Spatial moments of catchment rainfall: rainfall spatial organisation, basin morphology, and flood response

D. Zoccatelli¹, M. Borga¹, A. Viglione², G. B. Chirico³, and G. Blöschl²

¹Department of Land and Agroforest Environment, University of Padova, Italy
²Institut für Wasserbau und Ingenieurhydrologie, Technische Universität Wien, Vienna, Austria
³Dipartimento di Ingegneria Agraria, Università di Napoli Federico II, Naples, Italy

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Correspondence to: D. Zoccatelli (davide.zoccatelli@studenti.unipd.it)

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Abstract

This paper provides a general analytical framework for assessing the dependence existing between spatial rainfall organisation, basin morphology and runoff response. The analytical framework builds upon a set of spatial rainfall statistics (termed “spatial moments of catchment rainfall”) which describe the spatial rainfall organisation in terms of concentration and dispersion statistics as a function of the distance measured along the flow routing coordinate. The introduction of these statistics permits derivation of a simple relationship for the quantification of storm velocity at the catchment scale. The paper illustrates the development of the analytical framework and explains the conceptual meaning of the statistics by means of application to five extreme flash floods occurred in various European regions in the period 2002–2007. High resolution radar rainfall fields and a distributed hydrologic model are employed to examine how effective are these statistics in describing the degree of spatial rainfall organisation which is important for runoff modelling. This is obtained by quantifying the effects of neglecting the spatial rainfall variability on flood modelling, with a focus on runoff timing. The size of the study catchments ranges between 36 to 982 km². The analysis reported here shows that the spatial moments of catchment rainfall can be effectively employed to isolate and describe the features of rainfall spatial organization which have significant impact on runoff simulation. These statistics provide essential information on what space-time scales rainfall has to be monitored, given certain catchment and flood characteristics, and what are the effects of space-time aggregation on flood response modeling.

1 Introduction

Rainfall is a highly heterogeneous process over a wide range of scales both in space and time (e.g. Rodriguez-Iturbe et al., 1998; Fabry, 1996; Marani, 2005). Whether or not spatial heterogeneity of rainfall has an impact on catchment discharge and for
what reason, is a problem that has been often addressed in hydrology and that is still poorly understood. Many hydrological studies have focused on the role of rainfall space-time variability in catchment response, with the aim of developing a rationale for more effective catchment monitoring, modelling and forecasting (e.g., Naden, 1992; Obled et al., 1994; Blöschl and Sivapalan, 1995; Bell and Moore, 2000; Andréassian et al., 2001; Morin et al., 2006; Moulin et al., 2008; Saulnier and Le Lay, 2009; Gourley et al., 2011). From a practical perspective, it is important to know at what space-time scales rainfall has to be monitored, given certain catchment and flood characteristics, and what are the effects of space-time aggregations on model simulations (Berne et al., 2004).

An important feature frequently observed in these studies is that catchments act as space-time filters (Skojen and Blöschl, 2006) with specific dampening characteristics to the rainfall input. The filtering properties may be strong enough to efficiently smooth out some features of rainfall spatial variability. This means that only some specific characteristics of rainfall spatial organisation will eventually emerge as runoff spatial and temporal variability (Skojen et al., 2003). As stated by Obled et al. (1994) in a study focused on a rural 71 km$^2$ basin in Southern of France, “...it seems that the spatial variability of rainfall, although important, is not sufficiently organized in time and space to overcome the effect of smoothing and dampening when running off through this rural medium sized catchment”. Thus we believe there is a need to introduce measures to quantify the catchment filtering effect which, as a function of rainfall organization, basin scale and the heterogeneities embedded in the basin geomorphic structure, control the possible extent of the influence of rainfall spatial organisation on the hydrologic response. We distinguish here between rainfall spatial variability and organization. More specifically, by spatial organization we mean systematic spatial variation of rainfall with respect to certain basin geomorphic properties which directly control the runoff response. In this paper, the rainfall spatial organization is analysed with respect to the flow distance, i.e. the distance along the runoff flow path from a given point to the outlet.
Observational and modelling studies have shown that the river network geometry plays a central role in the structure of the catchment dampening properties (Naden, 1992; Woods and Sivapalan, 1999; Smith et al., 2002; Nicotina et al., 2008; Sangati et al., 2009). Runoff routing through branched channel networks imposes an effective averaging of spatial rainfall excess at equal flow distance, in spite of the inherent spatial variability. This implies that rainfall spatial organisation measured along the river network by using the flow distance coordinate may be a significant property of rainfall spatial variability when considering flood response modelling.

Various measures of rainfall organisation based on the flow distance coordinate have been introduced in the last decade. Smith et al. (2002, 2005), Zhang et al. (2001) and Borga et al. (2007), in a series of monographs on extreme floods and flash floods, systematically employed a scaled measure of distance from the storm centroid and a scaled measures of rainfall variability to quantify the storm spatial organisation and variability from the perspective of a distance metric imposed by the river network. Smith et al. (2004a) examined basin outflow response to observed spatial variability of rainfall for several basins in the Distributed Model Intercomparison Project (Smith et al., 2004b), by using, among other indexes, a rainfall location index based on the distance from the centroid of the catchment to the centroid of the rainfall pattern. They found that all basins except one had a very limited range of rainfall location index, with the rainfall centroid close to the catchment centroid. Interestingly, the catchment displaying the largest range of rainfall location index was also the one characterised by such complexities to suggest the use of a distributed model approach. A similar approach was taken by Syed et al. (2003) who evaluated the ability of simple geometric measures of thunderstorm rainfall in explaining the runoff response from a 148 km² watershed. They also used a location index similar to that introduced by Smith et al. (2004a). They observed that the position of the storm core relative to the watershed outlet becomes more important as the catchment size increases, with storms positioned in the central portion of the watershed producing more runoff than those positioned near the outlet or near the head of the watershed. Woods and Sivapalan (1999) proposed an analytical
method to identify the importance of different components of the hydrological cycle during storm events in humid temperate catchments. They expressed the mean catchment runoff time as a function of the distance from the centroid of the catchment to the centroid of the rainfall excess pattern measured along the flow distance coordinate. This term summarised the influence of the rainfall spatial variability on the timing of the flood peak discharge in their model. However, the extent of potential application of this method is limited by the assumption of multiplicative space-time separability for both rainfall and runoff generation processes. This implies that the storm event is assumed to be stationary, i.e., it does not move over the catchment. This assumption is relaxed in the analytical framework introduced by Viglione et al. (2010a), which describes the dependence of the catchment flood response on the space-time interactions between rainfall, runoff generation and routing mechanisms. Notably, this method affords examination of the effects of storm movement on runoff properties.

This paper builds upon and generalizes the work presented in Wood and Sivapalan (1999) and Viglione et al. (2010a), by introducing a set of statistics of spatial rainfall organisation measured along the flow distance which are relevant to the analysis of the runoff response. These statistics, termed “spatial moments of catchment rainfall”, provide a synthesis of the interaction between rainfall and basin morphometric properties. In this work we show, both analytically and empirically, how these statistics can be used to quantify the influence of spatial rainfall organization on flood hydrograph characteristics.

As part of this analysis, we show how the introduction of the spatial moments of catchment rainfall permits derivation of a simple relationship for the quantification of storm velocity at the catchment scale. The importance of storm movement on surface runoff has been investigated for nearly four decades (Maksimov, 1964; Surkan, 1974; Ogden et al., 1994; Singh, 1998; de Lima and Singh, 2002). However, to the best of our knowledge, these works are based on “virtual experiments” using idealized storm profiles and motion as input to watershed models. Results seem to support the conclusion that catchment response is sensitive to storm motion relative to catchment
morphology, depending on different processes and scales. With this work we aim to put forward a conceptual framework to isolate and quantify the “catchment scale storm velocity”, generated by imposing the observed space-time storm variability to the catchment morphological properties.

In the following developments, we disregard the differentiation between hillslopes and channel network to the total runoff travel time. While the methodological framework can be easily extended to include a hillslope term, we prefer here to focus on the interaction between the morphological catchment properties and rainfall organisation. On going investigations are aimed to examine the impact of varying the hillslope residence time on both the spatial moments of catchment rainfall and the catchment scale storm velocity.

The outline of the paper is as follows. In Sect. 2 we define the statistics termed “Spatial moments of catchment rainfall”. In Sect. 3 we show how these rainfall statistics can be related to the flood hydrograph properties. Section 4 is devoted to illustrate the derivation of the Spatial moments of catchment rainfall for five different extreme flood and flash flood events occurred in Europe in the period between 2002 and 2007, with catchment size ranging from 36 to 982 km$^2$. In Section 5 we perform numerical experiments in which modelled flood response obtained by using detailed spatial input is contrasted with the corresponding flash flood response obtained by using spatially uniform rainfall. Runoff model sensitivity to spatial organisation of rainfall is examined by exploiting the spatial rainfall statistics. Section 6 completes the paper with discussion and conclusions.

2 Spatial moments of catchment rainfall: definitions

Spatial moments of catchment rainfall provide a description of overall spatial rainfall organisation at a certain time $t$, as a function of the rainfall field $r(x,y,t)$ ($L^T^{-1}$) value at position $x,y$ and of the distance $d(x,y)$ ($L$) between the position $x,y$ and the catchment outlet measured along the flow path. The $n$-th spatial moment of catchment rainfall $p_n$
\((L^{n+1} T^{-1})\) is expressed as:

\[
p_n(t) = |A|^{-1} \int_A r(x,y,t)d(x,y)^n dA
\]

(1)

where \(A\) is the spatial domain of the drainage basin. The zero-th order spatial moment \(p_0(t)\) yields the average catchment rainfall rate at time \(t\).

Analogously, the \(g_n (L^n)\) moments of the flow distance are given by:

\[
g_n = |A|^{-1} \int_A d(x,y)^n dA.
\]

(2)

The zero-th order spatial moment of flow distance yields unity. Non-dimensional (scaled) spatial moments of catchment rainfall can be obtained by taking the ratio between the spatial moments of catchment rainfall and the moments of the flow distance, as follows, for the first two orders:

\[
\delta_1(t) = \frac{p_1(t)}{p_0(t)g_1}
\]

\[
\delta_2(t) = \frac{1}{g_2-g_1^2} \left[ \frac{p_2(t)}{p_0(t)} - \left( \frac{p_1(t)}{p_0(t)} \right)^2 \right]
\]

(3)

The first scaled moment \(\delta_1(\cdot)\) describes the location of the center of the mass of catchment rainfall with respect to the average value of the flow distance (i.e.: the catchment center of mass). Values of \(\delta_1\) close to 1 reflect a rainfall distribution either concentrated close to the position of the catchment center of mass or spatially homogeneous, with values less than one indicating that rainfall is distributed near the basin outlet, and values greater than one indicating that rainfall is distributed towards the periphery of the drainage basin.

The second scaled moment \(\delta_2(\cdot)\) relates to the spreading of the rainfall field about its mean position with respect to the spreading of the flow distances. Values of \(\delta_2\) close to 1 reflect a uniform-like rainfall distribution, with values less than 1 indicating that
rainfall is characterised by a unimodal distribution along the flow distance. As we will see below, values greater than 1 are generally rare, and indicate cases of multimodal rainfall distributions.

The spatial moments as defined in Eq. (3) describe the instantaneous spatial rainfall organization at a certain time $t$. Equations (1) to (3) can also be used to describe the spatial rainfall organization corresponding to the cumulated rainfall over a certain time period $T_s$ (e.g., a storm event). These statistics are termed $P_n$ and $\Delta_n$ and are defined as follows, respectively:

$$P_n = \frac{1}{T_s} \int_{T_s}^{T_s} p_n(t) \, dt$$

(4)

$\Delta_1$ and $\Delta_2$ are computed based on $P_n$ following Eq. (3), as follows

$$\Delta_1 = \frac{P_1}{P_0} g_1$$

$$\Delta_2 = \frac{1}{g_2 - g_1} \left[ \frac{P_2}{P_0} - \left( \frac{P_1}{P_0} \right)^2 \right]$$

(5)

### 2.1 Definition of catchment-scale storm velocity

Interestingly, the analysis of the evolution in time of the first scaled moment of catchment rainfall enables the calculation of the catchment-scale storm velocity along the river network, as follows:

$$V(t) = g_1 \frac{d}{dt} \delta_1(t)$$

(6)

Positive values of the storm velocity $V$ correspond to upbasin storm movement, whereas downbasin storm movement are related to negative values of $V$. The computation of the catchment-scale storm velocity according to Eq. (6) takes into account the overall space-time dynamics of the storm during its movement over the catchment, rather than reflecting the kinematics of specific storm elements across the basin.
A simple way to derive the mean value of $V$ over a certain time period is reported in the next sections.

### 3 Relationship between the spatial moments of catchment rainfall and the shape of the flood response

Viglione et al. (2010a) proposed an analytical framework (called $V2010$ hereafter) for quantifying the effects of space-time variability on catchment flood response. Viglione et al. (2010a) extended the analytical framework developed in Woods and Sivapalan (1999) to characterize flood response in the case where complex space and time variability of both rainfall and runoff generation are considered as well as hillslope and channel network routing.

In the $V2010$ methodology, the rainfall excess $r_e(x,y,t) \, [LT^{-1}]$ at a point $(x,y)$ and at time $t$ generated by precipitation $r(x,y,t)$ is given by

$$r_e(x,y,t) = r(x,y,t) \cdot c(x,y,t) \tag{7}$$

where $c(x,y,t) \, [-]$ is the local runoff coefficient, bounded between 0 and 1. $V2010$ characterizes the flood response with three quantities: (i) the catchment- and storm-averaged value of rainfall excess, (ii) the mean runoff time (i.e., the time of the center of mass of the runoff hydrograph at a catchment outlet), and (iii) the variance of the runoff time (i.e., the temporal dispersion of the runoff hydrograph). The mean time of catchment runoff is a surrogate for the time to peak. The variance of runoff time is indicative of the magnitude of the peak runoff. For a given event duration and volume of runoff, a sharply peaked hydrograph will have a relatively low variance compared to a more gradually varying hydrograph (see Woods, 1997, for details).

Since the aim of this study is to establish a relationship between the spatial moments of catchment rainfall and the flood response shape, we modified accordingly the $V2010$ methodology by assuming that the runoff coefficient is uniform in space and time, and that the hillslope residence time is negligible. Hence, in the following developments the
rainfall intensity and accumulation are used in place of the rainfall excess. Owning to this assumption, results obtained by this approach are likely to apply to heavy rainfall events characterized by large rain rates and accumulations. The runoff transport is described by using an advection velocity \( v \) \( (LT^{-1}) \) which is considered invariant in space and time. This hypothesis is consistent with the fact that for a given pattern of flow paths across the catchment, it is always possible to identify a single value of flow celerity such as the catchment response time in unchanged (Robinson et al., 1995; Saco and Kumar, 2002; D'Odorico and Rigon, 2003).

The analytical results are summarized below, by focusing on the elements which are essential to derive the relationship between the spatial moments and the characteristics of the flood response shape, i.e. the mean and the variance of runoff time and the catchment scale storm velocity. Catchment runoff time is treated as a random variable (denoted \( T_q \)), which measures the time from the storm beginning until a drop of water exits the catchment. Water that passes a catchment outlet goes through two successive stages in our conceptualisation: (i) the generation of runoff at a point (including waiting for the rain to fall), (ii) runoff transport. Each of these stages has an associated “holding time”, which is conveniently treated as a random variable (e.g., Rodriguez-Iturbe and Valdes, 1979). Since the water exiting the catchment has passed in sequence through the two stages mentioned above we can write

\[
T_q = T_r + T_c
\]

where \( T_r \) and \( T_c \) are the holding times for rainfall excess and runoff transport.

Mean catchment runoff time. Using the mass conservation property (see V2010) we can write the mean of \( T_q \) as

\[
E(T_q) = E(T_r) + E(T_c)
\]

In Eq. (8) we focus on the term \( E(T_c) \), which may be expressed as follows:

\[
E(T_c) = \frac{\int_0^{T_s} \left[ \int_0^A r(x, y, t)d(x, y)dA \right] dt}{AT_s P_0 v} = \frac{P_1}{P_0 v} = \Delta_1 g_1 \frac{1}{v}
\]

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where \( T_s \) is the duration of the storm event.

Therefore, Eq. (8) may be written as follows:

\[
E(T_q) = E(T_r) + \frac{\Delta_1 g_1}{\bar{E}T_1} + \frac{\Delta_1}{\bar{v}} + \frac{\Delta_1}{\bar{E}T_2}
\]  

Equation (10) has two terms: \( ET_1 \) and \( ET_2 \). The first term \( ET_1 \) represents the time from the start of the event to the centroid of the rainfall time series, and is independent from the rainfall spatial variability. For the conceptualization of \( E(T_r) \), which is not of interest here, we refer to V2010. The second term \( ET_2 \) represents the average time to route the rainfall excess from the geographical centroid of the rainfall spatial pattern to the catchment outlet. It is important to note here that the spatial distribution of the rainfall excess is the same as that of the rainfall pattern, since the runoff coefficient is assumed to be spatially uniform.

It is interesting to note that, from Eq. (10), the first time-integrated scaled moment represents the ratio between the routing time corresponding to the rainfall center of mass with respect to the catchment response time \( g_1/\bar{v} \):

\[
\Delta_1 = \frac{ET_2}{g_1/\bar{v}}
\]  

Analogously to \( \delta_1 \), the values of \( \Delta_1 \) are greater than zero, and are equal to one for the case of spatially uniform precipitation or for a spatially variable precipitation which is concentrated on the catchment centroid. Values of \( \Delta_1 \) less than one indicate that rainfall is concentrated towards the outlet, and values larger than one indicate that rainfall is concentrated towards the headwater portion of the basin. Based on Eq. (10), the expected effect of a less-than-one value of \( \Delta_1 \) is an anticipation of the mean hydrograph time. This means that when rainfall is concentrated towards the outlet, the hydrograph is anticipated relative to the case of spatially uniform rainfall. The opposite is true for rainfall concentrated towards the periphery of the catchment, with the hydrograph delayed relative to the case of a spatially uniform rainfall.
One should note that the storm velocity has no influence on $E(T_q)$. This is a direct consequence of the hypothesis that catchment response is fully kinematic, therefore it is influenced by the averaged spatial organization of the rainfall and not by the variability of the spatial organization within the storm.

Variance of catchment runoff time. The variance of $T_q$, which represents the dispersion of the hydrograph, is given by

$$\text{Var}(T_q) = \text{Var}(T_r) + \text{Var}(T_c) + 2 \text{Cov}(T_r, T_c)$$

(12)

We focus here on the terms $\text{Var}(T_c)$ and $2 \text{Cov}(T_r, T_c)$. For the conceptualization of $\text{Var}(T_r)$, which is not of interest here, we refer to V2010.

By using the concept of scaled spatial moments, $\text{Var}(T_c)$ may be written as follows.

$$\text{Var}(T_c) = \frac{\Delta_2}{v^2} \left( g_2 - g_1^2 \right)$$

(13)

For the case of $\text{Cov}(T_r, T_c)$ equal to zero, $\Delta_2$ represents the ratio between the added variance in runoff timing generated by rainfall spatial distribution, and the variance of the catchment response time. The values of $\Delta_2$ are greater than zero and take the value of one when the rainfall field is spatially uniform. When the rainfall field is spatially concentrated anywhere in the basin, the values of $\Delta_2$ are less than one. In the less frequent cases when the rainfall field has a bimodal spatial distribution, with concentration both at the headwaters and at the outlet of the catchment, the values of $\Delta_2$ are greater than one. It should be noted that, with the rainfall excess volume remaining unchanged, the effect of decreasing the variance of runoff time is to increase the flood peak. This shows that in general the parameter $\Delta_1$ is expected to have an influence on the runoff timing, whereas the parameter $\Delta_2$ should affect the shape of the hydrograph and then the value of the flood peak.
The role of catchment scale storm velocity is represented by the term of $\text{Cov}(T_r, T_c)$. By using rainfall weights, defined as

$$w(t) = \frac{\rho_0(t)}{P_0}$$

(14)

and based on $V2010$, the last term of Eq. (12) may be written as follows:

$$\text{Cov}(T_r, T_c) = \frac{\text{Cov}_t[T, \text{Cov}_{xy}(D, R)]}{vP_0} - \frac{\text{Cov}_t[T, p_0(t)] \text{Cov}_{xy}[D, P(x, y)]}{P_0} =$$

$$g_1 \frac{\text{Cov}_t[T, \delta_1(t)w(t)]}{v} - g_1 \frac{\text{Cov}_t[T, w(t)]}{v} = g_1 \frac{\text{Cov}_t[T, \delta_1(t)w(t)]}{v} - g_1 \frac{\text{Cov}_t[T, w(t)]}{v} = \frac{(g_1 + \Delta_1 g_1 - g_1)}{(g_1 + \Delta_1 g_1 - g_1)}$$

(15)

$$g_1 \left\{ \frac{\text{Cov}_t[T, \delta_1(t)w(t)]}{v} - \frac{\text{Cov}_t[T, w(t)]}{v} \right\}$$

where $\text{Cov}_t[\cdot]$ is the temporal covariance of the space-averaged terms, and $\text{Cov}_{x,y}[\cdot]$ is the spatial covariance of the time-averaged terms. Here we define the term “catchment scale storm velocity” $V_s$ as follows

$$V_s = g_1 \frac{\text{Cov}_t[T, \delta_1(t)w(t)]}{\text{var}[T]} - g_1 \frac{\text{Cov}_t[T, w(t)]}{\text{var}[T]} \Delta_1$$

(16)
Finally, the term $Cov(T_r, T_c)$ may be written as follows:

$$
Cov(T_r, T_c) = g_1 \left\{ \frac{Cov_t[T, \delta_1(t)w(t)]}{V} - \frac{Cov_t[T, w(t)]}{V} \Delta_1 \right\} = \frac{V_s}{\bar{v}} \text{Var}[T]
$$

(17)

Equation (16) shows that the velocity formulation is given by the difference between two slope terms of linear space-time regression. The first term describes the total storm motion, as related to the temporal evolution of the product of the weights of the precipitation $w(t)$ and of the centre of mass $\delta_1(t)$. The second term describes the temporal storm variability as related to the temporal evolution of the weights of the storm. The difference between these two terms describes the storm velocity $V_s$. In case of rainfall pattern characterized by the same spatial organization over time (as it may occur for stratiform rainfall events), $\delta_1(t)$ would be a constant and equal to $\Delta_1$. Hence, the two terms in Eq. (16) would be equal and opposite in sign, implying a null value for the storm velocity. $V_s$ takes positive (negative) values if the storm centre of mass moves upbasin (downbasin). As a result, for downstream moving storm the variance of catchment runoff time tends to reduce and therefore the peak discharge tends to increase, consistently with the findings from several investigations (Niemczynowicz, 1984; Ogden et al., 1995; De Lima and Singh, 2002).

4 Assessment of spatial moments of catchment rainfall

Assessment of spatial moments of catchment rainfall is reported for five extreme storms and ensuing floods which have been observed in Europe in the period between 2002 and 2007 (Fig. 1). The case studies are the following: Sesia at Quinto (North-western Italy, 982 km$^2$) occurred on 4 June 2002, Sora at Vester (Slovenia, 212 km$^2$), occurred
on 18 September 2007, Feernic at Simonesti (Romania, 168 km$^2$), occurred on 23 August 2005, Clit at Arbore (Romania, 36 km$^2$), occurred on 30 June 2006 and Grinties at Grinties (Romania, 51 km$^2$), occurred on 04 August 2007. The main features of the storms and ensuing floods are reported in Table 1. These storms were selected because of the various catchment sizes (ranging from 36 to 982 km$^2$), storm durations (ranging from 05:30 h to 21 h) and space-time variability which characterize the storm events. The data concerning the events were derived from the flash flood data archive developed in the frame of the EU Project HYDRATE (www.hydrate.tesaf.unipd.it) (Borga et al., 2010). The archive includes data from twenty-five major flash flood events occurred in various regions of Europe since 1994, with twenty events occurred since 2000. The hydro-meteorological data includes high-resolution rainfall patterns, flow type processes (either liquid flow or debris flow or hyperconcentrated flows) and hydrographs or peak discharges. Climatic information and data concerning morphology, land use and geology are also included in the database. These data enable the identification and analysis of the hydrometeorological causative processes and the individual reconstruction of the events by using hydrologic and hydraulic modelling.

For the five events, both original raw radar reflectivity values and raingauge data were made available for rainfall estimation. The quantitative precipitation estimation problem is particularly crucial and difficult in the context of flash-floods since the causative rain events may develop at very short space and time scales (Krajewski and Smith 2002; Bouilloud et al., 2010). The methodology implemented here for radar rainfall estimation is based on the application of correction procedures exploiting the understanding of radar observation physics. It is based on (1) detailed collection of data and metadata about the radar systems and the raingauge networks (including raingauge data from amateurs and from bucket analysis), (2) analysis of the detection domain and the ground/anthropic clutter for the considered case (Pellarin et al., 2002), (3) implementation of corrections for range-dependent errors (e.g. screening, attenuation, vertical profiles of reflectivity) and (4) optimisation of the rainfall estimation procedure by means of radar-raingauge comparisons at the event duration scale (Buoilloud et al., 2010). The
methodology was applied consistently in the same form over the five storm events.

Analyses of rainfall variability by means of the spatial moments is attempted here to isolate and describe the features of rainfall spatial organisation which have significant impact on runoff simulation. As such, spatial moments provide information to quantify hydrological similarities among different storms, and support the transfer of knowledge and exchange of estimation and analysis techniques. The rainfall spatial moments and the catchment-scale storm velocity were computed at each time step (either at 15-min or 30-min time steps) as time series, to examine the variability in time of the statistics. The time series of the first and second scaled moments of catchment rainfall are reported in Plate 1 and 2, together with the basin-averaged rainfall rate, the fractional coverage of the basin by rainfall rates exceeding 20 mm h\(^{-1}\) (this threshold has been selected to indicate a flood-producing rainfall intensity), and the storm velocity. The values of catchment scale storm velocity were computed based on Eq. (16) by computing the two linear regressions in a moving window with window size equal to the response time of the corresponding catchment.

The time series of the first scaled spatial moment \(\delta_1\) exhibit a relatively large variability, particularly in the Feernic case, with the first scaled moments varying from 0.6 to 1.6 in the first 80 min (with a clear upbasin storm motion, as reflected in the increasing values of the statistic) and then decreasing in the following three hours, where a downbasin storm motion can be recognized. A strong downbasin storm motion can be recognized even for the Grinties during the period of strong flood-producing rainfall, with values of \(\delta_1\) steadily decreasing from 1.2 to 0.7. The case of the Sesia river basin at Quinto, as well as that of Feernic, documents the striking effect of the orography on convection development, with a concentration of the flood producing rainfall on the headwaters and values of \(\delta_1\) ranging between 1.4 and 1.6 during the period of flood-producing rainfall. Examination of the values reported for Grinties shows that the spatial moments may take values quite far from one even in small basins. The values of \(\delta_2\) generally reflects the trend of \(\delta_1\), as expected, with small values of dispersion when \(\delta_1\) is both larger or smaller than one, and values of dispersion close to one when
δ₁ is also close to unity.

For three cases out of the five (Grinties, Sora and Sesia), the values of the catchment scale storm velocity are significantly different from zero. For the case of Grinties, the value of storm velocity is steadily around $-0.2 \text{ m s}^{-1}$ for the period of strong rain rates, reflecting the important downbasin motion reported for the rainfall center of mass. A similar velocity ($-0.3 \text{ m s}^{-1}$) is found for the event occurred on the Sora. An upbasin storm velocity value ranging between $0.3$ and $0.4 \text{ m s}^{-1}$ is reported for the case of Sesia at Quinto. This value is clearly consistent with the constant upflow of humid air that sustained the formation of convective cells over the steep topography of the basin. In the three cases, the values of the storm velocity are relatively small with respect to the flood celerity characterizing flash floods, which was quantified as $3 \text{ ms}^{-1}$ by Marchi et al. (2010) with reference to several flash floods in Europe. This may suggest that even for these cases the values of storm velocity may be not large enough to influence the flood hydrograph shape.

As a further step of the analysis, we examined the relationship between the statistics $\Delta_1$ and $\Delta_2$ (Fig. 2). The analysis is carried out by dissecting the five study catchments into a number of nested subcatchments (see Table 2), as a means to examine potential catchment scale effects on the relationship between $\Delta_1$ and $\Delta_2$. The subdivision into subcatchments was either based on earlier hydrological analyses (see Table 1) where post-flood observations were used to derive indirect peak discharges (Borga et al., 2008) or on availability of internal streamgauges. Details are reported in the papers describing the relevant case studies (Sangati et al., 2009; Zoccatelli et al., 2010; Zanon et al., 2010). This subdivision will be used also for the hydrological simulations in Sect. 5. Overall, 27 catchments were used for the computation of $\Delta_1$ and $\Delta_2$. The corresponding catchment size ranges between 5 and 982 km², with 9 catchments less than 50 km², 10 catchments ranging between 50 and 150 km², and 8 catchments larger than 150 km².
Inspection of this figure shows that in 16 cases out of 27 the value of $\Delta_1$ falls in a narrow interval around one ($0.95 < \Delta_1 < 1.07$). In 13 cases out of these 16 cases, $\Delta_2$ ranges between 0.9 and 1.02, indicating that generally $\Delta_2$ is close to one when $\Delta_1$ is also close to one. In these cases the first two scaled moments are virtually unchanged with respect to the spatially uniform rainfall case. However, it is interesting to note one case of Grinties, reporting a value of $\Delta_2$ around 0.7 in correspondence to a value of $\Delta_1$ equal to 1.03. This is one of the few cases in which a strong rainfall concentration corresponds spatially to the geomorphologic center of mass of the catchment. When $\Delta_1$ exceeds the upper bound of the interval (1.07), the corresponding value of $\Delta_2$ is lower than 0.9. There is only one case of $\Delta_2$ exceeding 1.1, indicating a case of multimodal spatial distribution of rainfall. More than half of the cases show values of $\Delta_1$ in the range 1.05–1.4, documenting the effect of orography on the spatial rainfall distribution. Indeed, one of the elements that favour the anchoring of convective system is the orography, which play an important role in regulating of atmospheric moisture inflow to the storm and in controlling storm motion and evolution (Davolio et al., 2006). Consistently with this observation, values of $\Delta_1$ less than 0.95 are not represented in the study floods.

As expected, all but two of the catchments with area less than 50 km$^2$ are characterized by values of $\Delta_1$ and $\Delta_2$ close to one. For these cases, we expect a limited impact of rainfall spatial organization on flood response. On the other side, six out of the eight cases with catchment area exceeding 150 km$^2$ are characterized by values of $\Delta_1$ larger than 1.2 and corresponding values of $\Delta_2$ less than 0.8. These values (corresponding to subcatchments of Sesia and Feernic) imply a strong concentration of rainfall towards headwater and a correspondingly low dispersion around the mean values. Accordingly with the analysis reported in this work, these characteristics should translate to a delayed and more peaky hydrograph, with respect to the one obtained by using spatially uniform rainfall.
5 Examination of runoff model sensitivity to rainfall spatial organization by using scaled spatial moments of catchment rainfall: the case of the timing error

In this section we quantify the effect of neglecting the rainfall spatial variability on the rainfall-runoff model application. Hydrologic response from the five storm events over the 27 subcatchment analysed in Sect. 4 is examined by using a simple spatially distributed hydrologic model. The distributed model is based on availability of raster information of the landscape topography and of the soil and land use properties. In the model, the runoff rate \( q(x, y, t) \) [LT\(^{-1}\)] at time \( t \) and location \( x, y \) is computed from the rainfall rate \( r(x, y, t) \) [LT\(^{-1}\)] using the Green-Ampt infiltration model with moisture redistribution (Ogden and Saghafian, 1997). The adopted formulation of the Green and Ampt model has been chosen because it provides a simple, but not simplistic (Barry et al., 2005) and yet physically-based description of the infiltration-excess mechanisms. A simple description of the drainage system response (Da Ros and Borga, 1997) is used to represent runoff propagation. The distributed runoff propagation procedure is based on the identification of drainage paths, and requires the characterization of hillslope paths and channeled paths. A channelization support area \( (A_s) \) [L\(^2\)] is used to distinguish hillslope elements from channel elements. The model includes also a linear conceptual reservoir for base flow modeling (Zoccatelli et al., 2010). The reservoir input is provided by the infiltrated rate computed based on the Green-Ampt method. Details about the application of the model to the individual events, its calibration and its verification are reported in the relevant papers (Sangati et al., 2009; Zoccatelli et al., 2010; Zanon et al., 2010).

In this first exploratory work we focus on the timing error (Ehret and Zehe, 2011), i.e. the difference in the timing of the centroid of the hydrographs obtained by using either spatially distributed or spatially uniform rainfall, and analyse the relationship between this kind of error and the \( \Delta_1 \) statistic. For each subcatchment, the flash flood response was simulated by using the actual rainfall spatial variability and then by using spatially
uniform precipitations, hence obtaining two different hydrographs. Moreover, in order to clarify the relative roles of transport paths and of heterogeneity in the runoff generation processes, we performed numerical experiments in which the infiltration is “turned off”, by assuming that the soil is impermeable.

The statistic $\Delta_1$ is expected to control the hydrograph timing error. For storms characterised by $\Delta_1$ larger than one, rainfall is concentrated towards the periphery of the catchment, with the hydrograph delayed relative to the case of a spatially uniform rainfall. The opposite is true for rainfall concentrated towards the outlet ($\Delta_1$ less than one); in these cases the hydrograph should be anticipated relative to the case of spatially uniform rainfall. A statistic, termed “normalised time difference” $dT_n$, is introduced to quantify the timing error between the two hydrographs. The normalised time difference $dT_n$ is computed by dividing the time difference between the two hydrograph centroids by the response time of the catchment $E(T_c)$, as follows:

$$dT_n = \frac{E(T_{q,\text{Dist}}) - E(T_{q,\text{Unif}})}{E(T_c)} \quad (18)$$

where $E(T_{q,\text{Dist}})$ and $E(T_{q,\text{Unif}})$ are the hydrograph centroids corresponding to the hydrographs generated by using spatially distributed rainfall (termed “reference hydrograph” hereinafter) and spatially uniform rainfall, respectively. A positive (negative) value of $dT_n$ implies a positive (negative) shift in time of the reference hydrograph with respect to the one produced by using uniform precipitation. It should be noted that Eq. (18) may written down by exploiting Eq. (10) as follows:

$$dT_n = \frac{E(T_{q,\text{Dist}}) - E(T_{q,\text{Unif}})}{T_c} = \frac{E(T_r) + \Delta_1 g_1 v - E(T_r) - g_1 v}{g_1 v} = \Delta_1 - 1 \quad (19)$$

The comparison between the two hydrographs is exemplified for the cases of Sesia at Quinto (982 km$^2$) and of Grinties at Grinties (52 km$^2$) in Fig. 3a,b, respectively. The storm event which triggered the Sesia flash flood was characterised by a strong concentration of rainfall towards the headwaters ($\Delta_1 = 1.33$, $\Delta_2 = 0.79$), which implies a
longer and more peaked catchment response with respect to that corresponding to the case of spatially uniform precipitation. Correspondingly, the simulated flood peak obtained by using spatially uniform rainfall is too early \((dT_n = 0.3)\) and its amplitude is too large with respect to the “reference” hydrograph. For the case of Grienties, the storm event was heavily concentrated over the catchment center of mass \((\Delta_1 = 1.03, \Delta_2 = 0.72)\), which has no implications in terms of response timing \((dT_n = 0.05)\) but translates to a much less peaked catchment response from spatially uniform rainfall with respect to the “reference”. Both cases show clearly the impact of neglecting the spatial distribution of rainfall in rainfall-runoff modelling even at small and moderate catchment scales.

To clarify the role of runoff transport processes alone on the sensitivity of runoff model to rainfall spatial organisation, we carried out a set of experiments by “switching off” the runoff generation model. We assume in this way that the soil is everywhere completely impervious. Results for the relationship between \(dT_n\) and \(\Delta_1\) for the various catchments are reported in Fig. 4a, whereas Fig. 4b displays the same results for various classes of catchment size. Both figures show a strong linear relationship between the two variables; the linear regression is as follows

\[
dT_n = 0.33\Delta_1 - 0.33; \quad r^2 = 0.96
\]  

\text{(20)}

Figure 4a, b are very interesting because they show that the relationship between \(dT_n\) and \(\Delta_1\) is able to capture in an efficient way the control on the timing error for catchment responses obtained in disparate conditions, in terms of catchment size and characteristics of the triggering storms. It is interesting to note that the increase of \(dT_n\) with increasing \(\Delta_1\) exhibits a slope equal to 0.33 instead of one equal to 1.0 as predicted by Eq. (19). This effect is due to the role of the hillslope residence time. Indeed, the hillslope processes are represented in the hydrological model and affect the value of \(dT_n\), but they are not represented in the values of \(\Delta_1\). Increasing the hillslope residence time reduces the sensitivity of the hydrological model to rainfall spatial organisation, as shown by Zoccatelli and Wuletawu (2010). In a graph of \(dT_n\)
versus $\Delta_1$ this leads to a reduction of the slope of the linear relationship between the two variables.

Figure 5a, b report the relationship between $dT_n$ and $\Delta_1$ when both runoff generation and transport processes are active (i.e., by considering the actual distribution of the infiltration parameters).

The linear regression is as follows

$$dT_n = 0.911\Delta_1 - 0.95; \quad r^2 = 0.82$$

and is characterized by a lower correlation coefficient with respect to the impervious case. Also in the case of Fig. 5a, b, two features are worth noting: first, the slope and the intercept of the linear regression are very close to those corresponding to Eq. (19); second, the range of values of $dT_n$ is much larger than for the impervious case. Both these effects are the result of the non linearity characterizing the rainfall to runoff transformation. Zoccatelli et al. (2010), in an investigation concerning three extreme flood events, showed that the non linearity in the rainfall-runoff transformation leads to a magnification of the values of the $dT_n$ statistics with respect to those obtained in the impervious case. Essentially, this means that when rainfall is either focused on the headwaters or on the outlet, the runoff exhibits an even stronger offset towards the periphery of the catchment as a result of the non linear hydrological processes implied in the runoff generation.

This effect leads both to a steepening of the linear relationship between $dT_n$ and $\Delta_1$ (which increases from 0.33 to 0.91), and to an increase in the range of values of $dT_n$. For the impervious case, $dT_n$ ranges between $-2.5\%$ to $13\%$, whereas for the pervious case it ranges between $-20\%$ to $36\%$. Overall, the combination of the results displayed in Figs. 4a, b and 5a, b shows that the effect of the rainfall-runoff transformation on the relationship between $dT_n$ and $\Delta_1$ balances almost exactly the effect of the hillslope residence time. On this basis, it seems that the intriguing overlapping between the theoretical analysis represented by Eq. (19) and the empirical results represented by Eq. (21) needs to be substantiated by means of a wider analysis based on other flood events and other catchments.
6 Discussion and conclusion

In this paper, we proposed a new set of spatial rainfall statistics which assess the dependence of the catchment flood response on the space-time interaction between rainfall and the spatial organization of catchment flow pathways. Termed “spatial moments of catchment rainfall”, these statistics describe the spatial rainfall organisation in terms of concentration and dispersion statistics as a function of the distance measured along the flow path coordinate. The introduction of the spatial moments of catchment rainfall permits derivation of the concept of catchment scale storm velocity, which quantify the up or down-basin rainfall movement as filtered by the catchment morphological properties relative to the storm kinematics. The work shows how the first two spatial moments afford quantification of the impact of rainfall spatial organization on two fundamental properties of the flood hydrograph: timing (surrogated by the runoff mean time) and amplitude (surrogated by the runoff time variance). The first spatial moment provides a measure of the scaled distance from the geographical centroid of the rainfall spatial pattern to the catchment centroid. The second spatial moment provides a scaled measure of the additional variance in runoff time that is caused by the spatial rainfall organization, relative to the case of spatially uniform rainfall.

The analysis reported here suggests that the rainfall statistics are effective in (i) describing the degree of spatial organisation which is important for runoff modelling and (ii) quantifying the effects of neglecting the spatial rainfall variability on flood modeling, with specific reference to the timing error. This is an essential aspect of this work, since our outcome clearly shows that catchment response is sensitive to spatial heterogeneity of rainfall even at small catchment size. The timing error introduced by neglecting the rainfall spatial variability ranges between –20 % to 36 % of the corresponding catchment response time.

We believe that the main strength of the statistics lies in a better understanding of the linkages between the characteristics of the rainfall spatial patterns with the shape and magnitude of the catchment flood response. This is a fundamental aspect, since
it enables evaluating the accuracy with which rainfall space and time distribution need to be observed for a given type of storm event and for a given catchment. It would be useful to check the rainfall statistics, and the methodology behind them, for a wider variety of catchments and events to explore whether the results obtained here can be extrapolated to other cases. The statistics should also be very useful for assessing and quantifying hydrological similarity across a wide range of rainfall events and catchments, within the broader framework of comparative hydrology.

Further research should also focus on the concept of the catchment scale storm velocity. The introduction of this concept permits assessment of its significance for actual flood cases and analyses of the space and time rainfall sampling schemes which are required for its adequate estimation for various catchment scales and configurations. There is also a need to extend the formulation of the spatial moments of catchment rainfall to incorporate the hillslope transit time as a way to conceptualise the impact of the hillslope system on the catchment’s filtering properties.

Finally, the rainfall statistics introduced in this paper could be used as an input to a new generation of semi-distributed hydrological models able to use the full range of statistics, and not only the mean areal rainfall, for flood modeling and forecasting. This will permit extending the capabilities of this class of hydrological models to rainfall events characterized by significant rainfall variability.

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Table 1. Flood cases considered in the study.

<table>
<thead>
<tr>
<th>Event</th>
<th>Date</th>
<th>Aggregation time</th>
<th>Area [km²]</th>
<th>Duration [hh:mm]</th>
<th>Rain cum. [mm]</th>
<th>Peak flow [m³ s⁻¹]</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sesia at Quinto</td>
<td>04/06/2002</td>
<td>30'</td>
<td>983</td>
<td>21:30</td>
<td>111</td>
<td>1358</td>
<td>Sangati et al. (2009)</td>
</tr>
<tr>
<td>Feernic at Simonesti</td>
<td>23/08/2005</td>
<td>15'</td>
<td>167</td>
<td>5:30</td>
<td>76</td>
<td>357</td>
<td>Zoccatelli et al. (2010)</td>
</tr>
<tr>
<td>Clit at Arbore</td>
<td>30/06/2006</td>
<td>15'</td>
<td>31</td>
<td>7:00</td>
<td>81</td>
<td>156</td>
<td>Zoccatelli et al. (2010)</td>
</tr>
<tr>
<td>Grinties at Grinties</td>
<td>04/08/2007</td>
<td>15'</td>
<td>52</td>
<td>7:00</td>
<td>67</td>
<td>89.5</td>
<td>Zoccatelli et al. (2010)</td>
</tr>
<tr>
<td>Sora at Vester</td>
<td>18/09/2007</td>
<td>30'</td>
<td>212</td>
<td>17:45</td>
<td>157</td>
<td>384</td>
<td>Zanon et al. (2010)</td>
</tr>
</tbody>
</table>
Table 2. Number and area ranges of sub-basins examined in each case study.

<table>
<thead>
<tr>
<th>Event</th>
<th>Date</th>
<th>Area [km²]</th>
<th>Number of sub-basins</th>
<th>Range of sub-basin areas</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sesia at Quinto</td>
<td>04/06/2002</td>
<td>983</td>
<td>9</td>
<td>75-983</td>
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<tr>
<td>Feernic at Simonesti</td>
<td>23/08/2005</td>
<td>167</td>
<td>9</td>
<td>5-167</td>
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<tr>
<td>Clit at Arbore</td>
<td>30/06/2006</td>
<td>31</td>
<td>2</td>
<td>12-31</td>
</tr>
<tr>
<td>Grinties at Grinties</td>
<td>04/08/2007</td>
<td>52</td>
<td>3</td>
<td>11-52</td>
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<tr>
<td>Sora at Vester</td>
<td>18/09/2007</td>
<td>212</td>
<td>4</td>
<td>31-212</td>
</tr>
</tbody>
</table>
Fig. 1. Study catchments and their location in Europe.
Plate 1. Precipitation analyses by using time series of precipitation intensity, coverage (for precipitation intensity > 20 mm h\(^{-1}\)), scaled first moment \(\delta_1(-)\), scaled second moment \(\delta_2(-)\) and storm velocity for the cases of Feernic, Clit and Grinties.
Plate 2. Precipitation analyses by using time series of precipitation intensity, coverage (for precipitation intensity $> 20 \text{ mm h}^{-1}$), scaled first moment $\delta_1(-)$, scaled second moment $\delta_2(-)$ and storm velocity for the cases of Sora and Sesia.
Fig. 2. Relationship between the time-integrated first and second scaled moment $\Delta_1$ and $\Delta_2$: (a) for the study catchments, (b) for specific classes of catchment area.
Fig. 3. Modelled flood hydrographs obtained by using spatially distributed and uniform precipitation, for the case of (a) Sesia at Quinto (983 km$^2$) and (b) Grinties at Grinties (52 km$^2$).
Fig. 4. Relationship between the normalised time difference $dT_n$ and the time-integrated first scaled moment $\Delta_1$ for impervious soils: (a) for the study catchments, (b) for specific classes of catchment area. The dashed line is the linear regression ($r^2 = 0.96$).
Fig. 5. Relationship between the normalised time difference $e dT_n$ and the time-integrated first scaled moment $\Delta_1$ for actual soils: (a) for the study catchments, (b) for specific classes of catchment area. The dashed line is the linear regression ($r^2 = 0.82$).