State updating of a distributed hydrological model with Ensemble Kalman Filtering: effects of updating frequency and observation network density on forecast accuracy

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Abstract

This paper presents a study on the optimal setup for discharge assimilation within a spatially distributed hydrological model. The Ensemble Kalman filter (EnKF) is employed to update the grid-based distributed states of the hourly HBV-96 model. Synthetic and real world experiments are carried out for the Upper Ourthe (1600 km²), a quickly responding catchment in the Belgian Ardennes. We assess the impact on the forecasted discharge of (1) various sets of the spatially distributed discharge gauges and (2) the filtering frequency. The results show that the hydrological forecast at the catchment outlet is improved by assimilating interior gauges. This augmentation of the observation vector improves the forecast more than increasing the updating frequency. In terms of the model states, the EnKF scheme is mainly changing the pdf’s of the two routing model storages. This also holds for situations, where the uncertainty in the discharge simulations is smaller than the defined observation uncertainty.

1 Introduction

Accurate and reliable hydrological forecasts have been a challenge in applied hydrology for decades. Better forecasts can be obtained through data assimilation (DA), by merging observations with model simulations (Reichle, 2008). This approach basically updates the model states with externally measured variables (Pauwels and De Lannoy, 2006) to obtain correct initial conditions for the next time step. Currently, most hydrological forecasting systems employ lumped hydrological models (with deterministic or manual state updating), but there is a clear tendency to move towards distributed models combined with hydrological ensemble forecasts, (e.g. Koren et al., 2004; Cole and Moore, 2009; Weerts et al., 2012). The main advantage of spatially distributed models is the possibility to force them with spatially measured data, which nowadays become more readily available due to rapid developments in telemetry. Distributed model states also resemble the real world observations (e.g. groundwater levels, soil moisture,
discharge) at the interior of the catchment more closely than lumped states over the whole catchment. Another advantage of applying distributed models is the ability to simulate and predict hydrological variables at interior locations within the catchment. For the future, techniques should be developed and tested on how to perform ensemble data assimilation using these models in real-time settings (Weerts et al., 2012; Liu et al., 2012).

Data assimilation methods used in hydrology can be divided into two classes: (1) sequential and (2) variational (e.g. Liu and Gupta, 2007). Sequential methods are mostly employed for state updating in hydrological models by assimilating observations sequentially. This analysis is performed for each step when the observations become available. Its impact depends on the uncertainties in both the observations and model states. Variational methods rather minimize a cost function over a simulation time window. At the beginning, a first-guess model is constructed, which is afterwards updated by creating an adjoint model which propagates backwards in time and incorporates the mismatch between the model and observations (Liu and Gupta, 2007).

A popular method often used in both meteorological and hydrological forecasting is the Ensemble Kalman Filter (EnKF) (Evensen, 2003, 2009). This sequential data assimilation method is an extension of the classical Kalman filter (KF), which was originally developed for linear systems (Kalman, 1960). The EnKF propagates an ensemble of model realizations through time and estimates the error covariance matrix from the ensemble statistics. The advantage of the EnKF to other data assimilation methods is computational efficiency, and easy and straightforward implementation within a data assimilation framework for both lumped and distributed models (Pauwels and De Lannoy, 2009).

Discharge measurements are the most widely used in-situ hydrological observations for model updating, since they reflect the local catchment wetness conditions and are often available at high temporal resolution (Pauwels and De Lannoy, 2006; Teuling et al., 2010), which is necessary for operational hydrological forecasting. Comprehensive analyses of discharge data assimilation into spatially lumped hydrological models
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was carried out by e.g. Pauwels and De Lannoy (2006); Weerts and El Serafy (2006); Pauwels and De Lannoy (2009). Among others, Aubert et al. (2003), Lee et al. (2011) and Lee et al. (2012) assimilated, in addition to discharges, in-situ soil moisture measurements. Furthermore, the water level is an example of an in-situ state variable, which can also be assimilated into a hydrodynamical model (e.g. Madsen and Skotner, 2005; Neal et al., 2007; Weerts et al., 2010).

Although the assimilation of remotely sensed data into operational hydrological models can improve model performance, this task is complicated (Pauwels and De Lannoy, 2009), because remotely sensed data usually have a higher uncertainty than other in-situ measurements. Nevertheless, several studies focus on assimilation of remotely sensed data into hydrological models, e.g. soil moisture (Pauwels et al., 2001, 2002; Moradkhani, 2008) and snow (Slater and Clark, 2006; Thirel et al., 2011). Remotely sensed water levels were assimilated into a hydraulic model by Giustarini et al. (2011).

So far, few studies have reported on data assimilation within spatially distributed hydrological models, both from a scientific and an operational perspective. Clark et al. (2008) assimilated discharges using the adjusted EnKF for a real world case only. Unfortunately, their approach was not tested and evaluated in a synthetic experiment, which would help to understand the behaviour of the simulated and updated probability density function (pdf) of the model states. Another EnKF study, by Blöschl et al. (2008), applied discharge assimilation into a real-time grid-based operational flash flood forecasting model. As Blöschl et al. (2008) did not elaborate on the importance of individual discharge gauges in the interior of the catchment, this remains an interesting question to address. However, more recently the positive effect of interior discharge gauges on hydrological forecast was described by Lee et al. (2011), who assimilated both discharge and/or in-situ soil moisture data using the variational method.

From an operational point of view it is also desirable to optimize the telemetry system, which delivers the observations to the forecaster. One of the interesting aspects is the frequency at which the observations become available. In most of the currently operational large scale flood forecasting systems, it is typical to separate the forecast
process into a state (i.e., carry-over or update) run, which is normally run once a day, and forecast runs which are run more frequently. The frequency of the state run is very much dictated by the frequency at which observed data becomes available (i.e., frequency of polling of the telemetry system). Hence it is interesting to test what the optimal updating frequency is, at which the hydrological data are to be assimilated into the forecasting model to obtain the most accurate forecast.

The objective of this study is to analyse the sensitivity of the DA framework to the number and the locations of discharge gauges, which are assimilated into a grid-based distributed operational hydrological forecasting model using the EnKF. The optimal updating frequency will be addressed as well.

The focus here lies mainly on the state run part of the forecast chain and therefore we leave out the meteorological forecasts and error modelling of the meteorological forecast, which is a research topic on its own. Using this approach we eliminate the large uncertainties in the meteorological forecasts and we can focus entirely on the uncertainty in the initial model states.

2 Material and methods

2.1 Catchment description and data availability

The hydrological simulations are carried out for the Upper Ourthe catchment, upstream of Tabreux (Fig. 1b), which drains an area of about 1600 km² (Berne et al., 2005). This catchment forms a tributary of the Meuse River basin, originating in the hilly parts of the Belgian Ardennes. The climatic conditions can be classified as rain-fed with irregular snow in winter and the runoff regime is highly variable with low summer discharges and high winter discharges (Leander et al., 2005). Relatively shallow soils in combination with significant elevation differences result in a quickly responding catchment. As such, the whole region represents a significant flood risk to the Netherlands (de Wit, 2008). Table 1 presents the catchment response time of the Upper Ourthe, which is defined
as the time between the event-based catchment averaged rainfall centroid and the discharge peak for individual discharge gauges. The catchment response time is about 30 h at Tabreux and about 11 h at the two most upstream gauges (Mabompre and Ortho). This indicates that it takes about 20 h of travel time within the main channel between the two upstream gauges and the catchment outlet.

Hourly precipitation data are available from 42 automatic rain gauges situated within the Belgian Ardennes region, from which 10 are located inside the Upper Ourthe catchment (Hazenberg et al., 2011). Discharge is measured at six different points at an hourly resolution. Next to that, temperature is obtained from the Saint Hubert meteorological station (Fig. 1b). The long term mean monthly values of potential evapotranspiration are assumed identical to those of the operational lumped HBV-96 model, derived from the St. Mihiel station in North-Eastern France.

2.2 Generation of a spatially distributed precipitation ensemble

A precipitation ensemble, a finite and discrete number of spatial realizations over time, represents the uncertainty associated with temporal as well as spatial variation of precipitation. This probabilistic model input approach enables hydrologists to more critically evaluate hydrological simulations and/or predictions. In the current paper we employed a time-dependent multivariate spatial conditional simulation method (Rakovec et al., 2012), which is further made conditional on preceding simulations. This method identifies, at an hourly time step, temporally coherent errors in spatial precipitation fields that are plausible from a hydro-meteorological perspective. Neglecting this temporal aspect could lead to underestimation of the overall uncertainty in the precipitation ensemble.

The theory of conditional (sequential Gaussian) simulations is fully explained by Goovaerts (1997). For a detailed description of this time-dependent simulation method using the gstat R package (Pebesma, 2004; Rossiter, 2007; R Development Core Team, 2011) we refer to Rakovec et al. (2012), who carried out synthetic experiment as well as analyzed three real rainfall events during winter 2002/2003 within
the Belgian Ardennes. Altogether, 27 rain gauges were used to simulate 64 ensemble members over a $100 \times 100 \text{ km}^2$ domain with $10 \text{ km}^2$ raster resolution (see Fig. 1b). In the current study we made the simulation of each realization conditional on 3 h of the corresponding previously simulated realization, which is a recommendation following from the results obtained by Rakovec et al. (2012).

### 2.3 Hydrological model

Currently, a spatially lumped HBV-96 model (Lindström et al., 1997) is used operationally by the Dutch authorities for flood forecasting of the Meuse River basin at and downstream of Sint Pieter at an hourly time step (Driessen et al., 2010). The Meuse River basin upstream of Sint Pieter ($\sim 21\,000\ \text{ km}^2$), the entrance point into the Netherlands, is conceptualized into 15 lumped HBV-96 sub-catchments of which the Upper Ourthe is one (Fig. 1a). The models for the Meuse River sub-catchments were calibrated for the period 1970–1984 and validated for the period 1985–1996 at a daily time step by Booij (2002). The precipitation inputs for the original HBV-96 models were derived from 39 rain gauges, of which only one station was located within the Upper Ourthe catchment. For operational purposes, the hourly HBV-96 models were derived and re-calibrated based on the work of Booij (2002) by van Deursen (2004). Comparison of daily and hourly model versions was carried out by Weerts (2007). Currently, the operational forecast derived with the lumped hourly HBV-96 models is used as a lateral inflow into a 1D-hydrodynamic model of the Meuse River. This lumped model does not employ sequential state updating, but is updated with discharge observations by means of an automated autoregressive moving average error correction method (Broersen and Weerts, 2005).

For this study, we have developed a grid-based spatially distributed HBV-96 based model within PCRaster, a computer language for constructing iterative spatio-temporal environmental models (Karssenberg et al., 2009; PCraster, 2012). Such a grid-based approach is a popular concept in applied hydrology (e.g. Koren et al., 2004; Blöschl et al., 2008; Cole and Moore, 2009; Thielen et al., 2009). For each $1 \times 1 \text{ km}^2$ grid cell
of the Upper Ourthe the operational HBV-96 model was implemented and is used as a benchmark case. Both model parameters and structure are taken identical to the lumped HBV-96 version used operationally except for the discharge routing, for which a kinematic wave model (Chow et al., 1988; PCraster, 2012) is used. We are aware that we did not yet take full advantage of distributed models, which allows to define spatially variable model parameters, since that is beyond the scope of the current study.

The structure of the model used in this study is shown in Fig. 2. For each grid cell the model consists of four model states: (1) snow (SN), (2) soil moisture (SM), (3) upper zone storage (UZ) and (4) lower zone storage (LZ). The dynamics of the model states are governed by the following model fluxes: rainfall, snowfall, snowmelt, actual evaporation, seepage, capillary rise, direct runoff, percolation, quick flow and base flow. The latter two fluxes force the kinematic wave model. This routing scheme calculates the overland flow using two additional model states, the water level (H) and discharge accumulation over the drainage network (Q). In this study we use a very similar routing setup as the one applied within the distributed hydrological CQ-flow model (Schellekens, 2006). The main drainage network is obtained using the 8-direction steepest descent algorithm based on a digital elevation model with a grid resolution of $1 \times 1 \text{km}^2$. Afterwards, the catchment is split between the channel and non-channel grid cells. The channel network is defined for the cells with Strahler stream order (Strahler, 1988) greater than 3. As such, only the major tributary of the Upper Ourthe was identified, which corresponds well to the channel network derived from a topographic map (for comparison see Figs. 1b and 4). A channel width is assigned based on a field survey and Google maps (2011) according to the Strahler stream order number, as shown in Table 2. For the non-channel cells, the channel width remains equal to the grid cell width. The channel width is then used to derive the water level, which defines together with the local topography gradient the gravity force, which is a driver for the river flow. Channel roughness coefficients were estimated by a sensitivity analysis.

To verify that the implemented spatially distributed version is well able to produce hydrological discharge simulations using the aforementioned precipitation ensemble
generator, Fig. 3 shows the observed discharge and both the spatially lumped and distributed HBV-96 discharge simulations. This figure shows a clear consistency between both HBV-96 simulations for the 5 month period (15 August 2002–15 January 2003), which will be further used in this study. The Nash-Sutcliffe (NS) model efficiency coefficient (Nash and Sutcliffe, 1970) between both HBV-96 simulations is about 0.99, which gives strong evidence of model similarity. Moreover, the NS’s between the observed and the ensemble of discharges using the grid-based version of HBV-96 is between 0.92 and 0.96 and the root mean square error (rmse) ranges between 9 and 12 m$^3$ s$^{-1}$ (Fig. 3). The NS between the observed and the simulated discharge using the lumped HBV-96 is about 0.96 as well. For completeness, additional statistics are summarized in Fig. 3. Even though the grid-based HBV-96 model was not recalibrated, the model performs very well.

2.4 Ensemble Kalman filter

The Ensemble Kalman filter (Evensen, 2003, 2009; Weerts and El Serafy, 2006) is a recursive Bayesian estimation method, which estimates the true probability density function of the model states conditioned on observations. Let us denote a dynamic state space system as:

$$x_k = f(x_{k-1}, \theta, u_{k-1}) + \omega_k \quad \omega_k \sim N(0, S), \quad (1)$$

where $x_k$ is a state vector at time $k$, $f$ is an operator expressing the model state transition from time step $k - 1$ to $k$ in response to the model input $u_{k-1}$ and time-invariant model parameters $\theta$. So $f$ is in fact the hydrological model. $\omega_k$ stands for system noise, normally distributed with zero mean and covariance $S$. This additive system noise incorporates the overall uncertainties in model structure, parameters and model inputs. One can expect some spatial patterns of model errors to be found in the covariance matrix $S$. However, quantification of $S$ for highly non-linear hydrological systems is a complicated task and therefore we keep it time-invariant.
The observation process is governed by Eq. (2):

\[ y_k = H(x_k) + v_k \quad v_k \sim N(0, R_k), \]  

(2)

where \( y_k \) is an observation vector derived from the model state \( x_k \) and the model parameters through the function \( H \) (in our case the kinematic wave model). \( v_k \) stands for additive observational noise, normally distributed with zero mean and covariance \( R_k \). For independent measurement errors between the observations in vector \( y_k \), we can assume \( R_k \) to be a diagonal matrix. As such, this simplification does not consider any dependency in model simulations for observation points located close to each other.

The idea of recursive Bayesian estimation is to construct a conditional density \( p \) for an ensemble of the state \( x_k \) given all available information up to and including the step \( k \): \( p(x_k|Y_k) \), where \( Y_k = (y_1, y_2, \ldots, y_k) \). This can be obtained using the Bayesian rules in two steps: forecast and update.

After the update of model states at time \( k - 1 \), the hydrological model is used to forecast model states at time \( k \) (Eq. 1). The grid-based model states form a matrix, which consists of \( N \) state vectors \( x_k \) corresponding to \( N \) ensemble members:

\[ X_k = (x^1_k, x^2_k, \ldots, x^N_k), \]  

(3)

where

\[ x_k = (SN_{1:m}, SM_{1:m}, UZ_{1:m}, LZ_{1:m}, H_{1:m}, Q_{1:m})^T, \]  

(4)

SN, SM, UZ, LZ, H and Q are the HBV-96 model states (Sect. 2.3), \( m \) gives the number of grid cells and \( T \) is the transpose operator. The ensemble mean

\[ \bar{x}_k = \frac{1}{N} \sum_{i=1}^{N} x^i_k \]  

(5)

is used to derive the model error for each ensemble member:

\[ E_k = (x^1_k - \bar{x}_k, x^2_k - \bar{x}_k, \ldots, x^N_k - \bar{x}_k). \]  

(6)
The ensemble estimated model covariance matrix $P_k$ is defined as

$$P_k = \frac{1}{N-1} E_k E_k^T.$$  \hfill (7)

When observations become available, the model states are updated as follows:

$$X_k^+ = X_k^- + K_k (y_k - H(X_k^-)),$$  \hfill (8)

where $X_k^+$ is the new updated (posterior) model state matrix and $X_k^-$ is the forecasted (prior) model state matrix. $K_k$ is the Kalman gain, a weighting factor of the errors in model and observations:

$$K_k = P_k H_k^T (H_k P_k H_k^T + R_k)^{-1},$$  \hfill (9)

where $P_k H_k^T$ is approximated by the forecasted covariance between the model states and the forecasted discharge, and $H_k P_k H_k^T$ is approximated by the variance of forecasted discharge (Houtekamer and Mitchell, 2001):

$$P_k H_k^T = \frac{1}{N-1} \sum_{i=1}^{N} (x_k^i - \bar{x}_k)(H(x_k^i) - \bar{H}(x_k))^T,$$  \hfill (10)

$$H_k P_k H_k^T = \frac{1}{N-1} \sum_{i=1}^{N} (H(x_k^i) - \bar{H}(x_k))(H(x_k^i) - \bar{H}(x_k))^T,$$  \hfill (11)

where

$$\bar{H}(x_k) = \frac{1}{N-1} \sum_{i=1}^{N} H(x_k^i).$$  \hfill (12)
2.5 Experimental setup

2.5.1 Synthetic experiment

A synthetic experiment has been carried out to examine the ability of the EnKF to update the grid-based hydrological model states via assimilation of the spatially measured discharge. The EnKF framework was applied to a 5-month period from 15 August 2002 to 15 January 2003, which includes a dry and a wet period. For reasons of clarity of the experimental setup, we did not update model parameters, which were kept constant. In an open loop simulation, i.e. without data assimilation, the model was initially forced with uncertain precipitation inputs with a simulation memory of 3 h (64 ensemble members) derived using time-dependent multivariate spatial conditional simulation (see Sect. 2.2 and Rakovec et al., 2012) and observed deterministic potential evapotranspiration and temperature. This produced an ensemble of simulated discharges from which one complete realization was randomly selected as the true discharge ($Q_{\text{true},k}$). To introduce discharge observation uncertainty, a normally distributed error $\nu_k$ (Eq. 2) with heteroscedastic variance was added to $Q_{\text{true},k}$ to obtain a synthetic observation $Q_{\text{obs},k}$:

$$Q_{\text{obs},k} = Q_{\text{true},k} + \nu_k \quad \nu_k \sim N(0,(0.1Q_{\text{true},k})^2) .$$

The discharge measurement error was defined similar to Thiemann et al. (2001) and Weerts and El Serafy (2006) and the ensemble size of 64 members corresponds to other studies as well, e.g. Pauwels and De Lannoy (2009). The model errors ($S$, Eq. 1) were obtained by perturbing the model states indirectly by uncertain precipitation input. Additional direct perturbation of the model storages, which was applied by e.g. Clark et al. (2008) was not considered, because this was considered to be beyond the scope of this study. That would make our example, which focuses on the input uncertainty only, even more complicated.

In the synthetic experiment we assimilated in total 5 discharge observation schemes, expressed by the vectors $y_k$, as shown in Fig. 4. The first case (A) is identical to
a lumped model for the Upper Ourthe, where only the most downstream observation is available. The second case (B) considers only the two most upstream discharge gauges. The third case (C) includes three additional observations upstream to case A. The fourth case (D) contains all six discharge gauges and the fifth case (E) includes an additional 12 imaginary gauges to the fourth case.

Moreover, the effect of the updating frequency, i.e. how often the observations become available and how often they are assimilated into the model, was analysed for updating frequencies of every 24 h, 12 h and 6 h.

The performance of the data assimilation machinery regarding discharge forecasting was then evaluated using the root mean square error (RMSE_{lt}):

\[
RMSE_{lt} = \sqrt{\frac{1}{MN_l} \sum_{i=1}^{M} \sum_{j=1}^{N_l} \left( Q_{\text{obs},lt}^{i,j} - Q_{\text{for},lt}^{i,j} \right)^2},
\]

(14)

where \(lt\) stands for lead time (\(lt = 1 \text{ h}, 2 \text{ h}, \ldots, 48 \text{ h}\)) and \(Q_{\text{for}}\) is a forecasted discharge vector of length \(N_l\). \(M\) is the number of hydrological forecasts, which were issued over the 5-month period. To allow comparison of the different updating frequencies between each other, the hydrological forecast was issued every 6 h, i.e. 4 times a day.

### 2.5.2 Real world experiment

In the real world experiment we applied the same model forcings as in the synthetic experiment (Sect. 2.5.1). The difference with the synthetic experiment was the assimilation of the real discharge observations \((Q_{\text{obs},k})\), which were perturbed by a normally distributed observation error with a variance of \((0.1 Q_{\text{obs},k})^2\). The size of the observation vector \(y_k\) was limited to cases A, B and D (Fig. 4).
3 Results

3.1 Synthetic experiment

3.1.1 Root mean square error of forecasted discharge

The long-term RMSE (Eq. 14) between the synthetic observed discharge and the forecasted discharge of the 64 ensemble members at Tabreux (Fig. 1b) for the three updating frequencies of 24 h, 12 h and 6 h is shown in Fig. 5. The forecasted discharge without data assimilation gives a constant RMSE of about 5 m$^3$s$^{-1}$, which corresponds to 16% error with respect to the mean simulation over the 5-month period. For the forecasted discharge with data assimilation, there is a reduction in RMSE for all discharge observation vectors, however the magnitudes differ as follows: (1) for the updating frequency of 24 h, the benchmark case A performs worst of all 5 cases except for the first couple of hours, during which the model transports the updated discharge from the two most upstream gauges (case B) to the catchment outlet. Moreover, there is a gradual decrease in RMSE for the cases with large number of assimilated gauges. (2) For the updating frequency of 12 h, there is even further reduction in RMSE for all the observation vectors. The largest reduction is achieved for case E, for which the RMSE at the lead time of 1 h is 1.4 m$^3$s$^{-1}$ (5% of the mean observed discharge). Additionally, case A (one gauge at the outlet) outperforms case B (two interior gauges only) for lead times up to about 20 h, which is in line with the channel travel time from the most upstream gauges to the outlet. (3) For the updating frequency of 6 h, there is not a pronounced improvement in RMSE. This can be expected, because within the 6 h between updating moments, hardly any rainfall is transformed into discharge, even at the most upstream gauges, as is shown in Table 1. Slightly improved forecasts are issued for cases A and B, however for case E the 6-h discharge assimilation even deteriorates the forecast performance at Tabreux in comparison to the 12-h updating frequency.
3.1.2 State updating

A further logical step in the analysis of the results is to have a look at the ability of the DA machinery to correctly update the model states. In other words to check if the setup of the EnKF can identify the pdf around the true model states. However, we recall, that there is not a single configuration of model states yielding one discharge value due to the fact that our system is spatially distributed.

We investigate the effect of the observation vector at the updating frequency of 24 h. We selected 2 locations (location 1 and 6 in Fig. 1b) within the catchment domain for which the simulated and updated model states are presented in Figs. 6 and 7. Figures 6 and 7 show in the top panel the open loop simulations for five model states with the highlighted true model states. For two time instants, on 3 November and on 2 January (dashed vertical line), we show the histograms of the 24 h lead time forecasted and the updated model states for the five discharge observation vectors (cases A, B, C, D and E, see Fig. 4). The true model states are indicated by asterisks. Recall that the true state for the synthetic experiment was a randomly selected sample. Note that the snow model states are not shown because there was no snow simulated during the 5-month period. We have chosen 3 November and 2 January, because both dates occur shortly before a discharge peak, although the wetness conditions of the catchment are different. The first and smaller peak is observed, when the model storages are rather dry. The second and larger peak occurs after an extensive rainy period, when the model states became fully saturated.

Figure 6 indicates that at the catchment outlet there is hardly any difference between the forecasted and updated model states in soil moisture (SM), upper zone storage (UZ) and lower zone storage (LZ) for all discharge observation vectors and for both dry and wet conditions. However, note that both the forecasted and updated pdf’s of SM, UZ and LZ tend to have more accurate peaks around the “true” values for a larger number of assimilated discharge gauges. This means that even though there is no clear difference between the forecasted and updated pdf’s at one time instant, its accumulation
over time makes it visible in the higher kurtosis. Therefore, it makes sense to update those rather insensitive model states. Furthermore, the EnKF is well able to identify the “truth” in two routing storages, the water level (H) and the water storage in the channel (Q) on 3 November as well as on 2 January. There is a clear shift of the updated histogram centroid towards the true value for all discharge observation vectors, except for case B, which remains unchanged. This caused by fact that case B (Fig. 4) consists of two discharge gauges, which are located far away from the catchment outlet. Furthermore it can be seen that the EnKF is well able to identify the two routing states even if the prescribed discharge observation error bands are larger than the ensemble spread of the forecasted discharge.

At the interior point (Fig. 7), similar to the catchment outlet (Fig. 6), there is no pronounced update in forecasted and updated histograms for soil moisture (SM), upper zone storage (UZ) and lower zone storage (LZ). The two routing states are again easier to identify. However, the ability of the EnKF to identify the true H and Q depends on the location of stations in the discharge observation vector. For the cases B, D, and E, which contain at least one gauge situated close to location 6, the updated histogram of H and Q approaches the true state and also its shape becomes narrower with higher frequency, i.e. more leptocurtic. On the other hand, for the cases A and C, which do not include gauges close to the location 6, no changes in H and Q histograms occur.

3.2 Real world experiment

3.2.1 Root mean square error of forecasted discharge

The long-term RMSE for the real world experiment is shown in Fig. 8. Similar to the synthetic experiment, all three discharge observation vectors assimilated into the model improve the forecasted discharge for lead times up to 48 h, except for case B, which slightly deteriorates the forecast performance in comparison with the forecasts without discharge assimilation for lead time longer than 30 h. The best results, meaning the lowest RMSE, are achieved by assimilating all six gauges (case D) for all updating
frequencies, although for longer lead times it approaches the benchmark case A (the outlet gauge only). The largest reduction was achieved by case D, for which the RMSE at the lead time of 1 h was about 6 m$^3$ s$^{-1}$ (20% of the mean observed discharge) for the updating frequency of 6 h.

Similar to the synthetic experiment, case A (one gauge at the outlet) outperforms case B (two interior gauges only) for all updating frequencies in the real world experiment. Moreover, we can observe for case B a rather constant RMSE during the first 20 h. This surprisingly steady RMSE may be explained by the assimilation effect of the most upstream gauges (locations 5 and 6), for which it takes about 20 h to reach the outlet (location 1). Although an increase in updating frequency from 24 h to 12 h improves the RMSE, further increase in the updating frequency from 12 h to 6 h yields more or less an equal RMSE, which corresponds to the synthetic experiment.

The short-term RMSE for an individual major flood peak, which was observed at the beginning of January 2003, is shown in Fig. 9. Because of the rather short period used in this analysis, the shapes of the RMSE are not smoothed out and the forecasted RMSE without updating is not constant over time either. The best forecast improvements are again achieved by assimilating all six discharge gauges (case D) for all updating frequencies for lead times up to about 15–20 h. For longer lead times, case B (only two upstream gauges) gives very similar RMSE to case D, because the added value of the more downstream gauges (1–4 in Fig. 1b) is filtered out after about 20 h, as shown in Table 1. It is worth mentioning that case B outperforms case A for lead times from 5 h to 20 h, which is contradicting the long-term statistics (Fig. 8). Additionally, with regard to the updating frequency, the scenarios of both 12 h and 6 h give very similar RMSE, which is lower than the updating frequency of 24 h.

### 3.2.2 State updating

Like in the synthetic experiment, hardly any change between the forecasted and the updated histograms is observed for soil moisture, upper zone storage and lower zone storage (Figs. 10 and 11), but visible changes can be seen in the routing storages,
water level and discharge. For the discharge observation vectors, which contain at least one gauge in the vicinity of the location of the state observation, there is a shift of the centroid of the histograms for discharge and the corresponding water level towards the uncertain discharge observation constrained by the error bars.

4 Discussion

The advantage of a grid-based hydrological model with grid-based routing over a lumped model without explicit routing (e.g. unit hydrograph-type) is that the modelled discharge is represented by spatially distributed model states, which quantify the volume of water within the channel network. This means that we do not have to explicitly consider any time delay between model states and discharge, as it would be needed in spatially lumped models using the retrospective EnKF (Pauwels and De Lannoy, 2006). Another advantage of a grid-based approach over a lumped one is that the spatially distributed discharge observations can be easily incorporated into the model states and make the forecast more accurate for longer lead times.

A novel approach of this study was the application of time-dependent multivariate spatial conditional simulations (Goovaerts, 1997; Pebesma, 2004; Rakovec et al., 2012) of hourly rain gauge observations, used to force the hydrological model in hindcasting mode. As demonstrated by Rakovec et al. (2012), this multivariate approach satisfies for each precipitation realization the requirement to have a coherent temporal evolution (required within the DA framework), unlike the time-independent univariate simulations. Using this precipitation ensemble generator we can achieve the goal, that the corresponding simulated spatially distributed model states inherit the temporal aspect of the rainfall fields. Finally, as an alternative to rain gauge observations, the precipitation ensemble could be derived from radar rainfall estimates from the C-band radar located in the catchment (Hazenberg et al., 2011), which is a possible topic of further studies.
This study provided also a closer look at the pdf’s of the forecasted and updated model states during two hydrologically different situations, while the majority of hydrological DA papers on state updating focus only on the forecasted and analysed discharge and do not address the importance of individual model states. In this study, mainly the pdf’s of the two routing model storages were changed, while the other model states (SM, UZ, LZ) were less sensitive to the EnKF scheme. However, with a larger number of assimilated discharge gauges, both the forecasted and updated pdf’s of SM, UZ and LZ had more accurate peaks around their true values. Therefore, it makes sense to update those rather insensitive model states. The reason for this behaviour might be the limited model structure, which is similar to other PCRaster operational hydrological applications like the LISFLOOD model (Salamon and Feyen, 2009), where the individual neighbouring model cells are not connected by means of interflow and regional groundwater flow but only drained by some sort of sheet flow via the routing states. This means that the SM, UZ and LZ model states are only controlled by the spatial variation of rainfall.

Finally, it is interesting to note that, unlike Clark et al. (2008), we were able to improve hydrological forecasts using the standard EnKF implementation in both synthetic and real world experiments compared to open loop simulations.

5 Conclusions and recommendations

We analysed the sensitivity of the data assimilation framework to the updating frequency, the number and the locations of interior discharge gauges, which were assimilated into a grid-based distributed hydrological forecasting model using the EnKF. The validation station of this study is Tabreux, which is located at the Upper Ourthe catchment outlet, Belgium.

In the synthetic experiment we showed that the hydrological forecast at the catchment outlet is improved by assimilating more interior gauges in terms of the forecasted root mean square error (RMSE). That is logical, because all the other discharge
observations contain information from upstream to improve the posterior forecast. In addition, the EnKF scheme mainly changed the pdf’s of the two routing model storages and this also holds for situations, where the uncertainty in the discharge simulations is smaller than the defined observation uncertainty. Moreover, with an increasing number of discharge observations, the centroid of the updated histograms within the observation error bounds was approaching the true value more closely and with smaller variance than for the less dense discharge observation networks.

In the real world experiment, the best results in terms of the RMSE were achieved using all observations, which includes all six discharge gauges and given the travel time of the catchment, the updating frequency of 12 h seems to be the most appropriate. Additionally, similar to the synthetic example, only the two routing model storages showed some sensitivity to the EnKF scheme in terms of the forecasted and updated histograms. We can conclude that the hydrological forecast at Tabreux can be improved by assimilating more upstream gauges using the EnKF data assimilation framework. This augmentation of the observation vector improves the forecast more than increasing the updating frequency.

For operational use we recommend to implement additional upstream gauges into the observation system, which enables an increase of the updating frequency and more accurate forecasts, if the polling frequency allows doing so. Another recommendation for future research is to have a closer look at alternative model structures (including re-calibration of spatially distributed model parameters) and their effect on the sensitivity of individual model states within the EnKF framework. The main limitation of the current model structure is that there is no flux between neighbouring cells except for the two routing model states. Alternatively, hydrological forecasts can be improved by applying other Kalman-type methods, e.g. the Ensemble Kalman Smoother (Evensen and Van Leeuwen, 2000), which calculates the analysis from several previous time steps up to the time of forecast, instead of mapping the instantaneous covariance between states and discharge (Clark et al., 2008), as shown for the standard EnKF in this study. Finally, additional in-situ observations can be considered to be assimilated.
into the spatially distributed model states, e.g. soil moisture and/or groundwater levels. The latter are believed to resemble point-wise the actual regional water storage more closely than soil moisture observations.

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Table 1. Catchment response time between the catchment averaged rainfall centroid and the discharge peak.

<table>
<thead>
<tr>
<th>Discharge gauge*</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upstream area [km²]</td>
<td>1620</td>
<td>1225</td>
<td>959</td>
<td>743</td>
<td>318</td>
<td>387</td>
</tr>
<tr>
<td>Time to peak [h]</td>
<td>31</td>
<td>26</td>
<td>21</td>
<td>13</td>
<td>11</td>
<td>11</td>
</tr>
</tbody>
</table>

* Location of discharge gauges is indicated in Fig. 1b.
Table 2. The channel width corresponding to Strahler stream order number.

<table>
<thead>
<tr>
<th>Strahler stream order number</th>
<th>Channel width [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>20</td>
</tr>
<tr>
<td>5</td>
<td>15</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
</tr>
</tbody>
</table>
Fig. 1. (a) The Meuse River basin upstream of Sint Pieter, the entrance point into the Netherlands, and its 15 sub-catchments (gray polygons) including the Upper Ourthe (black polygon). (b) Topographic map of the Upper Ourthe (OU, white line) including the river network (black lines), rain gauges (plusses), six river gauges (triangles and labeled with numbers: 1 = Tabreux, 2 = Durbuy, 3 = Hotton, 4 = Nisramont, 5 = Mabompre, 6 = Ortho) and the climatological station in Saint Hubert (circle). For completeness, two neighbouring sub-catchments Ambleve (AM) and Vesdre (VE) are shown, however, they were not analyzed in this study. The grey grid shows the $10 \times 10 \text{ km}^2$ pixel resolution of the rainfall generator, which will be further explained in Sect. 2.2. Projection is in the Universal Transverse Mercator (UTM) 31N coordinate system. After Hazenberg et al. (2011).
Fig. 2. Left: catchment discretization using a grid-based approach including the channel delineation. Arrows indicate flow direction. Right: schematic structure of the HBV-96 model for each grid cell. Model states are in bold and model fluxes in italics.
Fig. 3. Hourly hydrograph for the Upper Ourthe at Tabreux: observed (solid blue line); HBV-96 simulation using the grid-based spatially distributed version (light grey band for the 95 % confidence bounds of 64 ensemble members); HBV-96 simulation using the operational spatially lumped version (dashed black line). Histograms of the Nash Sutcliffe (NS) model efficiency coefficient and the root mean square error (rmse) between the observed and the ensemble of discharges using the grid-based version of HBV-96 are shown in the upper left corner.
Fig. 4. Five cases of the discharge observation vector $y_k$ of increasing spatial extent. Discharge gauges contained in the observation vector are plotted in black. Channel delineation using Strahler stream ordering plotted in white pixels.
Fig. 5. Synthetic experiment, simulation period from 15 August 2002 to 15 January 2003. Root mean square error at Tabreux for different discharge observation vectors. Forecast issued every 6 h. EnKF assimilation every 24 h (left), 12 h (center), 6 h (right).
Fig. 6. Synthetic experiment. Top: open loop model state simulations (gray lines) including the true states (black line) at location 1 in Fig. 1b. Bottom: forecasted (grey histograms) and analysed (dashed histograms) model states at location 1 for two dates (3 November and 2 January) and considering 5 discharge observation vectors (A, B, C, D, E). The true states are indicated by asterisks and the error bars represent the observation errors.
Fig. 7. Same as Fig. 6, but for location 6 in Fig. 1b.
Fig. 8. Real world experiment, simulation period from 15 August 2002 to 15 January 2003. Root mean square error at Tabreux for different discharge observation vectors. Forecast issued every 6 h. EnKF assimilation every 24 h (left), 12 h (center), 6 h (right).
Fig. 9. Same as Fig. 8, but for simulation period from 27 December 2002 to 8 January 2003.
Fig. 10. Real world experiment. Forecasted (grey histograms) and analysed (dashed histograms) model states at location 1 for two dates (3 November and 2 January) and considering 3 discharge observation vectors (A, B, D). The observed discharge is indicated by asterisks and the error bars represent the associated observation errors.
Fig. 11. Same as Fig. 10, but for location 6.