Introducing a rainfall compound distribution model based on weather patterns sub-sampling

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Abstract

This paper presents a new probabilistic model for daily rainfall, using sub-sampling based on meteorological circulation. We classified eight typical but contrasted synoptic situations (weather patterns) for France and surrounding areas, using a “bottom-up” approach, i.e. from the shape of the rain field to the synoptic situations described by geopotential fields. These weather patterns (WP) provide a discriminating variable that is consistent with French climatology, and allows seasonal rainfall records to be split into more homogeneous sub-samples. An exponential POT model is used to fit the distribution of each sub-sample. The distribution of the multi-exponential weather patterns (MEWP) is then defined as the composition, for a given season, of all WP sub-sample marginal distributions, weighted by the relative frequency of occurrence of each WP. The MEWP distribution appears able to fit various shapes of distributions using a simple and robust approach for asymptotic behaviour. It is a new contribution to the ongoing debate on the probabilistic tools used to study the asymptotic behaviour of extreme rainfall from observed records. The paper is illustrated throughout with the example of the Lyon (France) rainfall record for the period 1953–2005.

1 Introduction

The correct estimation of extreme rainfall quantiles is a critical stage in the estimation of extreme flood quantiles. In recent years, many approaches have been described in the hydrological literature to address this issue. Several solutions based on the extreme value theory use an asymptotic model to describe the stochastic behaviour of extreme value processes. In fact the extreme value theory is a sound approach and is widely used in several scientific disciplines including hydrology. Standard methodology for modelling extremes is based on the hypothesis of independence, stationarity and homogeneity. According to Coles et al. (2003), a false assumption of model homogeneity can lead to considerable underestimation of the probability of a disastrous
event. The standard approaches based on extreme value theory use generalized extreme value (GEV) distribution or generalized Pareto (GP) distribution, and have to deal with the difficulty of locally estimating the shape parameter on the basis of point data (Koutsoyiannis, 2004). The regional approach consists either in refining the analysis to homogeneous climatic zones, in which the shape parameter is considered to be constant (Madsen et al., 1995; Ribatet et al., 2007; Pujol et al., 2008), or in using indirect methods, i.e. methods based on stochastic simulation of rainfall events, such as the SHYPRE method (Arnaud et al., 2007), in which the parameters are estimated using a regional approach (SHYREG method, Arnaud et al., 2006).

To ensure the homogeneity of the sample, and to provide a more reliable estimate of extreme rainfall values, we propose a new approach using sub-sampling based on the classification of atmospheric circulation patterns. These patterns provide a discriminating variable that enables the rainfall record to be split into more homogeneous sub-samples in terms of meteorological genesis. An appropriate probabilistic model is proposed and fitted to each sub-sample. Finally, a compound distribution is obtained by composing all the marginal distributions.

The purpose of this paper is to describe a weather pattern classification for France and its associated rainfall probabilistic model used by Paquet et al. (2006) within the SCHADEX method. The SCHADEX method aims at estimating extreme flood quantiles (see Boughton and Droop, 2003 for a review) by the combination of a rainfall probabilistic model and a continuous conceptual rainfall-runoff model. The weather pattern classification, so-called EDF 2006, is described in Sect. 2 below. The need for seasonal and weather pattern sub-sampling is explained in Sect. 3. In Sect. 4, the rainfall probabilistic model based on this approach is introduced, in accordance with the extreme value theory, but with some additional assumptions. In Sect. 5 some of the statistical characteristic of each sub-sample and of the compound distribution are analyzed, mainly to evaluate the hypothesis of this study and the features of extremes quantiles of this distribution, which leads to a conclusion for this study. The paper is illustrated with the daily rainfall records from Lyons (SE France) for the period 1953–2005.
2 Weather patterns classification

2.1 Context

The relationship between large-scale atmospheric circulation and precipitation events has been studied for a long time (see Yarnal et al., 2001; Boé et al., 2008; Martinez et al., 2008, for a review), especially over Western Europe, and it has been demonstrated that analysing synoptic situation can provide significant information on heavy rainfall events (Littmann, 2000).

From this point of view, a classification based on a limited number of typical but contrasted synoptic situations (or weather patterns) is a useful tool to link rainfall events with its generating processes. In this section, we identify the weather patterns for France and the resulting classification of rainy days.

To define a daily synoptic situation over France and surrounding areas, we used a dataset that has already been optimised in previous works on quantitative precipitation forecast using the analogue method (Guilbaud et al., 1998; Obled et al., 2002):

– Geopotential height fields at 700 and 1000 hPa pressure levels, at 0 h and 24 h, defined on 110 grid points;

– Analysis centred on Southeastern France from 6.2° W to 12.9° E, and from 38.0° N to 50.3° N (gray frame in Fig. 2).

In this way, each day can be defined in the $\mathbb{R}^{440}$ mathematical space of the geopotential fields concerned (four fields defined on 110 points).

2.2 A “bottom-up” approach for the identification of weather patterns

In our classification process, “bottom-up” should be understood as firstly identifying the centroids of classes using our variable of interest (i.e. rainfall), and secondly projecting them into the $\mathbb{R}^{440}$ space of geopotential heights.
The whole classification process is summarized in Fig. 1, and consists of the following steps:

- **STEP 1.** To describe a daily precipitation field over France, 54 rainfall series for the period 1956–1996 are used. Among these records, 3086 days (21%) with a minimum average rain depth of 5 mm are considered as rainy days. We then normalize each local rain depth by the average precipitation of the day concerned, as a way of considering the “shape” of the rain field rather than its scale. Whether extreme rainfall events are observed or not, we only use information on where it is raining in our study area.

- **STEP 2.** A Hierarchical Ascendant Classification (HAC) is then performed on this population of rainy day shapes, as defined in a $\mathbb{R}^{54}$ space. The dendrogram of this HAC showed that seven rainy classes could be chosen, at this stage the remaining days (79% of days) are combined in a non-rainy class.

- **STEP 3.** During this step the centres of gravity (or centroids) of the eight classes are calculated in the $\mathbb{R}^{440}$ space of geopotential heights.

- **STEP 4.** Each day of the 1953–2005 period is attributed to the weather pattern (WP) whose centroid is the closest in the $\mathbb{R}^{440}$, using the Teweles-Wobus score (Teweles and Wobus, 1954) as measure of proximity between synoptic situations. This led to changes for some days in the period 1956–1996 that were already classified by the HAC of rain, specially WP8 days (see below for the definition of WP8). Note that the Teweles-Wobus distance is used because we want to focus on atmospheric circulation, whatever the mean height of the geopotential fields (we could also have used other distances e.g. correlation between fields).

The obtained WPs are illustrated in Fig. 2a by their mean 1000 hPa geopotential field at 0 h. For pedagogical reasons, the fields are presented in logical order in terms of atmospheric circulations, i.e. 2-1-3-7-4-6-5 and 8 (see Fig. 1 Step 3). For each WP (except for WP8) an arrow indicates the atmospheric flow of low layers induced by the average
synoptic fields. The size and the direction of the arrow are a qualitative indication of the strength and direction of the wind. Figure 2b shows the corresponding precipitation fields (ratio of WP mean to global mean precipitation) over Western Europe. For this purpose, we used a gridded version of the European Climate Assessment and Data (ECA&D) of mean daily precipitation (Haylock et al., 2008). The grid resolution is $0.5 \times 0.5^\circ$ and the data cover the period 1953 to 2005.

These patterns give a picture of the diversity of rainy synoptic situations over France. They were named in relation with the atmospheric circulation they favour. WP2 (Steady Oceanic), WP1 (Atlantic Wave) and WP3 (Southwest Circulation) correspond to westerly oceanic circulations, WP1 being the most rainy pattern over the study area. WP7 (Central Depression) and WP4 (South Circulation) correspond to Mediterranean circulations, which bring heavy rains to Southeastern France. WP6 (East Return) also corresponds to a Mediterranean circulation, but rain is generally limited to the Italian border and Eastern Pyrenees. WP5 (North East) is a continental circulation, and finally WP8 (Anticyclonic) shows no well-defined circulation, as expected for a non-rainy day. The occurrence statistics of the eight WPs are presented in Table 1. For the whole year, the most frequent WP is the Anticyclonic one (WP8), followed by the Steady Oceanic (WP2) and the South Circulation (WP4). However, these figures change with the season, for example WP2 is more frequent in winter, and WP8 in summer.

### 2.3 Discussion

A weather pattern classification is a tool that cannot be separated from its object: a classification dedicated to wind or fog will obviously be significantly different from the one presented here. Furthermore, with not much contrasted mathematical objects like geopotential fields, clustering techniques are sensitive to initiation centers as well as to the number of classes. It is thus almost impossible to assert that a given classification “is the best”, because for the same dataset, equivalent solutions can easily be obtained with slightly different options. More reasonably, a classification should be evaluated on its ability to propose a reasonable typology of the phenomenon concerned.
Two other available classifications were evaluated and compared to the one proposed here: the well-known Hess-Brezowsky classification (Hess and Brezowsky, 1952), because it is often used for comparison, and another French classification (Boé, 2007), which is also used for precipitation analysis. The latter classification in fact comprises four classifications of 8–10 classes, one for each season (DJF, MAM, JJA, SON). The discriminating power of the three classifications was checked for rain/no rain occurrence, using appropriate criteria like the Cramer test (Bardossy et al., 1995). This coefficient ranges between 0 (no dependence between the classification and the rain/no rain occurrence) and 1 (absolute dependence). This criterion is first computed on each of our 54 rainfall chronicles on the period 1953–1998 and then averaged to obtain a single value. The results of the comparison are presented in Table 2, and show that the present classification based on the eight WPs has good discriminating power for the rain/no rain occurrence.

In addition, the corresponding average rain fields are contrasted (Fig. 2b). In our opinion, one of the major advantages of this classification is that it remains applicable throughout the year, enabling flexible use. For example, in a recent study by Gottardi (2009), this classification was used to interpolate daily precipitation fields over French mountainous regions. It is now time to evaluate its interest for heavy rainfall distribution. We draw the reader’s attention to the fact that at no time do we consider the extreme nature of precipitation: we simply classify the days according to atmospheric circulation and its impact on the shape of the rain field over France.

3 Extreme value theory and sampling techniques

3.1 Sampling techniques for extreme values

The extreme value theory is based on the fundamental hypothesis that the random variable realizations (extreme daily rainfall in our study) are independent and identically distributed (i.i.d). Two standard sampling techniques are used to build samples of
extreme values complying with these hypotheses:

- **Block maximum (BM).** The maximum values within blocks of equal length of data are selected. The choice of block size can be critical as too small blocks can lead to bias and too large blocks generate too few block maxima, thus giving a large estimation variance (Coles, 2001). Usually the one-year block is used for daily discharges or rainfall data, leading to the annual maxima (AM). The block maxima approach is associated with the use of the GEV distribution, according to the first theorem in extreme value theory (Fisher and Tippet, 1928; Gnedenko, 1943).

- **Peaks over threshold (POT).** All the events exceeding a given threshold are selected (see Lang et al., 1999; Rosbjerg and Madsen, 2004, for a review). According to Coles et al. (2003), if daily series are available, POT sampling is better than AM sampling, because additional information on several large events that occur during the same year is taken into account. According to the extreme value theory (Pickands, 1975), the POT distribution obtained by sampling i.i.d. variables converges to the GP distribution.

To ensure independence of POT values, an additional criterion based on a minimum time space between two successive events is usually applied. In the present paper, we begin to introduce a new variable, called the “central rainfall”, which is, at a daily time step, rainfall exceeding 1 mm and greater than the quantity of rain on the preceding and following day. In fact, it is just an alternative way to ensure an independent sample of rainy values. We therefore selected POT values of “central rainfalls”. In the Lyon records, the so-called “central rainfalls” represent about 17% of all daily rainfall (63% of the days being non-rainy days, and the 20% remaining days thus having less rainfall than the preceding or following days).

However, the “identically distributed” quality of such samples is somewhat questionable: the main feature shared by the selected observations is their “extreme status” of being the yearly maximum, or greater than a specific threshold. This can be illustrated
by considering daily discharges of small mountain catchments where high values are commonly observed either in spring or autumn. In this case, two populations linked to very different hydrological processes (snowmelt or heavy rain runoff floods) are mixed by BM or POT sampling (Hirschboeck et al., 1987; Petrow et al., 2007), making the “identically distributed” hypothesis harder to ensure, and consequently the use of extreme value statistical theory more questionable.

Therefore, two complementary sub-sampling techniques for rainfall records are introduced here to more closely approach the i.i.d. hypothesis.

3.2 Seasonal sub-sampling

In most places in the world and in a wide range of climates, rainfall displays strong seasonal variability. At a given location, the frequency and intensity of rainfall is driven by the meteorological situation, whose genesis is strongly influenced by large scale seasonal factors, among which variation in solar input (incidence of sunlight, day length), sea surface temperatures, the position of long lasting high or low pressure centres etc. The factors that cause heavy rainfall events are numerous, various and complex, and they interact at different scales, but their seasonal variation pattern has a true climatological consistency. This is common sense in strong bipolar precipitation regimes (like monsoon), but is also true in temperate climates with more mixed influences. For example, heavy rains hitting the French, Spanish and Italian regions surrounding the Mediterranean Sea most likely occur during fall (September to November). This kind of pattern must be taken into account by appropriate seasonal sampling to produce more homogeneous sub-populations for extreme rainfall analysis (Lang and Desurosne, 1994; Djerboua and Lang, 2007). In extreme rainfall studies for France, we usually consider three to four non-overlapping seasons. Although these seasonal divisions make sense regionally and climatologically, they may vary significantly from one year to the next and from one place to another.

Figure 3 is a box plot of annual daily rainfall maxima for each month at Lyons. The seasonal pattern is rather common for daily rainfall in Southern France, with the highest
quantiles between September and November, and the lowest between May and August.

3.3 Weather pattern based sub-sampling

As indicated in Sect. 2.1, in Europe, the links between atmospheric circulation patterns and heavy rainfall events have been widely studied in various locations, with special focus on the Mediterranean area (Romero et al., 1999; Littmann, 2000; Martinez et al., 2008). The analysis domain on which the classification is built is generally wide (several degrees of latitude and longitude), and thus has regional implications. A discrimination of rainfall records based on such a classification is one way to gather observations according to similar generating meteorological processes, and hence progress toward the homogeneity of sub-samples. One application was described by Ramos et al. (2001) for the 30′ rainfall in Marseilles (France), showing two distinct asymptotic behaviours depending on the presence of a meso-scale convective system. This approach can also provide additional information about extreme rainfall events, thus enhancing probabilistic analysis (Klemes, 1993). To come back to the rainfall in Lyons, Fig. 4 is a box-plot of annual maxima for each weather pattern. The WP4 (South Circulation), WP7 (Central Depression), and to a lesser extent WP1 (Atlantic Wave), clearly have higher quantiles than the other weather patterns. We will now integrate these sub-samplings into a new rainfall probabilistic model.

4 A probabilistic model based on sub-sampling of weather patterns

4.1 Global formulation

Let $Y$ represents the hydrologic variable of interest such as daily rainfall (or central rainfall, see above). Let us now consider a range of seasons $i=1,...,S$, where $S$ is the number of seasons that allows appropriate seasonal division of the local precipitation regime ($S$ equal to 3 or 4 generally in France).
Let us also consider a range of weather patterns $j=1,\ldots,NWP$, where NWP is the number of weather patterns that provides a robust discrimination of the meteorological situations of the study region (for France, NWP equal to 8 for the classification presented in Sect. 2).

To build sub-samples based on seasons and weather patterns, the hydrologic variable $Y$ is partitioned into $S \cdot NWP$ variables, $Y_{NWP=j}^{S=i}$, with respect to seasons and weather patterns, as follows:

$$Y = \bigcup_{i=1}^{S} Y^i \text{ and } Y^i = \bigcup_{j=1}^{NWP} Y_{j}^i$$

(1)

According to the asymptotic theory, POT values of a daily rainfall sub-sample of season $i$ and WP $j$, assumed to have independent and identically distributed values, can be fitted with a GP distribution, which takes the form:

$$F_i^j(z) = \Pr\left[Z_i^j = Y^i_j - u^i_j < z\right] = 1 - \left(1 + \frac{z}{\xi_{ij}}\frac{Z}{\lambda^i_j}\right)^{-\frac{1}{\xi^i_{ij}}}$$

(2)

with a parameter space $\{\left(\lambda^i_j, \xi^i_j\right) : \lambda^i_j > 0, \xi^i_j \in \mathbb{R}\}$, and a threshold $u^i_j$.

As the set of seasonal POT values across all WP is the union of the POT values within each WP, the seasonal rainfall distribution is computed from a mixture distribution of GP distribution for each WP. This seasonal distribution takes the form:

$$F^i(z) = \sum_{j=1}^{NWP} F_j^i(z) \cdot p^i_j$$

(3)

where weight $p^i_j$ is the relative occurrence of each WP within season $i$. The global distribution is therefore computed from a mixture distribution of each seasonal distribution,
which takes the form:

\[ F(z) = \sum_{i=1}^{s} F_i(z) \cdot p^i \]  \hspace{1cm} (4)

where weight \( p^i \) is the relative occurrence of each season that is equal to the ratio of the number of events in the season to the total number of events.

4.2 Choice and parameterization of a marginal model for WP sub-sampled distributions

In this section we study the effect of seasonal and weather pattern sub-sampling on the maximum likelihood estimation of model parameters for Lyons. Figure 5 shows the mean residual life (MRL) plot (Davison and Smith, 1990; Coles, 2001; Coles et al., 2003) for the rain gauge at Lyons, for respectively the whole year, the autumn season, and for the WP4 days within the autumn season. The mean excess above the threshold should be constant, equal to the scale parameter \( \lambda \) for the case of exponential distribution (\( \xi = 0 \)), and should increase linearly with the threshold value for the Pareto distribution (\( \xi > 0 \)) (Shanbhag, 1970).

Figure 5c shows, for the “WP4 and autumn” sub-sampling, an MRL plot that is reasonably constant with respect to the threshold, around a value of 18 mm/24 h. Given this result, the use of an exponential model to describe the extreme behavior of this sub-sample appears to be appropriate. The same conclusion can be drawn for the other WP sub-samples. Figure 6 shows the examples of the autumn WP4, WP7 and WP1, with the MRL plots of each sub-sample and the corresponding fitted scale parameters. By providing more homogeneous samples, WP sub-sampling provides a new asymptotic view compared to the analysis carried out on all seasonal data. Furthermore, it has a parsimonious effect on the global probabilistic model.
4.3 Multi-exponential weather pattern distribution

Considering that the shape parameter $\xi^i_j$ is equal to zero, the seasonal distribution given in Eq. (3) takes the form:

$$F^i(z) = \sum_{j=1}^{\text{NWP}} F^i_j(z) \cdot p^i_j = \sum_{j=1}^{\text{NWP}} \left(1 - \exp\left(-\frac{z}{\lambda^i_j}\right)\right) \cdot p^i_j$$

(5)

This seasonal distribution is then named *multi-exponential weather pattern* (MEWP) distribution. To provide a continuous probabilistic description of the whole range of observed rainfall, the CDF of each sub-sample is extended below its threshold $u^i_j$ by a linear interpolation of empirical quantiles. Otherwise, the MEWP distribution would only be defined above the greatest threshold of all sub-samples.

In practice, selecting a threshold level $u^i_j$ is not an easy task. In order to avoid compromising the asymptotic characteristic of the fitted values – threshold too low – and to avoid enlarging the variance of the estimators – threshold too high –, $u^i_j$ was chosen equal to the 70% empirical quantile of each WP sub-sample. This choice of threshold was checked on MRL plots. It proved to be a good compromise for a wide dataset over France (almost 500 rainfall chronicles) (Garavaglia et al., 2009).

Table 3 shows the scale parameter $\lambda^i_j$, the threshold $u^i_j$ (corresponding to the 70% empirical quantile) and the weight $p^i_j$ for each WP of the four seasonal MEWP distributions (DJF, MAM, JJA, SON) for the Lyon rain gauge. The last line gives the weight $p^i$ for the four seasons used to compute the MEWP global distribution. As already underlined in Table 1, the WP4 frequency jumps from 6% to 20% between summer and autumn. Being the WP with the highest rainfall quantiles, it is part of the explanation of why autumn is the most risky season in most of Southeastern France. More generally, these results reveal significant variability of the scale parameter in relation with the WP and the season. We consider this variability as proof of the suitability of WP sampling: inappropriate sub-sampling would have produced randomly parsed samples of the whole record, with a rather uniform scale parameter for each sub-sample.
Conversely, the a posteriori analysis of the WP sub-sampling shows significant heterogeneity of precipitation extremes, confirming the relevance of this sub-sampling. Figure 7 illustrates the eight WP exponential distributions fitted on the Lyon fall rainfall records, for the period 1953–2005. The x-axis of these graphs shows the return level, $T(z)$, expressed in years, obtained from the density function $F(z)$, through the following expression:

$$T(z) = \frac{1}{1 - F(z)\frac{n}{N}}$$

(6)

where $n$ is the number of elements of the sub-sample concerned (e.g. daily rainfall in autumn and WP1) and $N$ is the number of years of the data (i.e. 53 for the period 1953–2005). We can now define the WP at risk within a given season as the WP associated with the greatest scale parameter (numbers in bold in Table 3). For winter and spring, it is WP7 (respectively $\lambda_7$ equal to 9.6 mm/24 h and 11 mm/24 h), whereas in summer and autumn it is WP4 (respectively $\lambda_4$ equal to 17.6 mm/24 h and 18.7 mm/24 h), showing a seasonal change. This result is fully consistent with the climatological characteristics of the Lyon area, with Mediterranean circulations causing the heaviest rainfall events, especially in autumn. The eight WP exponential distributions illustrated in Fig. 7 are combined in an autumn MEWP distribution (Fig. 8a) using the weight $p_i$ given in Table 3. Similarly the four seasonal MEWP distributions are combined in the global MEWP distribution illustrated in Fig. 8b, according to the seasonal weight $p_i$ given in Table 3.

### 4.4 Properties of the MEWP distribution

Two important features of this model should be underlined:

- a significant bend of the CDF for low to moderate return times can be represented, meaning non-exponential behaviour of distributions in the range of observable frequencies can be accounted for;
– for high and extreme quantiles (currently over 50 years of return period), the asymptotic behaviour becomes exponential, and is fully parameterized by the scale parameter and the relative frequency $p_j$ of the WP at risk and the season at risk.

However, the high flexibility of a probabilistic model, i.e. its ability to fit the largest observed values, often has the serious drawback of lacking robustness for estimations of extreme quantiles. In this connection, Fig. 9 illustrates the robustness of the proposed probabilistic model. The MEWP distribution was compared with the GP distribution for the Lyon record. Both models were fitted locally (using maximum likelihood criterion) on two samples: the autumn observations for the period 1953–2005, with and without the maximum observed event (101 mm rainfall in 24 h on 30 September 1958).

The estimate of the 1000-year return levels for daily rainfall is 125 mm with the GP distribution fitted on the complete record, and 106 mm with the GP distribution fitted without the observed maximum (15% less). For the MEWP distributions, these values are respectively 155 mm and 148 mm, i.e. only a 4% difference. This example illustrates to what extent the local fit of the GP distribution is influenced by the maximum values observed. In these conditions, the MEWP distribution is more robust than the GP distribution for the estimation of extreme rainfall events.

As mentioned above, the asymptotic behavior is influenced by the highest quantiles of the most severe WP distribution, here WP4. For this sub-sample, an exponential distribution is fitted over a threshold corresponding to the 70% empirical quantile (see Sect. 4.3), i.e. 47 values. Here the asymptotic scale parameter is estimated with these 47 values. As a comparison, the local fit of a GEV distribution would require the estimation of three parameters (shape, scale and position) on 53 annual maxima for the period 1953–2005.
5 Conclusions

The main features of the proposed MEWP approach can be summarized as follows:

- construction of a rain-oriented weather pattern classification to approach the meteorological genesis of heavy rains, over an area of mixed climatological influences;

- discrimination of a rainfall record based on this classification, allowing us to approach the i.i.d. hypothesis for the extreme values of each sub-sample;

- use of marginal exponential distributions for each sub-sample based on a given weather pattern;

- construction of a versatile compound distribution able to fit various shapes of empirical daily rainfall distributions up to the highest quantile observed, but with a simple and robust approach for asymptotic behavior.

Our main concern was to approach the “i.i.d.” hypothesis of the extreme value theory for extreme rainfall samples. Independence of extreme values is quite easy to ensure, but the homogeneity of sub-samples has to be checked indirectly:

- a priori, considering the discriminating power of the WP classification, it should be checked that the chosen classification minimizes deviation within classes, and maximizes it between classes;

- a posteriori, regarding the strong variability of rainfall asymptotic behaviours induced by the WP sub-sampling.

Based on relevant sub-sampling of rainfall observations, our study shows that the exponential distribution can reasonably be used to describe the asymptotic behaviour of each sub-sample. A combination of those exponential distributions based on regional climatology, can adequately fit rainfall distributions showing Pareto behavior (ξ>0) for
observable quantiles. In this connection, the behavior for observable quantiles is not necessarily transposable to extreme quantiles. The proposed sampling method and the associated probabilistic model were presented and illustrated using the daily rainfall record for Lyons, France. Of course, one example is not enough to assess the global robustness of the MEWP approach. A comprehensive statistical study of the approach, based on a large dataset of almost 500 rainfall time series, located in France, Swiss, Italy and Spain, with records covering 50 years, will be presented in a future paper.

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Interactive Discussion


Table 1. Yearly and seasonal statistics of occurrence for the eight WP (records for the period 1953–2005).

<table>
<thead>
<tr>
<th>Class</th>
<th>WP name</th>
<th>Year</th>
<th>Winter</th>
<th>Spring</th>
<th>Summer</th>
<th>Autumn</th>
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<tbody>
<tr>
<td>WP1</td>
<td>Atlantic Wave</td>
<td>7%</td>
<td>5%</td>
<td>7%</td>
<td>11%</td>
<td>7%</td>
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<tr>
<td>WP2</td>
<td>Steady Oceanic</td>
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<td>36%</td>
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<td>14%</td>
<td>21%</td>
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<tr>
<td>WP3</td>
<td>Southwest Circulation</td>
<td>8%</td>
<td>4%</td>
<td>7%</td>
<td>12%</td>
<td>8%</td>
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<tr>
<td>WP4</td>
<td>South Circulation</td>
<td>18%</td>
<td>19%</td>
<td>18%</td>
<td>10%</td>
<td>23%</td>
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<tr>
<td>WP5</td>
<td>Northeast Circulation</td>
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<td>7%</td>
<td>8%</td>
<td>6%</td>
<td>6%</td>
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<td>WP6</td>
<td>East Return</td>
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<td>5%</td>
<td>8%</td>
<td>6%</td>
<td>5%</td>
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<td>WP7</td>
<td>Central Depression</td>
<td>3%</td>
<td>2%</td>
<td>4%</td>
<td>3%</td>
<td>4%</td>
</tr>
<tr>
<td>WP8</td>
<td>Anticyclonic</td>
<td>28%</td>
<td>21%</td>
<td>26%</td>
<td>38%</td>
<td>26%</td>
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Table 2. Comparison of the discriminating power of three classifications for the occurrence of rain/no rain (average of statistics made on 54 rainfall records).

<table>
<thead>
<tr>
<th>Classification</th>
<th>Region</th>
<th>Number of classes</th>
<th>Cramer Coefficient</th>
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<td>Hess and Brezowsky (1952)</td>
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<td>Boé (2007)</td>
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<td>38</td>
<td>0.429</td>
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<tr>
<td>EDF 2006</td>
<td>France</td>
<td>8</td>
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</tr>
<tr>
<td>EDF 2006 (seasonal)</td>
<td>France</td>
<td>8×4</td>
<td>0.453</td>
</tr>
</tbody>
</table>
Table 3. Scale parameter $\lambda^i_j$, threshold $u^i_j$, weight $p^i_j$ for each weather pattern and the weights $p^i$ of the four seasonal MEWP distributions for Lyons. Numbers in bold represent the greatest scale parameter (WP at risk) within a given season.

<table>
<thead>
<tr>
<th></th>
<th>Winter (DJF)</th>
<th>Spring (MAM)</th>
<th>Summer (JJA)</th>
<th>Autumn (SON)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\lambda^i_j$ (mm/24 h)</td>
<td>$u^i_j$ (mm)</td>
<td>$p^i_j$</td>
<td>$\lambda^i_j$ (mm/24 h)</td>
</tr>
<tr>
<td>WP1</td>
<td>7.26</td>
<td>9.27</td>
<td>11%</td>
<td>9.76</td>
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<tr>
<td>WP2</td>
<td>4.28</td>
<td>7.00</td>
<td>50%</td>
<td>4.51</td>
</tr>
<tr>
<td>WP3</td>
<td>5.03</td>
<td>7.00</td>
<td>6%</td>
<td>9.63</td>
</tr>
<tr>
<td>WP4</td>
<td>6.29</td>
<td>11.40</td>
<td>15%</td>
<td>9.27</td>
</tr>
<tr>
<td>WP5</td>
<td>2.63</td>
<td>5.50</td>
<td>6%</td>
<td>6.49</td>
</tr>
<tr>
<td>WP6</td>
<td>9.45</td>
<td>9.46</td>
<td>3%</td>
<td>9.15</td>
</tr>
<tr>
<td>WP7</td>
<td><strong>9.64</strong></td>
<td>15.88</td>
<td>6%</td>
<td><strong>10.98</strong></td>
</tr>
<tr>
<td>WP8</td>
<td>2.39</td>
<td>2.84</td>
<td>2%</td>
<td>2.47</td>
</tr>
<tr>
<td>$p^i$</td>
<td>24%</td>
<td>27%</td>
<td>25%</td>
<td>24%</td>
</tr>
</tbody>
</table>
Fig. 1. WP classification flowchart.
Fig. 2. Average geopotential height at 1000 hPa of the eight WP (A) and the ratio of the mean WP to global mean precipitation (B).
Fig. 3. Box plot of the annual maxima for each month at Lyon rain gauge (records for the period 1953–2005).
Fig. 4. Box plot of the annual maxima for each weather pattern at Lyon rain gauge (records for the period 1953–2005).
Fig. 5. Mean residual life plot for all year (A), autumn season S-O-N (B) and WP4 days within autumn season (C) at Lyon rain gauge. Gray lines represent the 95% confidence interval.
Fig. 6. Mean residual life plot for WP4 (A), WP7 (B) and WP1 (C) days in the autumn season at Lyon rain gauge. Gray lines represent the 95% confidence interval. The dashed line highlights the fitted value of the scale parameter.
Fig. 7. Exponential distributions of the eight WP sub-samples for Lyon rain gauges (data from the period 1953–2005; autumn season).
Fig. 8. Autumn MEWP distribution (A) and global MEWP distribution (B) for the Lyon rain gauge (data from the period 1953–2005).
Fig. 9. Sensitivity of the extreme daily rainfall quantiles to the maximum value recorded by the Lyon rain gauge.