Water balance modeling of Upper Blue Nile catchments using a top-down approach

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Abstract

The hydrological behavior and functioning of twenty catchments in the Upper Blue Nile basin have been analyzed using a top-down modeling approach that is based on Budyko’s hypotheses. The objective is to obtain better understanding of catchment response for prediction in ungauged catchments. The water balance analysis using Budyko-type curve at annual scale reveals that the aridity index does not exert a first order control in most of the catchments. This implies the need to increase model complexity to a monthly time scale to include the effects of seasonal soil moisture dynamics. The dynamic water balance model used in this study predicts the direct runoff and other processes based on limit concept. The uncertainty of model parameters has been assessed using the GLUE (Generalized Likelihood Uncertainty Estimation). The results show that the majority of the parameters are reasonably well identifiable. Moreover, a multi-objective model calibration strategy has been employed within the GLUE framework to emphasize the different aspects of the hydrographs on low and high flows. The model has been calibrated and validated against observed streamflow time series and it shows good performance for the twenty catchments of the upper Blue Nile. During the calibration period (1995–2000) the Nash and Sutcliffe coefficient of efficiency for monthly flow prediction varied between 0.52 to 0.93 during high flows, while it varied between 0.32 to 0.90 during low flows (logarithms of flow series). The model is parsimonious and it is suggested that the resulting parameters can be used to predict monthly stream flows in the ungauged catchments of the Upper Blue Nile basin, which accounts about 60% of total Nile basin flow.

1 Introduction

The Blue Nile river emanates from Lake Tana in Ethiopia at an elevation of 1780 m a.s.l. Approximately 30 km downstream of Lake Tana, at the Blue Nile falls the river drops in to a deep gorge and travels about 940 km till the Ethiopian-Sudanese boarder (Conway,
1997). Despite its 60% of annual flow contribution to the Nile river (e.g. UNESCO, 2004; Conway, 2005), the research in the Blue Nile has suffered from limited hydrological and climatic data availability, which hampers an in-depth study of the hydrology of the basin.

The hydrology of the Upper Blue Nile basin was studied using a simple water balance model (e.g., Johnson and Curtis, 1994; Conway, 1997; Misra and Hata, 2006; Steenhuis et al., 2009) and more complex model using SWAT on Lake Tana sub-basin (Setegne et al., 2008). However, most of these studies were conducted on large scale to analysis the flow at the outlet at the Ethio-Sudan border. An understanding of the processes at sub-catchment level is generally lacking. Moreover, as a part of model uncertainty and parameter identifiability issues, multi-objective calibration searching for optimal parameter sets towards different objective functions was missing in previous studies.

Assessment of catchment water balance is a pre-requisite to understand the key processes of the hydrologic cycle. However, the challenge is more distinct in developing countries; where data on climate and runoff is scarce as in the case of Upper Blue Nile basin. In such cases, a water balance study can provide insight into the hydrological behavior of a catchment and can be used to identify changes in main hydrological components (Zhang et al., 1999).

In order to analyze the components of the catchment water balance, Budyko (1974) developed a framework linking climate to evaporation and runoff from a catchment. He developed an empirical relationship between the ratio of mean annual actual evaporation to mean annual rainfall and mean annual dryness index of the catchment.

The Budyko hypothesis has been widely applied in the catchments of the former Union of Soviet Socialist Republics (USSR). Similar studies were conducted worldwide using Budyko's framework (e.g., Milly, 1994; Koster and Suarez, 1999; Sankarasubramania and Vogel, 2002, 2003; Zhang et al., 2004; Donohue et al., 2007; Gerrits et al., 2009; Yang et al., 2009). All these studies improved Budyko framework by including additional processes. Zhang et al. (2001, 2008) and Yang et al. (2007), suggested that by assuming negligible storage effects for long term mean (>5 yr), the annual aridity index
(Φ = E₀/P) controls partitioning of precipitation (P) to evaporation (E) and runoff (Q). E₀ is the potential evaporation, while E is actual evaporation. By evaporation we mean all forms of water changes from liquid to vapor, i.e., soil and water evaporation plus transpiration and interception evaporation. This is usually termed evapotranspiration in literature. However, Sankarasubramania and Vogel (2002, 2003) argued that, the aridity index is not the only variable controlling the water balance at annual time scale, and that the evaporation ratio (E/P) is related to soil moisture storage as well. Their results improved by including a soil moisture storage index, which could be derived from the “abcd” watershed model. Milly (1994) showed that the spatial distribution of soil moisture holding capacity and temporal pattern of rainfall can affect catchment evaporation, but could be of small influence on annual time scale. Obviously, spatial and temporal variability of vegetation affects evaporation and hence the water balance. Thus it is more important to include the soil moisture storage for smaller spatial and temporal scales (Donohue et al., 2007). Zhang et al. (2004) hypothesized that the plant available water capacity coefficient reflects the effect of vegetation on the water balance. They developed a two parameter model which relates the mean annual evaporation to rainfall, potential evaporation and plant available water capacity to quantify the effect of long term vegetation change on mean annual evaporation (Ē) in 250 catchments worldwide and found encouraging results. Inspired by the work of Fu (1981), Yang et al. (2007) analyzed the spatio-temporal variability of annual evaporation and runoff in 108 arid/semi-arid catchments in China and explored both regional and interannual variability in annual water balance and confirmed that the Fu (1981) equation can provide a full picture of the evaporation mechanism at the annual timescale.

The distinct feature of the present study is that, we learned from the data starts with simple annual model in different sub-catchments with in the Upper Blue Nile basin based on Budyko framework and model complexity is increased to monthly time scale. Monthly water balance model developed by Zhang et al. (2008), based on Budyko hypotheses has been tested in 250 catchments in Australia with different rainfall regime across various geographical region and they obtained encouraging results. We have
applied this model in the Upper Blue Nile catchments due to its parsimony, only having four physically meaningful parameters and its versatility to predict streamflow and to investigate impacts of vegetation cover change on stream flow. For example Zhao et al. (2009) applied the model in Australian and South African catchments on a monthly time scale and meso-scale and to large scale to study land cover change impacts on streamflow. Moreover, in data scares environment like Upper Blue Nile basin, complex models which require more input data and large number of parameters, are not recommended and the model we used has fewer numbers of parameters, which is less prone to equifinality problem and its wide application in different climate has a merit to use it for Upper Blue Nile catchments.

The objective of this paper is building on the work of Budyko (1974), Fu (1981), Zhang et al. (2004, 2008), to investigate the overall catchment behavior and functioning of twenty catchments in the Upper Blue Nile on a monthly and annual scale and a spatial scale of meso-scale (catchment area between 10–1000 km$^2$) to large scale (catchment area $>1000$ km$^2$). The results could be applied to predict runoff from ungauged catchments.

2 Study area and input data

2.1 Study area

The Upper Blue Nile River is located in the highlands of Ethiopia (Fig. 1). The elevation ranges between 489 on the western side to 4261 m a.s.l. at Mount Ras Dashen in the north-eastern part. The catchment boundary and the drainage pattern have been delineated using ArcGIS 9.3 with a 90 m resolution digital elevation model of the NASA Shuttle Radar Topographic Mission (SRTM) obtained from the Consortium for Spatial Information (CGIR_CSI) website (http://srtm.csi.cgiar.org).

The climate in the Upper Blue Nile river basin varies from humid to semi-arid and it is mainly dominated by latitude and altitude. The influence of these factors determine
a rich variety of local climates, ranging from hot and arid along the Ethiopia-Sudan border to temperate at the highlands and even humid-cold at the mountain peaks in Ethiopia. According to present study for a period of (1995–2004) the mean annual temperature ranges from 13°C in south eastern parts to 26°C in the lower areas of the south western part. The National Meteorological Services Agency (NMA) defines three seasons in Ethiopia: rainy season (June to September), dry season (October to January), and short rainy season (February to May). The short rains, originating from the Indian Ocean, are brought by south-east winds, while the heavy rains in the wet season originate from the Atlantic Ocean with south-west winds (BCEOM, 1999; Yilma and Ulrich, 2004). The rainfall in the basin has a monomodal pattern. Mean annual values range between 924–1845 mm/yr during the period 1953–1987, and 70% of it concentrates between June and September (Conway, 2000). The dominant land cover in the basin is rainfed agriculture, i.e. cropland (26%) and grassland (25%). Wood and shrubland are minor compared to the other land cover types (Teferi et al., 2010). The soil type in the basin is dominated by Vertisol and Nitisol types (53%). The Nitisols are deep non swelling clay soils with favorable physical properties like drainage, workability and structure, while the Vertisols are characterized by swelling clay minerals with more unfavorable conditions. The basin geology is characterized by basalt rocks, which are found in the Ethiopian highlands, while the lowlands mainly composed of basement rocks and metamorphic rocks such as gneisses and marbles (ENTRO, 2007).

2.2 Input data

Monthly stream flow time series of 20 rivers covering the period 1995–2004, has been collected from the Ministry of Water Resources Ethiopia, Department of Hydrology. The quality of the input data has been checked based on comparison graphs of neighboring stations and also double mass analyses were carried out to check the consistency of the time series on a monthly basis. The missing data were filled in using regression analysis. Monthly meteorological data for the same period were obtained from the Ethiopian National Meteorological Agency (ENMA). The data comprises precipitation.
from 48 stations and temperature from 38 stations. Potential evaporation was computed using Hargreaves method with minimum and maximum average monthly temperature as input data (Hargreaves and Samani, 1982). Table 1 represents basic hydro-meteorological characteristics of the twenty catchments.

3 Methodology

3.1 The Budyko framework

Budyko-type curves were developed by many researchers in the past (Schreiber, 1904; Ol’dekop, 1911; Turc, 1954; Pike, 1964; Fu, 1981). The assumptions inherent to the Budyko framework are:

1. Considering a long period of time \((\tau \geq 5 \text{ yr})\), the storage variation in catchments may be disregarded (i.e. \(\Delta S \approx 0\))

2. Long term annual evaporation from a catchment is determined by rainfall and atmospheric demand. As a result, under very dry conditions, potential evaporation may exceed precipitation, and actual evaporation approaches precipitation. These relations can be written as \(\frac{E}{P} \rightarrow 0\), \(\frac{E}{P} \rightarrow 1\), and \(\frac{E_0}{P} \rightarrow \infty\).

Similarly, under very wet conditions, precipitation exceeds potential evaporation, and actual evaporation asymptotically approaches the potential evaporation \(E \rightarrow E_0\), and \(\frac{E_0}{P} \rightarrow 0\).

The Budyko (1974) equation can be written as:

\[
\frac{E}{P} = \left[\phi \tanh(1/\phi)(1 - \exp(-\phi))\right]^{0.5}.
\] (1)

The Budyko curve representing the energy and water limit asymptotes are shown in Fig. 2.
3.2 Catchment water balance model at annual time scale

The water balance equation of a catchment at annual time scale can be written as:

\[
\frac{dS}{dt} = P - Q - E
\]  

(2)

Where:
- \( P \) is precipitation (mm/yr)
- \( Q \) is the total runoff (mm/yr), i.e. surface runoff, interflow and baseflow
- \( E \) is actual evaporation (mm/yr)
- \( \frac{dS}{dt} \) is storage change over time (mm/yr)

By assuming storage fluctuations to be negligible over long time scales, i.e. decades, Eq. (2) reduces to:

\[
P - Q - E = 0
\]  

(3)

Equation (3) is known as a steady state or equilibrium water balance, which is controlled by available water and atmospheric demand controlled by available energy (Budyko, 1974; Fu, 1981; Zhang et al., 2004, 2008).

Among the different Budyko-type curves, we used Eq. (4) given by:

\[
\frac{E}{P} = 1 + \frac{E_0}{P} - \left[ 1 + \left( \frac{E_0}{P} \right)^w \right]^{\frac{1}{w}}
\]  

(4)

Details of the derivation of this equation can be found in (Zhang et al., 2004). The fact that this equation has one annual parameter \( w \) [-] which is a coefficient represents the integrated effects of catchment characteristics such as vegetation cover, soil properties and catchment topography on the water balance (Zhang et al., 2004). This equation would enable to model individual catchments on annual basis. Even though, the results from different Budyko-type curves are not presented here, prediction of annual runoff and evaporation were estimated better using Eq. (4) than the other curves. Moreover, most of Budyko-type curves including Budyko (1974) do not have calibrated parameter
and could not be applied on individual catchments (Potter and Zhang, 2009). The three performance measures Nash and Sutcliffe coefficient of efficiency, root mean squared error and mean absolute error used in the annual model is given by:

\[
\text{NSE} = 1 - \frac{\sum_{i=1}^{n} (Q_{\text{sim},i} - Q_{\text{obs},i})^2}{\sum_{i=1}^{n} (Q_{\text{obs},i} - \bar{Q}_{\text{obs}})^2}
\]  

(5)

\[
\text{RMSE} = \sqrt{\frac{1}{n} \times \sum_{i=1}^{n} (Q_{\text{obs},i} - Q_{\text{sim},i})^2}
\]  

(6)

\[
\text{MAE} = \left| \frac{1}{n} \times \sum_{i=1}^{n} (Q_{\text{obs},i} - Q_{\text{sim},i}) \right|
\]  

(7)

Where: \(Q_{\text{sim},i}\) is the simulated streamflow at time \(i\) [mm/yr], \(Q_{\text{obs},i}\) is the observed streamflow at time \(i\) [mm/yr], \(n\) is the number of time steps in the calibration period and the over bar indicates the mean of observed streamflow.

### 3.3 Catchment water balance model at monthly time scale

At annual time scale, the equilibrium water balance model performs poorly in most of the Upper Blue Nile catchments. This indicates that the impact of catchment water storage cannot be neglected at annual time scale and more complex models are required to predict accurately the water balance in these catchments (Zhang et al., 2008; Yang et al., 2009). Similar work by Sankarasubramania and Vogel (2002) indicates that the aridity index alone cannot predict annual evaporation and that a soil moisture index improved the prediction at annual time scale. Hence, increased model complexity was tested to consider the storage effect on monthly time scale for the Blue Nile catchments under consideration.
The dynamic water balance model developed by Zhang et al. (2008) has been used to simulate the monthly streamflow. The model has four parameters describing direct runoff behavior $\alpha_1$ [–], evaporation efficiency $\alpha_2$ [–], catchment storage capacity $S_{\text{max}}$ [mm], and slow flow component $d$ [1/month]. Zhang et al. (2008) stated that each parameter can be interpreted physically. For example the parameter $\alpha_1$ represents catchment rainfall retention efficiency and an increase in parameter $\alpha_1$ implies high rainfall retention and less direct runoff. The maximum soil water storage in the root zone ($S_{\text{max}}$) relates to soil and vegetation characteristics of the catchment. The parameter $\alpha_2$, relates to the evaporation efficiency, a higher evaporation ratio implies higher partitioning of available water to evaporation. The parameter $d$ represents the baseflow and the groundwater storage. The model uses rainfall and potential evaporation data as an input to simulate monthly streamflow. A schematic diagram for the dynamic water balance model is shown in Fig. 3.

Details of the equations in Fig. 3 and model descriptions are presented in the Appendix. This dynamic conceptual monthly water balance model has two storages: the root zone storage and groundwater storage act as linear storage reservoirs. State variables and fluxes are reasonably defined based on Eq. (4). Referring to Eq. (4) $w$ [–] is a model parameter ranging between 1 and $\infty$. For the purpose of model calibration Zhang et al. (2008) defined that $\propto = 1 - \frac{1}{w}$ and it implies that $\alpha$ [–] values vary between 0 and 1.

### 3.4 Parameter estimation, sensitivity and uncertainty assessment

In this study both manual and automatic calibrations have been done to estimate the best parameters set. All twenty catchments and Blue Nile at Kessie Bridge station and Ethiopian-Sudanese border were calibrated on data from 1995–2000, and validated with data sets from 2001–2004 using split sample test. For manual calibration and validation we used the EXCEL spread sheet model developed by Zhang et al. (2008). However in order to attain the optimal parameter set in the reasonable parameter space and to avoid the subjectivity of choosing parameters manually and for parameter uncertainty...
assessments we changed the EXCEL spreadsheet model into a MATLAB code. In automatic calibration the Generalized Likelihood Uncertainty Estimation framework (GLUE) by Beven and Binley (1992) was employed to constrain model parameters using the Nash and Sutcliffe efficiency as likelihood measure. A threshold value of 0.7 for all catchments has been considered for behavioral models. Monte Carlo simulations of 20,000 parameter sets have been randomly sampled to constrain the parameters in a feasible parameter space. For monthly model in this study, we also considered a multi-objective optimization within the GLUE framework using two objective functions directed towards high flows and low flows. Past research pointed out that calibration based on one single objective function often results in unrealistic representation of the hydrograph (Gupta et al., 1998; Fenicia et al., 2007). In a monthly model the following evaluation criteria have been used in this study:

\[
F_{HF} = \frac{\sum_{i=1}^{n} (Q_{\text{sim},i} - Q_{\text{obs},i})^2}{\sum_{i=1}^{n} (Q_{\text{obs},i} - \bar{Q}_{\text{obs}})^2} \quad (8)
\]

\[
F_{LF} = \frac{\sum_{i=1}^{n} [\ln(Q_{\text{sim},i}) - \ln(Q_{\text{obs},i})]^2}{\sum_{i=1}^{n} [\ln(Q_{\text{obs},i}) - \ln(\bar{Q}_{\text{obs}})]^2} \quad (9)
\]

Where: \( Q_{\text{sim},i} \) [mm/month], is the simulated streamflow at time \( i \), \( Q_{\text{obs},i} \) [mm/month], is the observed streamflow at time \( i \), \( n \) is the number of time steps in the calibration period and the over bar indicates the mean of observed streamflow. The objective function \( F_{HF} \) was selected to minimize the errors in high flows, and \( F_{LF} \) uses logarithmic values of streamflow and improves the assessment of the low flows.
4 Results and discussions

4.1 Annual water balance

The annual water balance has been computed for the twenty catchments of the Upper Blue Nile basin with the assumption that evaporation can be estimated from water availability and atmospheric demand. Equation (4) has been applied to compute E/P for the twenty catchments. In most of these catchments, Eq. (4) couldn’t predict the annual evaporation and stream flow adequately. The poor accuracy of the prediction is related to the effect of neglecting soil water storage (assuming dS/dt=0) and other error sources could influence the results as well, e.g. the uncertainty of measured rainfall and the estimation of potential evaporation, seasonality of rainfall, and non-stationary conditions of the catchment itself (e.g. land use/cover change).

For evaluating model performance, we used three statistical measures following recommendation by Legates and McCabe (1999). Using Eq. (4) we predict annual streamflow and evaporation and performance results are given in Table 2.

Figure 4 explains the application of Eq. (4) to predict regional long term annual mean water balance (1995–2004) of the twenty catchments plotted in one figure. It can be seen that the aridity index varies from 0.7 to 1.5, and the catchments fall between wet environment, when aridity index less than 1.0 and for dry environment when aridity index greater than 1.0. Sankarasubramania and Vogel (2003) classified the catchments in the US based on the aridity index range of 0–0.33 as humid, 0.33–1 as semi-humid, 1–2 as temperate, 2–3 as semi-arid, and 3–7 as arid. Accordingly, the Upper Blue Nile catchments can be classified as semi-humid to temperate.

Considering all twenty catchments, the regional mean annual water balance were adequately predicted by Eq. (4) with a single model parameter (w=1.8) the model gave reasonable performance for the Nash and Sutcliffe efficiency of 0.70, coefficient of determination 0.71 and mean absolute error of 144 mm/yr. As it can be seen in Fig. 4 different groups of catchments follow a unique curve with an independent w [–]
value. Hence, catchments have been grouped according to the patterns they follow the Fu’s curve, as Yang et al. (2007) used to classify 108 arid to semi-arid catchments in China. Moreover, Zhang et al. (2004) categorized catchments in Australia based on vegetation cover as forested and grassed catchments with a higher parameter $w$ values for forested catchments and a relatively less values of parameter $w$ for grassed catchments. For each group of catchments goodness-of-fit statistics as recommended by Legates and McCabe (1999) are shown in Table 3. Furthermore, the parameter $w$ was correlated well with some measurable climatic and catchment characteristics like wetness index, which is the ratio of mean annual precipitation to mean annual potential evaporation, catchment area, slope, drainage density, and aridity index. The correlation showing parameter $w$ with different catchment characteristics and climatic indices is given in Table 4.

From calibrated results it is noted that the parameter $w$ ranges between 1.4 and 3.6. This may suggest that the Upper Blue Nile catchments under consideration exhibit different catchment characteristics. Zhang et al. (2004) pointed out that smaller values of $w$ are associated with high rainfall intensity, seasonality, steep slope and lower plant available soil water storage capacity. However, it is difficult to represent these characteristics explicitly in a simple model (Zhang et al., 2004; Yang et al., 2007). Typically the result of the analysis from group-1 catchments in our study shows that a larger fraction of precipitation becomes surface runoff and it resulting in lower evaporation ratios.

### 4.2 Modeling streamflow at monthly time scale

Modeling at finer time scale (monthly and daily) requires the inclusion of soil moisture dynamics to accurately estimate the water balance. In a top down modeling approach, model complexity has to be increased, when deficiencies of the model in representing the catchment behavior is encountered (Jothityangkoon et al., 2001; Atkinson et al., 2002; Montanari et al., 2006; Zhang et al., 2008). In order to improve the prediction of the annual model, we computed the monthly dynamics of the water balance using
the model of Zhang et al. (2008). The applicability of this model was tested before on 256 Australian catchments in variable time scales and the model performed well in predicting streamflow.

4.2.1 Model calibration and validation

In this research both manual and automatic calibrations have been carried out. The monthly streamflows were calibrated over the period 1995–2000 and validated over the period of 2001–2004. In manual calibration and validation the Nash and Sutcliffe coefficient efficiency was used as a performance measure. The main objective of calibration is finding the optimal parameter set that maximizes or minimizes the objective function for the intended purposes. In the parameter identification process, different parameter sets were randomly sampled from a priori feasible parameter space shown in Table 5.

The dynamic water balance model was calibrated and validated for the twenty catchments and also at Kessie Bridge station (catchment area 64,252 km²) and at the Ethiopian-Sudanese border (catchment area 173,686 km²) to test the ability of the model in large spatial scale. During calibration Nash and Sutcliffe coefficients of efficiencies were obtained in the range of 0.52–0.95 during high flows and 0.33–0.93 during low flows. Similarly, during the validation period Nash and Sutcliffe efficiencies were obtained in the range of 0.55–0.95 during high flows and 0.12–0.91 during low flows. The model result reveals that during calibration, the model gave reasonable results in most of the catchments including the simulation at larger scale at Kessie Bridge station (NSE=0.95) and at the Ethiopian-Sudanese border (NSE=0.93). However, during the validation period in some catchments the low flows were not captured well by the model. Figure 5 shows the plot of observed and predicted streamflows using automatic calibration for well performing meso-scale catchments and Blue Nile at the larger scale at Kessie Bridge station and Ethio-Sudanese border.

The optimal parameter values obtained using GLUE framework together with parameters obtained manually are presented in Table 6. It is clearly demonstrated that
the parameter values differ and the performance of the model improved using a GLUE framework. The parameter $\alpha_1$ in majority of the catchments shows that the rainfall amount retained by the catchments is not significant and in studied catchments surface runoff is dominant. The evaporation efficiency parameter $\alpha_2$ values are higher in some catchments and it implies that higher partitioning of available water into evaporation.

The dotty plots used to map the parameter value and their objective function values as a means of assessing the identifiability of parameters are shown in Fig. 6. It can be seen that most of the parameters in the Upper Blue Nile catchments are reasonably well identifiable; however, the recession constant $d$ exhibit poor identifiability in majority of the catchments. Figure 7 shows the model performance in the objective function space of Pareto-optimal fronts resulting from the dynamic water balance model. From multi-objective optimization point of view, the Pareto-optimal solutions are all equally important to achieve a better model simulation. The Pareto based approach is also important to compare different model structures in a way that model improvement can be attained as the Pareto-optimal front progressively moves towards the origin of the objective function space (Fenicia et al., 2007).

From Fig. 7 one can notice that for different catchments the Pareto-optimal set of solution approaches to the origin differently. It is demonstrated that the model structure performing well at the larger scale than meso-scale catchments. As the objective function values get closer to the origin, the model structure better represent the hydrologic system.

5 Conclusions

The Upper Blue Nile catchment water balance has been analyzed at different temporal and spatial scales using Budyko’s framework. The analysis included water balance at mean annual, annual and monthly time scale, for meso- to large scale catchments. The Budyko-type curve (Fu, 1981) was applied to explore the first order control based on available water and energy over mean annual and annual time scales. The results
demonstrated that predictions are not well in the majority of the catchments at annual time scale. This implies that, at annual scale, the water balance is not dominated only by precipitation and potential evaporation at annual scale. Thus, increased model complexity to a monthly time scale is a must for a realistic simulation of the catchment water balance, and that is by including the effects of soil moisture dynamics. Parameters were identified using the Generalized Likelihood Uncertainty Estimation (GLUE) framework and the results showed that most of the parameters are identifiable and the model is capable of simulating the observed streamflow quite well. The applicability of this model was tested earlier in 250 catchments in Australia with different rainfall regimes across different geographical regions and results were encouraging (Zhang et al., 2008; Zhao et al., 2009). Similarly, in our Upper Blue Nile case, the model performs well in simulating the monthly streamflow of twenty catchments. Based on Budyko’s hypothesis, the dynamic water balance model on variable time scale is a parsimonious model.

With only four parameters it has the advantage of minimal equifinality. Despite the uncertainties in input data, parameters and model structure, the model gives reasonable results for the Upper Blue Nile catchments. It is recommended and suggested that future work should focus on the regionalization of the optimal parameter sets given in this paper for prediction of streamflow in ungauged catchments in the Upper Blue Nile basin.

**Appendix**

In partitioning rainfall, referring to Fig. 3, rainfall $P(t)$ at time step $t$ partitions into direct runoff $Q_d(t)$ and $X(t)$. $X(t)$ is a lumped water balance component known as catchment rainfall retention which consists of the amount of retained water for catchment water storage $dS/dt$, $E(t)$ and recharge $R(t)$.

$$P(t) = Q_d(t) + X(t) \quad (A1)$$
Where \( P(t) \), and \( Q_d(t) \) are monthly rainfall and direct runoff, respectively.

Analogous to Budyko’s hypothesis, Zhang et al. (2008) defined the demand limit for \( X(t) \) to be the sum of \( dS/dt \) and potential evaporation \( (E_0) \), which is termed as \( X_0(t) \) and the supply limit as \( P(t) \). If the sum of available storage capacity and potential evaporation is very large as compared to the supply, then \( X(t) \) approaches \( P(t) \) whereas if the sum of available storage capacity and potential evaporation is smaller than the supply, \( X(t) \) approaches \( X_0(t) \). This postulate can be written as:

\[
\frac{X(t)}{P(t)} \rightarrow 1, \text{ as } \frac{X_0(t)}{P(t)} \rightarrow \infty, \text{ for very dry conditions} \quad \text{(A2)}
\]

\[
X(t) \rightarrow X_0(t), \text{ as } \frac{X_0(t)}{P(t)} \rightarrow 0, \text{ for very wet conditions} \quad \text{(A3)}
\]

The catchment rainfall retention can be expressed as:

\[
X(t) = \begin{cases} 
P(t)F \left[ \frac{X_0(t)}{P(t)}, \alpha_1 \right], & P(t) \neq 0 \\
0, & P(t) = 0 
\end{cases} \quad \text{(A4)}
\]

where \( F [-] \) is the Fu-curve Eq. (4) and \( \alpha_1 \) is the retention efficiency, i.e. larger \( \alpha_1 \) value result in more rainfall retention and less direct runoff.

From Eqs. (A1) and (A4) the direct runoff is calculated as:

\[
Q_d = P(t) - X(t) \quad \text{(A5)}
\]

The water availability \( W(t) \) for partitioning can be computed as:

\[
W(t) = E(t) + S(t) + R(t) \quad \text{(A6)}
\]

Sankarasubramania and Vogel (2002) defined the evaporation opportunity as maximum water that can leave the basin as evaporation at any given time \( t \).

\[
Y(t) = E(t) + S(t) \quad \text{(A7)}
\]
For the partition of available water, the demand limit for $Y(t)$ is the sum of the soil water storage capacity ($S_{\text{max}}$) and potential evaporation $E_0(t)$ termed as $Y_0(t)$, while the supply limit is the available water $W(t)$. Analogous to Budyko’s hypothesis, Zhang et al. (2008) postulated as:

$$\frac{Y(t)}{W(t)} \rightarrow 1 \text{ as } \frac{Y_0(t)}{W(t)} \rightarrow \infty, \text{ for dry conditions}$$  \hspace{1cm} \text{(A8)}

$$Y(t) \rightarrow Y_0(t) \text{ as } \frac{Y_0(t)}{W(t)} \rightarrow 0, \text{ for wet conditions}$$  \hspace{1cm} \text{(A9)}

The evaporation opportunity can be computed as:

$$Y(t) = \begin{cases} W(t)F\left[\frac{E_0(t) + S_{\text{max}}}{W(t)}, \alpha_2\right], & W(t) \neq 0 \\ 0, & W(t) = 0 \end{cases}$$  \hspace{1cm} \text{(A10)}

From Eqs. (A6) and (A10) the groundwater recharge $R(t)$ can be estimated as:

$$R(t) = W(t) - Y(t)$$  \hspace{1cm} \text{(A11)}

To estimate the evaporation, the demand limit is $E_0(t)$ and the supply limit is $W(t)$, then $E(t)$ can be computed as:

$$E(t) = \begin{cases} W(t)F\left[\frac{E_0(t)}{W(t)}, \alpha_2\right], & W(t) \neq 0 \\ 0, & W(t) = 0 \end{cases}$$  \hspace{1cm} \text{(A12)}

Where $\alpha_2$ is a model parameter representing evaporation efficiency. Equation (A10) is similar with Eq. (4) with the exception that precipitation is replaced by water availability $W(t)$ to take into account the effect of catchment water storage.

Zhang et al. (2008) also emphasized that Eqs. (A8) and (A10) use the same parameter. This is due to the fact that the groundwater recharge is essentially determined from...
evaporation efficiency. As evaporation efficiency becomes larger i.e for large values of $\alpha_2$ the recharge is diminished.

From Eq. (A5), the soil water storage can be computed as:

$$S(t) = Y(t) - E(t)$$  \hspace{1cm} (A13)

The groundwater storage is treated as linear reservoir. Thus the base flow $Q_b(t)$ and the ground water storage $G(t)$ can be modeled as:

$$Q_b(t) = dG(t - 1)$$  \hspace{1cm} (A14)

$$G(t) = (1 - d\Delta t)G(t - 1) + R\Delta t$$  \hspace{1cm} (A15)

Where: $Q_b$, $G$, and $d$ are the baseflow [mm/month], groundwater storage [mm/month], and reservoir constant [1/month], respectively.

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**References**


Water balance modeling using a top down approach
S. Tekleab et al.

Legates, D. R. and McCabe, G. J.: Evaluating the use of “goodness-of-fit” measures in hydro-
Mishra, A. and Hata, T.: A grid based runoff generation and flow routing model for the Upper
Montanari, L., Sivapalan, M., and Montanari, A.: Investigation of dominant hydrological pro-
cesses in a tropical catchment in a monsoonal climate via the downward approach, Hydrol.
Ol’dekop, E. M.: On evaporation from the surface of river basins, Transactions on Meteorologi-
cal Observations. Lur-evskogo, Univ. of Tartu, Tartu, Estonia, 1911.
Pike, J. G.: The estimation of annual runoff from meteorological data in a tropical climate, J.
Potter, N. J. and Zhang, L.: Interannual variability of catchment water balance in Australia, J.
Schreiber, P.: ¨Uber die Beziehungen zwischen dem Niederschlag und der Wasserf ¨uhrung der
Fl¨usse in Mitteleuropa, Meteorol. Z., 21, 441–452, 1904.
Setegne, S. G., Srinivasan, R., and Dargahi, B.: Hydrological modelling in the Lake Tana basin,
Steenhuis, T., Collic, A., Easton, Z., Leggesse, E., Bayabil, H., Whitee, E., Awulachew, S.,
Adgo, E., and Ahmed, A.: Predicting discharge and erosion for the Abay Blue Nile with a
Teferi, E., Uhlenbrook, S., Bewket, W., Wenninger, J., and Simane, B.: The use of remote sens-
ing to quantify wetland loss in the Choke Mountain range, Upper Blue Nile basin, Ethiopia,
Turc, L.: Le bilan d’eau des sols relation entre la precipitation l’evappiration et l’ecoulement,
Development Report for Ethiopia, UN-WATER/WWAP/2006/7, World Water Assessment pro-
Yang, D., Shao, W., Yeh, P. J. F., Yang, H., Kanae, S., and Oki, T.: Impact of vegetation cover-
Yang, D., Sun, F., Liu, Z., Cong, Z., Ni, G., and Lei, Z.: Analyzing spatial and temporal variability
of annual water-energy balance in nonhumid regions of China using the Budyko hypothesis,
Zhang, L., Dawes, W. R., and Walker, G. R.: Predicting the effect of vegetation changes on
berra, ACT, 1999.
Zhang, L., Dawes, W. R., and Walker, G. R.: Response of mean annual evapotranspiration to
Zhang, L., Hickel, K., and Dawes, W. R.: A rational function approach for estimating mean
2004.
variable time scales based on the Budyko framework – model development and testing, J.
Zhao, F. F. and Lu, Z.: Effects of vegetation cover change on streamflow at a range of spatial
scales, 18th World IMACS /MODSIM Congress cairns, Australia, 13–17 July 2009.

<table>
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<th>Catchment number</th>
<th>Catchment name</th>
<th>Area (km²)</th>
<th>Long term mean annual values (mm/yr)</th>
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<th>E₀</th>
<th>Q</th>
<th>E</th>
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Table 2. Model performance using Eq. (4) for annual runoff and annual evaporation during the period (1995–2004).

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<th>Performance measure</th>
<th>Runoff</th>
<th>Evaporation</th>
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<tr>
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<td>min</td>
<td>max</td>
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<td>Nash and Sutcliffe efficiency</td>
<td>-2.37</td>
<td>0.84</td>
</tr>
<tr>
<td>RMSE (mm/yr)</td>
<td>62.50</td>
<td>298.30</td>
</tr>
<tr>
<td>MAE (mm/yr)</td>
<td>50.00</td>
<td>263.90</td>
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Table 3. Average goodness-of-fit-statistics (Nash and Sutcliffe efficiency, root mean squared error, and mean absolute error) for prediction of regional long term mean annual streamflow and mean annual evaporation using Fu’s curve.

<table>
<thead>
<tr>
<th></th>
<th>NSE</th>
<th>RMSE (mm/yr)</th>
<th>MAE (mm/yr)</th>
<th>Calibrated Parameter “w”</th>
</tr>
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<tbody>
<tr>
<td>All catchments</td>
<td>0.70</td>
<td>177.51</td>
<td>147.10</td>
<td>1.8</td>
</tr>
<tr>
<td>Group-1 catchments</td>
<td>0.87</td>
<td>76.36</td>
<td>64.03</td>
<td>1.5</td>
</tr>
<tr>
<td>Group-2 catchments</td>
<td>0.97</td>
<td>57.22</td>
<td>48.39</td>
<td>1.9</td>
</tr>
<tr>
<td>Group-3 catchments</td>
<td>0.85</td>
<td>58.23</td>
<td>42.93</td>
<td>2.5</td>
</tr>
</tbody>
</table>
Table 4. Correlation matrix of parameter $w$ with climatic and catchment characteristics.

<table>
<thead>
<tr>
<th></th>
<th>Group-1 catchments $w$</th>
<th>Group-2 catchments $w$</th>
<th>Group-3 catchments $w$</th>
<th>The whole twenty catchments $w$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
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<tr>
<td>Wetness Index</td>
<td>$-0.308$</td>
<td>0.028</td>
<td>$-0.064$</td>
<td>$-0.071$</td>
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<td>Catchment area</td>
<td>$-0.055$</td>
<td>$-0.324$</td>
<td>0.982</td>
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<tr>
<td>$\tan\beta$</td>
<td>0.411</td>
<td>0.429</td>
<td>$-0.097$</td>
<td>$-0.004$</td>
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<td>Drainage Density</td>
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<td>$-0.396$</td>
<td>$-0.758$</td>
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<td>Aridity Index</td>
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<td>$-0.290$</td>
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<td>$-0.006$</td>
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Table 5. Ranges of parameter values for the catchments modeled in the Upper Blue Nile basin.

<table>
<thead>
<tr>
<th>Catchment</th>
<th>$S_{\text{max}}$ [mm]</th>
<th>Lower/Upper bound</th>
<th>$\alpha_1$ [-]</th>
<th>$\alpha_2$ [-]</th>
<th>$d$ [1/month]</th>
</tr>
</thead>
<tbody>
<tr>
<td>GilgelAbay, Koga, Birr, Fetam, Neshi</td>
<td>100–600</td>
<td>0–1</td>
<td>0–1</td>
<td></td>
<td>0–1</td>
</tr>
<tr>
<td>Dura, GilgelBeles, Gumera, Megech, Rib, Robigumero, Robijida, Didessa</td>
<td>100–600</td>
<td>0.1–0.75</td>
<td>0.1–0.75</td>
<td>0–1</td>
<td></td>
</tr>
<tr>
<td>Chemoga, Beles, Guder</td>
<td>100–600</td>
<td>0.1–0.85</td>
<td>0.1–0.85</td>
<td>0–1</td>
<td></td>
</tr>
<tr>
<td>Muger, Temcha, Uke, Jedeb</td>
<td>100–600</td>
<td>0–0.9</td>
<td>0–0.8</td>
<td>0–1</td>
<td></td>
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</tbody>
</table>
Table 6. Comparison of automatic (GLUE) single objective and manually calibrated parameters of twenty selected Upper Blue Nile catchments during the period of (1995–2000).

<table>
<thead>
<tr>
<th>Catchment</th>
<th>Optimized parameters (GLUE)</th>
<th>Manually calibrated parameters</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>$S_{\text{max}}$</td>
<td>$\alpha_1$</td>
</tr>
<tr>
<td>Beles</td>
<td>538.09</td>
<td>0.46</td>
</tr>
<tr>
<td>Birr</td>
<td>253.62</td>
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</tr>
<tr>
<td>Chemoga</td>
<td>216.40</td>
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<td>Didessa</td>
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<tr>
<td>Upper Blue Nile</td>
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<tr>
<td>at the border</td>
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<td></td>
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Fig. 1. Study area. Numbers indicate modeled catchments.
Fig. 2. Budyko curve representing evaporation ratio is a function of aridity index.
Fig. 3. Schematic diagram of dynamic monthly water balance model structure.
Fig. 4. Ratio of mean annual evaporation ratio \( \left( \frac{\bar{E}}{\bar{P}} \right) \) as a function of Aridity Index \( \left( \frac{\bar{E}_0}{\bar{P}} \right) \) for different values of “w” using Fu (1981) curve.
Fig. 5. Observed and simulated streamflow during calibration period (1995–2000).
Fig. 6. GLUE dotty plots of some meso-scale catchments and Blue Nile at larger scale at Kessie Bridge and Ethio-Sudanese border.
Fig. 7. Pareto-optimal fronts of parameter sets at different meso-scale catchments and Blue Nile at the larger scale based on the selected objective functions.