Effects of rating-curve uncertainty on probabilistic flood mapping

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Abstract

Comprehensive flood risk assessment studies should quantify the global uncertainty in flood hazard estimation, for instance by mapping inundation extents together with their confidence intervals. This appears of particular importance in case of flood hazard assessments along dike-protected reaches where the possibility of occurrence of dike failures may considerably enhance the uncertainty. We present a methodology to derive probabilistic flood maps in dike-protected flood prone areas, where several sources of uncertainty are taken into account. In particular, this paper focuses on a 50 km reach of River Po (Italy) and three major sources of uncertainty in hydraulic modelling and flood mapping: uncertainties in the (i) upstream and (ii) downstream boundary conditions, and (iii) uncertainties in dike failures. Uncertainties in the definition of upstream boundary conditions (i.e. design-hydrographs) are assessed by applying different bivariate copula families to model the frequency regime of flood peaks and volumes. Uncertainties in the definition of downstream boundary conditions are characterised by associating the rating-curve used as downstream boundary condition with confidence intervals which reflect discharge measurements errors and interpolation errors. The effects of uncertainties in boundary conditions and randomness of dike failures are assessed by means of the Inundation Hazard Assessment Model (IHAM), a recently proposed hybrid probabilistic-deterministic model that considers three different failure mechanisms: overtopping, piping and micro-instability due to seepage. The results of the study show that the IHAM-based analysis enables probabilistic flood hazard mapping and provides decision makers with a fundamental piece of information for devising and implementing flood risk mitigation strategies in the presence of various sources of uncertainty.
1 Introduction

Many studies in the literature highlight how inundation hazard and risk assessments are affected by several sources of uncertainties which limit their reliability, (e.g. Merz and Thieken, 2005; Apel et al., 2004, 2008; Most and Wehrung, 2005; Hall and Solomatine, 2008). In this context, there is an agreement in the scientific community that a proper risk analysis should provide an indication of uncertainty, emphasizing how the identification of the optimal flood risk management strategy can be pursued only if all major sources of uncertainty are adequately taken into consideration and a quantification of their impacts is provided (USACE, 1992).

Uncertainty has always been inherent in flood assessment and considered in flood defence engineering by means for example of adoption of an adequate freeboard (Hall and Solomatine, 2008). The unavoidable presence of uncertainty can be attributed to the fact that flood risk evaluations are usually carried out for extreme events that are seldom observed, which makes the calibration of flood risk assessment models difficult, if not impossible (Apel et al., 2004). Under such circumstances, the evaluation of uncertainty sources is a pragmatic extension to conventional validation. Furthermore, Hall and Solomatine (2008) and Apel et al. (2008) emphasize this concept highlighting how the quantification of the uncertainty could help to judge the consistency and the reliability of hydraulic risk assessment as well as provide useful advice for future data collection or research activities in order to yield more reliable results. In a context where model calibration and validation is difficult due to consideration of extreme events or lack of data, Hall and Anderson (2002) and Hall (2003) suggest a transparent and comprehensive description of the cause-effect relationships adopted in the methodology and implemented in mathematical formulations. This is particularly relevant in the case of dike failure analysis, where the uniqueness of breaches reduces or even eliminates the possibility to calibrate and validate deterministic numerical models. Evaluation of possible scenarios could only be handled by means of casual models considering uncertainties in dike breach processes (Hall, 2003; Vorogushyn et al., 2010).
In practical applications, the evaluation of the floodable area is usually carried out in a deterministic fashion by means of hydraulic models that are first calibrated relative to a specific historical major flood event and then used to estimate flood extents relative to different (and typically higher) event magnitudes. This procedure, even when physically based and numerically complex models are considered (e.g. fully-2-D model, etc.), relies on the assumptions of (i) time and space stationarity of model parameters (i.e. roughness coefficients); (ii) capability of the model to correctly reproduce the hydraulic behaviour of the river and inundated floodplains and (iii) all hydraulic information (i.e. flow hydrographs, rating-curves) are error-free. In a context characterized by these sources of uncertainty the definition of probabilistic flood hazard and flood risk maps appear the most reasonably way to proceed. Di Baldassarre (2012) argues that there are at least three main reasons why probabilistic flood-hazard maps should be preferred to deterministic ones: (1) hydrological and hydraulic analysis are always affected by uncertainty, which often cannot be neglected; (2) a fair presentation of the results of any analysis should also quantify and illustrate the associated uncertainty, and this can be accomplished only in a probabilistic framework; (3) stakeholders and decision-makers should be provided by hydrologists with probabilistic inundation maps to guide and support the definition of flood mitigation strategies; when deterministic maps are produced it implies that a decision has already been made by hydrologists, who are hence no longer behaving like scientists, but rather as decision-makers themselves. As a result, the deterministic estimation of flood extension may involve inexact and dangerous consequences, especially if it is used for planning and development purpose in the flood-prone area. In flood risk research, a number of studies have already considered and classified various uncertainty sources based on the distinction between two types of uncertainty: (i) natural or aleatory uncertainty, associated with the natural variability of the phenomena of interest and (ii) epistemic uncertainty, resulting from an imperfect knowledge of the system (e.g. Apel et al., 2004; Hall and Solomatine, 2008; Merz and Thieken, 2005; Most and Wehrung, 2005).
Many previous studies analyzed the effect of uncertainty associated with roughness parameterisations of hydraulic models (Aronica et al., 2002; Bates et al., 2004; Pappenberger et al., 2005). Additionally, Pappenberger et al. (2006) analyzed the uncertainty in upstream and downstream boundary conditions when applied to flood inundation predictions with a 1-D flow model. Other authors considered additional uncertainties in flood hazard and risk chain including extreme value statistic (Apel et al., 2008; Merz and Thieken, 2009), dike breach processes e.g. breach locations and dimension (Apel et al., 2004; Vorogushyn et al., 2010, 2011) as well as flood damage estimations (Apel et al., 2008; Merz and Thieken, 2009; de Moel et al., 2011; Vorogushyn et al., 2012). They concluded that currently uncertainties in damage estimations and in extreme value statistics dominate the uncertainties in risk estimates, although this conclusion remains site-specific.

In the previous analyses of flood hazard, uncertainty in upstream and downstream boundary conditions was insufficiently explored for two-dimensional models along with other uncertainty sources. Our analysis focuses in particular on the uncertainty associated with rating-curves used as downstream boundary conditions, while the aleatory uncertainty related to the selection of a design hydrograph is taken into account referring to different flood hydrographs estimated with a bivariate approach. Even though literature reports several studies highlighting the global uncertainty affecting discharge measurements and rating-curve construction, (e.g. Domeneghetti et al., 2012; Di Baldassarre and Claps, 2011; Di Baldassarre and Montanari, 2009), the literature on the effects of rating-curves uncertainty of flood-hazard and -risk assessments is still sparse. Moreover, institutions and agencies in charge of hydroclimatic monitoring usually do not provide practitioners and users with indications on uncertainty associated with rating-curves. Conversely, rating-curves are usually utilised in a deterministic way although their sampling variability may be significant and may play a dominant role in practical applications (Domeneghetti et al., 2012).

Our study investigates the effects of the uncertainty of upstream and, in particular, downstream boundary conditions (i.e. rating-curves) used in a flood hazard model
chain on flood hazard estimations. We make use of the outcomes of a previous study on rating-curve uncertainty performed for the same river reach (Domeneghetti et al., 2012), in order to explore the impact of this uncertainty onto probabilistic flood hazard mapping. The present investigation was performed by setting up a hybrid probabilistic-deterministic flood hazard assessment model for the flood-prone areas located along a diked reach of the lower portion of the middle Po River. We discuss how the consideration of this uncertainty may impact flood management decisions compared to a deterministic specification of boundary conditions.

2 Methodology

Chains of models which describe fluvial inundation processes and flood damages are typically applied for flood hazard and risk assessment. Concerning this approach, each modelling step or chain link exhibits a number of inherent uncertainties which are summarized in Table 1 starting from a triggering event to the final inundation. Referring to some natural and epistemic sources of uncertainty (sources listed in italic bold in Table 1), the study aims at quantifying the contribution of different terms of uncertainty evaluating the feasibility and the amount of uncertainty reduction achievable adopting additional information or different procedure. Taking into account several uncertainty sources highlighted in bold (Table 1) in the probabilistic-deterministic modelling system IHAM (Inundation Hazard Assessment Model, Vorogushyn et al., 2010), we analyse the role of uncertain boundary conditions on flood hazard statements. IHAM was set up for a 50 km reach of the Po River (see Fig. 1) and is comprised of three main modules: an unsteady one-dimensional hydraulic model (1-D-model), a probabilistic dike breach model, which evaluates dike system stability under hydraulic load conditions, and a 2-D raster-based diffusive wave model (2-D raster-based model; Merz, 1996) for the simulation of floodplain flow in case of dike failure (see Fig. 2).

The 1-D model simulates the flood wave propagation in the river channel and over floodplains between dikes and computes the hydraulic load on flood protection dikes.
During the simulation, each discretised dike section is evaluated for failure due to overtopping, piping and slope instability due to seepage flow through the embankment (micro-instability; see Vorogushyn et al., 2009). In case of dike failure, the flood wave propagation after outflow through the breach into flood-prone area is modelled by means of the 2-D storage cell model. The simulation of the water exchange between river channel and floodplain, including the reverse flow, is incorporated by means of a continuous data exchange between modules, which are coupled at runtime.

The schematic structure of the IHAM model is shown in Fig. 2, which highlights the model core system (three coupled modules), and the pre- (input) and post- (output) processor phases. The modelling system is run in a Monte Carlo framework (MC) to address the considered sources of uncertainty (e.g. upstream and downstream boundary conditions) and the stochasticity of dike breaching processes.

IHAM model considers the uncertainty related to dike system stability implementing the Dike breach module (see Fig. 2 and Sect. 3.2) that evaluates the probability of dike failures upon loading hydraulic conditions provided by the 1-D model. Each section of the dike system with length of approximately 1.2 km is tested for dike stability based on the current load during the whole simulation. The probability of failure for a given hydraulic load is estimated through fragility curves (see e.g. Sayers et al., 2002) defined for each dike section for three failure mechanisms: overtopping, piping and micro-instability (Apel et al., 2004; Vorogushyn et al., 2009).

In case of single or multiple dike collapses, the development time and the final dimension of each breach are stochastically generated based on probability distribution functions fitted to historical observations (see Govi and Turitto, 2000, Sect. 3.2).

Limited knowledge about flow dynamics, errors on flow-rates measurements and inaccuracy related to the applied methodology for rating-curve estimation (epistemic uncertainties) are considered in a MC simulation. As a result, the IHAM model computes dike failure probabilities for the whole embankment system and provides probabilistic flood hazard maps for a flood prone area indicating the uncertainty bounds of spatial...
inundation characteristics. A more detailed description of the IHAM modelling system is provided by Vorogushyn et al. (2010).

In this paper the IHAM model system has been extended to analyse the effect of the uncertainty related to flood waveform and to downstream boundary conditions (rating curves) on dike and flood hazard mapping.

2.1 Uncertainty in upstream boundary conditions

Several studies in the literature highlight how flood frequency analysis plays a dominant role in the overall flood hazard uncertainty (see e.g. Apel et al., 2008; Merz and Thieken, 2009). In particular, an appropriate estimation of peak discharge and flood volume associated to a specific return period is important when flood hazard is related to dike stability evaluated upon loading condition (Vorogushyn et al., 2009). For piping and slope-instability, the peak water level and also the duration of dike impoundment, which is related to flood volume, are decisive. In the light of these considerations, the uncertainty in flood event estimation considering both flood peak and volume is addressed adopting different flow hydrographs as upstream boundary conditions in a Monte Carlo framework (see Sect. 3.4).

2.1.1 Uncertainty in downstream boundary condition

Domeneghetti et al. (2012) proposed a general numerical procedure for quantifying global uncertainty of stage-discharge relationships by using numerical hydrodynamic models. Referring to the Cremona river cross-section (see Fig. 1) and considering errors affecting river flow measurements (EU ISO EN 748:1997, 1997, ISO748:97), the authors applied two different procedures for rating-curve estimation, which they termed traditional and constrained approaches, quantifying the global uncertainty for each one of them (Fig. 3). The traditional approach constructs a given rating-curve relation by fitting a series of stage-discharge values observed within the range of measurable streamflows (i.e. 6000 m$^3$ s$^{-1}$ at Cremona, EU ISO EN 1100-2, 2010, ISO 1100-2:10).
The constrained rating curve is estimated by fitting measured discharge and water-level pairs and by concurrently honouring an estimate of the cross-section maximum discharge capacity retrieved from a simplified one-dimensional steady-state numerical model implemented around the Cremona cross-section (Domeneghetti et al., 2012). The reduction of the extrapolation error ensured by the constrained approach results in reduced bias and variability of the estimated rating-curves.

By means of one-dimensional (1-D) and quasi two-dimensional (quasi-2-D) models implemented for different reaches of the Po River, Domeneghetti et al. (2012) estimated the median (red dashed line in Fig. 3) and the 90 % confidence interval (thin black lines in Fig. 3) for both methodologies. In particular, left and right panels of Fig. 3 report the “true” or reference normal rating-curve (blue thick line) obtained at Cremona river cross-section from the compound of unsteady stage-discharge pairs (grey dots). Also, left panel of Fig. 3 reports the global uncertainty relative to the traditional approach. In this case, the extrapolation error associated with the utilization of the curve beyond the range of observed data introduces a significant deviation with respect to the reference normal rating-curve. The width of 90 % confidence interval and bias of the traditional rating curve clearly emerged from Fig. 3. Right panel of Fig. 3, similarly to the left one, reports the median rating-curve (red dashed line) and 90 % confidence interval relative to the constrained approach.

We analyse the impacts of the uncertainty in specifying the rating-curves on flood-hazard mapping and highlight the differences between traditional and constrained approaches to rating-curve construction, comparing them with the results that one would obtain by using a single deterministic median rating-curve.

3 Study area and model implementation

Our study considers a 50 km reach of the middle-lower portion of the Po River (Fig. 1), which spans from Piacenza (upstream gauge) to Cremona (downstream gauge). The reach can be characterised as a unicursal river having a width varying between 200
and 500 m and a wide floodplain area. The floodplain inside the major river embankments is partly cultivated and plots are additionally protected by a series of minor dikes (Castellarin et al., 2011).

### 3.1 1-D Model

The hydrodynamic simulation of the flood wave propagation along the study reach was carried out using a 1-D model based on the full Saint-Venant equations numerically solved with the classical implicit four-point finite difference scheme (Martin and Wool, 2001). The channel geometry was characterised by 29 cross-sections (Fig. 1) derived from a 2 m DTM recently provided by AdB-Po (2005), which combine information collected by means of LiDAR (data collected using two different laser scanners: 3033 Optech ALTM and Toposys Falcon II), multi-beam sonar survey for the navigable portion of the river and data retrieved by means of traditional ground survey of river cross-sections.

The cross-sections were extracted from the DTM following the rules for optimal cross-section spacing (Castellarin et al., 2009). The unsteady 1-D model was driven by a flow hydrograph and conditioned through a rating curve as a downstream boundary. The representation of tributaries was limited to the River Adda, which is the biggest along the considered reach of the Po River. The Adda contribution was modelled as a lateral inflow hydrograph as the tributary may alter appreciably the Po streamflow downstream of its mouth. Considering their negligible contributions during the major floods events experienced along the study reach in the 1994 and 2000 the Nure and Chiavenna streams were not considered as tributaries during flood simulations.

The 1-D model was calibrated for a flood event with an estimated return period of approximately 50 yr, which occurred in the Po River in October 2000. The October 2000 event reproduces the hydraulic behaviour of the study reach in case of extreme floods because all floodplains protected by the system of minor dikes were flooded during the event. The 1-D model was calibrated by manually adjusting the roughness coefficients to match the maximum water levels that were provided by the wrack marks along the
reach. The model calibration was conducted twice: (1) adopting the traditional median rating curve (Fig. 3, red dashed line of the left panel) and (2) using the constrained median relation (red dashed line on the right panel of Fig. 3).

The high water marks of October 2000 flood are accurately reproduced by the model, with a mean squared error (MSE) of 0.22 m and 0.28 m for the constrained and the traditional case, respectively. This error, even though not negligible, is still satisfactory, especially considering the intensity of the simulated flood event and simplifications adopted in the geometrical description of the riverbed (pure-1-D model and single roughness coefficient for main channel and lateral floodplains).

The calibrated Manning’s values mainly vary between 0.04 s m$^{-1/3}$ and 0.05, which are in good agreement with those estimated by previous studies on the same reach (see e.g. Castellarin et al., 2009, 2011; Domeneghetti et al., 2012; Di Baldassarre and Montanari, 2009).

### 3.2 Dike breach module

The main embankment system was discretized into several sections, each one with a length of 1.2 km resulting in 28 and 32 sections respectively for the right and left side of the embankment system (Fig. 1, lower left panel). During the simulation, each section is tested for dike stability using fragility functions, which provide the probability of dike-section failure upon hydraulic loading simulated by the 1-D hydrodynamic model. Fragility functions for each breach mechanism (overtopping, piping and micro-instability) were developed for each dike section based on investigations focussing on the geotechnical and geophysical characteristics of the embankment system, which were commissioned by the River Po Basin Authority (AdB-Po-GEOVIT, 2004; AdB-Po, 2001) or derived from the literature and summarised by Vorogushyn et al. (2010).

In case of dike failure, the breach width ($B_w$) is stochastically sampled through a Monte Carlo procedure from a truncated lognormal probability density function fitted to a series of historical observations in the Po river system (see Fig. 4, Table 2 and
Coratza, 2005). In particular, the truncated distribution is constrained by the minimum and maximum values of \( B_w \) observed in the Po River system (see Table 2).

Breach development time \( h_w \) was adopted in the range 0.5–4 h and assumed to follow normal distribution with mean of 2 and standard deviation of 1.5 h. The resulting values for breach times are comparable with those adopted in other studies conducted for the same or comparable rivers (e.g. Apel et al., 2004; Alkema and Middelkoop, 2005; Di Baldassarre et al., 2009; Vorogushyn et al., 2010; Han et al., 1998).

### 3.3 2-D model

In case of a dike failure, the flood propagation over the dike-protected floodplains is simulated by a 2-D raster-based model run on a 50 m \( \times \) 50 m resolution grid. The topographical information for the whole study area (Fig. 1; global extension 890 km\(^2\)) were retrieved from the ASTER GDEM (Advanced Spaceborne Thermal Emission and Reflection Radiometer – Global Digital Elevation Model; www.gdem.aster.ersdac.or.jp) and rescaled to the coarser grid resolution in order to reduce the computational load.

Considering the absence of detailed information on inundation extents experienced in the area of interest and to the uniqueness of breach event, the calibration of the 2-D raster-based model appeared a difficult task. Consequently, spatially distributed Manning’s roughness coefficients were assigned to each cell based on literature values (Vorogushyn et al., 2010; Chow, 1959) for respective land use classes retrieved from the CORINE land use classifications (COoRdination of Information on the Environment – Land Cover, 2006).

### 3.4 Development of flood scenarios and model simulations

In order to account for the flood volume, which can be relevant for the stability of flood protection structures (Vorogushyn et al., 2009; Klein et al., 2010), we applied a copula-based bivariate approach for the estimation of flood hydrographs for a specific return period.
The annual peak discharge ($q$) and the corresponding flood volume ($v$) considering a time-window of 30 days around the flood peak (10 days before the peak, rising limb, and 20 days after the peak, recession limb) were extracted from the mean daily flow series in the period from 1951 to 2008 at gauge Piacenza. The dependence structure of the couple of variables ($Q,V$) was described using a copula approach. Among several fitted copulas, the Gumbel copula provided the best fit to the empirical relationship between $Q$ and $V$ according to the selected criteria.

Considering $F_Q(q)$ and $F_V(v)$ the marginal distribution functions of $Q$ and $V$ variable (in the case study a GEV and a log-normal distribution, respectively), the relationship between the normal distributed variables $u = F_Q(q)$ and $v = F_V(v)$ could be expressed by means of the Gumbel copula (Eq. 1)

$$C_\theta(u,v) = e^{-[(\ln u)^\theta + (\ln v)^\theta]^{1/\theta}}$$  \hspace{1cm} (1)

where $\theta \geq 1$ is a dependence parameter estimated over the set of observation (Salvadori and De Michele, 2007).

The goodness-of-fit was tested through several criteria including RMSE, AIC method, Kolmogorov and Smirnov test and tests based on the empirical copula and on Kendall transform (Genest et al., 2009; Fermanian, 2005). Figure 5 illustrates the selected flood events associated with different return periods, “Tr”. A critical event is determined if either $Q$ or $V$ exceeds given thresholds defined through the copula function associated with an exceedance probability (i.e. “OR”-case in Salvadori and De Michele, 2007).

Focusing on a return period of 200 yr (i.e. the reference recurrence period adopted by AdB-Po for designing and verifying the main embankment system of the Po River, hereafter also referred to as Tr200), red dots of Fig. 5a indicate the different ($q,v$) pairs used to discretize the Tr200 contour line and considered in order to take into account the natural variability of flood hydrographs: each combination has the same bi-variate probability of occurrence and could be associated with a $Tr$ of 200 yr.

We retrieved the shape of synthetic flow hydrographs analysing the series of historical flood events recorded at Piacenza. Estimated base-flow was first subtracted from
each observed hydrograph, which was then divided by the maximum discharge obtaining a dimensionless hydrograph with unit peak-flow. We then computed the mean of all dimensionless flood hydrographs and we rescaled the resulting mean hydrograph to match peak discharges and flood volumes estimated through the bivariate analysis for the five Tr200 events (red hots in Fig. 5a, see Vorogushyn et al., 2010, for details). Figure 5b reports the five synthetic hydrographs obtained in the study and their corresponding empirical relative probability of occurrence ($P = 0.2$). IHAM was driven by the developed flood hydrographs taking into account their equiprobable occurrence. To investigate the effect of rating curve uncertainty on flood hazard estimation, flood scenarios were simulated adopting different downstream boundary conditions defined for the gauge Cremona. Approximately 8000 Monte Carlo simulations were run in total to propagate the uncertainty in upstream and downstream boundary conditions to flood hazard estimations. In particular, subsets of ~2000 runs were used to explore the effects of uncertainty on flood hazard mapping:

- **MedianT** subset; flow hydrographs were randomly selected as upstream boundary conditions, whereas median rating-curve for traditional approach (red dashed lines on left panel of the Fig. 3) was used as downstream boundary conditions.

- **MedianC** subset; same as before but adopting constrained median rating-curve as downstream boundary condition.

- **RandomT** subset; both upstream (i.e. flow hydrographs) and downstream (i.e. traditional rating-curves within the 90 % confidence interval) boundary conditions are stochastically sampled.

- **RandomC** subset; same as before by considering constrained rating-curves.
4 Results

Figure 6 reports results provided by the 1-D-model for the RandomT subset. The upper panel of Fig. 6 reports the minimum levee-crest elevation (red dashed line) for the study reach of the Po River and compares it with the median (black line) and the range of variability (grey dashed lines) of water surface simulated for the Tr200 event. In the lower panel of Fig. 6 the variability of water depth (i.e. the width of the range of variation shown in the upper panel) simulated for the RandomT subset (black line) is compared with the one obtained from the RandomC subset (dashed line).

Panel (a) of Fig. 7 reports the probabilistic flood-hazard map obtained running the IHAM model in a MC framework for the MedianT subset (~2000 runs). Colours along the dike system in Fig. 7a provide the likelihood of failure of each dike section (see also Fig. 1, lower left panel), while the probability of inundation of flood prone areas is indicated through a blue colour scale. Such a measure is calculated at each cell as the ratio between the number of simulations in which the cell is wet (i.e. water depth > 0 cm) and the total number of Monte Carlo runs.

The dike hazard map indicates the presence of critical sections along the entire length of the study area (Fig. 7a). Although the failure probabilities for majority of the sections are quite small ranging between 20–30 %, the results highlight a critical section in the embankment stretch located downstream of the Torrente Chiavenna tributary (red area in the left panel of Fig. 7a). In this case, a local depression on the dike crest results in a high probability of overtopping, leading to a remarkable value of the probability of inundation for the flood-prone area opposite to the tributary mouth (dark blue colour in Fig. 7a).

The probabilistic map in Fig. 7b reports the difference in probability of inundation arising from the consideration of the uncertainty bounds around the median traditional rating-curve, that is the difference between the inundation probability obtained for the MedianT and RandomT subsets. Figure 8a shows the probabilistic inundation map obtained for the MedianC subset (~2000 Monte Carlo runs), while Fig. 8b provides
the difference in inundation probability between probability inundation maps obtained for the MedianC and RandomC subsets (\(\sim 2000\) Monte Carlo runs each).

The map reported on Fig. 8b does not highlight tangible variations in the probability of flooding for the area outside the main embankments due to the consideration on rating-curve uncertainty: patchy variations shown in the map (Fig. 8b) seem to be ascribable to the stochastic definition of breaches dimension and development time. Figure 9 compares probabilistic flood-hazard maps computed on the basis of the RandomT, panel a, and RandomC, panel b, subsets (\(\sim 2000\) Monte Carlo runs each). Areas highlighted in the figure emphasize the difference between the two subsets. Even though the uncertainty in downstream boundary conditions is considered in both scenarios, the highlighted area appears to be inundated only in the case of the application of the traditional approach.

In all considered scenarios, overtopping was the only breach mechanism responsible for dike failures. None of the dike sections failed due to piping and micro-instability even though considering the variable flood volume for 200-yr flood events ensured by the bivariate approach (see Fig. 5).

5 Discussion

Results presented in the previous section clearly highlight the remarkable impact of the methodology applied for rating-curve construction and associated uncertainty on flood-hazard assessment, and in particular on dike breaching and inundation probability. The variability of rating-curves produces a significant uncertainty in flood probability estimation. Figure 7b shows for our case study that the deterministic (i.e. neglecting uncertainty, MedianT subset) utilization of a rating-curve constructed using a traditional approach (i.e. fitting the available discharge-water level observations) results in a significant underestimation of flooding probability. Lower panel of Fig. 6 clearly shows the impact of rating-curve uncertainty in terms of water levels along the downstream end of the study Po River reach (RandomT and RandomC subsets). The bias introduced
by the downstream boundary condition influences the simulated water levels upstream through a backwater effect for a remarkable distance (i.e. 20–25 km in this case).

As a general remark, it is worth noting here that the flooding probability could be underestimated or overestimated in other study areas depending on local conditions, yet we want to underline the significant bias that may affect the flood inundation estimates. The bias sign depends on the specific local conditions which may produce systematic underestimation, or overestimation, of the water levels in the downstream end of the considered river reach, affecting the overtopping probabilities of the main embankments. Referring in a deterministic way to a more precise technique for constructing the rating curve (e.g. the so-called constrained approach in Domeneghetti et al., 2012), or taking the variability (i.e. uncertainty) of a traditional rating-curve into account may significantly reduce the bias of the estimated inundation probability. Concerning the rating-curve uncertainty, the importance of how large the variability of the rating-curve is (i.e. width of the confidence interval; lower panel of Fig. 6) is shown in Fig. 8, relative to the unbiased rating-curve constructed with the constrained approach. In this case, the reduced uncertainty (i.e. narrow confidence interval and small extrapolation errors, see right panel of Fig. 3) leads to a limited effect on flood estimation and inundation assessment. The limited variability of rating-curves obtained by means of constrained approach entails a reduced variability in terms of water elevation along the river and this results in a more reliable evaluation of dike stability and likelihood of flood. Although we are aware that also this result could in part be associated with our case-study, the analysis reveals how the reduction of the extrapolation error could be a good strategy in order to reduce bias and uncertainty on flood hazard estimation when the uncertainty of the rating-curve cannot be considered (i.e. deterministic interpretation of the curve) and has to be neglected during flood hazard assessments for various practical limitations (e.g. when performing a real-time flood inundation modelling).

The two maps in Fig. 9 emphasize the effects on inundation probability estimates of bias on water levels that might be associated with a traditional approach to rating-curve construction relative to the constrained approach (see also Fig. 3). The comparison of
these maps highlights a possible misinterpretation on hazard estimation due to extrapolation errors associated with the curve fitting exercise. The highlighted cells (red ellipse) appear flooded only in the case of the application of the traditional approach (Fig. 9a), and this is a consequence of the better reproduction of the hydraulic behaviour of the Po River at Cremona cross-section ensured by the constrained rating-curve (see Fig. 3).

Concerning the scientific debate on probabilistic versus deterministic inundation maps, Fig. 10 provides a comparison of probabilistic flood maps obtained for the MedianC (panel a) and RandomC (panel b) subsets. Both subsets refer to the constrained method for constructing rating-curves, therefore the adopted downstream boundary condition is the most accurate between all considered cases, both in terms of possible extrapolation errors (i.e. limited bias) and global rating-curve uncertainty (i.e. limited confidence interval).

The maps illustrate the inundated area differentiating between the cells that are inundated for less than 1/4 of the runs (grey) and those that are inundated more frequently (black). The comparison between the two maps enables one to understand the differences in terms of inundated areas (i.e. grey and black areas) that originates from the uncertainty in the downstream boundary condition. This difference resulted to be significant in terms of flooded hectares in our case (∼805 ha) even though (1) the rating-curve uncertainty (i.e. confidence interval) is rather limited in this case (see right panel of Fig. 3) and (2) the areas where the differences are more pronounced are mainly located near the upstream end of the study reach (∼25–35 km upstream the downstream end, where the boundary condition is set). The indication resulting from the comparison described above is not necessarily associated with the utilization of probabilistic maps, similar results may also be from a comparison between two deterministic inundation maps. The added value of a probabilistic inundation mapping is the capability of probabilistic maps of representing the uncertainty of the output (e.g. grey areas in Fig. 10) in a very effective way. The uncertainty in the rating curve produces a difference in terms of potentially floodable area (i.e. difference between grey areas: ∼843 ha) which quantifies the additional uncertainty in the output of the study. Also, the
representation of the uncertainty associated with the output facilitates the interaction between scientists and decision-makers, who may or may not have a strong background in numerical-hydraulic modelling. Once adequately informed, decision-maker will decide how best to deal with this uncertainty (e.g. by including grey areas among the restricted areas in the spatial planning acts) and weight his/her decision by the probability of flooding.

Finally, concerning our particular case study, the analysis pointed out that dike stability is strongly controlled by peak discharges rather than flood volumes. Although, the variability of flood volume was explicitly considered in the flood hydrograph scenarios (Fig. 5), the embankment system resulted to be sensitive practically to overtopping failures only. Failures due to piping or micro-instability did not occur in the Monte Carlo runs in light of the remarkable thickness of the main river embankments (average riverside slope 1 : 2 or 1 : 3; average landside slope 1 : 5–1 : 6). This result is in agreement with what has been observed along the study reach of the Po River during the October 2000 flood event; the freeboard on both sides of the dike system in the proximity of the Torrente Chiavenna was limited to few centimetres (Coratza, 2005). Piping and micro-instability do not threaten the stability of the dike system according to the results of our simplified analysis, which mainly focuses on the uncertainty in downstream boundary conditions. However, evidences of sandboils along the study reach during recent flood events in 1994 (magnitude similar to the October 2000 event) and 2000 suggest a starting retrogressive erosion and the presence of a not negligible danger of piping (Coratza, 2005), and this aspect needs to be better investigated in future analysis which are out of the scope of the present study.

Given that piping phenomenon is strictly related to the presence of geological controls (e.g. buried sedimentary bodies) that may induce preferential flows and the duration of the flood event, a correct description of the buried geological controls and seepage evolution and of the retrogressive erosion assume an important rule for the consistency of the study. Focusing in particular on the latter aspect, Vorogushyn et al. (2009) revealed the high sensitivity of piping likelihood on the retrogressive erosion
velocity, emphasizing how further investigations on that aspect as well as a more comprehensive knowledge of dike properties (geometrical and geotechnical characteristics) could improve the accuracy and the trustworthiness of the analysis.

6 Conclusions

The debate relative to the deterministic and probabilistic approach for flood hazard estimation is still ongoing in the scientific community (Di Baldassarre, 2012; Di Baldassarre et al., 2010; Montanari, 2007). Providing flood probability maps for the flood prone area appears to be an efficient way to visualize the likelihood of flooding and it also offers additional information concerning the reliability of its estimation.

The scientific community is well aware of all risks associated with deterministic statements (i.e. binary, yes or no kind of statements) when the system under study is uncertain. Nevertheless, the output of numerical simulations as well as hydraulic and hydrological input data are often used in practice and applied regardless of their uncertainty. Probabilistic inundation maps are still scarcely adopted as additional assets by decision makers called to define flood protection strategy. This should mainly be attributed to a lack of specific guidelines as well as to a limited ability of the scientific community to communicate the meaningfulness and effectiveness of this kind of spatial representation of flood-hazard. We investigated the effects of the uncertainty in the definition of the downstream boundary condition given by the rating-curve on the flood probability estimation for a diked reach of the Po River. The evaluation was carried out with the IHAM model, which enables the evaluation of failure probability of the dike system under variable hydraulic conditions and for different breach mechanisms. The intrinsic uncertainty in flood hydrographs was considered referring to a bivariate approach, modelling the correlation structure of peak streamflow and flood volume by means of a copula approach.

Results of the analysis highlighted how rating-curves uncertainty significantly affects flood mapping assessment and, in particular, probabilistic flood mapping, when the
curves themselves are used as downstream boundary conditions. This aspect appears particularly relevant when the range of uncertainty for high flow rates becomes wide due to the extrapolation error introduced during rating-curve construction. In this context, the methodology used for rating-curve construction plays a fundamental role in the model chain for flood hazard assessment. Concerning this point, we investigated the effects in terms of dike breaching and inundation probability of two methodologies for rating-curve construction, referred in the study as traditional and constrained approaches (see also Domeneghetti et al., 2012). In the case of rating-curve constructed by means of a typical approach (e.g. traditional approach) the analysis shows through a series of Monte Carlo simulation experiments that neglecting the uncertainty associated with empirical rating-curve may lead to highly inaccurate, and therefore dangerous, inundation mapping. In this context, the study clearly pointed out how taking into account the rating-curve uncertainty through a probabilistic approach significantly enhances the reliability of the flood-hazard mapping. Also, the results of our analysis pointed out that limiting the extrapolation error while constructing empirical rating-curves (for instance by adopting an approach similar to the so-call constrained approach illustrated in Domeneghetti et al., 2012) significantly reduces the effect of uncertain boundary condition on the flood likelihood estimation. Additionally, the reduction of the bias possibly affecting rating-curves leads to a more reliable flood hazard estimation, reducing the risk of unfounded estimation of floodable area.

A probabilistic statement on flood-hazard, which incorporates a quantification of the uncertainty that affects the output of numerical hydraulic modelling, represents, in our opinion, a fundamental piece of information for decision-makers, when, for instance, they are called to define spatial development plans for a given area, or during a flood event, when they need to identify priorities in the organization of civil protection actions. Probabilistic flood inundation maps are the most natural and straightforward graphical representation of such a statement, and should always be preferred to deterministic inundation maps.
Acknowledgements. The authors are extremely grateful to the Interregional Agency for the Po River (Agenzia Interregionale per il Fiume Po, AlPO, Italy) and Po River Basin Authority (Autorità di Bacino del Fiume Po, Italy) allowing access to their high resolution DTM of River Po.

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9830


Table 1. Sources of uncertainty in flood hazard mapping grouped into natural and epistemic uncertainty (adapted from Apel et al., 2004); sources in italic are directly considered into the presented analysis.

<table>
<thead>
<tr>
<th>Modules</th>
<th>Natural uncertainty</th>
<th>Epistemic uncertainty</th>
</tr>
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<tbody>
<tr>
<td>(1) Hydrological analysis</td>
<td>annual maximum discharge; flow hydrograph form;</td>
<td>measurement error; limited time series length;</td>
</tr>
<tr>
<td>(2) Rating-curve</td>
<td>variation of river geometry in time;</td>
<td>limited time series length; method for rating-curve estimation; interpolation/extrapolation errors; error in model selection;</td>
</tr>
<tr>
<td>(3) Flood routing</td>
<td>variation of river geometry over time; geometrical variation over space;</td>
<td>numerical simplification; parameter calibration; measurements errors of levee geometry;</td>
</tr>
<tr>
<td>(4) Dike stability</td>
<td>variation of geotechnical parameters in space; final width and development time of levee breaches;</td>
<td>variability estimations of levee parameters (permeability, turf quality, material cohesion, etc.); formalization of dike breach processes; error in model selection;</td>
</tr>
<tr>
<td>(5) Flood dynamics</td>
<td>variability of surface roughness in space and time due to variable land use;</td>
<td>numerical simplification; DEM inaccuracy; parameter estimation;</td>
</tr>
</tbody>
</table>
Table 2. Width of dike breaches, $B_w$: statistics observed along the Po River in the period 1800–1951 (data from Coratza, 2005).

<table>
<thead>
<tr>
<th>$B_w$ statistics for the Po River</th>
<th>Obs. value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of historical breaches with observed $B_w$</td>
<td>84</td>
</tr>
<tr>
<td>Mean $B_w$ (m)</td>
<td>240</td>
</tr>
<tr>
<td>Median $B_w$ (m)</td>
<td>180</td>
</tr>
<tr>
<td>Min $B_w$ (m)</td>
<td>27</td>
</tr>
<tr>
<td>Max $B_w$ (m)</td>
<td>1200</td>
</tr>
</tbody>
</table>
Fig. 1. Study area. Upper left panel: Po River basin and study area (box); lower left panel: river cross-sections (grey lines) and levee system discretization for the left (green dots) and right (red dots) side; right panel: 2-D raster-based model extension (grey box) and not floodable area (yellow).
Fig. 2. Schematic structure of the IHAM model adopted for flood hazard estimation under uncertainty conditions.
Fig. 3. Rating curves estimated at the Cremona cross-section: normal rating-curve (blue line), median rating-curve (red dashed line), and corresponding 90% confidence intervals (black lines) for traditional (left panel) and constrained (right panel) approaches (Domeneghetti et al., 2012).
Fig. 4. Empirical frequency distribution of breach widths, $B_w$, observed along the Po River in the period 1800–1951 (bars) and fitted probability density distribution (red line; log-normal).
Fig. 5. Bivariate analysis: (a) level curves for the Gumbel copula for different return periods (black lines) and \((q,v)\) pairs adopted for the 200 yr event (red dots); (b) flow hydrographs corresponding to copula-based \((Q,V)\) pairs.
Fig. 6. Monte Carlo simulations; upper panel: range of variation (grey dashed line) and median (black line) water elevation profiles simulated for the RandomT subset along the Po River, compared with the minimum levee-crest elevation (red dashed line). Lower panel: water depth variability simulated along the Po River for RandomT (black line) and RandomC (dashed line) subsets.
Fig. 7. (a): probabilistic inundation map for the Tr200 event adopting the MedianT subset; (b): difference in probability of flooding adopting RandomT and MedianT subsets.
Fig. 8. (a): probabilistic flood hazard map for the Tr200 event adopting the *MediamC* subset; (b): difference in probability of flooding adopting *RandomC* and *MedianC* subsets.
Fig. 9. Probabilistic flood hazard maps for the Tr200 event obtained with variable traditional (left panel, RandomT subset) and constrained (right panel, RandomC subset) rating-curves.
Fig. 10. Probabilistic flood maps for MedianC (a) and RandomC (b) subsets: grey colour for flood probability up to a quarter (0–0.25), black colour for higher flood likelihood.