Estimate soil moisture using trapezoidal relationship between remotely sensed land surface temperature and vegetation index

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Abstract

The trapezoidal relationship between surface temperature ($T_s$) and vegetation index (VI) was used to estimate soil moisture in the present study. An iterative algorithm is proposed to estimate the vertices of the $T_s$~VI trapezoid theoretically for each grid, and then WDI is calculated for each grid using MODIS remotely sensed measurements of surface temperature and enhanced vegetation index (EVI). The capability of using WDI based on $T_s$~VI trapezoid to estimate soil moisture is evaluated using soil moisture observations and antecedent precipitation in the Walnut Gulch Experimental Watershed (WGEW) in Arizona, USA. The result shows that, $T_s$~VI trapezoid based WDI can well capture temporal variation in surface soil moisture, but the capability of detecting spatial variation is poor for such a semi-arid region as WGEW.

1 Introduction

In 1980s’, it was found that, land surface temperature ($T_s$) and the fraction of vegetation cover, which is represented by vegetation indices (e.g., NDVI), typically show a strong negative relationship (e.g., Goward et al., 1985; Nemani and Running, 1989). Such a relationship has been widely used to investigate the moisture condition of land surfaces. Several studies focused on the slope of the $T_s$/NDVI curve for providing information on vegetation and moisture conditions at the surface (e.g., Smith and Choudhury, 1991; Nemani et al., 1993). Their approach was later extended to use the information in the $T_s$/VI scatter-plot space, whose envelope is considered to be in either a triangular shape (e.g., Price, 1990; Carlson et al., 1994), or a trapezoid shape (e.g., Moran et al., 1994).

The idea of triangle $T_s$/VI space has been used to develop the so called “triangle method”, and has been applied by a lot of researchers (e.g., Gillies et al., 1997; Sandholt et al., 2002; Margulis et al., 2005; Tang et al., 2010). The “triangle” method fits the scatter-plot of observed vegetation index (VI) and land surface temperature ($T_s$) using...
a triangle. The central assumption of the triangle method is that, given a large number of pixels reflecting a full range of soil surface wetness and fractional vegetation cover, sharp boundaries (edges) in the data reflect real physical limits: i.e., bare soil, 100% vegetation cover, and lower and upper limits of the surface soil water content, e.g., completely dry or wet (field capacity), respectively. The dry and wet edges ultimately intersect at a (truncated) point at full vegetation cover. Then, based on the triangle, the relative value of surface soil water content and the surface energy fluxes at each pixel can be defined in terms of its position within the triangle. The advantage of the triangle method is its independence of ancillary data. The approach, however, has difficulty in defining the dry and wet edge, especially the dry edge. Even with a large number of remotely sensed observations, the boundaries of the triangle space are still hard to be well established, because on one hand, there are situations when VI–Ta points scatter in a close range such as during rainy season or in areas with a narrow VI range; on the other hand, the T∞–VI relationship is much more complicated at large scale than at local scale and may vary at different parts due to heterogeneity in land surface properties and atmospheric forcing. Furthermore, because the triangle space is established empirically, the soil moisture estimates according to such an empirical triangle using an image at one time are hard to be compared with those at another time.

Moran et al. (1994) proposed the idea of vegetation index/temperature (VIT) trapezoid, and the water deficit index (WDI) for evaluating evapotranspiration rates of both full-cover and partially vegetated sites. However, very few applications were found in the literature based on the idea of trapezoid T∞/VI space for estimating soil moisture. In the present paper, we will extend the idea of VIT trapezoid and WDI, for estimating soil moisture estimation using MODIS products. The method, referred to as the trapezoid method, will be described in detail in Sect. 2. Then the method will be applied to the Walnut Gulch Experimental Watershed in Arizona, USA, for which, the data used and data pre-process will be introduced in Sects. 3 and 4, and the results will be presented in Sect. 5. Finally, some conclusions will be drawn in Sect. 6.

2 Trapezoid method

2.1 The concept of \((T_s - T_a) - V_c\) trapezoid

Idso et al. (1981) and Jackson et al. (1981) proposed the CWSI (Crop Water Stress Index) for detecting plant water stress based on the difference between canopy and air temperature. It is designed for full-cover vegetated areas and bare soils at local and regional scales. To overcome the difficulty of measuring foliage temperature in partially vegetated fields, Moran et al. (1994) proposed to use the shape of trapezoid to depict the relationship between the surface temperature and air temperature difference \((T_s - T_a)\) vs. the fractional vegetation cover \((V_c)\), ranging from 0 for bare soil to 1 for full-cover vegetation) (Fig. 1), so as to combine spectral vegetation indices with composite surface temperature measurements to allow application of the CWSI theory to partially vegetated fields without a priori knowledge of the percent vegetation cover. Based on the trapezoid assumption and the CWSI theory, Moran et al. (1994) introduced the Water Deficit Index (WDI) for evaluating field evapotranspiration rates and relative field water deficit for both full-cover and partially vegetated sites. For a given pixel with measured surface temperature and air temperature difference, i.e., \((T_s - T_a)\), WDI is defined as:

\[
WDI = \frac{(T_s - T_a)_{\min} - (T_s - T_a)_r}{(T_s - T_a)_{\min} - (T_s - T_a)_{\max}} \tag{1}
\]

where \(T_a\) is air temperature; \(T_s\) is surface temperature; the subscripts min, max, and \(r\) refer to minimum, maximum, and measured values, respectively; and the minimum and maximum values of \((T_s - T_a)\) are interpolated linearly on the cold edge and warm edge of the \((T_s - T_a) - V_c\) trapezoid for the specific \(V_c\) value of the pixel. Graphically, WDI is equal to the ratio of distances AC/AB in Fig. 1.
2.2 Calculation of vertices of the \((T_s - T_a) - V_c\) trapezoid and its simplification: the \(T_s - V_t\) trapezoid

The theoretical basis of \((T_s - T_a) - V_c\) trapezoid is the energy balance equation, i.e.,

\[
R_n = G + H + \lambda E
\]

where, \(R_n\) is the net radiant heat flux density \((\text{W m}^{-2})\), \(G\) is the soil heat flux density \((\text{W m}^{-2})\), \(H\) is the sensible heat flux density \((\text{W m}^{-2})\), and \(\lambda E\) is the latent heat flux to the air \((\text{W m}^{-2})\) and \(\gamma\) the heat of vaporization \((\text{kJ/kg})\).

In their simplest forms, \(H\) and \(\lambda E\) can be expressed as:

\[
H = C_v (T_s - T_a)/r_a
\]

\[
\lambda E = [\Delta (R_n - G) + C_v (VPD)/r_a] / [\Delta + \gamma (1 + r_c/r_a)]
\]

where

- \(T_s\) and \(T_a\) are the land-surface and air temperature (K), respectively;
- \(C_v\) is the volumetric heat capacity of air \((1295.16 \text{ J K}^{-1} \text{ m}^{-3})\);
- \(VPD\) (vapor pressure deficit of the air) \((\text{hPa})\) is calculated as a difference between saturation vapor pressure \(e_s\) and actual vapour pressure \(e_a\) \((\text{hPa})\), given by (WMO, 2008)

\[
e_s = 6.112 \exp \left( \frac{17.62 T_s'}{T_s' + 243.12} \right)
\]

\(e_a = \mu e_s\), \((\mu\) is observed relative humidity\)

- \(\Delta\) is the slope of the curve of saturation water vapour pressure versus air temperature, calculated with (WMO, 2008)

\[
\Delta = 4098 \cdot e_s/\left(237.3 + T_s'\right)^2
\]

- \(\gamma\) the psychrometric constant \((\text{hPa}/\text{C})\), given by (WMO, 2008)

\[
\gamma = 0.646 + 0.0006 T_s'
\]

- \(r_a\) the aerodynamic resistance \((\text{s m}^{-1})\);
- \(r_c\) the canopy resistance to vapor transport \((\text{s m}^{-1})\);

Then, combining Eqs. (2), (3), and (4), we obtain the equation for temperature difference between air and land surface:

\[
(T_s - T_a) = \left[ r_a (R_n - G)/C_v \right] \left\{ \gamma \left( 1 + r_c/r_a \right) / [\Delta + \gamma \left( 1 + r_c/r_a \right)] \right\}
\]

As suggested by Moran et al. (1994), for the \((T_s - T_a) - V_c\) trapezoid, its four vertices correspond to (1) well-watered full-cover vegetation, (2) water-stressed full-cover vegetation, (3) saturated bare soil, and (4) dry bare soil. Using the energy balance equations, Moran et al. computed the values of the four vertices of the trapezoid as the following:

(1) For full-covered and well-watered vegetation (Point 1)

\[
(T_s - T_a)1 = \left[ r_a (R_n - G)/C_v \right] \left\{ \gamma \left( 1 + r_cm/r_a \right) / [\Delta + \gamma \left( 1 + r_cm/r_a \right)] \right\}
\]

where \(r_cm\) is the minimum canopy resistance, i.e., canopy resistance at potential evapotranspiration.
2.3 Calculation of the components in the formula for four vertices of $T_s \sim \text{VI}$ trapezoid

2.3.1 Aerodynamic resistance ($r_a$)

The water vapor aerodynamic resistance $r_a$ (s/m) can be estimated with the following equation (Brutsaert, 1982):

\[
r_a = \left[ \ln \left( \frac{z - d}{z_{0m}} \right) - \psi_m \right] \left[ \ln \left( \frac{z - d}{z_{0h}} \right) - \psi_h \right] / k^2 u_z
\]  

where

- $z$ is the height (m) above the surface at which $u_z$ and $T_a$ are measured (commonly 2 m);
- $u_z$ is wind speed (m s$^{-1}$), which could be measured directly;
- $d$ is displacement height (m), given by $d = 0.667h$, and $h$ is the height of vegetation (Garratt, 1992), which should be given as an input.
- $z_{0m}$ is the roughness lengths for momentum (m), given by $z_{0m} = h/8$ (Garratt, 1992). For bare soil surface, $z_{0m}$ is commonly taken to be 0.01 m (Shuttleworth and Wallace, 1985).
- $z_{0h}$ is the roughness lengths for heat (m), given by

\[
z_{0h} = z_{0m} / \exp \left( KB^{-1} \right)
\]  

Here, $KB^{-1}$ is a dimensionless parameter. Kustas et al. (1989) showed that $KB^{-1}$ is a linear function of the product of $u$ and $T_a - T_s$, given by

\[
KB^{-1} = S_{KB} \cdot u \cdot (T_a - T_s)
\]
where \( S_{KB} \) is an empirical coefficient, which varies somewhere between 0.05 and 0.25.

- \( k \) is the von Karman constant (\( k = 0.41 \));
- \( \psi_h \) and \( \psi_m \) are the stability corrections for heat and momentum (unitless). \( \psi_h \) and \( \psi_m \) are calculated differently depending on the atmospheric stability, which could be indicated by the Monin-Obukhov length \( L \), given by

\[
L = - \rho C_p u^2 T_a / (k g H)
\]

where \( g = 9.8 \text{ m/s}^2 \), \( k = 0.41 \), \( \rho \) is the air density \( \text{(kg/m}^3\text{)} \), \( C_p \) the air specific heat at constant pressure \( (1004 \text{ J kg}^{-1} \text{K}^{-1}) \), \( u_s = u_z k / [\ln(z/z_0m)] \).

For stable situations \((L > 0)\),

\[
\begin{cases}
\psi_m = -5 \left( z - z_{0m} \right) / L \\
\psi_h = -5 \left( z - z_{0h} \right) / L
\end{cases}
\]

For unstable conditions \((L \leq 0)\),

\[
\begin{cases}
\psi_m = 2 \ln \left( \frac{1 + x}{1 + x_0} \right) + \ln \left( \frac{1 + x^2}{1 + x_0^2} \right) - 2 \tan^{-1}(x) + 2 \tan^{-1} x_0 \\
\psi_h = 2 \ln \left( \frac{1 + y}{1 + y_0} \right)
\end{cases}
\]

where, \( x = \left[ 1 - 16(z - d) / L \right]^{1/4} \), \( x_0 = \left[ 1 - 16z_{0m} / L \right]^{1/4} \), \( y = \left[ 1 - 16(z - d) / L \right]^{1/2} \), \( y_0 = \left[ 1 - 16z_{0h} / L \right]^{1/2} \).

### 2.3.2 Net radiant heat flux density \((R_n)\)

Net radiation is defined as the difference between the incoming and outgoing radiation fluxes including both long- and shortwave radiation at the surface of Earth. Net radiant heat flux density \((R_n) \text{ (W m}^{-2}\text{)}\) can be expressed as:

\[
R_n = (1 - \alpha) R_s + \varepsilon_a \times \sigma \times T_a^4 - \varepsilon_s \sigma T_s^4
\]

where

- \( \alpha \) is surface shortwave albedo, which can be calculated as a combination of MODIS narrow band spectral reflectance values \((\alpha_1 \sim \alpha_7)\) (Liang et al., 1999), given by

\[
\alpha = 0.3973 \alpha_1 + 0.23821 \alpha_2 + 0.3489 \alpha_3 + 0.265 \alpha_4 + 0.1604 \alpha_5 - 0.0138 \alpha_6 + 0.0682 \alpha_7 + 0.0036
\]

- \( R_s \) is solar radiation, estimated jointly by solar constant, solar inclination angle, geographical location and time of year, atmospheric transmissivity, ground elevation, etc. The basic formula for estimating \( R_s \) is (Zillman, 1972):

\[
R_s = \frac{S_0 \cos^2 \theta}{1.085 \cos \theta + e_0 (2.7 + \cos \theta) \times 10^{-3} + 0.1}
\]

where \( S_0 \) is the solar constant at the atmospheric top \( (1367 \text{ w/m}^2) \), \( \theta \) the solar zenith angle, \( e_0 \) is the vapor pressure. In consideration of the effects of topography on the incident short-wave radiation \((R_s)\), the solar zenith angle \((\theta)\) is corrected using digital elevation model (DEM) data (Duffie and Beckman, 1991) with the following formula:

\[
\cos \theta = \sin (\delta) \sin (\phi) \cos (s) - \sin (\delta) \cos (\phi) \sin (s) \cos (r) + \cos (\delta) \cos (\phi) \cos (s) \cos (\omega) + \cos (\delta) \sin (\phi) \sin (s) \cos (r) \cos (\omega) + \cos (\delta) \sin (\gamma) \sin (s) \sin (\omega)
\]

where \( \phi \) is the latitude (positive in the north hemisphere); \( s \) is the slope, and \( r \) is the slope orientation, both derived from DEM; \( \delta \) is solar declination, and \( \omega \) solar hour angle, given by

\[
\delta = 0.409 \sin (2 \pi \cdot \text{DOY}/365 - 1.39)
\]
where DOY is the day of year, and \( t \) is the time when the satellite TERRA pass over the region.

\[- \varepsilon_a \text{ is the atmospheric emissivity estimated as a function of vapor pressure, given by Iziomon et al. (2003)}\]

\[\varepsilon_a = 1 - 0.35 \times \exp (-10 \times e_a/T_a)\]

\[- \varepsilon_s \text{ is surface emissivity often evaluated as a function of NDVI. For instance, } \varepsilon_s \text{ could be predicted for the 8–14\( \mu \)m spectral range from NDVI using } \varepsilon_s = 1.009 + 0.047 \ln (\text{EVI}) \text{ (Bastiaanssen et al., 1998). Among MODIS Land Surface Temperature and Emissivity products (MOD11), there are emissivity products for band 31 and 32, i.e. } \varepsilon_{31} \text{ and } \varepsilon_{32}. \text{ In the present study, we take the average of the two products to get surface emissivity, namely, } \varepsilon_s = (\varepsilon_{31} + \varepsilon_{32})/2.\]

In our algorithm, \( R_n \) is not directly solved with the Eq. (16), because \( T_s \) is considered as an unknown variable. Instead, we replace the term \( R_n \) in Eq. (6)–(9) with the Eq. (10) respectively, so that we get four quartic equations for \( T_s \) at four vertices separately. Then the quartic equations are solved with the iterative algorithm which is shown later in Sect. 2.4 and Fig. 2, by doing so, all the values of \( T_s \) for the four vertices are obtained.

### 2.3.3 Soil heat flux density \( G \)

\( G \) is normally considered to be linearly related to \( R_n \). Several studies have shown that the value of \( G/R_n \) typically ranges between 0.4 for bare soil and 0.05 for full vegetation cover (Choudhury et al., 1987). Idso et al. (1975) conducted some experiments investigating the impacts of water content on the net radiation ~ soil heat flux relationship over bare soil surface, and showed that \( G/R_n \) ranges from 0.2 for wet bare soil to 0.5 for dry bare soil.

### 2.3.4 Canopy resistance \( (r_c) \)

Canopy resistance \( (r_c) \), including \( r_{cm} \) and \( r_{cx} \) that refer to the minimum and maximum canopy resistance respectively, should be calculated for Point 1 and Point 2. According to Moran et al. (1994), \( r_{cm} \) in Eq. (6) is calculated with \( r_{sm}/\text{LAI} \) (LAI is the leaf area index, \( r_{sm} \) is minimum stomatal resistance), \( r_{cx} \) in Eq. (7) is calculated with \( r_{sx}/\text{LAI} \) (\( r_{sx} \) is maximum stomatal resistance).

Values of minimum and maximum stomatal resistance (\( r_{sm} \text{ and } r_{sx} \), respectively) are published for many agricultural crops under a variety of atmospheric conditions. Moran et al. (1994) suggested that, if values are not available, reasonable values of \( r_{sm} = 25 – 100 \text{ s/m} \) and \( r_{sx} = 1000 – 1500 \text{ s/m} \) will not result in appreciable error; we set \( r_{sm} = 25 \) and \( r_{sx} = 1500 \). Because LAI are mostly less than 8 (Scurlock et al., 2001), we set LAI = 8. Therefore, we have \( r_{cm} = 3.125 \) and \( r_{cx} = 187.5 \).

### 2.4 Iterative procedure for calculating \( T_s \)

Values of \( T_s \) for the four vertices are obtained by an iterative procedure for each pixel. An initial value of \( r_s \) is estimated by ignoring the two stability corrections, i.e., \( \psi_h \) and \( \psi_m \). With the initial \( r_s \), initial values of \( T_s \) are obtained with Eq. (6)–(9) for the four vertices. Then the iterative procedure is proceeded by iteratively changing \( H, KB^{-1}, r_s \), and in consequence, \( T_s \) until the value of \( T_s \) is table (i.e., the change of \( T_s \) is less than 0.1 k, and the change of \( r_s \) is less than 0.1 s/m). Normally, it takes 5 to 10 iterations.

While \( T_s \) is derived, \( R_n, G, H, \) and \( r_s \) for each vertex are obtained as well.

The iterative procedure is conducted distributedly based on pixels, that is, the trapezoid is constructed separately for each pixel, and each trapezoid has its own values of \( T_s \).
3 Case study area and data used

3.1 The Walnut Gulch Experimental Watershed

Data of the Walnut Gulch Experimental Watershed (WGEW) was used in the present study. The WGEW is defined as the upper 148 km$^2$ of the Walnut Gulch drainage basin in an alluvial fan portion of the San Pedro catchment in southeastern Arizona (Fig. 3). It was developed as a research facility by the United States Department of Agriculture (USDA) in the mid-1950s. This rangeland region receives 250–500 mm of precipitation annually, with about two-thirds of it as convective precipitation during a summer monsoon season. The potential evapotranspiration is approximately ten times annual rainfall. The runoff in the ephemeral streams is of short duration and is typically near critical depth. The topography can be described as gently rolling hills incised by steep drainage channels which are more pronounced at the eastern end of the catchment near the Dragoon Mountains. Soil types range from clays and silts to well-cemented boulder conglomerates, with the surface (0–5 cm) soil textures being gravelly and sandy loams containing, on average, 30% rock and little organic matter (Renard et al., 1993). The mixed grass-brush rangeland vegetation ranges from 20 to 60% in coverage. Grasses primarily cover the eastern half of the catchment, while the western half is bush-dominated.

3.2 MODIS data and ground observational data used

The moderate resolution imaging spectroradiometer (MODIS) instrument is very popular for monitoring soil moisture because of its high spectral (36 bands) resolution, moderate spatial (250–1000 m) resolution, and various products for land surface properties. All standard MODIS data products are freely available at NASA Land Processes Distributed Active Archive Center (URL: https://lpdaac.usgs.gov/lpdaac/). MODIS products used in the present study include: MOD09A1 land surface albedo data, MOD11A1 land surface temperature data, and MOD13A1 vegetation data. Details of the products we used here are listed in Table 1. We selected MODIS data of ten cloud-free days approximately evenly distributed in the period from January to December 2004. All the MODIS data are resampled to 500 m resolution.

Meteorological data required here include air temperature $T_a$, relative humidity $\mu$, and wind velocity $u$, observed approximately at the time (11:00 a.m.) when the satellite Terra passes over the WGEP region. The $T_a$, relative humidity $\mu$, and wind velocity $u$, are observed at three sites. We take the average of the observations at three sites for $\mu$ and $u$. Observations of $T_a$ are pre-processed, which will be discussed in Sect. 4.3. To evaluate the soil moisture estimation results, soil moisture observations at 16 sites and precipitation data at 87 sites are used. The locations of the 3 meteorological observation sites, 16 soil moisture observation sites and 87 rain gauging sites are plotted in Fig. 4. Because some gauging sites are located on the edge of the watershed, to include the observations at these sites for evaluation, our study area is slightly larger than the WGEW watershed.

In addition, SRTM digital elevation model (DEM) data and land cover data are used. All the ground data are obtained from the website of United States Department of Agriculture (USDA) Southwest Watershed Research Center (URL: http://www.tucson.ars.ag.gov/dap/).

4 Data pre-processing

4.1 Destriping for MODDIS albedo data (MOD09A1)

MODIS09A1 product includes albedo data of 7 bands. Surface shortwave albedo is calculating as a weighted summation of the albedo data of 7 bands, as shown in Sect. 3.2.2. It was found that the albedo data of the fifth channel has serious problem of bad strips, which would affect the accuracy of surface albedo calculation.
The strips in Band 5 data mostly are lines of one pixel in width, which are distinguishable from neighbouring pixels. To identify the strips, we firstly define the following two convolution kernels:

\[
k_1 = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad k_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}
\]

Then, we calculate \( KK = \text{convol}(D, k_1) \times \text{convol}(D, k_2) \), where \( \text{convol}(\bullet) \) is the convolution filtering function in IDL, and \( D \) is the data to be processed. A pixel in a strip is identified if \( KK > 0.001 \) for this pixel. For bad pixels, linear interpolation is applied to replace the bad values using the values of upside and downside neighboring pixels.

Besides the strips of one pixel width, there are also some strips with two pixels in width resulted from the process of projection conversion in Band 5 of MOD09A1 albedo product. Considering that the pixels in strips have normally higher values than normal, we identify pixels with values larger than 0.35 as “bad” pixels. Then we interpolate the bad pixels with neighbouring “good” pixels with the method of Delaunay triangle (using the program DEM_BAD_DATA_DOIT in IDL). In the same way, pixels with value of 0 are also treated.

With the above two procedures, the quality of Band 5 albedo product was significantly improved (see Fig. 5).

4.2 Denosing the MOD13A1 vegetation index data

When observing the land surface, MODIS is inevitably impacted by the variation of satellite orbital position, cloud coverage and other atmospheric effects. Although several methods (such as Maximum Value Composites or Constrained View Maximum Value Composite) have been applied to reduce the noise impacts the MODIS NDVI/EVI products, quite amount of noise still exist in the VI dataset, and filtering is still necessary when using them for constructing \( T_s \sim VI \) space.

Many methods are available to denoise the MODIS NDVI/EVI data. Jennifer (2009) compared several methods, and found that the asymmetric Gaussian, Double logistic, and 4253H twice filter perform very well in general. Therefore, one of them, i.e., 4253H twice filter (Velleman, 1980) was adopted here. The 4253H twice filter applies a series of running medians of varying window size, and a weighted average filter (e.g., Hanning filter), with re-roughing, to the EVI time series.

To perform the denoising process, a series of continuous EVI data over one year are required. Therefore, before we use the MODIS data selected for 10 dates, we used 25 consecutive 16-day composite EVI data (the last 16-day composite data in 2003, all 23 16-day composite EVI data in 2004, together with the first 16-day composite data in 2005) to conduct the denoising procedure. The effects of denoising for two randomly selected pixels are shown in Fig. 6, from which we see that, both low values and high values are smoothed.

4.3 Topographic correction of air temperature

With methods of estimating soil moisture using thermal satellite images, often both land surface temperature and ground-based air temperature observations are needed. When applying such methods to mountainous regions, terrain effects have to be taken into account because terrain would significantly affect both land surface temperature and air temperature. To avoid the problem of steeply sloping terrain, some authors just eliminated those pixels in mountainous part (e.g., Carlson et al., 1994), while in some other cases, land surface temperature was corrected (e.g., Hassan et al., 2007). In the present study, we go the opposite way, i.e., instead of correct land surface temperature, we correct the air temperature.

To make a successful air temperature interpolation, many factors should be taken into account, such as the difference in elevation between grid points and monitoring stations, temperature vertical gradient, geometric characteristics (slope, aspect) of each grid cell, and vegetation coverage. Moore et al. (1993) proposed a specific algorithm...
to calculate daytime temperature at different altitudes within a valley. Based on that, Bellasio et al. (2005) proposed a simplified equation in the form of

$$
T_i = T_b - \beta (z_i - z_0) + C \left( S_i - 1/S_i \right) \left( 1 - LAI_i/LAI_{max} \right)
$$

(17)

where $T_i$ is the unknown atmospheric temperature (K) at a $z_i$ altitude (m), $T_b$ is the measured atmospheric temperature (K) at a $z_0$ altitude (m), $\beta$ is the vertical temperature gradient (K m$^{-1}$), $C$ is a constant, $LAI_{max}$ and LAI$_i$ are, respectively, maximum leaf area index (LAI) and its value at $z_i$, and $S_i$ is the ratio between direct shortwave radiation on the actual surface (with its slope and aspect) and direct shortwave radiation on a horizontal free surface.

The above equation did not consider the impacts of wind. But according to the research of MccuTchan and Fox (1986), for their study area (an isolated, conical mountain with elevation ranging from 2743 to 3324 m), wind speeds greater than 5 ms$^{-1}$ negate any slope, elevation or aspect differences present at low wind speed. We approximate this wind effect with a coefficient $e^{-u/3}$ ($u$ is the wind speed), in sequence, obtain a modified equation of Eq. (X) as

$$
T_i = T_b - \beta (z_i - z_0) + C e^{-u/3} \left( S_i - 1/S_i \right) \left( 1 - LAI_i/LAI_{max} \right)
$$

(18)

Therefore, when there are air temperature observations at several sites, we can conduct air temperature correction in the following three steps:

1) Correct the observations to a flat plane at a base level

2) Interpolate temperature for each pixel $p$ using observations on the flat plane at the base level

Use the corrected air temperature observations $T^{(i)}_{a,b}$, to interpolate the air temperature for all pixels with a spatial interpolation method (e.g., the inverse distance weighting interpolation method) to get interpolated air temperature $T^p_{a,l}$ for each pixel $p$ on the flat plane at the base level.

3) Topographic correction for each pixel $p$ to its real position using Eq. (18), where $T_b$ is replaced by $T^p_{a,l}$. Here we set $LAI_{max} = 10$, $C = 2$ and $\beta = 0.0065$.

5 Application of $T_s$–VI trapezoid method to WGEW

5.1 Construct $T_s$–VI trapezoids

Reasonable shape of trapezoid is the essence of all the algorithms based on the $T_s$–VI relationship for estimating soil moisture. When construct with the algorithm described in Sect. 2, two parameters, i.e., $S_{KB}$ and $G/R_n$, are set by trial and error process. For the case study area WGEW, we set $S_{KB}$ to be 0.1 for both vegetated points (point 1 and 2) and bare soil points (point 3 and 4), $G/R_n$ to be 0.3 for wet bare soil, 0.4 for dry bare soil, and 0.05 for full vegetation surface.

To show the effectiveness of the calculation for the values of $T_s$ of four vertices, we plot the four vertices of the trapezoids constructed for all the pixels of the WGEW region in four days in four seasons in Fig. 7. All the estimated $T_s$ at each point are plotted in the form of box-and-whisker plot. The data points (solid dots) of $T_s$ vs. EVI are also plotted in the map. From Fig. 7, we see that the constructed trapezoids well characterize the $T_s$–EVI space, and basically all the $T_s$–EVI data points are set in the envelope of the trapezoids.
5.2 Calculation of WDI

Based on the constructed $T_s$-VI trapezoid for each pixel, using the MODIS $T_s$ and EVI data, we calculate the WDI for each pixel $p$,

$$\text{WDI}^{(p)} = \frac{T_s^{(p)} - T_{s,\min}^{(p)}}{T_{s,\max}^{(p)} - T_{s,\min}^{(p)}}$$

(20)

where $T_s$ is surface temperature obtained from MODIS; the subscripts min, max, and $r$ refer to minimum, maximum, and measured values, respectively; and the minimum and maximum values of $T_s$ are interpolated linearly on the dry edge and wet edge of the $T_s$-VI trapezoid for the specific VI value of the pixel.

5.3 Comparison with soil moisture observation and precipitation

Using the surface soil moisture observations at 16 sites, we evaluate WDI estimates in several ways: (1) compared separate WDI estimates with ground observations of all 10 dates (Fig. 8); (2) compare the average of WDI estimates with the average ground observations of 10 dates (Fig. 9); (3) compare the WDI estimates with ground observations of each date separately (Table 2).

From the scatter plot of WDI vs. observation in Fig. 8, we see that from the perspective of a whole year, WDI estimates derived with the $T_s$-VI trapezoid method has a negative correlation (correlation coefficient $R = -0.7232$) with surface soil moisture, which indicates that WDI estimates can be used to detect the temporal variation in soil moisture. Especially on the scale of the watershed, the average WDI is strongly negatively related (correlation coefficient $R = -0.9$) to the average soil moisture observation, as shown in Fig. 9. Although this is not a high correlation, considering that soil moisture in dry environment, such as in semi-arid area, exhibits high spatial variability and potentially rapid rates of temporal change in moisture conditions, the result is reasonably good.

The comparison between the WDI estimates with ground observations of each date (Table 2) shows that, there is basically no correlation between WDI estimates and surface soil moisture observations. This is partly because of the scale effect, i.e., point soil moisture observations are essentially different from grid averaged soil moisture estimates due to sub-grid variabilities, partly because of the poor capability of using WDI to detect the variation in soil moisture with low spatial variability. Similar phenomena have been observed by some other researchers as well. For instance, Pellenq et al. (2003) noticed that the point-to-point comparison between observations and simulations shows a poor correlation, but a good correlation is obtained when averaging the simulated and observed soil moisture over a length of 100 m. Comparing the distribution of soil moisture observations over the year with that observed instantaneously, we see that the coefficient of variation (CV) for all soil moisture observations at 16 sites in 10 days over a year is 0.771, much larger than the CV for observed soil in each day (ranging from 0.336 to 0.702, with a mean value of 0.528). In consequence, we can use WDI to detect the temporal variation in soil moisture, but it is hard to detect spatial variation in each day, especially for a small watershed with low spatial soil moisture variability.

Despite of the poor performance for characterizing the spatial variability of soil moisture with WDI, by a visual inspection of the WDI maps of the WGEW region of the 10 dates in Fig. 10, we can still see a clear spatial pattern of soil moisture distribution, which indicates that, to some extent, soil moisture variability could be depicted by WDI maps.

We analyzed the impacts of precipitation on soil moisture by calculating the correlation between WDI and antecedent precipitation (AP) of different number of days, and between soil moisture observation and AP of different number of days. The results are illustrated in Fig. 11, which show that WDI and soil moisture observation have similar levels of correlation with AP (one is positive, another is negative), and the maximum correlation occurs when approximately 10-day AP is taken into account. The scatter plot is shown in Fig. 12. The result indicates that, as expected, the temporal variation
of soil moisture (either reflected by ground observations, or by WDI estimates) is significantly dominated by precipitation process.

6 Conclusions

Considerable efforts have been put on using the relationship between soil moisture and index values derived from surface temperature-vegetation index (\(T_s - \text{VI}\)) space, which use optical and thermal RS data as input, to estimate soil moisture. In the present study, we simplified the trapezoidal relationship between the surface temperature and air temperature difference (\(T_s - T_a\)) vs. the fractional vegetation cover, which is proposed by Moran et al. (1994), to a \(T_s - \text{VI}\) trapezoid. The trapezoid is constructed separately for each pixel (grid). An iterative algorithm is proposed to estimate the vertices of the \(T_s - \text{VI}\) trapezoid theoretically. Then water deficit index (WDI) which is calculated based on the \(T_s - \text{VI}\) trapezoid is calculated for each grid using MODIS remotely sensed measurements of surface temperature and enhanced vegetation index (EVI). In the process of constructing the \(T_s - \text{VI}\) trapezoid, a data pre-processing procedure, including de-striping bad pixels, eliminating the noise contamination in EVI data, and, especially correcting the topographic effects for air temperature data, is conducted.

Using satellite-based MODIS data (land surface temperature data, EVI, etc.), and ground-based on-site soil moisture data and meteorological data (air temperature, relative humidity, and wind velocity) for the Walnut Gulch Experimental Watershed (WGEW) in Arizona, USA, the capability of using WDI to estimate soil moisture is evaluated using (1) a-soil moisture observations and (2) antecedent precipitation. The result shows that, \(T_s - \text{VI}\) trapezoid based WDI can well capture temporal variation in surface soil moisture, but the capability of detecting spatial variation is poor for such a semi-arid region as WGEW.

Acknowledgements. We are very grateful to USDA Southwest Watershed Research Center for providing observation data of the Walnut Gulch Experimental Watershed. The financial supports from China Postdoctoral Science Foundation (20080431062), the National Science Foundation of China (40771039) and the 111 Project (B08048) are gratefully acknowledged.

References


Table 1. MODIS data used in the present study.

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Table 2. Correlation coefficients between WDI estimates with surface soil moisture observations.

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Fig. 1. The hypothetical trapezoidal shape based on the relation between \((T_s - T_a)\) and the fractional vegetation cover \((V_c)\).

Fig. 2. Iterative procedure for calculating \(T_s\) of the four vertices of \(T_s\)-VI trapezoid.
Fig. 3. Digital elevation model (DEM) of Walnut Gulch Experimental.

Fig. 4. Locations of ground-based observation sites in WGEW.
Fig. 5. Comparison of Band 5 albedo images before (left) and after destriping.

Fig. 6. Effects of EVI denoising preprocessing for two randomly selected pixels.
Despite of the poor performance for characterizing the spatial variability of soil moisture with low spatial soil moisture variability, because of the poor capability of using WDI to detect the variation in soil moisture with low spatial variability, partly due to sub-grid variability, partly due to the scale effect, i.e., point soil moisture observations are essentially different from grid averaged soil moisture estimates due to sub-grid variability, partly due to rapid rates of temporal change in moisture conditions, the result is reasonably good.

Fig. 7. Constructed $T_s$–EVI trapezoids in four dates in four different seasons.

Fig. 8. WDI estimates vs. ground observations at 16 sites in 10 dates ($R$ is the correlation coefficient).
From the scatter plot of WDI vs. observation in Fig. 8, we see that from the perspective of a whole year, WDI estimates derived with the Ts-Vi trapezoid method has a negative correlation (correlation coefficient $R = -0.7232$) with surface soil moisture, which indicates that WDI estimates can be used to detect the temporal variation in soil moisture. Especially on the scale of the watershed, the average WDI is strongly negatively related (correlation coefficient $R = -0.9$) to the average soil moisture observation, as shown in Fig. 9. Although this is not a high correlation, considering that soil moisture in dry environment, such as in semi-arid area, exhibits high spatial variability and potentially rapid rates of temporal change in moisture conditions, the result is reasonably good.

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Fig. 11. Correlation coefficient ($R$) between (a) soil moisture observation and AP of different number of days, and (b) WDI and AP of different number of days.

Fig. 12. Scatter plot of (a) soil moisture observation and (b) WDI vs. 10-day AP ($R$ is the coefficient of correlation).