\pi \cdot d^3}{32}} = \frac{16 \cdot F_B \cdot l}{\pi \cdot d^3},$$

where:
- $M$ maximum moment [Nmm],
- $W$ maximum resistance [mm$^3$],
- $F_B$ breaking load [N],
- $l$ distance between the points of support [mm].
d diameter of the specimen at the point of load application [mm].

The stress up to the proportional limit $\sigma_{\text{elast}}$ [N/mm$^2$] can be obtained by the previous formula, replacing the breaking load $F_B$ [N] by the load up to the proportional limit $F_{\text{elast}}$ [N] or by "Hooke's Law," which determines:

$$\sigma_{\text{elast}} = E \cdot \epsilon_{\text{elast}},$$

where:

$E$ modulus of elasticity [N/mm$^2$],

$\epsilon_{\text{elast}}$ deformation up to proportional limit [-].

The moisture content $u$ [%] of the wood was calculated as:

$$u = \frac{m_u - m_o}{m_o} \cdot 100$$

where:

$m_u$ wet mass of the specimen [g],

$m_o$ dry mass of the specimen [g]. and the basic apparent specific weight of the wood $\rho$ [g/cm$^3$] as:

$$\rho = \frac{m_o}{V}$$

where:

$m_o$ dry mass of the wood [g],

$V$ volume of wood [cm$^3$].

The thickness of the bark was measured using a digital calliper, and the percentage of the bark $\%c$ [%] was determined as the ratio between the area of the bark ring and the total cross-section of the specimen:

$$\%c = \frac{A_t - A_o}{A_t} \cdot 100,$$
where:
$A_t$ total area (wood+bark) [cm$^2$],
$A_o$ area without bark [cm$^2$].

After preparation and staining of histological sections a microscope was used to determine the age of the samples.

A new value, the angle of flexibility, was introduced. It describes the whole deformation process of plants under load:

- angle of flexibility at proportional limit, $\alpha_{\text{elast}}$ [$^\circ$],
- angle of flexibility at the point of rupture, $\alpha_B$ [$^\circ$].

At the original non-inflected position (dashed horizontal line in Fig. 2), the bending point divides the specimen into two equal parts with an angle of zero to the horizontal and $180^\circ$ between the two. With the values of $l$ and $f$ at the proportional limit and at the breaking limit, it is possible to calculate the angle of flexibility ($\alpha$), respectively $\alpha_{\text{elast}}$ and $\alpha_B$ (Fig. 2).

$$\alpha = 2 \cdot \tan^{-1} \left( \frac{f}{l} \right)$$

where:
$l$ distance between the points of support [mm],
$f$ displacement (at the proportional limit or at the point of rupture, respectively) [mm].

3 Results and discussion

3.1 Basic apparent specific weight, moisture content and bark

The basic apparent specific weight and the moisture content of the wood define the basic material conditions at testing time. *Sebastiania schottiana*, *Salix humboldtiana*...
and *Salix × rubens* show some proportional relationship (tendency) between increasing diameter and the basic apparent specific weight and a slight inverse relationship to the moisture content of the wood. No such relationship tendency can be found for *Phyllanthus sellowianus*. The values of basic apparent specific weight of the wood of *Phyllanthus sellowianus* were relatively high compared to the other three species, probably because of its slower growth and the higher average age of the specimens of this species.

As expected, the thickness of the bark strongly correlates with the diameters and maintains a relatively constant percentage up to the highest diameters of this study. The two species that have a lower average bark thickness – *Phyllanthus sellowianus* and *Sebastiania schottiana* – consequently show a more discreet increase in the bark thickness with the increase in diameter. *Salix humboldtiana* represents the thickest bark from 5 mm up to 7 mm for stem diameters between 60 and 70 mm. Brüchert et al. (2003) finds that the higher initial bark/wood ratio and its decline with the development of the branch may collaborate toward the initial variations regarding the modulus of elasticity (*E*). For bending tests all samples were tested with the bark on.

### 3.2 Modulus of elasticity

Figure 3 indicates the force (*F*) needed to reach the proportional limit (solid line) or to reach the point of rupture (dashed line), for different diameters.

For the same diameter, *Phyllanthus sellowianus* is the most resistant species and *Salix humboldtiana* the most fragile. This means that for *Phyllanthus sellowianus* a higher amount of load is necessary to reach the point of elastic limit and the point of rupture, respectively.

Table 1 shows the results of calculating the modulus of elasticity for the different species arranged by diameter classes. The coefficient of variation [%] is shown in brackets alongside the average. The last column contains the coefficient of determination between the modulus of elasticity and the stem and branch diameter.

The calculated moduli of elasticity are comparatively lower than those determined by
Vollinger et al. (2000) for green stems and branches of five European species (*Alnus glutinosa* (L.) Gaertn., *Fraxinus excelsior* L., *Salix alba* L., *Salix caprea* L. and *Acer pseudoplatanus* L.). The authors obtained values between 6.900 and 10.200 N/mm$^2$ for the different species (with diameters of 40 to 100 mm). Based on parameter ($E$), the tested southern Brazilian species are visibly less rigid.

The modulus of elasticity (Table 1) declines with the increase in diameter. However, this suggestion of an inverse correlation between the modulus of elasticity and the stem diameter must be considered with caution due to the high coefficients of variation $CV$ and the low coefficients of determination. Vollinger et al. (2000) found similar results. Brüchert et al. (2003) conducted tests on *Alnus glutinosa* (L.) Gaertn. and *Alnus viridis* (Chaix) DC. of an age from 1 to 24 years and found a slight increase in the modulus of elasticity up to the fifth year. In older samples the modulus of elasticity remained relatively constant. Niklas (1992) notes that young cell walls are ductile, while older cell walls tend to be much more elastic and resilient.

### 3.3 Proportional and rupture limit

Deformation is irreversible for inert materials once the proportional limit has been passed. For plants this limit becomes less important, because they remain alive and have the ability to regenerate, even when the proportional limit is greatly exceeded. This phenomenon can be observed in riparian vegetation right after flooding. When plants have not returned to their original position after the water level has dropped, this means that their proportional limit has been exceeded. Yet, the plants still have the biological capacity to recover gradually and adapt their habit to the new local environmental conditions. Even after exceeding the rupture limit, plants with vegetative reproduction potential are able to regenerate.

Thus the limit of rupture is an important parameter, apart from the modulus of elasticity, to describe the flexibility of riparian vegetation stems and branches (Fig. 4). The dashed lines show the maximum limit of deformation and stress that the specimen can resist directly before the point of rupture. Compared to the elastic range,
the deformation behaviour at the rupture point between the different species is subject to greater variation. The two Euphorbiaceae (*Sebastiania schottiana* and especially *Phyllanthus sellowianus*) are capable of resisting large deformation and larger stresses prior to the point of rupture. *Salix humboldtiana* supports slightly less stress than *Salix×rubens*. *Salix humboldtiana*, however, shows a higher deformation in comparison to *Salix×rubens*.

### 3.4 Angle of flexibility

The dimensionless deformation $\epsilon$ can be understood as an expression of how the material behaves under load. The angle of flexibility up to the proportional limit ($\alpha_{\text{elast}}$) or up to point of rupture ($\alpha_B$) is another way to demonstrate this deformation.

The angle of flexibility at the proportional limit and the diameter do not correlate significantly. The $\alpha_{\text{elast}}$ area in Fig. 5 limits the range where potentially any species at any diameter reaches the proportional limit angle. Figure 5 also shows the relationship between $\alpha_B$ and the stem and branch diameter. A parameter that can be easily used in practice, the angle of flexibility at the point of rupture textit $\alpha_B$ represents the maximum angle to which a stem or branch of a particular species and diameter can be bent at the point of rupture. The results have to be considered as a base to compare the bending ability of the tested species.

The diagram shows that *Phyllanthus sellowianus* forms a larger angle of flexibility at the point of rupture than the other species. For example, while a 20 mm branch of *Phyllanthus sellowianus* can form a $45^\circ$ angle of flexibility before breaking, a branch of *Salix×rubens* of the same diameter only reaches $24^\circ$ before breaking.

Figure 6 summarises the angle of flexibility at the rupture point ($\alpha_B$) with the breaking load ($F_B$) required for the four species across the distribution of the stem and branch diameters. The nomogram shown in Fig. 6 cannot be used to reproduce the values at the proportional limit due to the lack of a relationship between the angle of flexibility and the diameter at this limit (Fig. 5).

For all species, the load required to reach the point of rupture increases with its
diameter, while its angle of flexibility at rupture point decreases. Thus, a branch of *Phyllanthus sellowianus* measuring 20 mm in diameter must be stressed up to a load of 670 N to hit the point of rupture and forms a 45° angle of flexibility. A branch of *Sebastiania schottiana* of the same diameter, with a load of 550 N, breaks at 36°. *Salix humboldtiana* forms a smaller angle of flexibility (32°) and requires a load of 400 N to reach the breaking point. For the same diameter, *Salix × rubens* – despite having an angle of flexibility at rupture point that is even lower than the previous species – needs a higher load (460 N) than *Salix humboldtiana* to reach a maximum 25° angle of flexibility at rupture point.

As demonstrated in Fig. 6, the flexibility gradually declines with the increase in diameter and age. *Phyllanthus sellowianus* is the species that supports the greatest stress and can be bent to higher angles of flexibility at the breaking point. Moreover, its growth rate is the lowest of all species investigated. The following diagram (Fig. 7) shows the relationship between the age of the stems and branches and their diameter.

The relationships shown in Fig. 7 can not be generalised due to different local environmental conditions, but can be taken as reference values for the growth rate. In a following step the growth rate (diameter and age) is related to the angle of flexibility at rupture point ($\alpha_B$) and shown in the nomogram in Fig. 8.

The relationship between age and stem diameter can be seen directly on the x- and y-axes, respectively. The angle of flexibility at the point of rupture is characterised by means of the marked areas. For example, in order to know the diameter of each species in the fourth year of age, trace a straight line parallel to the y-axis at the age of 4 years. At the point where this line crosses the straight line that defines the relations for *Phyllanthus sellowianus*, you will get a diameter of 16 mm and an angle of flexibility at the point of rupture of 50°. At the same age, *Sebastiania schottiana* has a diameter of 23 mm and a 33° angle of flexibility at the point of rupture (interpolated between the lines of 30° and 35°). *Salix × rubens* at 4 years shows a diameter of 32 mm and bends down up to 21°, while *Salix humboldtiana*, reaching a larger diameter (39 mm), maintains a 23° or 24° angle of flexibility at the point of rupture. The same procedure can
be applied starting from any diameter \((d)\), age \((Y)\) or angle of flexibility \((\alpha_B)\), for each species. For the species investigated, Fig. 9 provides useful information for planning and managing soil bioengineering work in this region.

4 Conclusions and final remarks

The modulus of elasticity \((E, \text{ Young's modulus})\) does not satisfactorily explain the flexibility of live branches for the assessment of soil bioengineering structures. The angle of flexibility up to the point of rupture \((\alpha)\) and the applied load proved to be more appropriate to characterise the stem flexibility of different species.

Using this parameter and the maximum breaking load \((F_B)\), the bending properties of the plants can be determined. Stem flexibility is a better criterion to identify a plant’s suitability to stabilise fluvial slopes. All the results must take into account the age of the plants. Based on the parameters \(\alpha_B, F_B\) and \(Y\), it can be affirmed that plants of a smaller (younger) diameter are more flexible, regardless of species. The results show that the flexibility of the stems and branches diminishes over time but not equally or proportionally for each of the species studied.

Considering that large plants can cause instability on fluvial slopes (overload, lever effect), there are sufficient arguments to justify interventions such as pruning or even coppicing individuals when flexibility is lost.

It was proven that *Phyllanthus sellowianus* and *Sebastiania schottiana* are very appropriate for the protection of fluvial slopes according to the criteria of stem flexibility, resistance to rupture (stem breakage), growth rate and plant size. Riparian forest stands of *Salix humboldtiana* and *Salix × rubens* need more frequent maintenance in order to preserve branch flexibility. According to the literature, any of the four species studied can excellently withstand and respond to the pruning of branches and coppicing of trunks.

Bending tests and the newly introduced “angle of flexibility” \((\alpha_B)\) parameter serve as parameters to compare the species. In actual practice plants are stressed and bent
in a way which is different from the bending in the laboratory and that needs further research work. In nature the stresses on plants during high floods act dynamically and therefore the plants’ response is quite different from the static load test at the laboratory. Furthermore, the influence of the bark and the anatomical characteristics of the wood can be helpful to clarify additional biomechanical characteristics of the riparian vegetation.

Acknowledgements. Work funded by the Commission for Development Studies at the OeAD GmbH.

References


Table 1. Average values of the modulus of elasticity [N/mm²] for the different diameter classes of each species. Coefficient of variation is shown in brackets ( ).

<table>
<thead>
<tr>
<th>Species</th>
<th>10–20 mm</th>
<th>20–30 mm</th>
<th>30–40 mm</th>
<th>40–50 mm</th>
<th>50–60 mm</th>
<th>60–70 mm</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phyllanthus sellowianus</td>
<td>4.513 (20)</td>
<td>3.793 (31)</td>
<td>3.329 (25)</td>
<td>3.028 (27)</td>
<td>–</td>
<td>–</td>
<td>0.27</td>
</tr>
<tr>
<td>Sebastiania schottiana</td>
<td>4.615 (26)</td>
<td>3.930 (29)</td>
<td>4.104 (31)</td>
<td>3.485 (12)</td>
<td>3.114 (18)</td>
<td>–</td>
<td>0.14</td>
</tr>
<tr>
<td>Salix × rubens</td>
<td>4.940 (35)</td>
<td>4.562 (29)</td>
<td>4.296 (25)</td>
<td>3.555 (34)</td>
<td>3.625 (21)</td>
<td>3.031 (11)</td>
<td>0.19</td>
</tr>
<tr>
<td>Salix humboldtiana</td>
<td>4.084 (40)</td>
<td>3.347 (19)</td>
<td>3.254 (12)</td>
<td>2.822 (33)</td>
<td>2.419 (12)</td>
<td>2.155 (24)</td>
<td>0.35</td>
</tr>
</tbody>
</table>
Drawing on the results from a previous study which focused on the vegetative reproduction potential of suitable plants for soil bioengineering (SUTILI et al., 2007), the next step involved quantifying the biomechanical behaviour of plants under specific load. The quantification of these processes is a crucial point establishing standards and function related dimensioning of soil bioengineering techniques from an engineering point of view. The application of plants in river engineering projects is based on a proven knowledge about the interaction of plants and flow.

The hydraulic interaction of flow with plants depends not only on geometrical properties (e.g. stem and branch diameter, length and leaf density), but also on the dynamic response of plants under flood conditions (stem/branch/leaf bending and reduction of plant height; e.g. FATHI-MOGHADAM & KOUWEN, 1997; OPLATKA, 1998; GERSTGRASER, 2000; MEIXNER et al., 2004, RAUCH, 2005, RHIGETTI & ARMANINI, 2002; MUSLEY & CRUISE, 2006; McBRIDE et al., 2007). Mechanical properties such as flexural stiffness, modulus of elasticity and plastic deformation are indicators to assess the impact of plants on hydraulic conditions. The modulus of elasticity, the proportional limit, as well as the deformation and stress up to the point of rupture are shown in a typical stress $\times$ deformation diagram (Figure 1). The first part of the diagram is a straight line that determines the modulus of elasticity and the limit of elastic behaviour. From this point the deformation becomes plastic and continues up to the point of rupture ($B$ in Figure 1) in a non-proportional way.

![Typical stress $\times$ deformation diagram](image)

**Fig. 1.** Typical stress $\times$ deformation diagram.
where:

\[ A_t \] total area (wood + bark) [cm²],
\[ A_o \] area without bark [cm²].

After preparation and staining of histological sections a microscope was used to determine the age of the samples.

A new value, the angle of flexibility, was introduced. It describes the whole deformation process of plants under load:

- angle of flexibility at proportional limit, \( \alpha_{\text{elast}} \) [°],
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At the original non-inflected position (dashed horizontal line in Figure 2), the bending point divides the specimen into two equal parts with an angle of zero to the horizontal and 180° between the two. With the values of \( l \) and \( f \) at the proportional limit and at the breaking limit, it is possible to calculate the angle of flexibility (\( \alpha \)) respectively, \( \alpha_{\text{elast}} \) and \( \alpha_B \) (Figure 2).

\[
l \cdot f \tan \alpha = 22
\]

where:

\[ l \] distance between the points of support [mm],
\[ f \] displacement (at the proportional limit or at the point of rupture, respectively) [mm].

Fig. 2. Bent specimen, indicating the variables used in calculating the angle of flexibility (\( \alpha \)).
Fig. 3. Relationship between diameter (\(d\)) and load (\(F\)). Solid line: \(F_{\text{elast}}\) at the proportional limit; dashed line: \(F_B\) at the point of rupture.
Fig. 4. Deformation and stress at proportional limit (solid line) and at rupture limit (dashed line) for the four species.
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Fig. 5. Relationship between the diameter ($d$) and the angle of flexibility at the point of rupture ($\alpha_B$). The band at the bottom of the graph shows the area of distribution of the angles of flexibility at the proportional limit ($\alpha_{elast}$).
11 mm branch of *Phyllanthus sellowianus* can form a 45° angle of flexibility before breaking, a branch of *Salix × rubens* of the same diameter only reaches 24° before breaking.

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For all species, the load required to reach the point of rupture increases with its diameter, while its angle of flexibility at rupture point decreases. Thus, a branch of *Phyllanthus sellowianus* measuring 20 mm in diameter must be stressed up to a load of 670 N to hit the point of rupture and forms a 45° angle of flexibility. A branch of *Sebastiania schottiana* of the same diameter, with a load of 550 N, breaks at 36°. *Salix humboldtiana* forms a smaller angle of flexibility (32°) and requires a load of 400 N to reach the breaking point. For the same diameter, *Salix × rubens*—despite having an angle of flexibility at rupture point that is even lower than the previous species—needs a higher load (460 N) than *Salix humboldtiana* to reach a maximum 25° angle of flexibility at rupture point.

As demonstrated in Figure 6, the flexibility gradually declines with the increase in diameter and age. *Phyllanthus sellowianus* is the species that supports the greatest stress and can be bent to higher angles of flexibility at the breaking point. Moreover, its growth rate is the lowest of all species investigated. The following diagram (Figure 7) shows the relationship between the age of the stems and branches and their diameter.

**Fig. 6.** Regression of $\alpha_B$ (angle of flexibility at the point of rupture) against log $F_B$ (breaking load) for the four species, within the studied diameter classes.
Fig. 7. Relationship between the diameter \((d)\) and the age \((Y)\) of stems and branches.

<table>
<thead>
<tr>
<th>Species</th>
<th>(R^2)</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Phyllanthus sellowianus</em> (a)</td>
<td>0.82</td>
<td>(Y=0.24d+0.24)</td>
</tr>
<tr>
<td><em>Sebastiania schottiana</em> (b)</td>
<td>0.83</td>
<td>(Y=0.13d+1.04)</td>
</tr>
<tr>
<td><em>Salix × rubens</em> (c)</td>
<td>0.58</td>
<td>(Y=0.12d+0.15)</td>
</tr>
<tr>
<td><em>Salix humboldtiana</em> (d)</td>
<td>0.79</td>
<td>(Y=0.09d+0.47)</td>
</tr>
</tbody>
</table>
Fig. 7: Relationship between the diameter ($d$) and the age ($Y$) of stems and branches. The relationships shown in Figure 7 cannot be generalised due to different local environmental conditions, but can be taken as reference values for the growth rate. In a following step, the growth rate (diameter and age) is related to the angle of flexibility at rupture point ($\alpha_B$) and shown in the nomogram in Figure 8.

Phyllanthus sellowianus
Sebastiania schottiana
Salix humboldtiana
Salix rubens

Fig. 8: Diameter ($d$) and angle of flexibility up to the point of rupture ($\alpha_B$) at different ages ($Y$). The relationship between age and stem diameter can be seen directly on the x- and y-axes respectively. The angle of flexibility at the point of rupture is characterised by means of the marked areas. For example, in order to know the diameter of each species in the fourth year of age, trace a straight line parallel to the y-axis at the age of 4 years. At the point where this line crosses the straight line that defines the relations for Phyllanthus sellowianus, you will get a diameter of 16 mm and an angle of flexibility at the point of rupture of 50°. At the same age, Sebastiania schottiana has a diameter of 23 mm and a 33° angle of...