Operational hydrological data assimilation with the Retrospective Ensemble Kalman Filter: use of observed discharge to update past and present model states for flow forecasts

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Abstract

This paper describes the use of the Retrospective Ensemble Kalman Filter (REnKF) to assimilate streamflow data in an operational flow forecasting system of seven catchments in New Zealand. The REnKF updates past and present model states (soil water, aquifer and surface storages), with lags up to the concentration time of the catchment, to improve model initial conditions and hence flow forecasts. To our knowledge, this is the first time the REnKF has been applied in an operational setting, for a distributed model running over large catchments. We found the REnKF overcame instabilities in the standard EnKF which were associated with the natural lag time between upstream catchment wetness and flow at the gauging locations. The forecast system performance was correspondingly improved in terms of Nash Sutcliffe score and bounding of the measured flow by the model ensemble. We present descriptions of filter design parameters and explanations and examples of filter behaviour. The paper provides information and guidance valuable for other groups wishing to apply the REnKF for operational forecasting.

1 Introduction

Many river systems are susceptible to floods which occur rapidly after rainfall events; meaning that effective flood warning systems must use numerical weather prediction (NWP) forecasts to provide sufficient warning time. By coupling NWP and hydrological models, river flows may be forecast several days into the future. When hydrological models are used in such a capacity, forecast errors may occur due to uncertainties in model structure (e.g. unmodelled processes), model parameters, initial conditions (e.g. catchment wetness state determined from previous model runs) and input data (e.g. rainfall and temperature forecasts). The accumulation of such errors can be reduced by data assimilation, i.e. the use of observations to correct model states. The difference between modelled and observed flows, together with a quantification of the
errors in each, can be used to determine an optimal update to model states and hence to improve future forecasts.

Data assimilation in hydrological models is a relatively recent advance, but one which has been enthusiastically taken up, with various approaches being used: refer to recent reviews by Reichle (2008) and Liu et al. (2012), and references therein. The observations used to update model states can include river flows (Seo et al., 2003), soil moisture (Brocca et al., 2010; Flores et al., 2012), snow covered area and snow water equivalent (Clark et al., 2006; Zaitchik and Rodell, 2009; Andreadis and Lettenmaier, 2006), and satellite observations of discharge (Neal et al., 2009; Andreadis et al., 2007). Sequential use of different observation types is also possible, for example Aubert et al. (2003) assimilated streamflow and soil moisture observations. The model states which are updated include in-channel water volume (Ricci et al., 2011), soil water (Lee et al., 2011), groundwater (Zhou et al., 2011; Clark et al., 2008) and snow water equivalent (De Lannoy et al., 2012; Slater and Clark, 2006). Data assimilation methods used include the Extended Kalman Filter (EKF), Ensemble Kalman Filter (EnKF), square root EnKF, Particle Filter (PF) and variational data assimilation (see the review by Liu et al., 2012). Weerts and El Serafy (2006) compared the performance of the EnKF and PF with residual resampling, and found that the EnKF was most robust in cases of high uncertainties in model structure or inputs, or low number of ensemble members.

Data assimilation is being incorporated into operational flow forecast systems. In France, Météo-France runs an ensemble streamflow prediction system which assimilates streamflow data to update soil moisture in a distributed model. The system uses the Best Linear Unbiased Estimator (BLUE) (Thirel et al., 2010a, b). The same weather ensemble prediction system is used to drive a lumped soil-moisture-accounting type rainfall-runoff model at Cemagref (now Irstea), which uses discharge to update the routing store (Randrianasolo et al., 2010). Komma et al. (2008) showed how the EnKF could be applied operationally in the Kamp catchment in Austria. Their system updated runoff directly, with soil moisture and storage reservoirs updated using a similarity
method. Seo et al. (2009) report on the operational procedure used at the US National Weather Service where data assimilation is carried out using a Variational method. Generic data assimilation tools including EnKF are also available, which have been coupled with some popular hydrological models such as HBV and SOBEK (Weerts et al., 2010).

A significant challenge for data assimilation in hydrology is the natural time lag between catchment state and river flow (i.e. the time of concentration), especially in large catchments. This means that an update of distributed model states at the same time as the flow observations may not be physically realistic and can lead to under- or over-shoot in flow correction at later timesteps (Mendoza et al., 2012). This problem led some authors to reject updates to model states having this characteristic, e.g. groundwater stores; in favour of fast flow pathways such as surface runoff or in-channel water volume (Randrianasolo et al., 2010; Berthet et al., 2009). In the Ovens River catchment, Australia, Li et al. (2011) found that updating soil moisture states resulted in a lagged response in discharge leading to a poorer model performance, but also slower degradation of the forecast accuracy.

A solution was proposed by Pauwels and De Lannoy (2006) in the form of the Retrospective Ensemble Kalman Filter (REnKF). This filter uses an iterative approach to update model states preceding a flow observation, with lags up to the concentration time of the catchment. Other solutions which allow for the time delay in hydrological systems are the lagged particle filter proposed by Noh et al. (2011) and the Ensemble Kalman Smoother proposed by Evensen and van Leeuwen (2000) and improved in efficiency by Ravela and McLaughlin (2007).

This paper describes an implementation of the REnKF at NIWA (National Institute for Water and Atmospheric Research) for operational flood forecasting in New Zealand. To our knowledge, this is the first time the REnKF has been applied in an operational setting, for a distributed model running over large catchments. The aim of this paper is to present (1) a thorough exploration of the filter set-up decisions and parameter choices (2) quantification of the performance of the REnKF in a variety of catchments,
including comparison with the EnKF (3) investigation of filter behaviour including error parameter sensitivity, ensemble spread, perturbation size and magnitude of updates by lag time, location and model state. Our aim is to provide information and guidance valuable for other groups wishing to apply the REnKF for operational forecasting.

2 REnKF implementation

2.1 Algorithm

Our implementation of the Retrospective Ensemble Kalman Filter (REnKF) runs as a wrapper around the EnKF as implemented by Clark et al. (2008). The REnKF cycles through prior model states and updates each one based on a streamflow observation at the current timestep. In this iterative approach, the earliest states (greatest time before the observation time) are updated first, and the model is re-run to calculate the updated streamflow predictions; then the states one timestep closer to the current are updated, and the model again re-run, etc. (see the algorithm below). The main features of the implementation involve storing prior forcing data and model states for later retrieval during the assimilation process. The subroutine also writes model states and associated fluxes to output files after they have been altered for the last time, as the model steps forward in time past the point at which those steps could be further altered by the retrospective assimilation scheme.

An important parameter of the RenKF is the Lag Time, N (see Sect. 2.2.5). This parameter controls the maximum timestep difference between a streamflow observation and a prior model state that can be updated during the assimilation of that observation. To describe the REnKF process in more detail, assume that a simulation has reached a timestep $t$ ($t$ refers to the end of the timestep interval). At this timestep, the forcing data for the previous $N$ timesteps have been stored in temporary memory as well as model states at timestep $t - N$. The retrospective assimilation process for this timestep, $t$, is as follows:
i. Save forcing data for current timestep in temporary memory.

ii. Load model states at time step $t - N$ from temporary memory.

iii. Run model from time $t - N$ to $t$ to get the model estimate of the streamflow observations at time $t$.

iv. Set a retrospective timestep counter $j$ equal to zero, used in the following steps.

v. Load model states at timestep $t - (N - j)$.

vi. Assimilate streamflow observation at time $t$, using the EnKF to update model states at time $t - (N - j)$.

vii. If $j = 0$, write model states to output files at time $t - N$.

viii. If $j = 1$, store model states to temporary memory needed at step (ii) for next timestep.

ix. Run the model through one timestep from time $t - (N - j)$ to time $t - (N - j) + 1$.

x. If $j = 0$, write model fluxes for timestep $t - (N - j) + 1$ to output files.

xi. Store model states at time $t - (N - j) + 1$ to temporary memory to use in the next iteration at (v).

xii. Run the model from timestep $t - (N - j) + 1$ to $t$ to get the model estimate of the streamflow observation at $t$.

xiii. Repeat steps (v) to (xii) for $j = 1, 2, ..., N - 1$.

xiv. For $j = N$, repeat (vi).
Retrospective assimilation for streamflow observation at timestep $t$ is completed. Advance model from $t$ to $t+1$. The process is then repeated for all remaining timesteps. This implementation increases the number of model single-timestep executions by a factor of $F(N) = N(N+3)/2$, which gives $F(12) = 90$ and $F(24) = 324$. It is therefore clear that this implementation of the REnKF is computationally intensive.

### 2.2 Assimilation parameters

The REnKF requires a number of parameters in order to execute the EnKF updating step: these are described here. Refer to Sect. 4 (Results) for details on the choice of parameter values. As with all data assimilation schemes, the EnKF optimises the model behaviour according to the trade-off between errors in the model and errors in the observed data. Therefore specification of the magnitude of these two error sources directly determines the statistical optimality of the filter.

#### 2.2.1 Observation error

Errors in streamflow observations derive from errors in river stage measurement, and errors in the rating curve used to transform stage to discharge. The latter include errors in stage and velocity gaugings, assumption of a particular form of stage-discharge relationship, extrapolation beyond the maximum gauging and cross-section change (McMillan et al., 2010; Di Baldassarre and Montanari, 2009; Westerberg et al., 2011). The EnKF assumes that errors in streamflow are normally distributed. We follow Clark et al. (2008) who improved filter performance by transforming observed and modelled streamflow to log space before computing the Kalman gain. Hence, the standard deviation of the observed error is assumed proportional to the log discharge, with a proportionality constant $\varepsilon_{\text{obs}}$ which must be specified (Eq. 1)

\[
\log(q_{\text{true}}) - \log(q_{\text{obs}}) \sim N\left(0, \varepsilon_{\text{obs}} \cdot \log(q_{\text{obs}}) \right)^2 \tag{1}
\]
In addition, the Ensemble Square Root Filter variant of the EnKF is used, which uses a modified Kalman gain to remove the need to perturb the observations (for further details see Clark et al., 2008). This is advantageous as the observation perturbation magnitude was found to be a very sensitive parameter (Moradkhani et al., 2005).

2.2.2 Model error

The EnKF quantifies model error by using the variance of streamflow predictions from an ensemble of model realisations. Model errors can be caused by uncertainties in model inputs, model structure and parameter values. The number of ensemble members must be specified: we used 50 members which has been found to be sufficient (Clark et al., 2008; Moradkhani et al., 2005). The ensemble is created using stochastic perturbations of forcing and state variables: in this case we perturb the forcing precipitation depth and state variables for soil moisture and depth to the water table. For a detailed description of the approach to perturbing the ensemble refer to Clark et al. (2008).

The perturbations are parameterized as fractional error parameters for precipitation ($\epsilon_p$), change in soil moisture between subsequent timesteps ($\epsilon_s$), and baseflow ($\epsilon_z$). Errors are assumed to follow a uniform distribution $U[-\epsilon_q \cdot q_i, +\epsilon_q \cdot q_i]$ where $\epsilon_q$ is the error parameter corresponding to flux $q_i$. The derived errors are then applied to the precipitation depth, soil moisture state variable, and depth to water table (by inverting the baseflow equation in the latter case):

$$p' \sim p + U[-\epsilon_p \cdot p, +\epsilon_p \cdot p]$$

$$(2a)$$

$$s'_t \sim s_t + U[-\epsilon_s \cdot (s_t - s_{t-1}), +\epsilon_s \cdot (s_t - s_{t-1})]$$

$$(2b)$$

$$z' \sim -m \left\{ \log \left[ \frac{q_b + U[-\epsilon_z \cdot q_b, +\epsilon_z \cdot q_b]}{K_0 \cdot m} \right] + \lambda \right\}$$

$$(2c)$$
By specifying the perturbations in this way, model errors are permitted to be large when model fluxes are large (e.g. during storm events) and small when model fluxes are small (e.g. during drier periods). This approach differs from some previous studies where error variances are defined to be temporally constant (Reichle et al., 2002; Crow and Van Loon, 2006), but corresponds more closely to the modeller’s conceptualization of error magnitudes, for example that rainfall errors can be approximated by a multiplicative error term (McMillan et al., 2011).

2.2.3 Spatial and temporal correlation of model error

In our application of the REnKF, stochastic perturbations are applied to each subcatchment of the watershed model, and at each model timestep. The perturbations are correlated in space and time to represent spatial dependency of forcing errors and temporal persistence of model errors. The correlation is introduced by using Gaussian random fields parameterised by the decorrelation time \( \tau \) and the correlation length \( L \): these parameters are required for each perturbation (rainfall, soil moisture, depth to water table). The method is described by Clark et al. (2008) and uses techniques from Clark and Slater (2006) and Evensen (2003). Figure 1 shows an example of the precipitation perturbation patterns from the Grey catchment where the spatial correlation length for is set to 10 km vs. 65 km.

The perturbation correlations play a part in controlling the spread of the flow ensemble at the catchment outlet. Where perturbations are sustained in time, the ensemble spread is increased due to the memory of the catchment. Where spatial correlation length is large compared to the catchment size, many subcatchments undergo perturbations of the same sign, which increases ensemble spread. The last effect is also influenced by the geometry of the river network.
2.2.4 Model updates

The modeller must specify which model states should be updated as part of the REnKF assimilation. Following Clark et al. (2008), we allow the filter to update the soil storage, aquifer storage and surface (routing) storage for all subcatchments in the basin.

2.2.5 Lag time and stride

The Lag Time is an REnKF-specific parameter which determines the maximum timestep difference between a streamflow observation and a prior model state that can be updated during the assimilation of that observation. Lag Time can be conceptualised as the time of concentration of the catchment, i.e. the maximum time taken for rainfall occurring at the headwaters to affect streamflow at the gauging site. An initial estimate of the Lag Time could therefore be made a priori, although our experience showed that similar lag times performed well across a wide variety of catchments (refer to Sect. 4.1). An optional stride parameter, $M$, could be used to reduce running time by updating only every $M$-th set of prior states, however initial tests showed that this approach was detrimental to model performance and hence the option was not used in this study.

3 NZ operational system for NWP and flood forecasting

The aim of this work was to develop and evaluate an operational system for flood forecasting, with streamflow data assimilation provided by the REnKF. The system requires a regional NWP model for New Zealand (NZ) coupled to a hydrological model. An overview of this system is provided here.
3.1 Numerical weather prediction

The regional NWP model NZLAM (NZ Local Area Model) forecasts the atmospheric state over New Zealand for 48 h ahead. NZLAM is a local implementation of the UK Met Office Unified Model System, and derives its lateral boundary conditions from the global Unified Model. NZLAM is warm cycled on a 6 h basis, using initial conditions from previous runs. The model resolution is currently 12 km, using a rotated lat/lon grid with 324 × 324 points in the horizontal and 70 levels in the vertical, up to 80 km height. NZLAM incorporates a full 3DVAR data assimilation system (Lorenc et al., 2000) of observations from land, ship, and upper air stations, drifting buoys, aircraft, and satellites. Model forecasts of meteorological variables including precipitation and temperature are provided at an hourly time step.

3.2 Hydrological model

NZLAM forecasts provide input to a hydrological model, which simulates soil moisture, groundwater levels and river flow over the period of the NWP forecasts. We use the TopNet model (Fig. 2), which combines TOPMODEL concepts of sub-surface storage controlling the dynamics of the saturated contributing area and baseflow recession (Beven and Kirkby, 1979; Beven et al., 1995) with a kinematic wave channel routing algorithm (Goring, 1994). This approach leads to a model that can be applied over large watersheds using smaller sub-basins within the large watershed as model elements (Ibbitt and Woods, 2002; Bandaragoda et al., 2004). Complete model equations are provided by Clark et al. (2008). TopNet is routinely used for hydrological modelling applications in New Zealand, and uses nationally-available information on catchment topography, physical and hydrological properties. This information is derived from a digital river network (River Environment Classification; Snelder and Biggs, 2002), 30 m Digital Elevation Model, and land cover and soils databases (Land Cover Database; Land Resource Inventory; Newsome et al., 2000). For the applications described here, Strahler 3 subcatchments of typical size 10 km² are used.
TopNet uses 7 calibrated parameters for each subcatchment. To reduce the dimensionality of the parameter estimation problem, initial values for the parameters were estimated from the sources described. The spatial distribution of the parameters was then preserved, and the values were adjusted uniformly using a spatially constant set of parameter multipliers. Calibration used precipitation and climate data from Tait et al. (2006) who interpolated data from over 500 climate stations in New Zealand across a regular 0.05° latitude-longitude grid (approximately 5 km × 5 km). The precipitation data was bias-corrected using a water balance approach (Woods et al., 2006). These data are provided at daily time steps, and are disaggregated to hourly data before use in the model. The parameter values used here are those in current use for the operational forecasting system. The calibration methods varied by catchment according to the responsible hydrologist, and consisted of a semi-automatic method using either Monte Carlo simulation (2 catchments), or the ROPE (RObust Parameter Estimation) calibration method (Bardossy and Singh, 2008) (5 catchments) to obtain a small ensemble of possible parameter sets. This was followed by review by a hydrologist to determine a single preferred set based on visual inspection of the model simulation results.

### 3.3 Streamflow assimilation

The hydrological model is run on a 6 h cycle to mirror the NWP forecast input. Each time a new NZLAM forecast becomes available (0 h, 6 h, 12 h, 18 h), the hydrological model is run forward for 48 h to provide river flow forecasts to the end-user with as little latency as possible. Real-time hourly telemetered observations of streamflow then become available to the system. At forecast time + 5 h (i.e. 5 h, 11 h, 17 h, 23 h), the hydrological model is re-run in assimilation mode. The REnKF uses available observations to update the model states, and hence provide optimal initial conditions ready for the next forecast run in the 6 h cycle.
3.4 Forecast delivery

The weather and hydrological forecasts are made available to end users through the environmental forecasting tool “EcoConnect” which includes a web delivery application (Uddstrom, 2011). Graphical model output includes maps of NWP output, and location-specific forecasts for climate variables, rainfall and flow. The flow forecasts show observed flow up to current time, followed by predicted flow for a 48 h future time period. Flow forecasts from previous NZLAM cycles can be viewed to check for consistency between forecasts. The ensemble spread can optionally be superimposed on the forecast, as an indication of model error.

3.5 Test catchments

The operational system currently includes 7 catchments which are not influenced by hydropower operations and hence maintain natural flows. All these catchments were used to test the REnKF data assimilation method. Figure 3 shows the location of these catchments within NZ and Fig. 4 shows a close up of each catchment including major rivers and elevation. The catchments span a diverse range of topography, land cover and climate: these physical characteristics are summarised in Table 1.

4 System results

4.1 Flow forecasting results

To test the REnKF flow forecasting, the system was run in hindcast mode for the water year 1 April 2011–31 March 2012. This relatively short time period was used in order to limit model running time (a one-year run for the Grey catchment took approximately 9 days using an Intel Xeon CPU E5540, 2.53 GHz). The hydrological model runs on a 6 h cycle, with a 48 h forecast produced at each cycle. Therefore to create a continuous series for performance assessment, we concatenated the first 6 h of each forecast.
The system was run in three modes: (1) free running ensemble, no assimilation (2) EnKF assimilation (3) REnKF assimilation. The results from the REnKF assimilation for all catchments are shown in Fig. 5. The flow simulations are shown for an example period of 2 months containing some of the larger flow events of the water year. Rank histograms are also given, showing the quantile location of the measured flow value within the model ensemble. The simulation parameters are given in Table 2: parameters were kept constant across catchments apart from correlation length which was adjusted with catchment size. Correlation times and lengths were reduced for the Whirinaki where ensemble spread was otherwise too great. Lag time was set at 12 h which our experiments showed provided good results for a wide range of catchment sizes, after comparison of lag times from 0–24 h (not shown). The exception was in a catchment with pumice soils and hence a very damped response (not included in the 7 catchments used here), where a lag time of 24 h improved filter behaviour. This suggests that optimal lag time might be controlled by catchment soil and geological characteristics rather than size.

The results of the simulations in terms of (1) Nash Sutcliffe (NS) score of ensemble median and (2) Percentage of time the flow measurement lies within the ensemble bounds, are given in Tables 3 and 4 respectively. A graphical example of the differences between the free running ensemble, EnKF and REnKF, and their corresponding rank histograms, are given for the Pomahaka catchment in Fig. 6. The first column (“Single run”) of Table 3 shows the NS score of each model when run deterministically. The models are running in validation mode, i.e. outside of the time period used for calibration, and the performances vary considerably, providing a wide and challenging range of test conditions for the REnKF. Table 3 also shows that the NS scores for the “Ensemble” run (i.e. without assimilation) can be considerably worse than the underlying “Single run” model. This is the result of drift in the free-running ensemble. We discuss this point and its implications further in Sect. 4.1 “Flow forecasting results”.

There are several points to note resulting from these simulations. Importantly, the performance measures in Tables 3 and 4 show that the REnKF provides consistently
good performance across all catchments. This is true even when the original model had poor NS performance. The consistency of the REnKF is in contrast to the EnKF assimilation without time lag, which in 2/7 cases gives a higher NS score than the REnKF, but in other cases is significantly worse. The reasons are illustrated in Fig. 6, which shows that the EnKF can cause spikes and instabilities in the flow ensemble. We will return to the reason and nature of these instabilities in Sect. 4.4. Hence, although Fig. 6 shows that the rank histogram is flatter for the EnKF than the REnKF (which is typical of all catchments: not shown), there is a higher percentage of time where the observations are outside the bounds of the EnKF ensemble (refer to the dark outer bars of the rank histograms in Fig. 6, and numerically for all catchments in Table 3).

The spread of the ensemble clearly varies between catchments. Empirically, we found a narrower spread relative to flow magnitude was typical of larger catchments, even when using similar assimilation parameters (Table 2). This may be caused by the cumulative effect of perturbations in many subcatchments tending to be smaller than those in one subcatchment, as individual perturbations cancel each other out, even where spatially correlated perturbations are used. We also note a trade-off in ensemble spread, where a wide spread at low flow values is required in order that the high flow spread is sufficiently wide to enable the filter to correct for the large model errors that can occur. This behaviour might be mitigated by increasing the dependence of the perturbation size on the flow magnitude in the REnKF filter design. Because our system is designed mainly for flood warning purposes, we chose assimilation parameters which gave a good ensemble spread during flood events. When a sufficient ensemble spread is achieved, our results showed that this can degrade model performance under free-running conditions. For example, this is shown as bias in the rank histogram for the free-running ensemble for the Pomahaka (Fig. 6, upper row). This situation could occur where there are gaps in the observations available for model updates. Therefore the reliability of telemetered observations should be considered when setting ensemble spread. The degradation of performance is caused by asymmetries, thresholds or nonlinearities in the model which can cause ensemble drift, such as the lower bound
of zero flow, or nonlinearity of response when the water table reaches the surface. We return these points in Sects. 4.2 and 4.3, where the model sensitivity and update behaviour are discussed in more detail.

4.2 Sensitivity to error parameters

During set-up of our operational flow forecasting system, we carried out sensitivity experiments to determine the role and preferred values for each parameter of the REnKF system. Examples of our results are shown here to demonstrate to the reader the significance of each parameter, and to provide guidance for future implementations of similar systems.

The fractional error parameters $\varepsilon_p$ (precipitation), $\varepsilon_s$ (soil moisture) and $\varepsilon_z$ (depth to water table) control the spread of the model ensemble in terms of state variables and hence the ensemble spread in flow predictions (refer to Eqs. 2a–c). An example for the Grey catchment is shown in Fig. 7.

Figure 7 shows that the change in ensemble spread is greatest for Depth to Water Table, and smaller for Precipitation and Soil Moisture. These results are mirrored in the effect on the flow ensemble spread. Figure 8 shows an example where the model without assimilation (dotted line) underpredicts the observed flow (solid line). Changing the soil moisture fractional error parameter $\varepsilon_s$ (left column) has little effect on the ensemble spread (grey lines) and the model ability to correct the simulated flow values. Similarly, changing the precipitation parameter $\varepsilon_p$ has little effect (not shown). In contrast, changing the depth to water table parameter $\varepsilon_z$ (right column) has a large effect on the ensemble spread and consequently on the model predictive ability.

Weerts and El Serafy (2006) note that specifying the error model is the most difficult part of applying a Kalman or Particle Filter. Moradkhani et al. (2005) used a fractional error of 0.1 for all state variables but found that the ensemble spread was relatively insensitive to this value, compared to the observed value perturbation. Clark et al. (2008) used fractional errors of 0.2 (rainfall), 0.1 (soil moisture) and 0.05 (depth to water table), noting that it is difficult to attribute total model error to individual sources based only on
streamflow observations. Data assimilation applications favour over-estimation of the ensemble spread, to avoid excessive reliance on model predictions over observed values (Crow and van Loon, 2006). This effect can be especially severe during recession periods where model predictions converge.

Our results showed that using the EnKF set-up of Clark et al. (2008), the fractional error of depth to water table is the most sensitive parameter. We hypothesize that this is due to the different methods used to perturb the model states: for precipitation and soil moisture the flux is perturbed, whereas depth to the water table is modified by perturbing the baseflow and then inverting the baseflow equation. The latter approach changes the store volume as well as the flux. An additional investigation (not shown) demonstrated that if we forced a larger soil moisture ensemble spread, model flow predictions worsened. A possible reason is that empirical correlations between model soil moisture and flow do not indicate a causal relationship. Instead, they could have a common cause (rainfall depth), while elevated flows are controlled by the water table depth via saturation excess flow. Crow and Van Loon (2006) similarly found that assimilation of surface soil moisture observations to constrain deeper root-zone soil moisture in a land surface model could degrade model performance when model error assumptions were not appropriate.

4.3 Update behaviour

We selected some large rainfall events to investigate the update behaviour of the filter, i.e. where and when updates were made to model states. It is difficult to visualise filter update behaviour since it is high dimensional: two spatial dimensions (subcatchment location), observation time, time lag and update size. Each of these applies to each model state to be updated. For clarification, the update at observation time \( T_O \), lag time \( T_L \), refers to the change made to the model state at time \( T_O - T_L \) as a result of the information gained from the observation at time \( T_O \).

By plotting the observation time against lag time \( T_L \), and colouring according to update size, we can examine (1) which flow observations lead to the greatest model state...
updates and (2) at what lag times these updates occur. Figure 9 shows an example for the Pomahaka catchment, for updates to the water table depth in two catchments, one at the gauging site and one in the headwaters. From this figure we see that the gauging catchment undergoes greater updates than the headwater catchment. This can be expected since correlations between the water table depth in a subcatchment and flow at the gauge will be greater where the locations of these two measurements are closer. We also see that larger updates are often made at the maximum time lag allowed by the filter (12 h in this case). This reflects the design of the filter whereby the “first pass” (at the maximum lag) removes gross model errors, and then the model simulation is progressively fine-tuned at subsequent (smaller) lag times.

The dependence of model update size on lag time was compared in the general case for the Pomahaka catchment (Fig. 10), by taking an average over all observation times and all subcatchments (absolute update is used so that positive and negative updates do not cancel each other). This confirms the pattern whereby update size increases with lag time, however the updates to depth to water table and routed flow storage are relatively greater at shorter lag times than the updates to soil moisture storage. A comparison can also be made of the size of updates between the different model states. The size of the soil moisture update is up to two orders of magnitude smaller than the updates to routed flow or water table storages. This reflects both smaller correlations between soil moisture and streamflow, and smaller ensemble spread. However, as noted in the section “Sensitivity to Error Parameters”, increasing the ensemble spread and hence the update size tended to degrade the model forecast. The updates to the water table are greatest, and approximately three times the magnitude of the updates to routed flow. This behaviour may be partly dependent on the hydrological model structure; for example in this case the TopModel formulation means that water table depth controls saturation excess flow which is an important contribution to storm flow volumes.

To understand spatial patterns of updates, we also plotted the size of update in each subcatchment. Figure 11 shows an example, again from the Pomahaka, just prior to the
flood event illustrated in Fig. 9. The figure shows the updates for different lag times (i.e. different time before observation), relating to the same observation time of 06:00 a.m., 23 February 2012. Updates are typically largest at the largest lag time, but updates are made throughout the catchment at all lag times. At the shortest lag times, updates tend to be concentrated along the main stem of the river network (rivers of Strahler Order 4 and above are shown on the figure). This type of spatial organisation is consistent with our expectation that only subcatchments able to quickly affect flow magnitude would be updated at short lag times.

4.4 Numerical artefacts under the EnKF

The flow forecasting results above showed that one reason for improved performance of the REnKF over the EnKF was due to artefacts in the model behaviour under the EnKF. These could take the form of “spikes” in the forecast flow (Fig. 12a), or oscillatory behaviour (Fig. 12b). Spike-type artefacts were found by Mendoza et al. (2012), using a similar implementation of the TopNet model with EnKF data assimilation for a Chilean basin.

Our investigation showed that same underlying mechanism is responsible for both such artefacts. As described in the introduction, each catchment has a natural lag time between rainfall and the associated rise in river flow. This means that subcatchments distant from the gauging site may have model state variables (soil moisture, depth to water table, surface storage) which are not strongly correlated with the flow measured at the gauge at the same timestep. At times the correlation can become negative, even strongly negative, according to the random perturbations applied. Hence, for example, a model which predicts too high a flow, may try to correct that error during the filtering step by adding, rather than subtracting, water from the groundwater store in the distant subcatchments. The additional water causes a spike flow response in subsequent timesteps. This is the situation shown in Fig. 12a. In less severe situations, the correlation may be positive, but not representative of the stronger connection between current model state in remote subcatchments and future flow at the gauge. In this sit-
uation, oscillatory behaviour (Fig. 12b) could be produced before the system settles to the observed flow value. The oscillating updates to the model state “depth to water table” which cause this behaviour are shown in Fig. 13. Both these types of behaviour are resolved by the REnKF which both allows for a time lag between cause (change in water storage) and effect (change in flow), and essentially “tests” the Kalman Filter update by running the model forward in time to recalculate the simulated flow.

5 Conclusions

In this paper we demonstrated how the REnKF could be implemented for an operational flow forecasting system. We described the filter design decisions facing the user, and described the filter parameters and their effect on the filter behaviour. The performance of the system was compared with a deterministic model, with a free-running ensemble and with the standard EnKF which does not account for lag times between model states and streamflow at the catchment outlet.

Following our investigation, we can make several comments and recommendations for future users of the REnKF or similar systems in a hydrological context. For the filtering steps we used the implementation of the EnKF described by Clark et al. (2008) which perturbs model precipitation, soil moisture and depth to water table. We found that of these, the error parameters controlling perturbations of depth to water table were the most sensitive in terms of model ensemble spread and performance. Similarly, the filter updates the soil moisture storage, depth to water table and surface (routing) storage, and the updates to the depth to water table were largest. Updates to the surface storage were within the same order of magnitude. In contrast, soil moisture storage updates were small, and experiments to increase the ensemble spread and update size were shown to degrade model performance. This result suggests that a future implementation of the REnKF could reasonably remove the soil moisture storage from the model states to be perturbed or updated. This would increase the efficiency of the filter, and is unlikely to reduce forecast quality.
Our results showed that in cases where large model errors could occur (including due to error in forecast rainfall), then a large ensemble spread in modelled streamflow was required for good REnKF performance. This could be achieved most directly by increasing the fractional error parameter or correlation length of the depth to water table perturbations. The decorrelation time was less important since this time is typically longer than the time between model updates. However, the need for a large ensemble spread during high flow periods had to be balanced against creating too wide a spread at low flows, and a possible deterioration of performance where the free-running ensemble was used.

In all, we found a significant improvement in model forecasts when using the REnKF, which was able to overcome the instabilities of filter behaviour found with the EnKF. Across seven diverse catchments, the REnKF scored consistently highly against the performance measures of NS score for the ensemble median, and percentage time that the observed flow fell within the ensemble bounds. We would therefore recommend the REnKF concepts to other research groups wishing to use telemetered streamflow data for assimilation into a hydrological model.

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References


De Lannoy, G. J. M., Reichle, R. H., Arsenault, K. R., Houser, P. R., Kumar, S., Verhoest, N. E. C., and Pauwels, V. R. N.: Multiscale assimilation of Advanced Microwave Scanning Radiometer-EOS snow water equivalent and Moderate Resolution Imaging Spectroradiome-


Table 1. Catchment characteristics.

<table>
<thead>
<tr>
<th>Name</th>
<th>Waihua</th>
<th>Whatarea</th>
<th>Whirinaki</th>
<th>Taramakau</th>
<th>Motueka</th>
<th>Pomahaka</th>
<th>Grey</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area [km²]</td>
<td>50</td>
<td>450</td>
<td>510</td>
<td>880</td>
<td>1760</td>
<td>1870</td>
<td>3820</td>
</tr>
<tr>
<td>Average Stream Slope [°]</td>
<td>1.31</td>
<td>5.67</td>
<td>1.12</td>
<td>1.97</td>
<td>1.41</td>
<td>0.49</td>
<td>0.84</td>
</tr>
<tr>
<td>Land Cover [%]</td>
<td>Forest</td>
<td>97</td>
<td>34</td>
<td>93</td>
<td>43</td>
<td>64</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>Pasture</td>
<td>1</td>
<td>0</td>
<td>4</td>
<td>4</td>
<td>18</td>
<td>73</td>
</tr>
<tr>
<td></td>
<td>Scrub/Tussock</td>
<td>2</td>
<td>22</td>
<td>3</td>
<td>35</td>
<td>17</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td>Rock/Ice/Other</td>
<td>0</td>
<td>44</td>
<td>0</td>
<td>18</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Mean annual Snow Water Equivalent [m]</td>
<td>0.0</td>
<td>0.9</td>
<td>0.0</td>
<td>0.2</td>
<td>0.0</td>
<td>0.0</td>
<td>0.1</td>
</tr>
<tr>
<td>Mean annual precipitation [m]</td>
<td>1.83</td>
<td>11.4</td>
<td>1.61</td>
<td>6.29</td>
<td>1.67</td>
<td>0.96</td>
<td>3.95</td>
</tr>
<tr>
<td>Mean annual runoff [m]</td>
<td>1.14</td>
<td>9.35</td>
<td>0.90</td>
<td>5.72</td>
<td>1.07</td>
<td>0.49</td>
<td>3.17</td>
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</table>
Table 2. REnKF Model Parameters used for each catchment.

<table>
<thead>
<tr>
<th>Catchment</th>
<th>Precip</th>
<th>Soil</th>
<th>Baseflow</th>
<th>Precip</th>
<th>Soil</th>
<th>Baseflow</th>
<th>Precip</th>
<th>Soil</th>
<th>Baseflow</th>
<th>Lag (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Waihua</td>
<td>24</td>
<td>120</td>
<td>0.2</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>0.2</td>
<td>0.4</td>
<td>0.2</td>
<td>12</td>
</tr>
<tr>
<td>Whirinaki</td>
<td>1</td>
<td>24</td>
<td>0.2</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>0.2</td>
<td>0.4</td>
<td>0.2</td>
<td>12</td>
</tr>
<tr>
<td>Motueka</td>
<td>24</td>
<td>120</td>
<td>0.2</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td>0.2</td>
<td>0.4</td>
<td>0.2</td>
<td>12</td>
</tr>
<tr>
<td>Grey</td>
<td>24</td>
<td>120</td>
<td>0.2</td>
<td>65</td>
<td>65</td>
<td>65</td>
<td>0.2</td>
<td>0.4</td>
<td>0.2</td>
<td>12</td>
</tr>
<tr>
<td>Taramakau</td>
<td>24</td>
<td>120</td>
<td>0.2</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>0.2</td>
<td>0.4</td>
<td>0.2</td>
<td>12</td>
</tr>
<tr>
<td>Whataroa</td>
<td>24</td>
<td>120</td>
<td>0.2</td>
<td>25</td>
<td>25</td>
<td>25</td>
<td>0.2</td>
<td>0.4</td>
<td>0.2</td>
<td>12</td>
</tr>
<tr>
<td>Pomahaka</td>
<td>24</td>
<td>120</td>
<td>0.2</td>
<td>75</td>
<td>75</td>
<td>75</td>
<td>0.2</td>
<td>0.4</td>
<td>0.2</td>
<td>12</td>
</tr>
</tbody>
</table>
**Table 3.** Nash Sutcliffe score for single model run and ensemble median (free-running, EnKF and REnKF), calculated for time period 1 April 2011–31 March 2012.

<table>
<thead>
<tr>
<th>Catchment</th>
<th>Single run</th>
<th>Free Ensemble median</th>
<th>EnKF median</th>
<th>REnKF median</th>
</tr>
</thead>
<tbody>
<tr>
<td>Waihua</td>
<td>0.6167</td>
<td>0.5846</td>
<td>0.8973</td>
<td>0.8251</td>
</tr>
<tr>
<td>Whirinaki</td>
<td>0.4277</td>
<td>0.0051</td>
<td>0.9338</td>
<td>0.8307</td>
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<tr>
<td>Motueka</td>
<td>0.4379</td>
<td>0.2786</td>
<td>−1.1640</td>
<td>0.8982</td>
</tr>
<tr>
<td>Grey</td>
<td>0.4747</td>
<td>−0.5613</td>
<td>0.7302</td>
<td>0.8828</td>
</tr>
<tr>
<td>Taramakau</td>
<td>0.3759</td>
<td>−1.1020</td>
<td>0.7970</td>
<td>0.8305</td>
</tr>
<tr>
<td>Whataroa</td>
<td>0.1499</td>
<td>−0.1190</td>
<td>0.5353</td>
<td>0.6649</td>
</tr>
<tr>
<td>Pomahaka</td>
<td>0.0836</td>
<td>0.2116</td>
<td>0.7298</td>
<td>0.8583</td>
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</table>
Table 4. Percentage of time flow measurement lies within ensemble bounds, calculated for time period 1 April 2011–31 March 2012.

<table>
<thead>
<tr>
<th>Catchment</th>
<th>Ensemble</th>
<th>EnKF</th>
<th>REnKF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Waihua</td>
<td>0.9993</td>
<td>0.9961</td>
<td>0.9943</td>
</tr>
<tr>
<td>Whirinaki</td>
<td>0.9933</td>
<td>0.9999</td>
<td>0.9965</td>
</tr>
<tr>
<td>Motueka</td>
<td>0.9928</td>
<td>0.9450</td>
<td>0.9857</td>
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<tr>
<td>Grey</td>
<td>0.5295</td>
<td>0.8969</td>
<td>0.9628</td>
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<tr>
<td>Taramakau</td>
<td>0.3876</td>
<td>0.9714</td>
<td>0.9902</td>
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<tr>
<td>Whataroa</td>
<td>0.9869</td>
<td>0.9161</td>
<td>0.9826</td>
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<tr>
<td>Pomahaka</td>
<td>0.8878</td>
<td>0.9462</td>
<td>0.9857</td>
</tr>
</tbody>
</table>
Fig. 1. Example maps of fractional precipitation perturbation across the Grey catchments with spatial error correlation of 10 km (left panel) and 65 km (right panel).
Fig. 2. Schematic representation of the waterbalance component of TopNet (adapted with permission from Bandaragoda et al., 2004).
**Fig. 3.** Location of the 7 test catchments within New Zealand.
Fig. 4. Close up view of each of the test catchments, with the river network and elevation shown.
Fig. 5. REnKF simulations for all catchments, showing (Left) Ensemble simulations for a 2-month period 1 October 2011–1 December 2011 (Right) Rank histograms of location of measured flow within the model ensemble, calculated for the year 1 April 2011–31 March 2012. Dark bars at locations 0/1 show observations outside the model ensemble.
Fig. 6. Simulations for the Pomahaka catchments, showing (Left) Ensemble simulations for a 2-month period 1 October 2011–1 December 2011 (Right) Rank histograms of location of measured flow within the model ensemble, calculated for the year 1 April 2011–31 March 2012. Dark bars at locations 0/1 show observations outside the model ensemble. The rows represent (Upper) Free running ensemble with no assimilation (Middle) EnKF assimilation (Lower) REnKF assimilation.
Fig. 7. Ensemble spread during an example REnKF model run for (a) Cumulative Precipitation (b) Soil Moisture (c) Depth to Water Table with increasing fractional error parameter. Series shown for the Grey catchment, average values over all subcatchments.
Fig. 8. Streamflow ensemble spread during an example REnKF model run showing a comparison of fractional error parameters for (Left) Soil Moisture and (Right) Depth to Water Table. Series shown for the Grey catchment at the gauging location. Ensemble members are shown in grey; observed streamflow in black, modelled streamflow without assimilation as dotted line.
Fig. 9. Size of REnKF update to model state “Depth to Water Table” for the Pomahaka catchment, by observation time and lag time. Updates are shown for the gauging station reach (upper panel) and a selected headwater reach (middle panel). Observed and modelled flows at the observation times are shown for comparison (lower panel).
Fig. 10. Comparison of size of REnKF update by model state and lag time for the Pomahaka catchment, during the flood event shown in Fig. 8. Updates values shown are the mean value over time and subcatchments.
Fig. 11. Size of REnKF update to model state “Depth to Water Table” for the Pomahaka catchment, by subcatchment and lag time, at observation time 23 February 2012 06:00:00.
Fig. 12. (A) Spike in modelled flow for the Waihua River when using the Ensemble Kalman Filter (B) oscillations in modelled flow for the Motueka River when using the Ensemble Kalman Filter.
Fig. 13. Updates to depth to water table for each ensemble member for a sample reach in the Motueka. Modelled and measured flow is shown for comparison. Lags between updates and flow response cause oscillations during this period.