Determination of spatially varying Van der Burgh’s coefficient from estuarine parameter to describe salt transport in an estuary

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1. General comments

The paper is a continuation of previous work presented by Shaha et al. (2010) (*Using flushing rate to investigate spring-neap and spatial variations of gravitational circulation and tidal exchanges in an estuary*, Hydrol. Earth Syst. Sci., 14, 1465-1476). In the present paper the objective is to link the estuarine parameter $Q$, following from the theory by Hansen & Rattray (1965, 1966), to the Van der Burgh coefficient $K$ as introduced by Savenije in his salt intrusion model (e.g. 1993, 2005). The authors are to be praised for undertaking such a task because, if completed successfully, this would greatly attribute to a physical explanation of the Van der Burgh constant. The latter is presently considered as some sort of shape factor with a specific value for an estuary but ‘constant’ for different hydrodynamic conditions.

The authors have thoroughly analyzed the different modes for salt transport in the various regions of the Sumjin estuary for spring as well as neap tides. However, it is my view that a direct relation between $Q$ and $K$ has not yet been convincingly shown. This raises the question whether the application of a spatially varying $K$, representing the variation of the relative contributions of tide-driven and density-driven mixing processes along the estuary, is feasible. In this respect I will discuss in Section 2 (Specific comments) the derivation of equation (5) that relates $Q$ with $K$. For situations that both $Q$ and $K$ are functions of $x$ this relationship appears to be different.

The paper has been well-structured. However, text has been repeated several times using almost the same phrasing. Examples will be given in Section 2 (Specific comments). In contrast, I feel that some text is missing such as the computational method for $Q$. Although this has been described in the previous paper I think that given the objective of the present paper this should be summarized here. Further improvement of the text is necessary.

Section 3 contains some typing errors.
2. Specific comments

The basic assumption on which the paper relies is that a direct relationship is assumed between \( v \) and \( K \) according to Eq. (S5)\(^1\). This equation is later extended to Eq. (S6) which I will discuss further. If \( v \) and \( K \) are constant the derivation is straightforward as described in the paper. However, if both parameters are functions of \( x \) the result becomes different.

Eq. (S2) is written as:

\[
\frac{\partial D(x)}{\partial x} = \frac{Q}{A(x)}
\]  

(1)

and Eq. (S3) as:

\[
\nu(x) = \frac{D'_x(x)}{D(x)}
\]  

(2)

where \( \nu \) is explicitly given as a function of \( x \).

Following the approach by Shaha and Cho for constant \( \nu \) Eq. (1) and Eq. (2) result in:

\[
\frac{\partial [\nu(x)D(x)]}{\partial x} = \frac{Q}{A(x)}
\]  

(3)

or

\[
\frac{\partial D(x)}{\partial x} = \frac{1}{\nu(x)} \frac{Q}{A(x)} - \frac{D(x)}{\nu(x)} \frac{\partial [\nu(x)]}{\partial x} = \left\{ \frac{1}{\nu(x)} - \frac{D(x)A(x)}{\nu(x)} \frac{\partial [\nu(x)]}{\partial x} \right\} \frac{Q}{A(x)}
\]  

(4)

Eq. (S1) is written as:

\[
\frac{\partial D(x)}{\partial x} = K(x) \frac{Q}{A(x)}
\]  

(5)

so that the relation between the Van der Burgh constant \( K \) and the estuarine parameter \( \nu(x) \) for the case with spatially varying \( v \) and \( K \) becomes:

\[
K(x) = \frac{1}{\nu(x)} \left[ 1 - \frac{D(x)A(x)}{Q} \frac{\partial [\nu(x)]}{\partial x} \right]
\]  

(6)

Comparison of this equation with Eq. (S5) (or Eq. (S6)) shows that there is no simple relationship between \( v \) and \( K \). The authors should clarify whether and how this is captured by their analysis.

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\(^1\) S5 refers to Eq. (5) in the paper by Shaha and Cho.
Secondly, the authors propose an empirical relationship between $v$ and $K$ according to Eq. (S6). To my view this is only to mask the fact that $1/K$ and $v$ have non-overlapping ranges ($1/K > 1$ and $0<v<1$) which remains unexplained. Although the authors give some possible causes for this inconsistency (non-constant cross-section, non-linear salinity distribution) the introduction of Eq. (S6) does not solve this inconsistency except that $K$ as derived from $v$ is now scaled to values less than 1.

It is further noted, that with Eq. (S6) and $0<v<1$ $K$ only varies between $1/e$ (= 0.37) and 1. This is strange as the lower bound for $K$ is theoretically 0. Savenije (2005) found values for $K$ less than 0.37 for various estuaries, which can never be deduced from Eq. (S6). This poses some questions on the validity of the equation.

Specific comments on the text are as follows.

I suggest that the authors briefly summarize the computational method to derive the estuarine parameter $v$ from the characteristics of the estuary (although this has been discussed in a previous paper). As in the present paper a relationship is being explored between $v$ and $K$ it seems to me crucially to understand the method applied for $v$ so that $K$ can be derived in a predictive way.

I would also stress that all hydrodynamic and geometric characteristics as used for the computations of the dispersion coefficients and salinities should be presented in a table. This includes parameters as: converging lengths for the width and cross-sectional area, water depth, salinity in the estuary mouth ($S_0$), water depth, river discharge (which is given in the present paper), tidal range in the estuary mouth, roughness coefficient etc.. This will be very valuable for other researchers.

Sometimes I had the notion that text was repeated in the paper. Examples are:
- Page 8784, line 16-19: the text is almost similar to the text on Page 8788, line 8-11.
- Page 8787, line 15-17: the text is almost similar to the text on Page 8789, line 13-15.

Section 2: please refer to a table with details on parameters used for the computations (see also above).

Page 8785, line 27: decrease to 12 and 8% or decrease with 12 and 8% (probably the last).

Page 8786, line 25: a linear decrease of the salinity is also assumed for each individual section in the estuary?

Page 8787, line 15: until this line the discussion was generic. However, that $v$ is determined at the mouth between Narragansett Bay (please indicate in Figure 1!) and the adjacent sea is specifically for this estuary. I would suggest to move this line to Section 4 where the methodology is really applied to the Sumjin estuary (in fact this sentence is already there, see line 13-15).
Page 8788, line 16 and 17: please define the dimensionless diffusion length scale and the tidal dissipation length scale, because in this way it is not very instructive to the reader.

Page 8789, line 5 and 6: what is meant with “… supports the K-based dispersion equation…”.

Page 8789, line 24: “…is almost entirely dominant landward of 6 km…”. I assume that landward should be seaward? This has been formulated wrongly several times hereafter.

Page 8789, line 25: “This length also shows well consistency…”. Please replace with: “This length is consistent with …”. Please indicate for which values of $\frac{\delta S}{S}$ well-mixed or stratified conditions are present. It would be nice to show this parameter in a figure (it is not included in the paper by Shaha and Cho (2009)).

Page 8790, line 8: “Landward from 6 to 21 km…”. I assume that between 6 and 21 km is meant.

Page 8790, line 12: Please define potential energy anomaly (in words or by means of an equation), because in this way it is rather vague to the reader.

Page 8791, line 2: which calculation is meant (the previous text?) and in what way is there consistency?

Page 8791, line 10: Seaward in stead of landward? See also Page 8792, line 3 and other lines in the text.

Page 8791, line 15: Note that according to Eq. (S6) $K$ can never be less than 0.37 (~0.4).

Page 8791, line 17: “… during neap tide”. Thus the preceding text was for spring tides? Please indicate more clearly.

Page 8791, line 20-21: Please note that $K$ values of 0.25 and 0.3 can never be computed with Eq. (S6) using $0<\nu<1$.

Page 8791, line 27: this is apparently for spring tide

Page 8793, line 23-24: The text is repeated from Page 8792, line 4-6.

Page 8794, line 8: “To be sure …”. The meaning is unclear.

Page 8794, Eq. (S7): it is noted that this equation follows from the transport equation and Eq. (S1), see Savenije (2005). Thus there is nothing new.

Page 8795, line 16: Landward should be seaward? See also line 25.

Page 8796, lines 14-19: This sentence should be reformulated; it is too long and the meaning is unclear.
3. Technical corrections

Some typing errors etc. are given below.

Page 8787, line 23: leads to

Page 8790, line 18: dominates.

Page 8792, line 2: “is entirely dominated” should be “entirely dominates”?

Page 8792, line 17: “was the dominant mechanism”

Page 8793, line 6: exit should be exists.

Page 8794, line 10: The dispersion is the maximum… (remove the).

Page 8794, line 21: coincides should be coincide.

Page 8795, line 22: increase should be increases.

Page 8795, line 9: Figure 4a does not exist. Is there a reference to Figure 5?

Figures 2 and 3: Please indicate units along vertical axis and use different symbols for both lines because the symbols used are difficult to distinguish.

Figures 4-7: Please indicate units along vertical axis.

Figures 6 and 7: It is not clear whether S/S0 relates to observed or computed values. The legend D/D0-S/S0 is the same for two lines.

Please include a Figure showing the observed salinity distribution for the conditions presented (spring, neap, summer, winter).