A GIS- Independent method for Stream-Order-Law Ratios Prediction in Catchments for Runoff Estimation using GIUH Model

Pouyan Keshtkaran[1*], Touraj Sabzevari[2]

[1] Department of Civil Engineering, Estahban Branch, Islamic Azad University, Estahban, Iran

* To correspond with the author, please e-mail: Pouyan_keshtkaran@yahoo.com

Tel: 00989173132092

Fax: 009871132360352

[2] Department of Civil Engineering, Estahban Branch, Islamic Azad University, Estahban, Iran

e-mail: t_sabzevari@iauest.ac.ir, Tooraj.sabzevari@gmail.com

Tel: 00989177151596

Abstract

Estimation of direct runoff in ungauged catchments is of great importance in the design of hydraulic structures. The geomorphologic instantaneous unit hydrograph (GIUH) technique uses geomorphologic parameters to estimate catchment runoff. In this research, regression equations were developed based on the morphometric characteristics of some catchments such as the area, length and slope of the main river, in order to estimate the stream-order-law ratios and geomorphologic characteristics of other catchments with no need for GIS or a digital elevation model. These equations were verified in the Gagas, Heng-Chi and Kasilian catchments. In this study, the effect of stream-order-law ratios on the rate of runoff in the Kasilian catchment was examined, and the sensitivity of runoff rate to each ratio was analysed. The GIUH model was assessed using two cases: a GIS-supported method and the method proposed in this paper. The mean errors of the regression equations in the estimation of the ratios R_B, R_L, R_A, R_S and R_{SO} in the three catchments studied were 4.7%, 23.5%, 7.1%, 41.3%, and 22.9%, respectively. The direct runoff hydrographs for the Heng-Chi and the Kasilian catchments were computed using the GIUH model and compared with the observed direct runoff. The results indicate that the

errors in peak discharge for four rainfall-runoff events in the case of the proposed method were, on average, 10% more than those obtained using GIS-supported GIUH. The results of GIUH analysis for the two cases were very similar. The mean coefficient of efficiency of the model was computed as 0.87.

Key words: GIUH; GIS; Stream-order-law ratios; Geomorphologic parameters; Runoff

1. Introduction

Estimation of design flood in catchments is an important issue in design of flood control structures. Most catchments of the world are ungauged and statistical methods which strongly rely on rainfall-runoff data are not efficient; rainfall-runoff models are therefore employed to estimate runoff. GIUH is a rainfall-runoff model for estimating runoff in ungauged catchments based on the geomorphologic parameters (GP) of the catchment.

The concept of GIUH was proposed by Rodriguez-Iturbe and Valdes (1979). They suggested an instantaneous unit hydrograph (IUH) model in which the time to peak and peak flow of the catchment were functions of geomorphologic features. The GIUH model was extended and used by other scientists in different catchments (e.g., Gupta et al., 1980; Rodriguez-Iturbe and Valdes, 1982; Lee and Yen, 1997; Kumar and Kumar, 2008; Sabzevari and Norouzpoor, 2014). An alternative approach was devised by Lee and Yen (1997). The travel times for different orders of overland areas and channels were derived using the kinematic-wave theory and then substituted into the GIUH model to develop a kinematic wave-based GIUH model for watershed runoff simulation. Lee and Chang (2005) proposed a GIUH model for the estimation of surface and subsurface flow of catchments. In their research, special importance was given to the separation of surface flow from subsurface flow in catchments. Sabzevari et al. (2013) modified the model presented by Lee and Chang (2005) for the estimation of surface and subsurface flows of the Kasilian catchment. The geomorphologic parameters of the catchments are calculated by GIS softwares such as ArcGis and hydrologic extensions such as ArcHydro. For this purpose, a digital elevation model (DEM) of the catchment is necessary. Stream networks are delineated and GP such as the number of streams, lengths, slopes, and drainage areas in each order of streams are obtained out based on stream orderings.

The stream-order-law ratios (SOLR) are calculated based on geomorphologic information and vice versa. The equations governing the GIUH model can be developed based on SOLR or GP (Kumar and Kumar, 2008). According to the GIUH model offered by Yen and Lee (1997), the travel times of overland regions and streams could be calculated using stream-order-law ratios prior to IUH estimation.Studies of stream orderings of catchments were first introduced by Horton (1932, 1945). Later, modifications were made to Horton's method by Strahler (1952, 1957, 1964), leading to a new method of ordering. Sherve (1966) concluded that the Strahler stream numbers generally gave more accurate results for natural stream networks than did the Horton stream numbers. Horton-Strahler's laws were extensively used in geomorphological applications to classify river systems (e.g., Raff et al., 2003; Reis, 2006), to establish relationships with the fractal nature of channel networks, as detailed by Rodríguez-Iturbe and Rinaldo (1997) (e.g., Beer and Borgas, 1993; La Barbera and Roth, 1994; Rodríguez-Iturbe et al., 1994), and to characterize scale properties (Claps et al., 1996; Peckham and Gupta, 1999; Veitzer and Gupta, 2000; Dodds and Rothman, 1999, 2001).

The use of GIS tools is one of best ways of calculating the geomorphologic parameters (Sarangi et al., 2003; Obi Reddy et al., 2004; Valeriano et al., 2006; Ozdemir and Bird, 2009). The high resolution of DEM provides a more accurate prediction of GP and runoff when used with GIUH models. Although DEM is offered globally today by the Shuttle Radar Topography Mission, some hydrologists prefer to apply simpler models of rainfall-runoff due to their lack of familiarity with GIS or being time-consuming of extraction of GP. In this research, a set of equations is presented that are derived from the geomorphological information of a number of catchments where the SOLR and GP can be calculated, based on parameters such as the catchment's area and main river length. These data are applied as input parameters to the GIUH model and are used in predicting the surface and subsurface runoff of the catchment.

The primary aims of this research are:

(1) To present equations which can predict the stream-order-law ratios of catchments on the basis of length, slope of the main stream and area of the catchment;

(2) To apply predicted stream-order-law ratios to estimate direct runoff of ungauged catchments by means of the GIUH method.

2. GIUH model

Surface runoff from overland regions moves, through stream networks, to the outlet of the catchment. If a catchment is ordered via Strahler ordering scheme, the paths of water travel from overland regions to the outlet are specified. Each flow path is composed of various states, the first of which is the overland region, and the remainder of which are the streams. The probability of water motion along a certain path $w: x_{o_i} \rightarrow x_i \rightarrow x_j \rightarrow \ldots \rightarrow x_\Omega$ is expressed as:

$$P(w) = P_{OA_i} P_{x_{oi}x_i} P_{x_i x_j} \ldots P_{x_k x_\Omega} \quad (1)$$

where P_{OA_i} is the initial state probability of a raindrop moving from the *i*th order overland region to the *i*th order stream. This can be approximated as the ratio of the *i*th order overland area to the total catchment area. $P_{x_{oi}x_i}$ is the probability of the raindrop moving from the *i*th order overland region (x_{o_i}) to the *i*th order stream, and is equal to one. $P_{x_i x_j}$ is the transitional probability of the drop moving from the *i*th order stream (x_i) to the *j*th order channel (x_j). The number of streams at each order, and the way in which they are connected to each other, determine the probabilities in Eq. (1).

The value of the IUH of a watershed comprising different runoff paths is given by Eq. (2) (Rodriguez-Iturbe and Valdes, 1979).

$$u(t) = \sum_{w \in W} [\, f_{x_{o_i}}(t) * f_{x_i}(t) * f_{x_j}(t) * \ldots * f_{x_\Omega}(t)]_w \times P(w) \quad (2)$$

where $f_{x_k}(t)$ denotes the travel time probability density function (PDF) in state x_k with a mean travel time value (T_{x_k}), and the function f is the IUH of any state x_k calculated by the formula $f(t) = (1/T_{x_k})\exp(-t/T_{x_k})$. The PDF is a function of the travel time in each state in the overland regions and streams. The asterisk (*) denotes a convolution integral. $w \in W$, where W is $W = \langle x_{o_i}, x_i, x_j, \ldots, x_\Omega \rangle$, i = 1, 2, 3, … , Ω and t is the time.

To solve Eq. (2), the Laplace transformations could be used. In the derivation of GIUH, the computation of travel time forms the most complex part of the work, since its value depends on the GP of the catchment.

The ordinates of DRH for the catchment were estimated by combining the effective rainfall hyetograph with the derived IUH. The equation for estimation of DRH is:

$$Q(t)=\int_0^t u(t-\tau)I_e(\tau)d\tau \tag{3}$$

where I_e is the excess rainfall and $u(t)$ is the catchment IUH.

2.1. Travel time of overland planes and streams

According to the kinematic wave theory, the travel time of an overland plane depends on the length, slope, Manning coefficient, and excess rainfall intensity. Eq. (4), proposed by Yen and Lee (1997), gives the travel time of the *i*th overland plane.

$$T_{X_{oi}}=\left(\frac{n_0 A P_{OA_i}\sum_{i=1}^{\Omega} R_L^{i-\Omega}}{2a^{1/2} S_{c\Omega}^{b/2} L q_L^{m-1} R_B^{\Omega-i} R_L^{i-\Omega} R_S^{b(i-\Omega)/2}}\right)^{1/m} \tag{4}$$

where R_B, R_L, R_A, and R_S are the bifurcation ratio, stream-length ratio, stream-area ratio, and stream-slope ratio, respectively; A is the area of the catchment; a and b have values 5.463 and 1.083 respectively; q_L is the excess rainfall intensity; n_0 is the Manning's roughness coefficient for overland flow; and $S_{c\Omega}$ is the slope of the highest order stream. The constant m can be recognized as 5/3 from Manning's equation, and L is the sum of the mean lengths of the streams of different orders.

The travel time of the *i*th-order channel in each path is obtained, based on its GP, from Eq. (5) (Yen and Lee 1997):

$$T_{X_i} = \frac{B_\Omega L R_L^{i-\Omega} R_B^{\Omega-i} \sum_{i=1}^{i} R_L^{i-\Omega}}{q_L A P_{OA_i} (\sum_{i=1}^{\Omega} R_L^{i-\Omega})^2} \left[\left(h_{co_i}^m + \frac{q_L A P_{OA_i} n_c \sum_{i=1}^{\Omega} R_L^{i-\Omega}}{B_\Omega S_{c_\Omega}^{1/2} R_S^{(i-\Omega)/2} R_B^{\Omega-i} \sum_{i=1}^{i} R_L^{i-\Omega}} \right)^{1/m} - h_{co_i} \right] \tag{5}$$

where h_{co_i}, the inflow depth of the ith-order channel due to water transported from upstream reaches, is given by:

$$h_{co_i} = \left(\frac{q_L n_c A (R_B^{\Omega-i} R_A^{i-\Omega} - P_{OA_i}) \sum_{i=1}^{\Omega} R_L^{i-\Omega}}{S_{c_\Omega}^{1/2} B_\Omega R_S^{(i-\Omega)/2} R_B^{\Omega-i} \sum_{i=1}^{i} R_L^{i-\Omega}} \right)^{1/m} \tag{6}$$

where n_c is the Manning coefficient of the stream, and B_Ω is the width of the stream. The value of h_{coi} is equal to zero for i=1.

3. Horton-Strahler stream-order-law ratios

As can be observed from Eqs. (5) and (6), stream-order-law ratios (SOLR), and particularly R_S, R_A, R_L and R_B, are of great importance in runoff estimation using the GIUH model. These affect the travel times, IUH and DRH; they are also calculated according to the GP. For this purpose, the stream network is delineated by means of GIS. In the GIS, the streams are ordered using the Horton-Strahler method, and the number, length, and slope of the streams are computed at each order. In order to obtain the coefficients R_S, R_A, R_L, and R_B, the equations presented in Table (1) are used.

Table 1: Equations related to the Horton-Strahler stream-order-law ratios

Equation Number	Equation	Description
(7)	$R_B = N_{i-1} / N_i$	N_i : Number of ith-order channels
(8)	$R_L = \overline{L_{c_i}} / \overline{L_{c_{i-1}}}$	$\overline{L_{c_i}}$: Length of ith-order channels
(9)	$R_A = \overline{A_i} / \overline{A_{i-1}}$	$\overline{A_i}$: Mean area of catchment of order i
(10)	$R_S = \overline{S_{c_i}} / \overline{S_{c_{i-1}}}$	$\overline{S_{c_i}}$: Mean slope of ith-order streams

As a result of experiments in the natural catchments, the following ranges are observed: $3 \le R_B \le 5$ and $1.5 \le R_L \le 3.5$. The slopes of the streams and overland planes for different catchments at each order are different. The mean values of these slopes at each order take a considerable time to compute using GIS, especially for large catchments.

In this research, a new slope ratio, the overland slope ratio (R_{SO}), is introduced. It is given in terms of the mean slope of the overland plane by:

$$R_{SO} = \overline{S_{o_{i-1}}} / \overline{S_{o_i}} \tag{11}$$

where $\overline{S_{o_i}}$ is the mean slope of the *i*th-order overland plane. In this research, we aim to clarify the relationship between R_{SO} and the other SOLR.

In this paper, a technique for computing stream-order ratios using regression equations is presented. These equations, obtained by regression methods, are based on statistical analysis of information from catchments possessing known geomorphologic attributes. The application of these equations in performing computations will be described in subsequent sections.

To study the relationship between SOLR, knowledge of the GIS-based SOLR (that is, the SOLR derived from GIS) of several natural catchments is required. This research uses information obtained from twelve catchments in various countries. Table (2) shows the GIS-based SOLR, with the stream order ratios of the case study catchments. The catchments of Long Chi, Long Men, Chaukhutia, Al-Malaqi, Debarwa, Gherghera, San-Hsia, Al-Badan, and Al-Faria were used for training and the estimation of the regression equations. The catchments of Gagas, Heng-Chi and Kasilian were used for the verification of the regression equations.

The columns in Table (2) show, from left to right, the catchment name and reference, stream order (*i*), number of streams, mean stream length, mean stream area, mean stream slope, mean overland slope, R_B, R_L, R_A, R_S, and R_{SO}.

The Heng-Chi catchment is located in northern Taiwan and has an area of 53 km^2. The Gagas catchment lies in the middle and outer range of the Himalayas in the Uttarakhand state of India and has an area of 506 km^2. The Kasilian Catchment is located between longitudes 53° 18$'$ E and

53° 30′ E and latitudes 35° 58′ N and 36° 7′ N in the north of Iran, and has an area of 67.8 km^2. Fig. (1) shows the Gagas, Kasilian and Heng-Chi catchments.

Table 2: Geomorphologic characteristics of twelve case study catchments

Catchment	Geomorphologic parameters										
Name	Order	N_i	$\overline{L_i}$	$\overline{A_i}$	$\overline{S_c}$	$\overline{S_o}$	R_B	R_L	R_A	R_S	R_{SO}
1. Gagas	1	121	1.74	3.02	0.172	0.810	4.8	2.4	5.4	0.4	2.6
(Kumar and Kumar, 2008)	2	23	3.04	18.58	0.141	0.655					
	3	6	7.63	79.22	0.041	0.172					
	4	1	23.4	506	0.017	0.065					
2. Heng-Chi	1	30	0.66	1.043	0.087	0.450	3.3	2.6	4	0.6	1.1
	2	6	2.74	6.919	0.050	0.419					
(Lee and Chang, 2005)	3	2	1.6	19.9	0.012	0.349					
	4	1	4.97	53.23	0.012	0.347					
3. Kasilian	1	42	1.6	0.915	0.241	0.345	3.5	1.5	4.3	0.4	1.1
	2	11	1.79	4.813	0.070	0.297					
(Sabzevari et al., 2013)	3	3	2.45	20.75	0.047	0.263					
	4	1	4.65	67.8	0.008	0.261					
4. San-Hsia	1	69	0.92	1.15	0.161	0.314	4.2	2.9	5	0.4	1.1
	2	16	2.08	4.99	0.092	0.203					
(Chang and Lee, 2008)	3	3	3.88	18.15	0.037	0.364					
	4	1	17.8	125.9	0.013	0.293					
5. Al-Badan	1	41	1.38	1.37	0.170	0.140	4	1.5	4.5	1	1.7
	2	6	3.2	10.12	0.092	0.062					
(Shadeed et al., 2007)	3	2	5.03	40.73	0.140	0.051					
	4	1	3.17	85	0.135	0.029					
6. Al-Faria	1	49	1.03	0.937	0.154	0.117	4	1.5	4.3	1.1	1.6
	2	8	2.12	6	0.085	0.058					
(Shadeed et al., 2007)	3	3	3.5	19.4	0.161	0.033					
	4	1	2.62	64	0.125	0.031					
7. Al-Malaqi	1	62	1.92	1.81	0.146	0.140	9	1.3	17	0.8	4.3
	2	16	2.61	5.83	0.122	0.063					
(Shadeed et al., 2007)	3	1	3.21	185	0.081	0.010					

8. Debarwa	1	23	2.26	5.6	0.032	0.135	4.9	3	6	0.6	1.2
	2	6	4.2	27.8	0.018	0.091					
(Alemngus and Mathur, 2014)	3	1	17.7	195	0.010	0.098					
9. Gherghera	1	58	2.45	5.9	0.027	0.136	2.9	1.4	3.3	0.9	1.4
	2	14	4.19	30.6	0.018	0.087					
(Alemngus and Mathur, 2014)	3	5	10.2	101.0	0.010	0.064					
	4	2	4.47	259.9	0.016	0.025					
	5	1	4.19	525.7	0.011	0.117					
10. Long Chi	1	46	1.13	2.5	0.210	0.444	3.7	2.4	4	0.6	1.1
	2	10	3.45	11.8	0.124	0.487					
(Shuyou et al., 2010)	3	4	3.19	32	0.073	0.514					
	4	1	9.94	141.8	0.054	0.364					
11. Long Men	1	58	1.31	2.74	0.560	0.256	4	2.2	4.7	0.9	1.8
	2	13	2.48	12.3	0.560	0.123					
(Shuyou et al., 2010)	3	3	9.33	77.11	0.560	0.056					
	4	1	8.18	246.8	0.385	0.056					
12. Chaukhutia	1	134	1.41	2.27	0.191	0.910	5.3	2.5	5.7	0.5	2.4
	2	31	2.65	12.28	0.123	0.567					
(Kumar, 2014)	3	7	7.21	60.18	0.041	0.174					
	4	1	20.7	452.3	0.019	0.074					

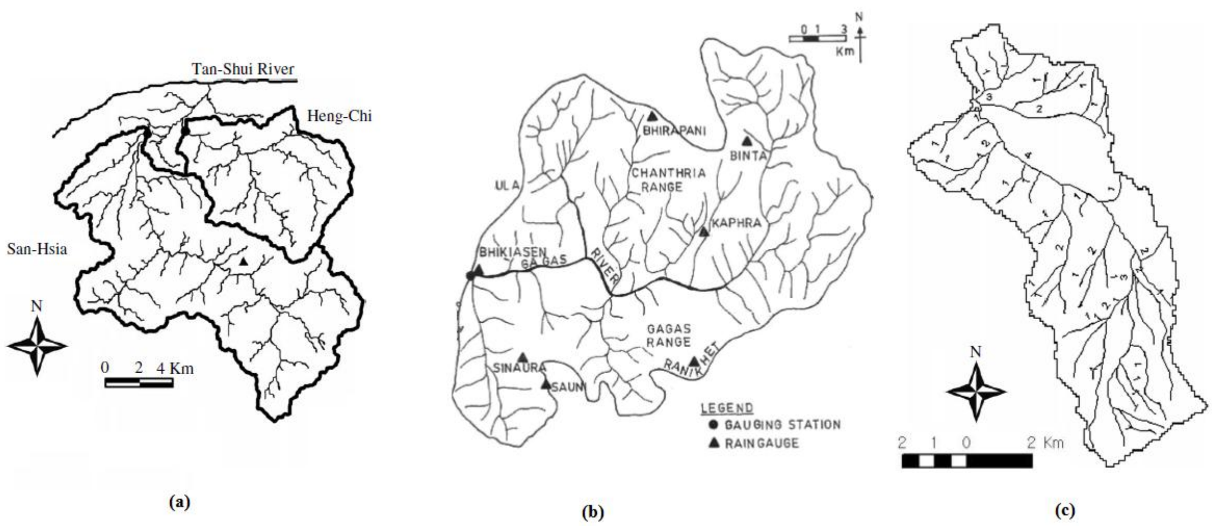

Figure 1: Drainage network map:

a) Heng-Chi catchment; b) Gagas catchment; c) Kasilian catchment

4. Prediction of Horton-Strahler SOLR

To estimate the bifurcation ratio of a catchment, information concerning 37 catchments with areas between 1 km^2 and 600 km^2 and with known values of R_B and area were used. Through the use of Statistical Package for the Social Sciences (SPSS) software (Norusis, 1999, Mohamoud and Parmar, 2006), and using the information from these 37 catchments, the optimum relationship was obtained as:

$$R_B = 0.0027A + 3.47 \tag{12}$$

The correlation coefficient of the fitted equation is 0.8 and the real mean bifurcation ratio of the catchments is 4. Figure 2 shows the fitted linear regression.

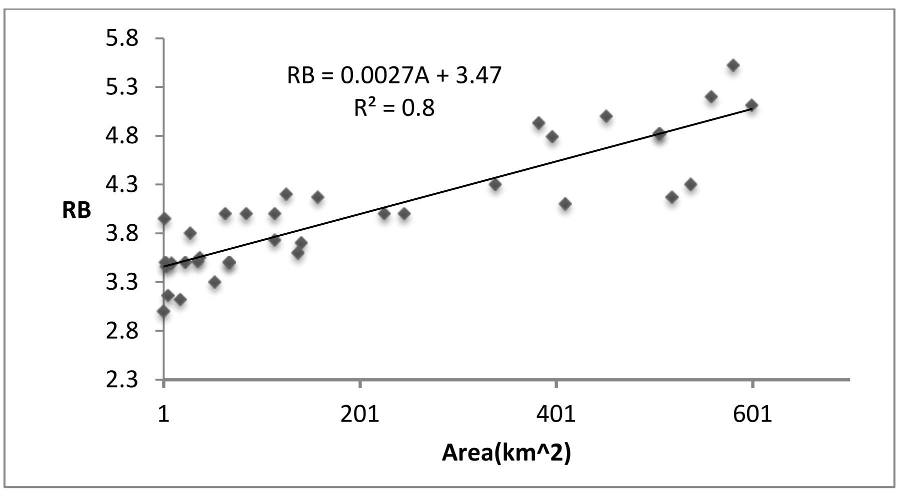

Figure 2: Linear regression between bifurcation ratio and catchment area

Eq. (12) indicates that in small catchments with an area of less than 600km^2 the value of R_B falls between 3.47 and 4. It is suggested that Eq. (12) should be applied to catchments of areas lower than 600km^2. It should be noted that, in regard to Eq. (7) and R_B, the values of N_i are calculated for $i \le \Omega$, where Ω is the maximum order of the catchment. $N_{i=\Omega} = 1$ is assumed, and $N_{i-1} = R_B N_i$, $i \le \Omega$ (Horton, 1945).

In this study the length ratio R_L was assumed to be a function of the main stream length and the area of the entire catchment.

Among all 37 catchments, in 12 catchments required information (the length of main river, area, slope and SOLR) was available, so 9 catchments were used for estimation of equations and 3 catchments for validation.

The fitted regression equation for the nine selected catchments shown in Table (2) is as follows:

$$R_L = 2.59 L^{0.41} \ A^{-0.2} \tag{13}$$

The correlation coefficient is equal to 0.91.

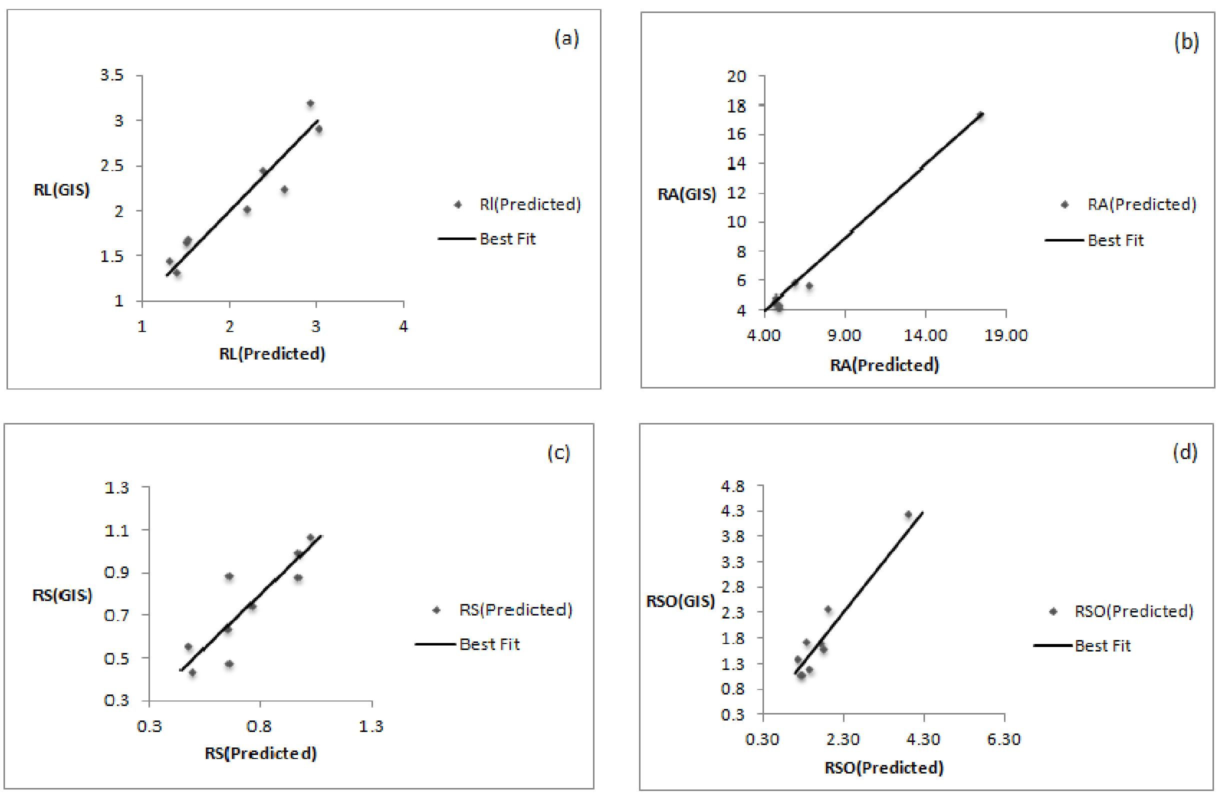

Figure 3: GIS-based stream-order-law ratios against predicted SOLR

Fig. 3(a) shows the R_L values calculated from Eq. (13) compared to the GIS values of R_L . Based on Eq. (8) and R_L, the values of $\overline{L_{c_i}}$ are calculated for $i \le \Omega$. $\overline{L_{c\Omega}} = L$ is assumed, and $\overline{L_{c_{i-1}}} = \overline{L_{c_i}} / R_L$, $i \le \Omega$ (Horton, 1945).

The area ratio (R_A)was assumed to be a function of the bifurcation ratio and the length ratio, with fitted equation:

$$R_A = 0.597 R_B^{1.553} R_L^{-0.177} \tag{14}$$

The correlation coefficient is 0.99.

Fig .3(b) shows the R_A values calculated from Eq. (14) compared to the real value.

$\overline{A_\Omega} = A$ is considered, and $\overline{A_{i-1}} = \overline{A_i} / R_A$, $i \le \Omega$ (Schumm, 1956).

Stream slope ratio was assumed to be a function of R_B, R_L, and R_A. Eq. (15), with correlation coefficient 0.79, represents the fitted regression relation for the data.

$$R_S = 1.198 R_B^{1.26} R_L^{-0.97} R_A^{-1.04} \tag{15}$$

A nonlinear regression equation consisting of the parameters R_B, R_L, R_A, and R_S was used to calculate the slope ratio of the overland plane with the fitted relation:

$$R_{SO} = 0.366 R_B^{2} R_L^{-0.58} R_A^{-0.66} \tag{16}$$

The correlation coefficient of Eq. (16) is 0.93, and no strong correlation between R_{SO} and R_S was observed.

Figs. 3(c) and 3(d) show the values of R_S and R_{SO} obtained from Eqs. (15) and (16) in comparison with the real values. From Eqs. (16) and (11), the slope of the overland planes of the catchment can be obtained. It should be noted that Eqs. (12) to (16), which are obtained from information on the nine catchments, may be calibrated by adding more data. Given that, for all catchments, the lengths of the main rivers and the areas are known, the ratios R_B, R_L, R_A, R_S, and R_{SO} can be calculated from Eqs. (12) to (16).

5. Prediction of geomorphological information

The catchment area and the length and slope of the main river can be determined from the topographic maps of the catchment (scale 1:25000 to 1:50000). If a catchment has a maximum stream order of Ω, this implies that the stream is located at the end of the catchment with mean slope $(\overline{S_{c_\Omega}})$ and mean slope of the lateral overland planes $(\overline{S_{o_\Omega}})$. For instance, Fig. 4 shows a small catchment with three sub-catchments (I, II, III). The maximum stream order is two $(\Omega = 2)$. Sub-catchment III is created with two lateral overland planes and stream III is positioned at the end of the main catchment. Fig. 4 shows the mean slope of the stream III $(\overline{S_{c_2}})$ and the mean slope of the two lateral overland planes $(\overline{S_{o_2}})$.

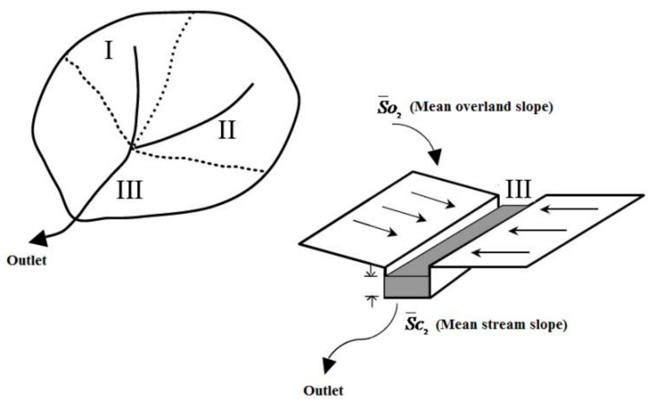

Figure 4: Catchment with maximum stream order two

If the values of $\overline{S_{c_\Omega}}$, $\overline{S_{o_\Omega}}$, R_S and R_{SO} are known, the values of $\overline{S_{c_i}}$ and $\overline{S_{o_i}}$ are computable from Eqs. (10) and (11) for lower orders, $i < \Omega$ $(\overline{S_{c_{i-1}}} = \overline{S_{c_i}} / R_S$, $\overline{S_{O_{i-1}}} = \overline{S_{O_i}} R_{SO})$.

To calculate the value of $P_{x_i x_j}$ in Eq. (1), the following equation is used:

$$P_{x_i x_j} = N_{i,j} / N_i \qquad (17)$$

where $N_{i,j}$ is the number of ith order streams contributing flow to the jth order stream, and N_i is the number of ith order channels. The value of N_i can be calculated by the bifurcation ratio, although to obtain the parameter $N_{i,j}$, the following equation is suggested:

$$N_{i,j} = 2N_i \exp(-0.64j) \quad (18)$$

which is obtained through nonlinear regression of the stream network data based on the geomorphologic parameters of the Kasilian and the Gagas catchments. The catchments possessing DEM must be delineated by stream network and ordered using GIS software; however, calculation of $N_{i,j}$ must be done manually and rendered by the GIS operator, and this is a time-consuming and difficult task.

6. Effect of ratios R_B, R_L, R_A, R_S and R_{SO} on DRH

In the previous section of this study, empirical equations for geomorphologic ratios were presented. In this section, we apply the GIUH model to carry out sensitivity analysis on these ratios and examine their effects on DRH and peak flood. For this analysis, information from the Kasilian catchment was utilized.

Fig. (5a) illustrates the effect of bifurcation ratio on DRH for the Kasilian catchment on 4th May, 1993.

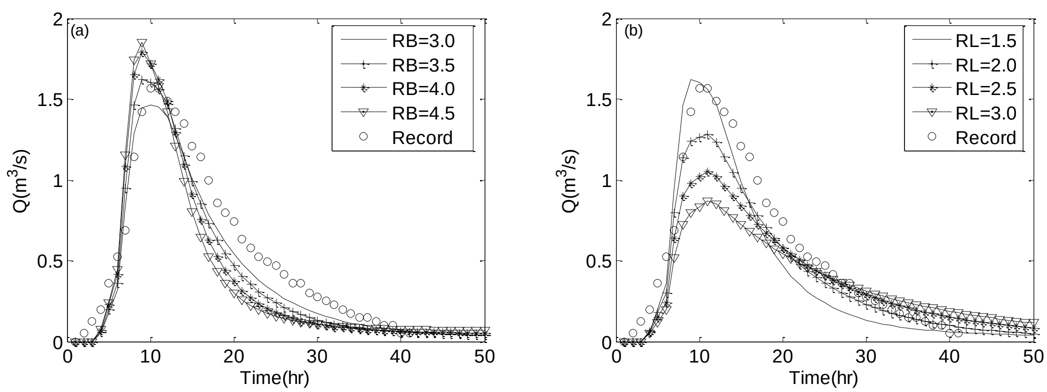

Figure 5: Effect of R_B and R_L on direct runoff hydrograph

For 4th May 1993 (Kasilian catchment)

Values for the bifurcation coefficient of 3, 3.5, 4, and 4.5 with 0.5 unit increments were considered for the Kasilian catchment, and the number of streams and the values of the input parameters into the GIUH model were computed and inserted into the model. The effect of R_B on the shape of the hydrograph and the peak of the runoff is shown in Fig. 5(a). The results of the model are compared with those of recorded runoff hydrographs.

To evaluate the effect of R_B on the peak flow, the following equation for relative sensitivity was used:

$$S_r = \frac{O_2 - O_1}{P_2 - P_1}(\overline{P} / \overline{O}) \qquad (19)$$

where O and P represent the specific outputs and parameters respectively of the model. Therefore, S_r gives the percentage change in O for a 1% change in P. $\overline{P}$ and $\overline{O}$ are given by $(P_1+P_2)/2$ and $(O_1+O_2)/2$ respectively. The results confirm that the lowest computational error in peak discharge relative to the observed peak discharge was shown at R_B=3.5 with an error of 3.5%. The actual R_B for the Kasilian catchment is also 3.5. The mean relative sensitivity of R_B, as derived from Eq. (19), is 0.56.

Fig. 5(b) shows the effect of R_L on DRH for same event. The values of this ratio were taken to be 1, 1.5, 2, and 2.5 with a 0.5 increment. The results show that R_L=1.5 gives the least error in peak discharge, with 3.6% error value. The actual R_L of the catchment is 1.46, and the mean relative sensitivity of R_L amounts to 0.92. Larger values of R_L give higher peak errors. The runoff is affected more by the length ratio than the bifurcation ratio, as can be observed in Fig. (5). The next section of this paper is dedicated to the effects of the area ratio on the peak of the runoff. The values of area ratio considered ranged between 3 and 6, with 1 unit increment values. Fig. 6 shows the effect of area ratio on DRH.

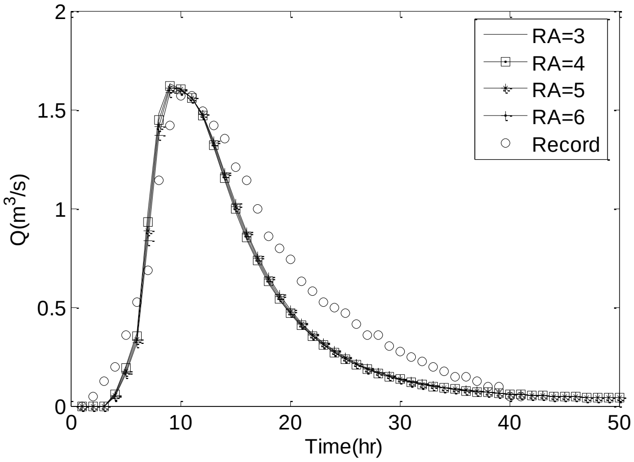

Figure 6: Effect of area ratio on direct runoff hydrograph

For 4th May 1993 (Kasilian catchment)

The results show that the area ratio has a small effect on the runoff peak, so that alterations of this ratio do not noticeably influence the shape of hydrograph and the flood peak.

Fig. 7(a) shows how R_S affects DRH for the values 0.1, 0.4, 0.7, and 1 with an increment of 0.3.

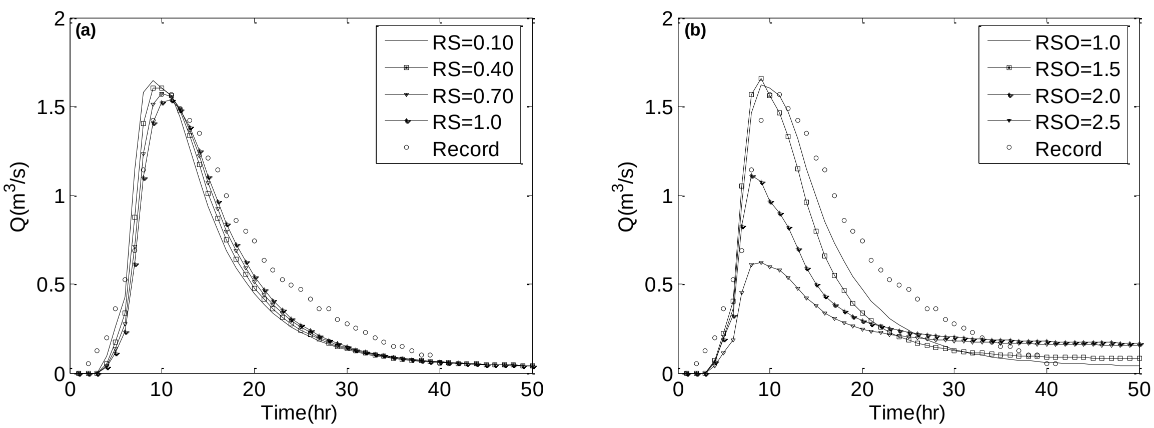

Figure 7: Effect of R_S and R_{SO} on direct runoff hydrograph

For 4th May 1993 (Kasilian catchment)

The lowest error is 0.47, which corresponds to the ratio (0.7), while the actual slope ratio of the Kasilian catchment is 0.38. In addition, the mean relative sensitivity ratio is 0.042. The results indicate that this parameter also has little effect on runoff peak.

Figure 7(b) shows the influence of R_{SO} on DRH for values of 1, 1.5, 2, and 2.5 with an increment of 0.5. The lowest error is seen at a ratio of 1, with a 3.54% error value, while that of Kasilian catchment would be 1.1, and the mean relative sensitivity ratio 1.33. These results indicate that the parameter R_{SO} has a substantial effect on runoff peak.

The overall results show the relative sensitivity of DRH to R_B, R_L, R_A, R_S, and R_{SO} to be 0.56, 0.92, 0.01, 0.042, and 1.33, respectively. The strongest effects correspond to the R_{SO}, R_L R_B, R_S, and R_A respectively.

7. Verification

7.1. Validation of stream-order-law ratios relationships

In the previous sections, equations were presented for the estimation of stream-order-law ratios based on GP in nine different catchments worldwide. For verification of the results of the regression equations, the GP of three catchments, Gagas, Heng-Chi, and Kasilian, were applied.

Table (3) lists the GP and stream-order-law ratios of the three selected catchments using Eqs. (12) to (16). Table (3) also provides the values of stream order ratios and their computational errors.

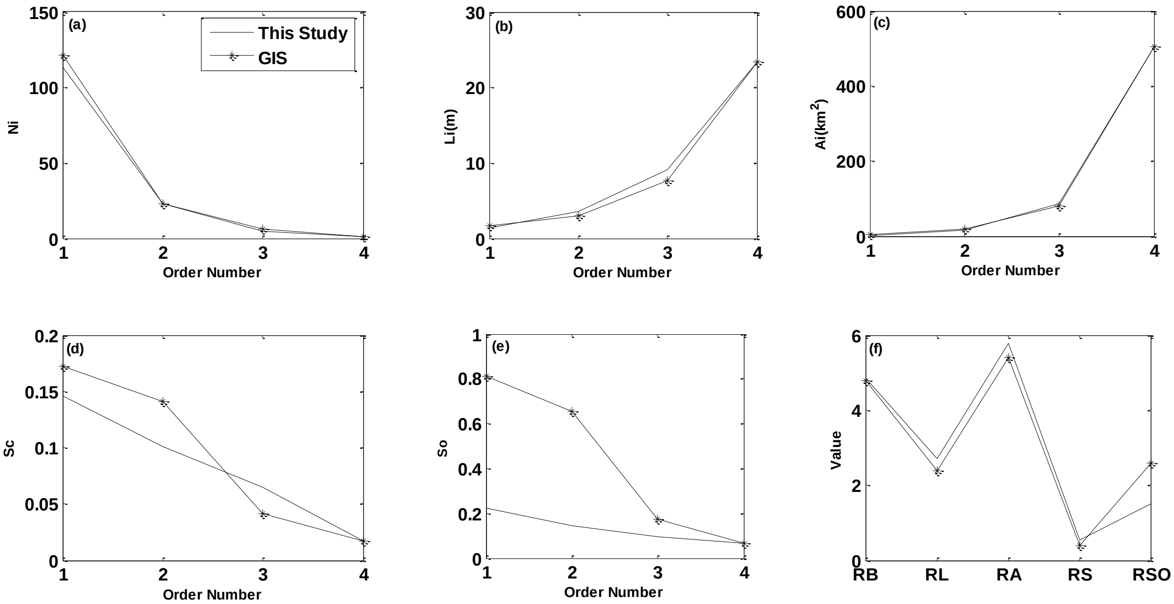

Figure 8: Verification of geomorphological parameters in the Gagas catchment

In order to calculate the error related to the stream-order-law ratios, Eq. (20) below was used:

$$Error\% = 100 * (R_P - R_{GIS}) / R_{GIS} \qquad (20)$$

where R_P is the predicted stream-order ratio and R_{GIS} is the GIS-based stream order ratio.

Table 3: Calculated geomorphological parameters of the Gagas, Heng-Chi, and Kasilian catchments

Catchment Name	Predicted Geomorphologic parameters										
	Order	N_i	$\overline{L_i}$	$\overline{A_i}$	$\overline{S_c}$	$\overline{S_o}$	R_B	R_L	R_A	R_S	R_{SO}
1. Gagas	1	113	1.38	2.6	0.146	0.222	4.84	2.72	5.78	0.53	1.5
	2	23	3.54	15.1	0.101	0.147					
	3	5	9.10	87.5	0.065	0.098					
	4	1	23.40	506.0	0.017	0.065					
GIS Results							4.80	2.40	5.40	0.40	2.60
%Error							0.40	13.7	7.6	21.0	41.4
2. Heng-Chi	1	47	0.32	1.0	0.104	0.654	3.61	2.26	3.80	0.68	1.2
	2	13	1.34	3.7	0.060	0.530					
	3	4	2.43	14.0	0.031	0.429					
	4	1	4.97	53.2	0.012	0.347					
GIS Results							3.30	2.60	4	0.60	1.10
%Error							9.4	13.7	5.0	13.3	9.1
3. Kasilian	1	49	0.49	1.1	0.109	0.563	3.65	2.09	3.92	0.72	1.3
	2	13	1.03	4.4	0.073	0.436					
	3	4	2.19	17.3	0.038	0.337					
	4	1	4.65	67.8	0.008	0.261					
GIS Results							3.5	1.5	4.3	0.4	1.1
%Error							4.3	43.2	8.8	89.5	18.2

Figs. (8) to (10) show the GIS-based and computational GP of the three case study catchments.

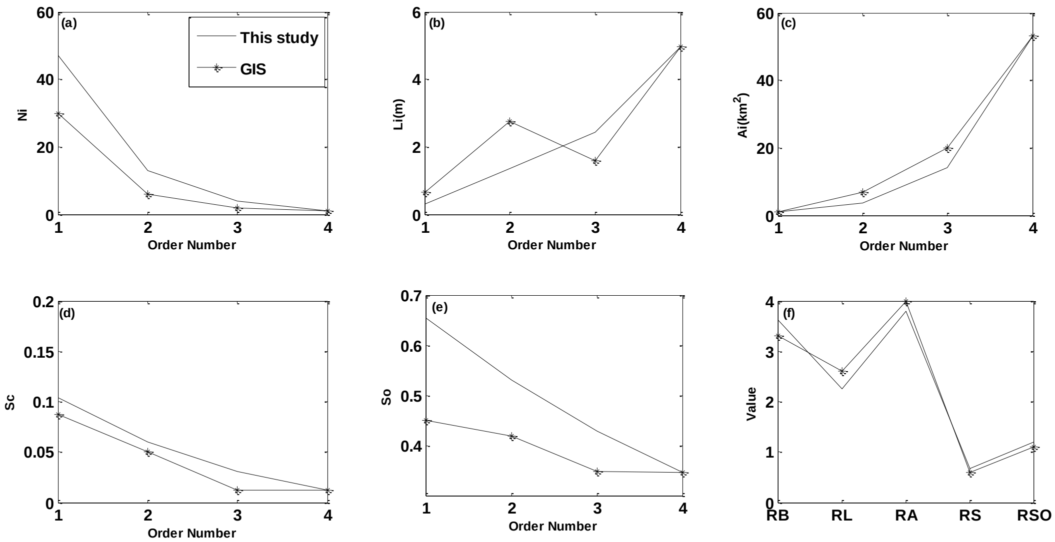

Figure 9: Verification of geomorphological parameters in the Heng-Chi catchment

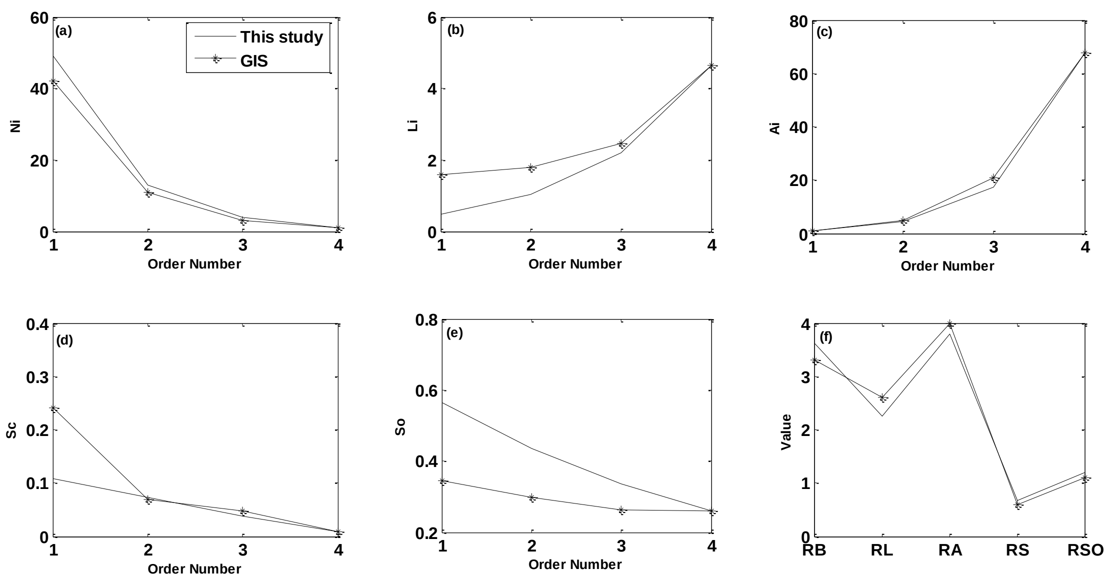

Figure 10: Verification of geomorphological parameters in the Kasilian catchment

The mean errors of the regression equations for the estimation of R_B, R_L, R_A, R_S, and R_{SO} in the three selected catchments are, respectively, 4.7%, 23.5%, 7.1%, 41.3%, and 22.9%.

The greatest errors of the model emerged in the estimation of, respectively, R_S, R_L, R_{SO}, R_A, and R_B. As can be observed in Fig. 7(a), the stream slope ratio has a small effect on runoff, and its error can therefore be ignored. Regarding the high sensitivity of the length and overland slope ratios, these errors range from 23% to 24%, and it is recommended that the joint effects of all the ratios on DRH of the selected catchments be considered.

7.2. Validation of the catchment's direct runoff

In the previous sections, the influences of SOLR on runoff were considered separately, and the GP of the three catchments were estimated using the regression equations. To investigate the accuracy of the estimations in more detail, it is more effective to estimate the DRH using the GIUH model. We therefore turn to verification of the predicted direct runoff for the two catchments, taking into consideration the information from the excess rainfall hyetograph and the recorded runoff from the Kasilian and the Heng-Chi catchments.

The GIUH model was applied to two cases: one in which the SOLR were GIS-based, and the other in which the empirical regression equations developed in this paper were used for the Kasilian and the Heng-Chi catchments. The results of the model in each case were compared with those of observed and recorded runoff. Since observed runoff and rainfall data for the Gagas catchment were not available, this catchment was dispensed with for the verification phase. Figure 11 shows the results of the GIUH model for DRH estimation for the Kasilian catchment for two events, on 10th May 1992 and 4th May1993.In addition, Fig. (12) shows those in the Heng-Chi catchment for two events, in July 1996 and October 2000.

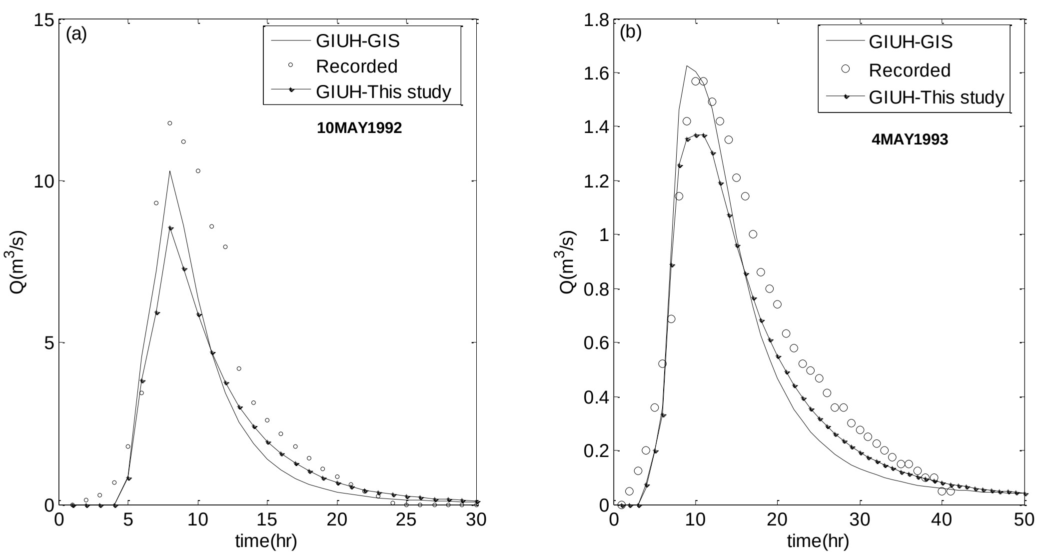

Figure 11: Estimated direct runoff hydrograph for the Kasilian catchment

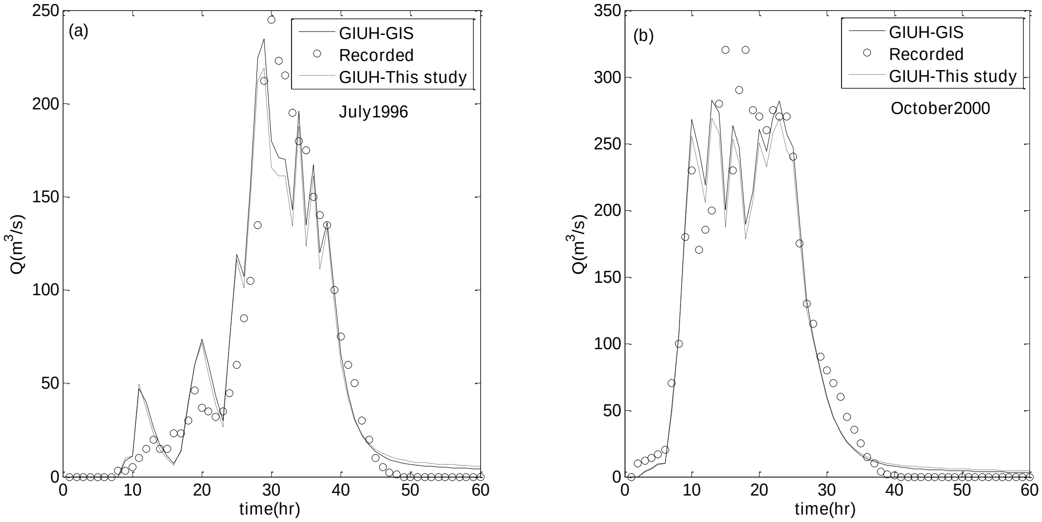

Figure 12: Estimated direct runoff hydrograph for the Heng-Chi catchment

To validate the suitability of the model for the Kasilian and Heng-Chi catchments, three common statistical measures were used: the coefficient of efficiency (*CE*), root mean square error (*RMSE*), and relative error in peak (*REP*).

Estimation of these three parameters was carried out using the following equations:

$$CE = 1 - \frac{\sum_{t=1}^{n}[Q_r - Q_s]^2}{\sum_{t=1}^{n}[Q_r - \overline{Q_r}]^2} \qquad (21)$$

$$RMSE = \left[\frac{1}{n}\sum_{i=1}^{n}(Q_r - Q_s)^2\right]^{0.5} \qquad (22)$$

$$REP = 100 \times [Q_{p_s} - Q_{p_r}] / Q_{p_r} \qquad (23)$$

where Q_r is the recorded discharge at time *t*; Q_s is the simulated discharge at time *t*; $\overline{Q_r}$ is the mean recorded discharge during the storm event; *n* is the number of discharge records during the storm event; Q_{p_s} is the peak discharge of the simulated hydrograph; and Q_{p_r} is the recorded peak discharge.

Table (4) gives the values of *REP*, *CE*, and *RMSE* for the two selected catchments, calculated using both the GIS-supported model and the method proposed in this study.

Table 4: Validation of results of the GIUH model

July1996	*REP*%	*CE*	*RMSE*
GIS	4.18	0.87	24.54
This study	10.62	0.86	25.44
October 2000			
GIS	11.81	0.93	31.22
This study	15.99	0.92	32.25
10 May 1992			
GIS	12.68	0.81	1.13
This study	27.33	0.76	1.26
4 May 1993			
GIS	3.5	0.87	0.10
This study	12.6	0.91	0.10

It can be concluded that the computational error of the values of the runoff peak (*REP*%) that can be inferred from the results of the method proposed in this paper for the four rainfall-runoff events are, on average, 10% greater than the error resulting from the actual information (GIS-supported). As can be seen from Figs. (11) and (12), the results of the GIUH model in the two cases, using GIS and empirical equations, are very similar. *CE* and *RMSE* are also similar in value. The mean *CE* of the model was computed for the four events as 0.87, which is a satisfactory value.

8. Summary and conclusion

In this research, experimental equations are presented for the calculation of geomorphologic and stream-order-law ratios of watersheds of less than 600 km^2 in area. These equations were developed using a nonlinear regression method fitted to the stream-order-law ratios of nine different worldwide catchments. The equations were applied for verification purposes to three other selected catchments, and the results were compared with those calculated from GIS.

The geomorphologic and stream-order-law ratios of three catchments, Gagas, Heng-Chi, and Kasilian, were determined based on the experimental equations given in this research, and were compared with their actual results. The average errors of the model in the estimation of R_B, R_L, R_A, R_S, and R_{SO} in the three case study catchments were 4.7%, 23.5%, 7.1%. 41.3%, and 22.9%, respectively.

The sensitivity to runoff of the bifurcation ratio (R_B), length ratio (R_L), area ratio (R_A), stream slope ratio (R_S), and overland slope ratio (R_{SO}) in the Kasilian catchment were investigated. The relative sensitivities of the ratios R_B, R_L, R_A, R_S, and R_{SO} are shown to be 0.56, 0.01, 0.92, 0.042, and 1.33, respectively. The greatest effects were evident in the overland slope ratio, length ratio, and the bifurcation ratio, and the lowest effect was seen in the area and stream slope ratios.

The direct runoff hydrograph was estimated using GIUH with regard to the geomorphologic data computed for the three catchments, and this was then compared with the observed values.

Finally, the estimated stream-order-law ratios were input into the GIUH model and the values for the direct runoff hydrograph of two catchments, Kasilian and Heng-Chi, were calculated and compared with those of the observed direct runoff. The results show that the computational error values of runoff peak (*REP*%) for the four rainfall-runoff events are, on average, 10% greater than the error resulting from actual information (GIS-supported). The results of the GIUH model in the two cases, both with and without GIS, are very similar. *CE* and *RMSE* in the two cases also have similar values. The mean coefficient of efficiency of the model was computed for the four events as equal to 0.87.

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