Extended Water Allocation System Model Formulation

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This supplemental material presents the complete optimization program for the extended Water Allocation System model in standard form: the objective function followed by nine primary and five additional constraints. Notation follows Fisher et al. (2005) and Rosenberg et al. Model extensions include and allow return flows from agriculture, brine waste from desalination, multiple water quality types to meet a minimum in-stream flow requirement, fixed-increment infrastructure capacity expansions, and are shown in constraints 1 and 6 – 9.

Objective Function

\[
\max Z = \sum_e \text{prob}_e \left[ \frac{b_{ide} \left( \sum_q QD_{idqe} \right)^{a_{ide}+1}}{\alpha_{ide} + 1} - \sum_{i} \sum_{q} \sum_{s} c_{s_{qps}} QS_{qs} - \sum_{q} \sum_{p} \sum_{j} c_{tr_{qjpe}} QTR_{qjpe} \right. \\
- \sum_{i} \sum_{d} \sum_{q} c_{e_{phe}} QDC_{ide} - \sum_{i} \sum_{d} \sum_{q} c_{e_{ide}} QDC_{ide} \bigg] \\
- \sum_{p} \sum_{j} \sum_{q} c_{extr_{pjg}} XTR_{qj} - \sum_{i} \sum_{q} \sum_{s} c_{xs_{qgs}} XS_{qs} - \sum_{i} \sum_{q} \left( c_{xtw_{iq}} XTW_{iq} + c_{xl_{iq}} XL_{iq} \right) \\
- \sum_{i} \sum_{d} c_{xcon_{id}} XCON_{id}
\]

Subject to (constraints)

1. Continuity equation (mass balance) at each district for each quality in each event

\[
\sum_d QDC_{idqe} = \left( \sum_i QS_{iqse} + \sum_d QTW_{idqe} + \sum_p QTR_{qpe} - \sum_p QTR_{qpe} \right) \cdot \left( 1 - d_{l_{iq}} - XL_{iq} \right), \forall i, q, e
\]

2. Continuity equation (mass balance) at each node for each quality in each event
1. \[
\sum_{p} QTR_{qpe} = \sum_{p} QTR_{qpe}, \forall n, q, e
\]

2. 3. Treated waste-water comes from water demanded

3. \[
\sum_{q} QTW_{ldqe} = PR_{ld} \sum_{q} QDC_{ldqe}, \forall i, d, e
\]

4. 4. Lower limit on demand for each water use sector in each district in each event

5. \[
\sum_{q} QD_{ldqe} \geq \left( \frac{P_{\text{max}}}{b_{ldc}} \right)^{\frac{1}{\alpha_{ae}}}, \forall i, d, e
\]

6. 5. User conservation reduces real water demanded without loss of economic benefit

7. \[
QDC_{ldqe} = QD_{ldqe} \cdot \left( 1 - p\text{con}_{0, id} - XCON_{i, d} \right), \forall i, d, q, e
\]

8. 6. Brine waste generated is a fraction of desalinated water

9. \[
QB_{qse} = \sum_{q, s, e} QS_{qse, qse}, \forall i, q, e, s = \text{desal}
\]

10. 7. Supply expansions limited to fixed increments

11. \[
X_{i} = qS_{\text{int,erval} i} \cdot XSLEV_{i}, \forall i, q, s
\]

12. 8. Conveyance expansions limited to fixed increments

13. \[
XTR_{qpe} = qtr_{\text{int,erval} qpe} \cdot XTRLEV_{qpe}, \forall q, p, j
\]

14. 9. Expected flow of one or more water quality types along a conveyance link must meet a minimum required flow

15. \[
\sum_{e} \left( prob_{e} \cdot \sum_{q, r, (p, j)} QTR_{qpe} \right) \geq qtr_{\text{min,flow} p, j}, \forall p, j
\]

16. 17. With the following bounds

17. \[
Q_{s_{qpe}} \leq qS_{0, qpe} + XS_{qpe}, \forall i, q, s, e \quad qS_{0, qpe} + XS_{qpe} \leq qS_{\text{max, qpe}}, \forall i, q, s
\]

18. \[
PR_{ldc} \leq pr_{\text{max, id}}, \forall i, d, e \quad QS_{qpe} \leq qS_{\text{flow, qpe}}, \forall i, q, s, e
\]

19. \[
QTR_{qpe} \leq qtr_{0, qpe} + XTR_{qpe}, \forall q, p, j, e \quad qtr_{0, qpe} + XTR_{qpe} \leq qtr_{\text{max, qpe}}, \forall q, p, j
\]
\[ QTR_{qij} \geq qtr_{\min \, q}, \forall q, p, j, e \quad \text{pcon}_{0 \, id} + XCON_{id} \leq \text{pcon}_{\max \, id}, \forall i, d \]

\[ \sum_d QTW_{idq} \leq qtw_{0 \, iq} + XTW_{iq}, \forall i, q \quad dl_{0 \, iq} + XL_{iq} \leq dl_{\max \, iq}, \forall i, q \]

and all variables positive.

Variables are:

- \( Z \) = net benefit from water in millions of dollars;
- \( QB_{iqe} \) = brine waste volume generated in district \( i \) in event \( e \) of quality \( q \) in \( 10^6 \, \text{m}^3 \);
- \( QS_{iqse} \) = volume supplied by source \( s \) of quality \( q \) in district \( i \) in event \( e \) in \( 10^6 \, \text{m}^3 \);
- \( QD_{idqe} \) = volume of quality \( q \) demanded by sector \( d \) in district \( i \) in event \( e \) in \( 10^6 \, \text{m}^3 \);
- \( QDC_{idqe} \) = volume demanded after conservation in \( 10^6 \, \text{m}^3 \);
- \( QTR_{qipje} \) = volume of water quality \( q \) transferred from \( p \) to \( j \) in event \( e \) in \( 10^6 \, \text{m}^3 \);
- \( QTW_{idqe} \) = sector \( d \) wastewater treated to quality \( q \) in district \( i \) in event \( e \) in \( 10^6 \, \text{m}^3 \);
- \( PR_{dse} \) = percent of sector \( d \) wastewater treated in district \( i \) in event \( e \) in fraction;
- \( XS_{iqs} \) = Size of supply expansions for source \( s \) of quality \( q \) in district \( i \) in \( 10^6 \, \text{m}^3 \);
- \( XSLEV_{i, q, s} \) = Number of supply expansions implemented at district \( i \) of quality \( q \) for source \( s \) in integers \([0, 1, 2, \ldots]\);
- \( XTR_{qpi} \) = Size of conveyance expansions from point \( p \) to \( j \) of quality \( q \) in \( 10^6 \, \text{m}^3 \);
- \( XTRLEV_{qpi} \) = Number of conveyance expansions implemented of quality \( q \) from \( p \) to \( j \) in integers \([0, 1, 2, \ldots]\);
- \( XTW_{id} \) = Size of wastewater reuse plant expansions in district \( i \) for quality \( q \) \( 10^6 \, \text{m}^3 \);
- \( XL_{iq} \) = Leak reduction program expansion in district \( i \) for quality \( q \) in fraction;
- \( XCON_{id} \) = User conservation program expansion in district \( i \) for quality \( q \) in fraction;

Indices are:

- \( p \) = points (districts and nodes);
- \( i \) = district;
n = nodes;

d = water sector (urban, industrial, or agricultural);

s = supply source or step;

q = water quality type (fresh, recycled water);

e = events (water supply availability / demand)

Parameters are:

$\alpha_{ide}$ = exponent of inverse demand function for demand $d$ in district $i$ in event $e$;

$b_{ide}$ = coefficient of inverse demand curve for demand $d$ in district $i$ in event $e$;

$bf_{iq}$ = brine fraction that represents the volume of brine generated for each 1 m$^3$ of desalinated water produced [unitless];

$ce_{ide}$ = unit environmental cost of water discharged by demand sector $d$ in district $i$ in event $e$ in $\$ m^{-3}$;

$cr_{idqe}$ = unit cost to treat sector $d$ waste in district $i$ to quality $q$ in event $e$ in $\$ m^{-3}$;

$cs_{iqse}$ = unit cost to supply new water of quality $q$ from source $s$ in district $i$ in event $e$ in $\$ m^{-3}$;

$ct_{qjpse}$ = unit cost to transport water quality $q$ from point $p$ to $j$ in event $e$ in $\$ m^{-3}$;

$cx_{siqs}$ = annualized capital cost to expand source $s$ of quality $q$ in district $i$ in $\$ m^{-3}$;

$cx_{triqs}$ = annualized capital cost to expand conveyance capacity of quality $q$ from point $p$ to $j$ in $\$ m^{-3}$;

$cx_{twiq}$ = annualized capital cost to expand wastewater treatment capacity to quality $q$ in district $i$ in $\$ m^{-3}$;

$cx_{conid}$ = annualized capital cost to expand user conservation program in district $i$ for sector $d$ in $\$ fraction^{-1}$;

$cx_{lq}$ = annualized capital cost to expand leak reduction program in district $i$ for quality $q$ in $\$ fraction^{-1}$;
\( dq_i(q) \) = set of source water quality types that, when desalinated in district \( i \), generate brine of quality \( q \) [unitless];

\( p_e \) = probability of event \( e \) in fraction;

\( p_{\text{max id}} \) = maximum price of water from demand sector \( d \) in district \( i \) in $ \text{m}^{-3}$;

\( pr_{\text{max id}} \) = maximum percent of waste from demand sector \( d \) in district \( i \) that can be treated in fraction;

\( qs_0_{iqs} \) = existing capacity of source \( s \) of quality \( q \) in district \( i \) in \( 10^6 \text{ m}^3 \);

\( qs_{\text{flow iqe}} \) = availability of source \( s \) of quality \( q \) in district \( i \) in event \( e \) in \( 10^6 \text{ m}^3 \);

\( qs_{\text{interval iqs}} \) = fixed interval to expand source capacity \( s \) of quality \( q \) in district \( i \) in \( 10^6 \text{ m}^3 \);

\( qs_{\text{max iqs}} \) = maximum capacity for source \( s \) of quality \( q \) in district \( i \) in \( 10^6 \text{ m}^3 \);

\( qt(p,j) \) = set of water quality types whose flows can count towards the minimum required flow along the conveyance link from \( p \) to \( j \) [unitless];

\( qtr_0_{qpj} \) = existing conveyance capacity for quality \( q \) from point \( p \) to \( j \) in \( 10^6 \text{ m}^3 \);

\( qtr_{\text{interval qpj}} \) = fixed interval to expand conveyance capacity of quality \( q \) from \( p \) to \( j \) in \( 10^6 \text{ m}^3 \);

\( qtr_{\text{min qpj}} \) = minimum required flow of quality \( q \) from point \( p \) to \( j \) in \( 10^6 \text{ m}^3 \);

\( qtr_{\text{minmq pqj}} \) = minimum required flow from point \( p \) to \( j \) that multiple water quality types must satisfy on average in \( 10^6 \text{ m}^3 \);

\( qtw_0_{iq} \) = existing capacity to treat water to quality \( q \) at district \( i \) in \( 10^6 \text{ m}^3 \);

\( pcon_0_{id} \) = reduction in use associated with existing conservation programs for sector \( d \) in district \( i \) in fraction;

\( pcon_{\text{max id}} \) = maximum reduction in use from conservation programs for sector \( d \) in district \( i \) in fraction;

\( dl_0_{iq} \) = existing leak rate for quality \( q \) in district \( i \) in fraction;

\( dl_{\text{max id}} \) = maximum reduction in leakage rate for quality \( q \) in district \( i \) in fraction;

**Additional Constraints**

10. Total demand consists of paid water and unaccounted-for losses
\[ QDC_{idqe} = QD_{paid_{idqe}} + r_{q_{e}} P_{r_{unpaid_{ide}}}, \forall i, d, q, e. \]

11. Demand for certain water quality types must be less than a specified quantity

\[ \sum_{d} QDC_{idqe} \leq q_{rec_{max_{iq}}}, \forall i, q, e \]

12. Demand for certain water quality types must be less than a specified percentage of total demand.

\[ \sum_{d} QDC_{idqe} \leq p_{rec_{max_{iq}}} \sum_{d, q_{e}} QD_{id_{q_{2}}, e}, \forall i, q, e \]

13. Use from a pool of sources must be less than a specified quantity

\[ \sum_{q} indep_{cips} QS_{idqe} \leq q_{shared_{ce}}, \forall c, e \]

14. Minimum required allocation to each sector

\[ \sum_{q} QDC_{idqe} \geq q_{required_{ide}}, \forall i, d, e \]

**References**
