Uncertainty in computations of the spread of warm water in a river – lessons from Environmental Impact Assessment

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Abstract

The present study aims at evaluation of sources of uncertainty in modelling of heat transport in a river caused by the discharge coming from a cooling system of a designed gas-stem power plant. This study was a part of Environmental Impact Assessment and was based on two-dimensional modelling of temperature distribution in an actual river. The problems with proper description of the computational domain, velocity field and hydraulic characteristics were considered in the paper. An in-depth discussion on the methods of evaluation of dispersion coefficients in the model comprising all four components of the dispersion tensor was carried out. Numerical methods and their influence on final results of computations were also discussed. All computations were based upon a real case study performed in Vistula River in Poland.

1 Introduction

A prerequisite for the construction of a new industrial plant is an Environmental Impact Assessment (EIA for brevity), understood as a formal process used to predict the environmental consequences of any development project. In case of gas-stem power plants, among others, one definitely needs to evaluate the threats caused by heated water discharged into a river and it obviously should be a part of EIA. The question of performance of natural system (a river in the considered case) begs the question of the quality and relevance of the used model and the evaluation criteria. It is rather obvious that the credibility of the computations performed within EIA depends strongly on the available data. The practice, however shows that such data in many cases is of limited value and, in spite of this, quantitative results from relevant models are expected. In other words time constraints and lack of information require that the EIA must rely exclusively on expert skills and opinions. Moreover, planning of additional measuring campaigns during EIA may become impossible in the given time. Therefore the discussion of the usefulness of the obtained results is of crucial importance. The authors
aim to share their experience in the use of an up to date model on the spread of warm water jet in a river in the light of the scarcity of proper data. It is quite a common practice that professionals gain personal experience, particularly with respect to the models developed by themselves, they have good feeling how their parameters and boundary conditions should be assigned to yield the satisfactory results and at the same time they realize that their model may fail under some circumstances and an evaluation of such circumstances constitute, in fact, an empiric estimate of the uncertainty of the simulated results.

Beven (2007a) claims that in non-ideal cases (i.e. nearly all real applications) non-statistical (epistemic) uncertainties may dominate. They are e.g. bias and nonstationarity in input errors, model structural errors and commensurability errors (where a variable or parameter in a model is different to an equivalent quantity that can be measured in the field). In practice it is almost impossible to separate different sources of aleatory (statistical) and epistemic uncertainties unless very strong assumptions are made (Beven, 2007b; Beven et al., 2011). Anyway assessing the sources of uncertainty in respect to the discussed hydrological processes is crucial for its correct analysis (e.g. Catari et al., 2011; Hughes et al., 2011).

Only a few sources of uncertainty, but those which significantly influence the results, will be studied herein. It is by no means a formal uncertainty analysis – even simple goodness-of-fit criteria cannot be applied because no observed values are at our disposal to compare them with the predicted dependent variables. In uncertainty analysis there are obviously other possibilities, for example with use of Monte-Carlo type simulations that are capable to predict model output variability by examining variable distributions resulting from a large number of deterministic model runs, with input variables sampled from appropriate distributions (e.g. Kochanek and Tynan, 2010; Scharffenberg and Kavvas, 2011; Shen et al., 2012). The question, however arises what are those appropriate distributions in the case considered in this study. Moreover the computational expense of multiple model runs would definitely be too high.
One source of uncertainty is insufficient knowledge of analyzed phenomena. In addition we have to consider errors introduced by the models used in the calculations, which always simplify the described phenomena. For various reasons these models cannot take into account all the variables controlling the phenomena, also they usually have to simplify the problem to make it practically solvable. Possible numerical errors caused by the applied numerical algorithms have to be considered as well.

Very often a way out in EIA process is the analysis of extreme cases in which the natural environment is endangered most. This way one may obtain relatively safe solutions, i.e. such solutions that the real behavior of the system should appear even more environmentally friendly. This way the end-users of the model results can be sure that if the solutions to the model fulfill their requirements then the real dependent variables are more likely not to exceed their threshold values.

The present study draws from the case study computations aimed at building scenarios of the spread of heated water discharged from a designed gas-stem power plant on the Vistula River below Włocławek town (Kalinowska et al., 2012). Such heated water constitutes a real environmental problem in many other situations and therefore is often called thermal pollution. It is usually a side effect of operation of power plants, chemical industries, and other hydraulic engineering facilities using water from rivers and open channels in cooling process. Authors encountered a number of problems significantly influencing the credibility of the results and it motivated them to share this experience with the readers. In the study a two-dimensional temperature field behind warm water discharge was sought.

Similar mathematical models as used in this study are useful in EIA of completely different nature, e.g. in the studies of the quality ground water treated as a fundamental source of drinking water reservoirs (Aniszewski, 2011) and therefore further consideration maybe useful for relatively big research and engineering community.
2 Mathematical model of heat transport

In developing models there is always a trade off and as simplification of the described process certain details may be excluded. Although in general situation temperature distribution in rivers should be described by three-dimensional (3-D) heat transport equation, its reductions to two (2-D) or even one dimensional (1-D) situations are considered in practice. The main obstacles to progress in 3-D approach is the lack of knowledge of the realistic detailed 3-D velocity field and high computational expense. In the so-called mid-field region the depth-averaged heat transfer models are relevant (Rodi et al., 1981; Seo et al., 2010; Szymkiewicz, 2010):

\[
h(x) \frac{\partial T(x,t)}{\partial t} = \nabla (h(x)D(x) \cdot \nabla T(x,t)) - \nabla (h(x)v(x) \cdot T(x,t)) + q; \tag{1}
\]

where: \( t \) – time, \( x = (x, y) \) – position vector, \( T(x,t) \) – depth-averaged water temperature, \( h(x) \) – local river depth, \( v(x) \) – depth-averaged velocity vector, \( D(x) \) – heat dispersion tensor, \( q \) – source function describing additional heating or cooling processes. This equation represents the temperature change in time due to heat flows: that is carried with average velocity, carried by velocity fluctuations (turbulent heat conduction), transmitted by the deviation of velocity and temperature across the depth (dispersion of heat) and related to heating or cooling additional sources processes.

The dispersion tensor that appears in the equation due to the depth-averaging represents additional, significant transport mechanism, which is not a physical process but only a consequence of averaging of the equation. By analogy with the turbulent heat exchange, it is assumed that dispersion flow is proportional to the average temperature gradient (Rowiński, 2002; Rutherford, 1994). This proportionality is determined by the so-called dispersion coefficients \( (D_{xx}, D_{xy}, D_{yx}, D_{yy}) \). It is worth noting that similar molecular and turbulent heat conduction coefficients are orders of magnitude smaller than the dispersion coefficients. Usually the molecular coefficients are omitted and the turbulent ones are omitted or included in the dispersion tensor.
Depth-averaging results in model simplifications. Water in rivers is usually well-mixed and temperature is nearly uniform from surface to bottom (Allan, 1995), but nevertheless we should always remember that some variations caused by external environment (e.g. surface water temperature exchange with the atmosphere, ground waters tributaries, etc.) may occur. Sources and sinks of heat energy may play an important role in Eq. (1) but in the case considered herein we did lack the information on all the components. In principle they embrace the exchange with the atmosphere and with the ground waters through e.g. shortwave and longwave radiation, latent heat transport, ground heat flux etc. To some extent we may assume that this additional term affects both the heated water and the ambient water in more or less the same way and since we are interested in the difference between the temperatures in the river and in the warm water jet, the source term may be neglected in computations. But this simplification is definitely a potential source of an error that may arise in computations. On the other hand side inclusion of this term with guessed coefficients of proportionality would introduce much more serious errors.

3 Solution to 2-D heat transport equation

To solve the heat transport equation described in the previous section we need to know: the geometry of the river, the two-dimensional velocity field, the boundary and initial conditions, the full dispersion tensor, and if we want to include any additional sources (e.g. temperature exchange with the atmosphere) substantial additional information is required (like meteorological data). We may encounter serious problems in the process of data collecting and acquisition which frequently lead to simplifications affecting to some extent the final solutions. When solving typical academic problems (especially in laboratory) one usually tries to gather all necessary data and consider all possible processes which affect the solution. In real situation when dealing with EIA such precise approach is often practically impossible. Measurements of potentially necessary data are usually limited due to the time constraints, huge costs and many technical
restrictions. Recently Piotrowski et al. (2011) have analysed methods for analysis of the fate of pollutants over long distances (far field) with a view to applications on un-gauged rivers, i.e. those for which little hydraulic or morphometric data are available. In the present study a different approach – limited to the so-called mid-field – is proposed and the situation assumed is a bit more optimistic. However, the obtained solutions are biased by errors that we should be aware of. We do have to have in mind what aspect is of crucial importance for the decision-maker (orderer of EIA) and a good point within EIA exercise is that frequently it may be enough to show the possible range of the values of interest (and not the precise numbers!). Knowing the minimum, the maximum and the mean possible values we may be in the position to prepare realistic scenarios (including the most significant extreme cases).

Below the selected problems that may be encountered in the preparation of a scenario of the spread of the heated water below the discharge from a power plant are discussed. Various aspects from an actual EIA study (selected just for the purpose of example) will be considered herein.

### 3.1 Problem solved

This study is based upon the computations of the spread of the heated water discharged from a designed gas-stem power plant located near Włocławek town in the Vistula River. The variability of water temperature depending on four various locations of the differently designed exit pipes was of particular interest within the study. The water is supposed to be discharged with constant intensity of 14 m$^3$s$^{-1}$ and temperature higher by 7 $^\circ$C than the temperature of ambient water. The adopted water flow in the river was the averaged low-flow: $Q = 334$ m$^3$s$^{-1}$. The detailed description of the study area, the discussion of the computations and final results for different variants may be found in (Kalinowska et al., 2012).
3.2 Geometry

The first serious problems are encountered already at the level of the definition of the computational domain. It is nothing new in hydraulic computations but the problem is not often revealed by modellers when they publicise their results. It is worth noting that characterisation of the computational domain for natural rivers is kind of an art even in 1-D situations when 1-D velocity field is the main concern of the modeller (see e.g. Rowiński et al., 2005b).

The crucial inconvenience is related to insufficient number of the measured cross-sections which requires special interpolation procedures between the consecutive profiles. Taking for granted that cross-sections were carefully chosen accounting for all the sensitive points (like river bends, dunes, river narrowness etc.), which obviously is not always the case, the bathymetry and the computational grid can be described mathematically. In the considered case 21 cross-sections of the Vistula River (between the 690 250 and 718 200 km of the Vistula River) were available. The problem with them was that they were measured relatively long time ago, namely in 1994, using electroacoustic probe positioned in Cartesian coordinated system. The data obviously does not truly represent todays situation. An example of the variation in the bed elevation in the yet earlier period (between 1971 and 1994) for the selected cross-section is shown in Fig. 1. This change is significant and it comes up to the value of 2 m and one may expect that a significant change could occur after 1994 as well.

Since the preliminary computations have shown that the analysed stretch of the river may be reduced (transverse mixing is faster than initially expected), only 9 cross-sections have been used for final calculations. The average distance between the cross-sections turned out to be as large as ca. one kilometre and obviously some changes in bed elevations were omitted.

It is quite obvious that the number of the measured cross-sections was too small for adequate mapping of the shape of the river channel and therefore it was necessary to use a topographic map and to create (by interpolating procedures) additional
cross-sections before the final grid and bed profile are generated. The two-dimensional computation grid and the bed profile have been prepared using the CCHE_MESH generator developed by NCCHE – National Center for Computational Hydroscience and Engineering (Zhang, 2005a). This is user-friendly mesh generator for generating structured quadrilateral meshes based on the bed topography and the bed elevation data. The grid has been then used to calculate the velocity field by means of CCHE2D model, also developed by NCCHE. A different type of the grid was required by the River Mixing (RivMix) model, developed by Authors, used to compute the temperature distribution (Kalinowska and Rowiński, 2008). This is often the case when two different computational models are used. Then all necessary data have to be transformed from the computational domain of one model to the domain of another one. This procedure should be performed carefully, because this is another stage where additional error may be introduced. Proper interpolation procedures (Delaunay interpolation) have been used to transform the data in the analysed case from one grid into the other one. This way a number of sets of grids with different grid steps have been prepared. For the final computations and analysis presented in the paper the grid with the grid steps $\Delta x = \Delta y = 10 \text{m}$ has been chosen. Such grid steps, together with an appropriate choice of numerical method, give us fast and accurate enough solution in the considered situation. Figure 2 presents the resulting interpolated water depth on the chosen grid. The water depth has been computed for the selected discharge and the water level elevation. It should be noted that their change would definitely affect the solutions. In this case study it was impossible to perform calculations for different ranges of possible water levels and discharges and therefore it was assumed to be enough to carry out the calculations for reasonably low water levels to capture the environmentally most severe situation.

It is also important to note the two-dimensional models’ limitations. Since 2-D depth-averaged models can be only used after the complete vertical mixing occurs, we cannot interpret any results in the initial stage in the direct neighbourhood of the discharge (so-called near-field zone). According to the procedure proposed in
(Jirka and Weitbrecht, 2005) this initial distance in the considered case reaches about 75 m from the discharge site.

### 3.3 Velocity field

The knowledge of a relatively accurate two-dimensional velocity field is a precondition to solve the two-dimensional heat transport equation (Eq. 1). In reality we usually do not have enough measurements, especially when dealing with 2-D case; therefore it is necessary to model the velocity profile. There are numerous software packages available, but nevertheless it is essential to bear in mind that solution of the momentum equations is not straightforward. It is necessary to know what kind of equations are solved in the applied software and what kind of numerical algorithms are being used. Then we may have some expectations on the eventual errors committed. In the considered case the CCHE2D – two-dimensional depth-averaged, unsteady turbulent open channel flow model has been used (Altinakar et al., 2005; Jia and Wang, 2001; Ye and McCorquodale, 1997; Zhang, 2005b). The model is based on the depth-averaged Navier-Stokes equations. The details on the velocity field are given in (Kalinowska et al., 2012) and will not be discussed herein. However, one needs to remember that the correct evaluation of the velocity field is crucial for further computations and it is burdened with all the problems that turbulence modelling, choice of the so-called closure hypotheses etc. bring. The resulting velocity magnitude is presented in Fig. 3 and further in the paper, for simplicity, will be assumed as the true one.

### 3.4 Initial and boundary conditions

The nature of differential equations causes that one needs to a priori know the initial and boundary conditions for the problem solved. In this study an average natural river temperature of the considered area has been taken as the initial condition. It was assumed that the depth-averaged 2-D river temperature was the same within the entire domain area. We do realize that in reality the river temperature may change in space...
or in time due to day-night or seasonal changes. Taking into account those changes would require yet more experimental data. Since we are interested in water temperature distribution after the release of the heated water (more precisely: increase of the water temperature values), calculations may be performed with regards to the relative temperature ($\Delta T$) and may readily be scaled to the river temperature at a given time of a day or a season.

In the considered case the continuous discharge of $14$ m$^3$ s$^{-1}$ of water heated by $7^\circ C$ in relation to ambient water was assumed in the computations. It was assumed that the river bank had a temperature equal to the initial temperature of ambient water, and the boundary conditions were defined in the way not allowing the river water to heat the bank but the bank could possibly cool down the water. Such assumption was caused by the lack of information on the bank temperature but in principle could locally influence the results. It may particularly be important when the source of the heated water is located on the river bank and the difference between the bank and the source temperature is large.

There are numerous possible ways of discharging the heated water into the river and the localization of the exit pipes, minimizing the negative environmental impact, constitutes the key element of EIA. Different variants have been analyzed in the considered case according to their locations and the method of discharge: the point-like continuous discharge or discharge along straight exit pipe containing nozzles for uniform distribution of heated water. Four of those variants – two point-like with different localization and two straight exit pipes $14$ m and $28$ m long were presented in (Kalinowska et al., 2012).

Numerical implementation of the way of discharge is again only a poor approximation of the real conditions. Solving numerically the heat transport equation, the continuous solution domain has to be replaced with a discrete domain and the line segment pipe within the considered grid is represented by discrete points in the grid nodes located along the segment. In case of the grid with steps of $10$ m, the $14$ m line segment is represented by just $2$ points (located near the beginning and the end of the segment) and $28$ m line segment is replaced by $3$ points (near the beginning, the centre and the
end of the segment). Such approximation is obviously a major simplification. Additional serious simplification is made in the process of calculation of the effecting temperature ($T_E$) at a relevant discharge site. Since the single grid cell of dimension of $\Delta x$ by $\Delta y$ is the smallest fragment of the river that we can consider, it was assumed that in such single cell surrounding the point of discharge the heated water mixed immediately with the river water. Taking into account the volume of the river water and the heated water in the considered cell we calculated the effective temperature after the discharge based on a simple expression:

$$T_E = \frac{Q_W T_W + Q_Z T_Z}{Q};$$

(2)

where: $T_W$ – the temperature of the ambient water, $T_Z$ – the temperature of the discharged heated water, $Q_W$ – river flow at the source, $Q_Z$ – heated water flow at the source, $Q$ – the resultant (summary) flow discharge. The river flow discharge at a given point at the source was determined by the relationship:

$$Q_W = h_Z v_Z \Delta x;$$

(3)

where: $h_Z$ – the water depth at the source, $v_Z$ – water velocity at the source, $\Delta x$ – grid spacing. Simple calculation shows that the effective temperature obviously depends strongly on the assumed grid spacing and the effective temperature naturally tends to $7^\circ C$ when the grid approaches zero (see Table 1).

The above assumptions involve no loss of generality – they influence the results of computations mostly in the direct surrounding of the discharge point, area that cannot be interpreted anyway, since must be modeled using 3-D equations.

### 3.5 Dispersion coefficients

Dispersion coefficients controlling the rate of mixing are essential to solve Eq. (1) and at the same time their determination is extremely complex. In general case in Cartesian
coordinates the dispersion coefficients form a non-diagonal dispersion tensor with four dispersion coefficients:

\[
D = \begin{bmatrix} D_{xx} & D_{xy} \\ D_{yx} & D_{yy} \end{bmatrix}.
\] (4)

Those four dispersion coefficients should be computed on the basis of the so-called longitudinal \(D_L\) and transverse \(D_T\) dispersion coefficients. It is crucial to compute the dispersion tensor \(D\) in the proper way, otherwise unrealistic results may easily be obtained. Authors in their earlier studies (Kalinowska and Rowiński, 2008; Rowiński and Kalinowska, 2006) presented how erroneous ways of the simplifications in the treatment of dispersion tensor often met in literature, result in misleading concentration distributions revealed both in their shapes as well as the concentration values. In brevity, the proper way to obtain the full tensor \(D\) is rotation of a diagonal tensor \(D_D\) containing the above mentioned \(D_L\) and \(D_T\) coefficients:

\[
D = R(\alpha) \cdot D_D \cdot R^{-1}(\alpha);
\] (5)

where: \(R = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}\) – rotation matrix, \(\alpha\) – angle between the flow direction and \(x\)-axis, \(D_D = \begin{bmatrix} D_L & 0 \\ 0 & D_T \end{bmatrix}\). The erroneous ways of tensor simplification could be different, but they usually boil down to somehow negligence of the off-diagonal components: \(D_{xy}, D_{yx}\). Figure 4 presents the distribution of the relative temperature \(\Delta T\) (the difference between the temperature of ambient water and the actual river temperature) in case of the point-like continuous discharge in the middle of the channel along the cross-sections located ca. 250 and 500 m from the discharge point. The results have been obtained for four different methods of computation of dispersion tensor \(D\): using the proper method defined by Eq. (5) and marked as I; using simplified methods marked as II (the off-diagonal elements of dispersion tensor are omitted), III (dispersion coefficients \(D_L\) and \(D_T\) are treated as a vector), and IV (the diagonal elements of dispersion...
tensor $D_{xx}$ and $D_{yy}$ are simply replaced by $D_L$ and $D_T$, the off-diagonal are treated as 0). The difference between the results could be easily observed. The increase of the temperature in the considered case in the middle of the channel is much bigger (almost 3 times) while using the proper way of dispersion tensor computation (I), which could be very important in case of EIA. Also the mid-field zone is much larger in this case.

In case of the simplified method IV, the cloud of thermal pollution is pushed to the left river bank. Detailed definition and analysis of the simplified methods could be found in (Rowiński and Kalinowska, 2006).

Longitudinal and transverse dispersion coefficients depend on many factors related to the geometry of the channel, dynamics and turbulence of the flow. Due to their significance and difficulties associated with their determination those coefficients are subject of many discussions in literature, but there are still a lot of questions and misconceptions concerning their values. The best source of information about the dispersion coefficients for the actual river is above all a tracer experiment, but usually in practical applications – since such experiment is expensive and time consuming – impossible to be carried out. Moreover, the practice shows that tracer tests are feasible in one-dimensional situations only and they allow for the determination of just the longitudinal dispersion coefficients (Deng et al., 2001, 2002; Guymer, 1998; Kumar and Dalal, 2010; Rowiński et al., 2008; Sukhodolov et al., 1998). In case when such tracer tests are not available, the reliable estimation of dispersion coefficients becomes extremely difficult and can be a source of large uncertainty. Universal formulae for those coefficients are sought in many research centres and they are attempted to be related to the known hydraulic parameters, such as averaged depth $H$, width $B$ and velocities $U$, shear velocities $U_*$ and the channel sinuosity $S$ or the radius of curvature $R$ of the considered channel. That search is of rather limited universality and often the formulae working for one channel do not hold for the others. Another problem for the modellers is that many papers proposing or applying such formulae do not discuss their limitations or ways of their determination. Following selected review articles presenting expressions for transverse dispersion coefficient (e.g. Deng et al., 2001; Jeon et al., 2007;
Seo and Baek, 2008), the values of $D_T$, obtained from them for the considered area of Vistula River are plotted in Fig. 5.

Bulk hydraulic parameters were used in all formulae and one can readily note differences that those various formulae produce. Those differences are caused by a number of factors. In principle various formulae are constructed for different types of rivers and flow conditions and they take into account different processes and different variables. For example the formula proposed by Fischer (1967) considers the transverse turbulent diffusion only. The formula was built for an artificial uniform and straight channel with constant depth. Other formulae take into account both the transverse turbulent diffusion and the transverse dispersion. In the Rutherford formula (Rutherford, 1994) it is assumed that dispersion effect is significantly larger than turbulent diffusion which is omitted in considerations. Note also other problems that should be tackled while using these formulae. Some of them include, for example, geometrical parameters such as sinuosity index or curvature radius of the channel. Those values are easily measurable in laboratory conditions but in natural rivers their determination may become extremely difficult and not unique. Meanders and bends may change significantly from one section to another and then the values of the dispersion coefficients should also change. Single values assumed for the whole river reach may introduce serious errors. Moreover, in practice the division of the river reach into sections with constant sinuosity may become intractable. The situation turns out to be not much simpler when the mean river width or depth are taken into account. In the considered herein case the average width of the channel is about 400 m. The values of $D_T$ coefficient obtained with use of the average river width are presented in Fig. 5 in green. The geometry of the considered Vistula River reach is such that at some cross-sections the river narrows to the values of 300 m or even to 200 m. Then the values of $D_T$ change but those changes are revealed in dependence on the chosen formula (see Fig. 5 blue and red bars).

In case of longitudinal dispersion coefficient $D_L$ the number of formulae presented in literature is extremely large. Wallis and Manson (2004) review and discuss many of them. Numerous and successful attempts pertain also to other methods like e.g.
artificial intelligence methods allowing to estimate this coefficient on the basis of the known hydraulic parameters (see e.g. Kashefipour et al., 2002; Piotrowski et al., 2011, 2006; Rowiński et al., 2005c; Tayfur and Singh, 2005).

It turns out that the differences between the values of the obtained $D_L$ with use of different methods or expressions are larger than in case of the coefficient $D_T$. The differences in the obtained $D_L$ by means of various methods often differ by orders of magnitude. There is also another crucial problem related to the basics of the definition of coefficient $D_L$. The longitudinal dispersion coefficient appearing in 2-D equation is not the same creature as the longitudinal dispersion coefficient in 1-D equation and this fact is very often forgotten in scientific considerations. The values of dispersion coefficient relevant in 1-D situation are very often adopted to 2-D models which on one hand side is often done unconsciously and on the other hand side it is practical approach since usually only values from 1-D approach are available. Such operation introduces additional and eventually quite serious errors.

Let us illustrate the above problem. In the practical case study considered in this paper there was no experimental data that could be used to calibrate the model to obtain the proper values of coefficients $D_L$ and $D_T$. As expected different relationships for dispersion coefficient resulted in significantly different values of $D_L$ and $D_T$. In the study the most general relationship for dispersion coefficients (Czernuszenko, 1990; Sawicki, 2003) was used:

$$D = a h u_*;$$

were: $D$ – longitudinal or transverse dispersion coefficient, $u_*$ – bed shear velocity, $a$ – dimensionless parameter that theoretically may assume values from a relatively large range. Taking into account the reasonable maximum and minimum values of parameter $a$ (based on the experience gained in alike rivers) we were in position to analyse the environmentally most severe cases, which is actually the main task of EIA. The most probable value of $a$ has been also analysed and presented in the study. The choice of $a$ is not free from subjectiveness.
The value of the dimensionless parameter $a$ is not the same for longitudinal and transverse dispersion coefficient. Usually the ranges of $a$ for $D_L$ and $D_T$ are the following (e.g. Rutherford, 1994; Sawicki, 2003):

\begin{align}
30 \leq D_L/hu_* \leq 3000; \\
0.15 \leq D_T/hu_* \leq 0.9.
\end{align}

Note relatively large range of dimensionless parameter $a$. On top of that any averaging of the value of $h$ – either within the entire river reach or even between two given cross-sections may introduce a substantial error. Another problem of quite fundamental nature is the determination of the bed shear velocity (see Rowiński et al., 2005a).

In this study the bed shear velocity has been calculated based on the following basic expression:

$$u_* = \sqrt{\frac{\tau}{\rho}};$$

where: $\tau$ – total shear stress, $\rho$ – water density, the calculation assumes $\rho = 1000 \text{ kg m}^{-3}$. The distribution of the total bed shear stress was calculated with use of the CCHE2D model based on the turbulent Reynolds stresses.

In the present study a number of simulations for different parameters $a$ have been performed. The variant with the point-like continuous warm water discharge of $14 \text{ m}^3 \text{ s}^{-1}$, located in the middle of the channel has been chosen to illustrate the problem. Figure 6 presents the 2-D temperature fields for three selected values of parameter $a$ used to calculate the longitudinal dispersion coefficient $D_L$ ($a = 100, a = 500, a = 1000$). Plots in the top row of Fig. 6 present the temperature for the whole area under consideration whereas plots in the bottom row zoom in the area surrounding the point of release. Note that plots present the relative temperature $\Delta T$ – the difference between the actual river temperature and the temperature of ambient water. Figure 7 allows for accurate quantitative analysis of the differences among the distributions of
water temperature calculated for various parameters \( a \) and it presents the values of the temperature at the cross-sections located respectively at 100, 250 and 500 m from the point of discharge. For the convenience of the reader those cross-sections are also marked in Fig. 6 (bottom row). Note that in case of large values of parameter \( a \) the temperature cloud slightly propagates also upstream.

Visible differences in the results reveal the importance of the selection of the dispersion coefficient. Its proper choice may be guaranteed by relevant tracer tests performed in situ but as mentioned earlier in the paper it is usually undoable simply because of environmental concerns (Rowiński and Chrzanowski, 2011). Therefore potentially extreme values of \( a \) are considered to discuss the least favourable variant that may occur in the river reach. In comparison to longitudinal dispersion the selection of transverse dispersion coefficient in analyzed case does not influence the results so significantly and therefore computations of different variants are not presented.

Further, similarly to previous work (Kalinowska et al., 2012) the results of computations are presented for the longitudinal and transverse dispersion coefficients calculated for \( a \) equal to 500 and 0.6 respectively using the averaged values of water depth (1.53 m) and the bed shear velocity (0.045 m s\(^{-1}\)). In such case the dispersion coefficients values are: \( D_L = 34.425 \text{ m}^2 \text{s}^{-1} \) and \( D_T = 0.041 \text{ m}^2 \text{s}^{-1} \). In case of using local values of the depth and the bed shear velocity the distributions of dispersion coefficients are not uniform and have been presented in Fig. 8.

Note the differences in the solutions obtained with use of the averaged and local values of the depth and the bed shear velocities (see Fig. 9).

The difference in results in the selected case of continuous discharge of 14 m\(^3\) s\(^{-1}\) of heated water along straight exit pipe, 14 m long, located near the left bank of the river is not significant. The maximum difference between the results is equal to 0.34°C. But situation may possibly change in case of different hydraulic parameters.
3.6 Numerical solution

An important issue in the discussion of the model results is the mathematical consistency between continuum and discrete variants of partial differential equations used to represent the heat transport in a river. In principle in academic studies an important step would be to compare numerical solutions obtained from calculations performed on successively refined meshes or grids to a reference. In the case considered herein the exact solution of the advection-diffusion equations is not known to define such reference. Therefore geometrically simpler problems are considered to assess the numerical solutions. In the evaluation of the numerical methods applied the key problems to be assessed are (Kalinowska and Rowiński, 2004, 2008): numerical diffusion and numerical dispersion, instability and limitation of the computation time. The first one – numerical diffusion – results in faster spreading of thermal pollution. It is quite known effect but since it may be difficult to observe in real applications, usage of numerical schemes that may generate such error has to be conscious. Numerical dispersion error may cause non-physical oscillations, but the effect is easier to notice in the obtained results. The details of the errors arising from the use of various numerical methods applied to Eq. (1) are given in (Kalinowska and Rowiński, 2007). In that study a two-dimensional model of the spread of passive pollutants in flowing surface water – RivMix (River Mixing Model) allowing to choose between four different numerical schemes has been applied. In this study RivMix model was adopted in order to simulate the spread of thermal pollution and an accurate and fast Alternative Direction Implicit (ADI) method has been selected to solve the Eq. (1). The proper time ($\Delta t = 1$ s) and grid steps ($\Delta x = \Delta y = 10$ m) have been finally chosen to ensure the detailed and fast enough solution. But before the time and spatial steps were chosen several numerical tests had been performed. The influence of the increasing model resolution on the final results is a subject of many studies in environmental fluid mechanics problem (see e.g. Ziemiański et al., 2011).
4 Conclusions

Mathematical models are suitable and powerful tools in Environmental Impact Assessment whenever influence of the new hydraulic constructions on natural environment have to be considered. Those tools cannot of course replace relevant observations and measurements and in the light of lacking data and impossibility of model calibration, understanding of various sources of uncertainties are crucial for the assessment of model credibility. This study was based upon a case investigation of the spread of heated water discharged into a river from a designed gas-stem power plant. It was analysed how model simplifications may influence the final results. Particular attention was paid to methods of evaluation of dispersion coefficients. It was shown that in natural rivers all components of a dispersion tensor should be taken into account to qualitatively reflect the proper shape of temperature distributions. All the results considerably depend on the 2-D velocity field, hydraulic and morphometric characteristics of the flow, particularly the bed shear stresses should be mentioned in this respect. One should also remember about the significance of the choice of a numerical method that might introduce unphysical phenomena in the final results.

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References


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Spread of warm water in a river

M. B. Kalinowska and P. M. Rowiński


Table 1. The effective temperature in the single cell at the source in case of the point-like continuous discharge located at point $Z = (1850, 775)$ for different sizes of grid cell.

<table>
<thead>
<tr>
<th>$\Delta x = \Delta y [m]$</th>
<th>$T_E [^\circ C]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.6</td>
</tr>
<tr>
<td>5</td>
<td>5.3</td>
</tr>
<tr>
<td>10</td>
<td>4.3</td>
</tr>
<tr>
<td>25</td>
<td>2.7</td>
</tr>
</tbody>
</table>
Fig. 1. Selected cross-section of Vistula River with marked bed profiles in years 1971 and 1994. Significant changes at the bed profiles are visible.
Fig. 2. The water depth for the considered reach of the Vistula River, computed for the averaged low-flow discharge.
Fig. 3. The velocity magnitude for the considered reach of the Vistula River, computed for the averaged low-flow discharge.
Fig. 4. Distribution of temperature ($\Delta T$) in case of the point-like continuous discharge in the middle of the channel along the cross-sections located ca. 250 and 500 m from the discharge point: I – with the proper method of dispersion tensor computation; II – with simplified method in which the off-diagonal elements of dispersion tensor $D_{xy}$ and $D_{yx}$ are omitted; III – with simplified method in which dispersion coefficients $D_L$ and $D_T$ are treated as a vector; IV – with simplified method in which the diagonal elements of dispersion tensor $D_{xx}$ and $D_{yy}$ are simply replaced by $D_L$ and $D_T$, the off-diagonal are treated as 0.
Fig. 5. Transverse dispersion coefficient for the considered reach of the Vistula River calculated using several formulae (taking into account different hydraulic parameters). For the convenience of the reader, the lower part of the chart was also presented in a different scale.
Fig. 6. Two-dimensional temperature distribution in case of continuous discharge of $14 \text{ m}^3 \text{s}^{-1}$ of warm water at point $Z_1 = (1850, 800)$ for different values of dimensionless coefficient $a$: 100, 500 and 1000.
Fig. 7. Temperature distribution in case of continuous discharge of 14 m$^3$ s$^{-1}$ of warm water at point $Z_1 = (1850, 800)$ for different values of dimensionless coefficient across the cross-sections located at 100, 250 and 500 m from the discharge.
Fig. 8. The longitudinal $D_L$ and transverse $D_T$ dispersion coefficients for the considered reach of the Vistula River, computed for the local values of river depth and bed shear velocity.
Fig. 9. Two-dimensional temperature distribution results for continuous discharge of 14 m$^3$ s$^{-1}$ of heated water along straight exit pipe, 14 m long, located near the left bank of river in case of using averaged (left chart) and local values of depth and shear velocity (middle chart) for computing dispersion coefficients and difference between them (right chart). The beginning and end of the exit pipe are located respectively at points: $Z_{3B} = (1730, 690)$, $Z_{3E} = (1740, 700)$. 