Response to the reviewers

I appreciate very much the comments that were very helpful in improving the manuscript. Below I have copied all the comments and have inserted my replies in italics. I believe that this revision adequately addresses most concerns. Nonetheless, since this is a major revision, minor revisions may still be needed. I hope that the Reviewers will be available for a subsequent revision round.

Before addressing the comments one-by-one, I would like to address thoroughly the main concern of the reviewers. It seems that the main concern of the reviewers is that the results and conclusions as presented in the manuscript are either not novel or irrelevant. Later on I will refer to the response to this main concern as the main response.

Main response:

To address main concern of the reviewers, I have revised the manuscript in the following ways. Firstly, I have changed the title. Following the comments, I realized that the original title is misleading (A consistent implementation of the dual node approach for coupling surface-subsurface flow and its comparison to the common node approach). Namely, it may imply that this implementation is a novelty which is not the case. The new title is: New insights into the differences between the dual node approach and the common node approach for coupling surface-subsurface flow. Secondly, I have revised the manuscript thoroughly to make clearer what those insights are. Also, I have decreased the number of model scenarios and figures. I hope that this will help to keep more focus on the most important insights provided in this study and to make the manuscript easier to read.

Nonetheless, I disagree that that most of the conclusions in this study are not novel. To convince the Reviewers, I here summarize the novel insights provided in the revised manuscript:

1) According to commonly held views the common node approach is more physically-based. This view is based on the idea that the dual node approach introduces an additional parameter in the form of a coupling length. Typically, this coupling length is thought of as the thickness of layer in between the surface and the subsurface domain. I show that the dual node approach can be formulated such that this coupling length is fully defined by the grid topology. This results in what I call a consistent dual node approach. In this approach, the coupling length is not a non-physical model parameter.

2) In comparison to the common node approach the head continuity as implemented in the consistent dual node approach is more correctly formulated in the consistent dual node approach

3) The manuscript explains in detail the comparison between the common node approach and the consistent dual node approach. Such a comparison has not been published as far as I know. Instead, most comparison studies between the dual node approach and the common node approach...
approach are based on alternative dual node approaches. The comparison in this study considers accuracy as well as numerical efficiency. I think that this comparison is a valuable contribution to the scientific literature.

4) Although the scheme is not new, the scheme is typically not being recognized as a dual node approach. Instead, it is taken as being equivalent to the common node approach (An and Yu, Kumar et al.). Even if it is recognized as a dual node scheme, it is not recognized that this is a particularly advantageous scheme (Panday and Huyakorn). This study shows that models that already use this scheme should display some advantages in terms of accuracy as well as efficiency with respect to models that use the common node approach.

5) The manuscript explains that to understand how the approaches work, it is important to consider the meaning of nodal values. Although, this may seems trivial to a numerical modeler, it seems that this point is often overlooked. Moreover, the consistent dual node approach is derived in the manuscript from basic flow equations using finite differences. Again, this may seem trivial to a numerical modeler, but I think it is important because it leads to a conceptualization of the dual node approach, which is very different from the one in existing literature. Namely, it is not an approach that needs an additional non-physical model parameter. The manuscript also provides an explanation how the dual nodes can be separated in a vertex-centered scheme to obtain a consistent dual node scheme.

Except for not presenting new results or conclusions, another comment is that the differences between the approaches are irrelevant because they all yield accurate results if the spatial resolution is chosen carefully.

It is indeed true that all coupling approaches can yield accurate results if the vertical discretization is sufficiently fine and if the coupling length (when using a dual node approach) is carefully chosen. I made revisions such that this point is acknowledged clearer and to be more objective. It is also true that the advantages in accuracy are limited to simulating excess infiltration using a relatively coarse grid. Thus it is true as mentioned by Reviewer 1 that problems related to a difference in accuracy are not a problem if the model setup is defined carefully. However, in my opinion this does not mean that the difference in accuracy when using a relatively coarse resolution is not interesting or irrelevant. In essence, from a practical point of view, I can understand the argument why this could be irrelevant. Namely, one would wish a very fine vertical discretization irrespective of the chosen coupling approach to simulate accurately the movement of saturation fronts. But there are different coupling schemes in use today to solve surface-subsurface flow. I think it is important to understand when and why these schemes yield different results. If scientists cannot answer this question, I think that that would be a problem.

Moreover, the idea that a dual node approach can be more accurate than a common node approach is new and the current consensus is very different. Namely, the commonly held view is that the maximum accuracy of the dual node approach is reached when it mimics a common node approach. Also, studies based on models that use a consistent dual node approach (i.e. An
and Yu, Kumar et al.) did not recognize that there is a difference in accuracy with respect to the common node approach. In the study of An and Yu it was found that their model is less sensitive to the vertical discretization in comparison to ParFlow (which uses the common node approach). However, they did not recognize that this difference is related to using a different coupling approach.

The difference in accuracy also shows the consequence of how the head continuity is formulated in the common node approach. Namely, this formulation is only correct if the discretization is very fine and thus if a coarser vertical resolution is used, the common node approach can become less accurate than the consistent dual node approach (i.e. the formulation of head continuity in the consistent dual node approach is correct irrespective of the vertical discretization).

Overall, it seems that the disagreement on whether the difference in accuracy is relevant or not, depends on whether one takes a practical or theoretical viewpoint. Obviously, I look at it from a more theoretical perspective.

But suppose for the sake of argument that the difference in accuracy is completely irrelevant to the scientific community. (Although as argued I do not think that is true). Then I still do not see why this would make the manuscript irrelevant in its totality. Namely, there is also the case of numerical efficiency. And the manuscript indicates quite clearly that the consistent dual node approach can be advantageous in terms of efficiency. Moreover, the case of accuracy can also be regarded as simply being an important issue to be considered when comparing two different approaches.

Anonymous Referee #1

Received and published: 12 May 2017

I have carefully read the manuscript called “A consistent implementation of the dual node approach for coupling surface-subsurface flow and its comparison to the common node approach” by Rob De Rooij. This paper raises important issues regarding the application of integrated hydrological models through the examination of the possible influence of the coupling strategy and the vertical discretization. It especially investigates the following scientific questions (i) what is the proper coupling length to be used for the so-called dual node approach; (ii) how to formulate the dual node approach to conserve the physically based nature of the model; (iii) how does the coupling strategy influence the simulated dynamics when the vertical resolution is coarsened and (iv) how do the common node and the dual node approaches compare on synthetical test cases.

I appreciate the careful reading and interest of the reviewer.

Before going to my comments of the paper, I want to stress out that these issues are critical and barely discussed in the integrated hydrologic modeling literature. Integrated hydrologic models are more and more used to investigate hydrologic behaviors but the questions of the appropriate
scale, spatial resolutions (both horizontal and vertical), the crucial modeling choices that are to be made (coupling length for instance) and their effect on the simulated dynamics are too often forgotten although in my opinion of primary importance. I especially believe that there is a need to keep the physical meaning of integrated hydrological models through the use of appropriate spatial resolutions. This point is made very clear in the paper and is in a way the starting point of the research presented.

*I agree with the reviewer that it is important to keep the physical meaning of a model through appropriate spatial resolutions. Indeed, this becomes a critical issue if non-linearity is significant. And this is typically the case in integrated surface-subsurface models.*

The consistent dual node approach proposed in the paper is clearly exposed and is a way to properly account for infiltration, especially in partially ponded cells. This approach for coupling allows preserving the physics of infiltration across the land surface if numerical parameters and spatial resolution are chosen adequately. A detailed analysis on the surface and subsurface pressure values, on the infiltration flux and on the time to ponding is provided. This analysis demonstrates the added-value of this method mainly (and only?) to describe the infiltration excess process. Although the issues tackled are of interest and the method proposed seems appropriate, I have serious concerns with the paper and I am not sure that the material presented is enough for a research paper. It seems that the added value of the approach proposed is not so important compared to the classical coupling approaches if the classical approaches are used in a relevant way. I hope that the following comments will somehow help improving the manuscript and maybe help in the publication process.

**Major comments:**

1. One of my major concern deals with the fact that most of the conclusions of the research proposed in this paper are not novel and already documented in the literature. For instance, it has already been demonstrated that when using a proper discretization both coupling approaches gives very similar results and that a relatively small coupling length needs to be used with the dual node approach to conserve the physical meaning. It is true that integrated models tend to be used out of their proper application domain with coarse vertical discretization but it is more than intuitive that the vertical resolution should be small to properly capture the non-linear dynamics of infiltration fronts (especially when infiltration excess occurs). If the integrated models are properly applied, most of the questions that are tackled in the paper are not a problem anymore. In a way, the paper aims at determining which method is the less inaccurate (see line 554 to 556) when using a coarse vertical discretization, which is in a way irrelevant as both approaches are acceptable when using a proper resolution. These comments are illustrated through the conclusion that is short and not so much informative.

*I have addressed this concern by acknowledging in the conclusions more clearly that the advantages in terms of accuracy of the consistent dual node approach versus the common node approach are indeed limited.*
Although I understand the reviewer’s concern, I do not agree that the most conclusions are already documented nor that the differences in between the approaches are in a way irrelevant. I refer to my main response as to why I disagree.

(2) The second main concern is linked to the tone and the phrasing of the paper that are not always adapted especially when reference models of the literature – i.e. Hydrogeosphere, MODHMS or Parflow – are criticized. I acknowledge that the coupling in Parflow is not well described in Kollet and Maxwell (2006) and that as a consequence some important aspects of Parflow turn out to be unclear. But I don’t feel like there is a need to point out in details what the author think is not done properly by others. Once again, if an integrated model is used carefully with proper discretization and coupling length, it will produce consistent (with the physics) results regardless if it is a common node or a dual node approach. As a consequence, it is preferable to highlight what the consistent dual node approach brings than to denigrate the other approaches. I think that part 5 should be removed or at least strongly modified.

I have changed the explanation of the coupling in ParFlow. However, Section 5 (now section 4.3) is in my opinion essential. This section does not aim to denigrate other models. Namely, the shortcomings of inconsistent dual node approaches have already been discussed elsewhere. As such I do not heavily criticize other models here. I merely contrast the shortcomings with the consistent dual node. Nonetheless, I have tried to change the tone and the phrasing in this section.

(3) I have serious concern about the result regarding the numerical efficiency. First I don’t understand the arguments presented at the beginning of the part 7.2 that directly link the infiltration rate and the gradient across land surface with the numerical efficiency. It is a problem for me as all the following discussion on the efficiency is related to that argument. I feel like this point should be explained better. Moreover, the efficiency of the resolution is highly linked to the numerical procedure (numerical scheme, time integration,….) that is used to solve the common node approach. In the paper by De Rooij (2013) it is explained that the model uses a dual node approach. But the common node approach is not described. Either I missed something or this should be detailed somewhere so that the reader can have all the needed information. Finally, for some test cases the difference in the number of Newton iteration is rather limited making it difficult to say in a general way that the dual node approach is more efficient than the common node approach.

I have made major revisions in the discussions to explain better the differences in efficiency. Moreover, I also added an explanation about how the common node approach is implemented.

(4) Regarding the efficiency, I also believe that the tighter the coupling, the more difficult the resolution will be. Considering the experience I have in the domain, it is much harder to impose continuity through a common node type of approach than to impose a first order coupling through a dual node approach (if the numerical resolution is the same). As a consequence, it is for me logical that convergence is harder to obtain for some test cases with the common node approach.
In the literature the difference in efficiency has indeed been explained in terms of tight or less tight coupling (i.e. Ebel et al.). I think that my explanations of why the consistent dual node approach can be more efficient are more detailed and add some significant understanding on why the efficiency can be different. Namely, it is shown that this difference can be tied to how fast water depths are changing at the moment of ponding. These rates are different depending on the approach.

(5) The paper is quite clear but some parts are too long. This makes the paper sometimes hard to read. Part 4 is an example. This part is very long and the first conclusions are deceiving – i.e the proper implementation has already been proposed by other (Line 240) and the proposition of a numerical trick to properly implement dual node in vertex-centered scheme (line 256 to 259). Maybe this can be improved.

I shortened this section considerably. I have tried to re-phrase this part a bit to be clearer.

(6) The part that presents the results is also hard to follow. I believe that there are too many test cases presented and that all of them are not needed. The saturation excess test cases may be removed as they are only illustrative for the efficiency. Maybe only the infiltration excess should be kept as it is for this process that the added-value of the method proposed is the most important. The consequence of multiple test cases per hydrological processes is that the reader has to jump from one figure to another which is not convenient at all. The number of figures presenting the results is also quite high.

Instead of removing the saturation excess cases, I have removed the column experiments. In fact, the hillslope experiments are enough to make my points.

(7) Regarding hydrological processes, it seems that the differences between both approaches are very small when dealing with the saturation excess process, which is the dominant process of streamflow generation in most temperate region. The main problems/conclusions are linked to the infiltration excess process. The findings for both processes are rather limited as (i) for saturation excess both approaches are OK and (ii) it is well-known that using the Richards equation infiltration excess cannot be properly capture with a 20 cm or a 50 cm resolution.

I acknowledge more clearly that indeed, the advantage in accuracy is limited. But the fact that a dual node approach can be at least or more accurate in comparison to the common node approach is significant in my opinion as it illustrates that it matters how the head continuity is implemented. Moreover, the findings in this study contrast to commonly held views, according to which the dual node approach is only more efficient with respect to the common node approach at the expense of accuracy. Namely, the consistent dual node approach can be more efficient as well as more accurate for certain simulation scenarios. See also my main response for additional arguments why I think the difference in accuracy is actually quite relevant.

The point of the manuscript is not only that the consistent dual node approach can be more accurate. Efficiency is also considered. In a more general sense, the manuscript simply compares in detail the consistent dual node approach with a common node approach. The fact
that both approaches can yield similar results and that the differences in accuracy or efficiency are not extreme does not make this comparison study irrelevant.

(8) The coupling between surface and subsurface strongly depends on the numerical schemes used for resolution. This point is clear on the paper (especially through the explanations related to figure 1) but the paper – although using 2 different schemes – is not exhaustive. Some published models using other resolution schemes are built using a properly implemented dual node approaches and this point should be fairly mentioned somewhere.

I only found that the model of Kumar et al. is also in essence based on a consistent dual node approach. In addition, I also point out that CATHY as well as the model of Morita and Yen share characteristics with the consistent dual node approach.

(9) I am a bit uneasy with the concepts of elegance and generality when considering physically-based modelling. In my opinion, the main question is whether the modelling approach chosen allows for a proper description of the physics considered. I believe that it is an endless debate to determine which approach is the more elegant or the more general and I would suggest the author to remove the sentences related to that and focus on the accuracy and/or the efficiency that are can be somehow measured.

Corrected in the revised manuscript.

Other comments:

- Some parts of the paper are only about interpretation and as a consequence are very subjective. See for instance from line 274 to line 283.

Removed

- Line 45: hillslopes not hill slopes

Corrected


Corrected

- Line 60: the interface is not always saturated. Its property is constant but saying that it is always saturated can be misunderstood regarding the infiltration process.

Corrected

- From line 191 to line 196: this part is not clear and needs to be improved. To my knowledge and in most of the integrated models mentioned in the paper, when a cell is not ponded, all the rainfall infiltrates. When the cell is ponded or partially ponded, infiltration occurs under the ponded area. I agree that infiltration under the non-ponded fraction of a partially ponded area should be theoretically accounted for, but the sentences in the paper could lead to misunderstandings.
Rephrased paragraph

- Line 223: I don’t understand why it is mentioned here that the surface head can be used as a Dirichlet boundary condition. I agree that it can be done but not in the context of a coupling through a dual node approach. Maybe this is linked to the implementation of the common node approach.

Corrected

- Line 326: typo - Figure 1c

Corrected

- Line 365-368: Repetition of things already said from line 274 to 283

Removed

- Line 395-397: I quickly checked in de Rooij et al (2013) and this paper only describe the dual node approach for coupling. Some results with the common node approach are presented later in the paper. The way the common node approach is implemented should be presented somewhere.

Corrected in revised manuscript, added explanation in section 5 (numerical experiments)

- Line 464 to 478: this part does not bring anything to what is already well known and described in the literature. Just say that the reference is computed using a fine resolution.

This is not completely true (already known), because I compare with a consistent dual node approach which is different. But the overall idea does indeed remain the same. I have shortened the paragraph.

- Line 498-500: Please explain before in the paper how the inconsistent dual node approach was implemented.

I have removed this approach from the experiments (Note that it would be quite simple. Namely a simple change in elevation heads of the surface nodes).

- It is strange that figure 2 d and 4d shows so different results. We would expect that the behavior between different coupling approach/resolution provides same trends regarding the reference and it’s not the case. Can you explain?

These figures are removed. But the difference is related to a difference in the effective rainfall rate, Namely, when simulating excess infiltration, the inconsistent dual node approach requires a water depth greater than the coupling length for top-down saturation to occur. Thus, if the effective rainfall rate is large enough then this is more likely to be the case.

- Test cases with excess infiltration: even though the dual node approach displays “more desirable behavior” (line 521), the results with coarse discretizations are far from the reference. Meaning that a consistent implementation of the dual node approach is not sufficient enough if the resolution is not well chosen.
I have changed the phrasing. But, the finding that the consistent dual node approach is less sensitive to the vertical discretization remains a significant insight. Indeed, I think that it can be argued well that the dual node approach displays more desirable behavior. Namely, ponding starts before the topmost subsurface node is saturated. Since this represents a value at some depth below the surface, it is logical to assume that this node should reach fully saturated conditions some time after reaching fully saturated conditions at the surface. Of course, this does not mean that the consistent dual node approach is accurate for any discretization. More in terms of comparing the two approaches I think it is fair to say that the consistent dual node approach displays more desirable behavior.

- Figure 10 c and 10 d: it is hard to say who the best is between the common node and the dual node. Needs to be discussed.

I have removed the simulations with the coarsest discretizations. Also because Reviewer 2 stated that such a coarse discretization is rarely used.

- Figure 13: why is there so much difference for this test case only? When the discharge are so close and match pretty well, the efficiency seems very different between the coupling approaches.

*Added further and better explanation.*

- Line 538-539 (excess infiltration): all the simulations are far from the reference. The argument presented in this sentence is not valid in my opinion.

*Corrected*

- Line 553: typo “underestimates or overestimates”

*Corrected*

- Line 671: Figure 9 not 10

*Corrected*

- Line 635: Figure 6 not 7

*Corrected*

Anonymous Referee #2
Received and published: 16 May 2017

R. deRooij (RdR) presents the dual node approach for coupling surface and groundwater flow including a comparison to the common node approach and other dual node approximations based
on synthetic numerical experiments and also numerical measures (i.e. number of non-linear iterations).

I have two major points of concern with the manuscript. While I like and appreciate the effort by RdR to clarify general misperceptions and confusion of different common and dual nodes approaches, the manuscript reads more like a reckoning with numerical, hydrologic scientific software than a research paper. It is important to keep in mind that we are dealing with a highly non-linear problem ultimately cast in discrete mathematics that a computer can understand. As such there will always be ambiguities and errors. For example, I was always wondering, how these models handle the following situation. Imagine the following thought experiment of model with a cell-centered grid, where the top layer is just under tension saturation. Adding an incremental amount of water will switch the pressure value at the cell center from some negative value to $\sim \frac{dz}{2}$. A dual node right at the land surface interface would switch from some negative value to $\sim 0$. In both cases surface runoff is initiated. Thus, there is something like a discontinuity in pressure due to the discrete mathematics, which will lead to errors under both excess infiltration and saturation conditions for both the dual and common node approach, which can only be resolved with very high spatial discretization. This can be nicely seen, in my opinion in the results of the numerical experiments presented here and have been shown before in publications related to the simulation of coupled groundwater-surface water flow and the development of integrated hydrologic scientific software. Looking at the results presented here, these types of problems are still not resolved by the proposed dual node approach, and probably never will be because of the limitations of discrete mathematics.

The idea or objective of this paper is not to find a panacea for all these problems. Instead I show how the consistent dual node approach compares to the common node approach, which has not been done to the best of my knowledge. While I use similar experiments as previous studies, the results are thus novel. I use similar experiments as it is common practice to make comparisons on benchmark tests if available.

I have changed the tone of the paper to make it look less than a reckoning with other models. Nonetheless, to make clear that this paper contains novel insight, I do need to discuss the differences with respect to other numerical models.

I hope that the new figures in the manuscript will help the reviewer in finding an answer to his thought experiment. In general, the pressure head will never make an abrupt jump as long there is a specific storage greater than zero. Instead the pressure head can change very fast from 0 to a value equal to half the thickness of the topmost cell. How fast this change occurs depends on the coupling scheme as it is now explained in more detail in the manuscript. Also, when using the consistent dual node approach ponding will only start if the infiltrability is exceeded. The computation of this infiltrability depends on the vertical discretization. But in case of excess saturation, this does not matter since the ponding is merely governed by the time it takes to saturate the subsurface which depends on the initial water content and the applied flux rate.

Therefore, because of numerical aspects, it is also not appropriate to compare directly the non-linear iterations for both coupling schemes. The common and dual node implementation are
different discrete approaches that of course will exhibit different non-linear convergence, and,
second, it is not clear from the presentation how the common node approach has been
implemented by RdR.

I have added an explanation of how the common node approach is implemented in the model
code. However, since all the flow computations (except for the exchange flow) are identical, I
think it is fair to compare the non-linear iterations. I have added a remark that the iterations do depend on how the model code is constructed. But that dependency is equal for both approaches as they are implemented in the same code. Moreover, I explain in greater detail why there are differences in efficiency. Since they can be tied to how abrupt the pressure heads are changing near the surface at the moment of ponding, I think that any model will encounter similar problems (i.e. more iterations and smaller time steps if the changes are more abrupt).
Considering the concern of comparing the number of iterations, I would be interested if the
Reviewer has alternative ideas of measuring the efficiency.

My second concern is related to the RdR’s dual node approach, which is not novel. As the author acknowledges himself that “Nonetheless, their [An, H., and S. Yu (2014)] approach is actually a properly implemented dual node approach practically similar to the one proposed in this paper.” Thus, it appears that main contribution of the manuscript is the discussion of the difference between the common and dual node approach and clarification of some of the applied concepts in different scientific hydrologic software.

While I feel this is a valuable contribution to the scientific literature, the manuscript requires major revisions and a more objective discussion. After all, for example, figure 2 suggests that for coarse spatial resolution both the common and dual node approach are quite far off the reference simulation. But in the past ten years or so, model implementations improved and a spatial discretization of 0.5m at the land surface is rarely used in todays models that I read about.

I have tried to strike a more objective tone and to make clearer what the paper is about. I acknowledge in the revised conclusion section that the advantage in accuracy is limited. The point that is being made in the paper is that the dual node approach should be perceived more positively in comparison to the common node approach. Namely, the common view is that a) the common node approach is more physically based, b) the common node approach is more accurate (i.e. the common view is that a dual node approach is most accurate when it mimics a common node approach), c) the dual node approach can be more efficient but at the expense of accuracy vis-à-vis the common node approach. This paper shows that this is very different when using a consistent dual node approach. Namely, in the dual node approach the head continuity is actually more properly formulated, the approach is at least as accurate as the common node approach and is often more efficient without a trade-off in accuracy. That the approach is not new (which is acknowledged in the paper) does not change the fact that these are significant new insights.

I also removed the spatial discretization of 0.5 m and now set the coarsest discretization to 0.2 m. Again, I do not pretend that the consistent dual node approach is always better or that it can be used with a very coarse vertical discretization. But it is interesting that the consistent dual
node approach is less sensitive to the vertical discretization when simulating excess infiltration and that it can be more efficient for excess saturation as well as excess infiltration.
A consistent implementation of New insights into the differences
between the dual node approach and the common node approach
for coupling surface-subsurface flow and its comparison to the
common node approach

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22 Key points

23 Surface-subsurface flow coupling
Abstract

Commonly, the common node approach and the dual node approach for coupling are two widely applied approaches to couple surface-subsurface flow. The surface-subsurface flow is conceptualized as a hydraulic separation of the surface and the subsurface by a distinct interface with a given thickness. Since such an interface is not supported by field observations, it has been argued that the dual node depends on a non-physical parameter in the form of an ill-defined interface thickness. As such, the alternative common node approach is considered to be a more general and a more elegant approach since it is based on the physical principle of head continuity along the surface-subsurface interface. In this study, however, it is argued that if properly implemented, then the dual node approach is actually the more general, the more elegant as well as the more accurate approach. This insight is obtained by considering that the topmost subsurface nodal values represent the mean values within discrete control volumes and by deriving the dual node approach from equations that govern infiltration and infiltrability. Both approaches are analyzed for cell-centered as well as vertex-centered finite difference schemes. It is shown that the dual node approach should be conceptualized and implemented as a simple one-sided first-order finite-difference to approximate the vertical subsurface hydraulic gradient at the land surface and that there is no need to assume a hydraulic separation between the two flow domains by a distinct interface. Whereas a consistent properly implemented, this results in a consistent dual node approach in which the coupling length is related to grid topology. In this coupling approach the coupling length is not to be interpreted as a non-physical model parameter. Although, this particular coupling approach is technically not new, the differences between this consistent dual node approach and the common node approach have not been studied in detail. In fact, this coupling scheme is often believed to be similar to the
common node approach. In this study it is illustrated that in comparison to the common node approach, the head continuity at the surface-subsurface interface is formulated more correctly in the consistent dual node approach. Numerical experiments indicate that the consistent dual node approach is in agreement with the physical principle of head continuity at the land surface, it is shown that the common node approach is not. Studies that have compared the two coupling approaches have been based on improperly implemented dual node approaches. As such, this study presents a re-evaluation of how the common node compares to the less sensitive to the vertical discretization when simulating excess infiltration. It is also found that the consistent dual node approach. Cell-centered as well as vertex-centered schemes are considered can be advantageous in terms of numerical efficiency.

1 Introduction

There exists a variety of hydrogeological problems, such as the hydrologic response of hill slopes and river catchments, which requires an integrated analysis of surface and subsurface flows. This has led to the development of physically-based, distributed parameter models for simulating coupled surface-subsurface flows. Well-known examples of such models include MODHMS [Panday and Huyakorn, 2004][Kollet and Maxwell, 2006; Panday and Huyakorn, 2004], InHM [Ebel et al., 2009], HydroGeoSphere [Therrien et al., 2010], CATHY
Typically, subsurface flow is governed by the Richards’ equation whereas surface flow is either governed by the kinematic wave or the diffusive wave equation.

The coupling between subsurface and surface flow may be either based on the common node approach [Kollet and Maxwell, 2006] or on the dual node approach [Ebel et al., 2009; Panday and Huyakorn, 2004; VanderKwaak, 1999]. In the common node approach coupling is formulated by a continuity in head between surface and subsurface nodes. The dual node approach is based on formulating an exchange flux between the surface and subsurface nodes. Typically, the dual node approach is conceptualized as a hydraulic separation of the surface and the subsurface by a saturated interface with a given thickness [Liggett et al., 2012]. The thickness of this interface defines a coupling length between the dual nodes to formulate the discrete exchange flux between the dual nodes.

It has been argued that the coupling length represents a non-physical model parameter, because there is often no evidence to support the existence of a distinct interface between the two flow domains [Kollet and Maxwell, 2006]. As such it appears that the common node approach is a more general coupling approach [Kollet and Maxwell, 2006]. Considering that smaller coupling lengths tend to improve the accuracy of the dual node approach, as such it appears that the common node approach is a more physically based coupling approach [Kollet and Maxwell, 2006; Liggett et al., 2012]. It has also been found that accurate simulations based on the dual node approach typically require a very small coupling length [Ebel et al., 2009; Liggett et al., 2012; Liggett et al., 2013]. It also seems that the common node approach is generally more accurate. Namely, in the limit as the coupling length goes to zero, the dual node approach mimics the common node
Since it is known that the dual node approach mimics the common node in the limit as the coupling length goes to zero [Ebel et al., 2009], it thus seems that the dual node approach is most accurate if it mimics the common node approach. Nonetheless, it has been argued that the dual node approach remains an attractive alternative coupling approach since it offers more flexibility than the common node approach. Namely, while it can mimic the common node approach, the dual node approach offers the possibility to simulate a less tight coupling of surface-subsurface flow which results in increased computational efficiency [Ebel et al., 2009]. It has been illustrated that both the dual node approach as well as the common node approach are sensitive to the vertical discretization near the surface [Liggett et al., 2012; Sulis et al., 2010].

In this study it is illustrated that if the dual node approach is properly implemented as well as properly conceptualized, then the dual node approach is actually the more general, more elegant as well as the more accurate approach. This is a significant finding particularly since this contradicts the findings of other studies in which the common node is commonly regarded as a more general and more elegant approach [Dawson, 2008; Kollet and Maxwell, 2006; Liggett et al., 2012; Liggett et al., 2013]. To arrive at a properly implemented or consistent dual node approach the dual node approach is derived from basic flow equations. Moreover, to develop and understand the consistent approach, it is crucial to realize that the topmost subsurface nodes should ideally represent values at the centroids of discrete control volumes. It is shown that the dual node approach should not be conceptualized as a distinct interface across which an exchange flux is computed. Instead the dual node approach should be interpreted as a one-sided finite difference approximation of the vertical hydraulic gradient at the land surface in which the coupling length is defined by the grid geometry. Moreover, whereas the consistent dual node
approach is in agreement with the principle of head continuity at the surface-subsurface interface, it can be shown that the common node approach is not.

In this study, both coupling approaches are considered for cell-centered as well as vertex-centered finite difference schemes. Theoretical considerations as well as This analysis starts with the crucial observation that that the topmost subsurface nodal values as computed by the finite difference schemes represent the mean values within the topmost discrete control volumes. Numerical experiments indicate that the dual node approach when properly implemented is often more accurate as well as more computationally efficient than the common node approach, particularly if the vertical discretization is relatively coarse. This is an important finding because using a relatively coarse vertical discretization is common practice in regional coupled surface-subsurface models [Jones et al., 2008; Kollet and Maxwell, 2008; Srivastava et al., 2014]. The numerical experiments to compare the coupling approaches are carried out with the model code DisCo [de Rooij et al., 2013]. It is shown that the dual node approach should be interpreted and implemented as a one-sided finite difference approximation of the vertical hydraulic gradient at the land surface. This yields a consistent dual node scheme in which the coupling length is defined by the half the thickness of the topmost subsurface cells. The scheme of An and Yu [An and Yu, 2014] as well as the scheme of Kumar et al. [Kumar et al., 2009] are essentially very similar to this consistent dual node scheme. In the work of Panday and Huyakorn [Panday and Huyakorn, 2004], one of the suggestions to define the coupling length is to use half the thickness of the topmost subsurface cells, which yields a consistent dual node scheme. While the idea that the coupling length can be based on the grid topology is not new [Panday and Huyakorn, 2004], the idea that it must be related to grid topology to obtain a consistent approach is a significant new insight. Namely, since the coupling length in the consistent dual node approach
is not to be interpreted as the thickness of a layer that separates the subsurface from the surface, the consistent dual node approach is not automatically less physically based than the common node. In fact, as explained in this study in comparison to the common node approach the implementation of a head continuity at the surface-subsurface interface is formulated more correctly in the consistent dual node approach.

The current consensus about how the dual node approach compares to the common node approach is based on alternative dual node approaches which as explained in this study are different from the consistent dual node approach. In this study the consistent dual node approach is compared in detail with the common node approach. It is shown that if the vertical discretization is sufficiently fine, then the common node approach and the consistent dual node approach are equally accurate. However, when simulating excess infiltration the consistent dual node approach is found to be less sensitive to the vertical discretization in comparison to the common node approach. This advantage in accuracy is related to the fact that head continuity is more correctly formulated in the consistent dual node approach. Moreover, it is also shown that the consistent dual node approach can be advantages in terms of numerical efficiency when simulating runoff due to both excess saturation as well as excess infiltration. The finding of this study show that the consistent dual node approach compares more positively with respect to the common node approach than other dual node approaches.

2 Interpretation of nodal values

As explained later on, a correct interpretation of nodal values is crucial for understanding the dual and common node approach for coupling surface-subsurface flow. Moreover, both coupling approaches depend on the configuration of surface and topmost subsurface nodes near the land
surface. This configuration depends on whether cell-centered or vertex-centered schemes are used.

In this study both type of schemes will be covered, but for simplicity only finite difference schemes are considered.

In both cell-centered as vertex-centered schemes the flow variables such as the heads and the saturation are computed on nodes. In vertex-centered schemes these nodes coincide with the vertices of mesh, whereas in cell-centered schemes the nodes coincide with the cell centers. When employing a finite difference scheme, nodal values correspond to the mean value within surrounding discrete control volumes. In cell-centered finite difference schemes these discrete volumes are defined by the primary grid cells. In vertex-centered finite difference schemes these discrete volumes are defined by the dual grid cells. Ideally, the mean values in the discrete control volumes are derived by applying the midpoint rule for numerical integration such that their approximation is second-order accurate. Therefore, the nodal values should ideally represent values at the centroid of the surrounding discrete control volume [Blazek, 2005; Moukalled et al., 2016]. In that regard, a cell-centered finite difference scheme is thus more accurate than a vertex-centered finite difference scheme. Namely, in cell-centered finite difference schemes the nodal values always correspond to the centroids of the cell whereas in vertex-centered finite difference schemes nodes and centroids (of the dual cells) do not coincide at model boundaries and in model regions where the primary grid is not uniform. It is well-known that this mismatch between nodes and centroids can lead to inaccuracies since the mean values within affected discrete volumes are not computed by a midpoint rule [Blazek, 2005; Moukalled et al., 2016].

Typically, vertex-centered schemes for simulating coupled surface-subsurface flow are based on mass-lumped finite element schemes [Liggett et al., 2012] and not on finite difference schemes. However, with respect to coupling surface-subsurface flow there is actually no difference
between a mass-lumped finite element scheme and a vertex-centered finite difference scheme. Simi-
lar as in vertex-centered finite difference schemes, the nodal values in mass-lumped finite ele-
ment schemes define the mean values inside dual grid cells [Zienkiewicz et al., 2005]. Moreover, the coupling approaches establish one-to-one relations between surface and topmost subsurface nodes which do not depend on whether a finite difference or a finite element approach is being used. Thus, a less complicated vertex-centered finite difference scheme may be used to provide insights in the coupling approaches as used in mass-lumped finite element schemes.

3 Common node approach

The common node approach defines a head continuity between the topmost subsurface nodes and the surface nodes. This continuity requires that the topmost subsurface nodes and the surface nodes are co-located at the land surface such that there exists a continuity in the elevation head. This requirement is automatically full-filled in vertex-centered schemes. Figure 1a illustrates the configuration of common nodes in ParFlow, a cell-centered scheme [personal communication Maxwell, R. in relation to previous work of the author [De Rooij et al., 2012]]. Figure 1c illustrates the configuration of common nodes for vertex-centered schemes. This configuration is similar to the configuration as used in HydroGeoSphere [Therrien et al., 2010]. However, in cell-centered schemes such as ParFlow the co-location of nodes is less straightforward. Also, the basic explanation that the pressure head continuity is assigned at the top cell of the subsurface domain at the boundary between the two domains [Kollet and Maxwell, 2006; Maxwell et al., 2009; Sulis et al., 2010] is ambiguous since the location of the land surface with respect to the top cell is not specified. Nonetheless, since ParFlow is a cell-centered scheme where the topmost subsurface node is located at the center of the top cell, it follows that the surface node is located at the center of the topmost subsurface cells as depicted in Figure 1a such that the land surface is located at the
center of the topmost subsurface cell. This is the correct configuration as applied in ParFlow [personal communication Maxwell, R. in relation to previous work of the author [De Rooij et al., 2012]]. It can be argued that the additional subsurface volumes that extent above the land surface do not drastically affect the timing of runoff. Namely, once the topmost subsurface node reaches fully saturated conditions, the amount of additional water that can be stored in those volumes is relatively small as long as the specific storage assigned to the topmost cell is relatively small.

Since the location of the land surface in ParFlow is somewhat unclear, some studies have inferred that ParFlow uses a completely different nodal configuration. For example, it has been inferred that the topmost subsurface nodes in the ParFlow model are placed on top of the topmost subsurface cell such that they are co-located with the surface nodes [Liggett et al., 2013]. An and Yu [An and Yu, 2014] infer that the surface and subsurface nodes are not co-located at all and the surface nodes are located at the top face of the topmost subsurface cells and that the topmost subsurface nodes are located at the center of the topmost subsurface cells.

Considering that nodal values represent ideally the mean values within discrete control volumes as described in Section 2, it can be argued that the head continuity as implemented in the common node approach is not in agreement with the physical principle of head continuity at the land surface. Namely, the common node approach enforces a continuity between surface heads at the land surface and the mean subsurface heads within the topmost subsurface discrete control volumes which have a finite thickness. This is different from enforcing a continuity between surface heads and subsurface heads within an infinitesimal thin subsurface layer directly below the land surface. As such inconsistent behavior is expected when using the common node approach.

To effectively remove this inconsistency a is only numerically correct if the topmost subsurface cells are very fine vertical discretization is required near the land surface.
4 Consistent Dual node implementation approach

4.1 Basics

Figure 1b and 1c illustrate the classical arrangement of surface and subsurface nodes in cell-centered and vertex-centered finite difference schemes, respectively. Commonly, the dual node approach is expressed in terms of an exchange flux $q_e$ [LT$^{-1}$] computed as [Liggett et al., 2012; Panday and Huyakorn, 2004]:

$$q_e = f_p \frac{K_z}{l} (h_s - h_{ss}) q_e = f_p \frac{K_z}{l} (h_s - h_{ss})$$

(1)

where $h_s$ and $h_{ss}$ are the hydraulic heads [L] associated with the surface node and the topmost subsurface node, respectively, $f_p$ [-] the fraction of the interface that is ponded and $l$ the coupling length [L]. The ponded fraction of the interface is typically defined by a function that varies smoothly between zero at the land surface elevation and unity at the rill storage height which defines the minimum water depth for initiating lateral overland flow [Panday and Huyakorn, 2004]. In equation (1) the term $f_p \frac{K_z}{l}$ is commonly referred to as the first-order exchange parameter, where first-order means that the exchange flux depends linearly of the hydraulic head difference.

Typically, equation (1) is not derived as a numerical approximation of basic flow equations that govern the exchange flux, but is merely presented a numerical technique to couple two different flow domains [Ebel et al., 2009; Liggett et al., 2012]. Subsequently, the dual node approach is conceptualized by interpreting equation (1) as an expression that describes groundwater flow across a distinct interface separating the two flow domains [Ebel et al., 2009;
Evidently, if the coupling length is assumed to be a non-physical parameter, then it follows that equation (1) cannot be derived from basic flow equations.

### 4.2 Consistent dual node approach

In the following, however, it is illustrated that the dual node approach can and should be derived from basic equations that describe infiltration into a porous medium. This derivation is inspired by but slightly different from the work of Morita and Yen [Morita and Yen, 2002].

Before deriving the dual node approach from equations that describe infiltration, it is worthwhile to point out that above formulation of an exchange flux implies that infiltration only occurs across the ponded fraction of the surface-subsurface interface. This is not correct, because rainfall typically results in infiltration across non-ponded areas. Although this issue is not a crucial problem since the ponded fraction will typically increase during rainfall, it is more elegant to account explicitly for infiltration across non-ponded areas. This is relatively straightforward since before ponding occurs the infiltration rate equals the rainfall rate if the rainfall rate is smaller than the infiltrability and is limited to the infiltrability otherwise [Hillel, 1982] and such a computation is also used by others [Morita and Yen, 2002]. In the approach presented here the surface cell can be partially ponded whereas in the work of Morita and Yen [Morita and Yen, 2002] a surface cell is either ponded or non-ponded.

Using Darcy’s Law, the infiltration rate at the ponded land surface \( q_{s \rightarrow ss} \) [LT\(^{-1}\)] can be written as a function of the vertical subsurface hydraulic gradient at the land surface:

\[
q_{s \rightarrow ss} = \left( k_z K_z \frac{\partial h}{\partial z} \right)_{z = z_i} = K_z \frac{\partial h}{\partial z} \bigg|_{z = z_i} = \left( k_z K_z \frac{\partial h}{\partial z} \right)_{z = z_i} = K_z \frac{\partial h}{\partial z} \bigg|_{z = z_i} \tag{2}
\]
where $h$ the hydraulic head [L], $z$ the elevation head [L], $k_r$ the relative hydraulic conductivity [-]

$K_z$ the saturated vertical hydraulic conductivity [LT^{-1}] and $z_s$ the elevation head at the land surface.

The relative hydraulic conductivity is unity because equation (2) applies to the ponded land surface which implies fully saturated conditions at the land surface (i.e. ponding means $p_s > 0$, where $p_s$ is the pressure head at the surface). Similarly, the infiltrability [LT^{-1}], defined as the infiltration rate under the condition of atmospheric pressure [Hillel, 1982], can be written as:

$$I = \left( k_z K_z \frac{\partial h}{\partial z} \right)_{z=z_s, p_s=0} = K_z \frac{\partial h}{\partial z} \bigg|_{z=z_s}$$

(3)

The relative hydraulic conductivity is again unity because the saturation equals unity under atmospheric conditions ($p_s = 0$). The infiltration rate at non-ponded land surface $q_{atm\rightarrow ss}$ [LT^{-1}] can be expressed as:

$$q_{atm\rightarrow ss} = \min \left( \max \left( I, 0 \right), q_R \right)$$

(4)

where $q_R$ is the effective rainfall rate (i.e. the infiltration rate is limited by either the infiltrability or the available effective rainfall rate). The total exchange flux across the surface-subsurface interface can now be written as:

$$q_e = f_p q_{s\rightarrow ss} + \left(1-f_p\right) q_{atm\rightarrow ss}$$

(5)

To approximate the vertical subsurface hydraulic gradient in equations (2) and (3), it is crucial to recognize that according to the principle of head continuity at the land surface, the surface hydraulic head at a surface node must also represent the subsurface head at the land surface at that location. Thus, the surface hydraulic head can be used as a Dirichlet boundary condition for the subsurface flow domain. Moreover, it is also crucial to recognize that since the subsurface hydraulic heads at the topmost subsurface nodes are ideally associated with
the centroids of the topmost subsurface discrete control volumes, these head values do not
represent values at the land surface but at some depth below the land surface. Because the
subsurface hydraulic heads at the dual nodes can be and should be associated with a different
elevation, the vertical subsurface head gradient between the dual nodes can be approximated by a
standard finite difference approximation. If this approximation is being used to approximate the
gradient at the land surface in equations (2) and (3), then this approximation is by
definition a one-sided first-order finite difference. Defining the coupling length by
\[ l = \Delta z - l = \Delta z_{dn} \]
where \( \Delta z - \Delta z_{dn} \) is the difference in the mean elevation head associated with the dual nodes, the
infiltration rate and infiltrability can thus be computed with the following one-sided finite
difference approximation:

\[
K_z \frac{\partial h}{\partial z} \bigg|_{z=z_s} \approx \frac{K_z}{l} (h_s - h_{ss}) \\
K_z \frac{\partial h}{\partial z} \bigg|_{z=z_i} \approx \frac{K_z}{l} (h_s - h_{ss})
\]

(6)

The above definition of the coupling length \( l = \Delta z - l = \Delta z_{dn} \) ensures a proper approximation of the
vertical gradient in elevation head at the land surface:

\[
\frac{\partial z}{\partial z} \bigg|_{z=z_s} = \frac{\Delta z}{l} = 1 \\
\frac{\partial z}{\partial z} \bigg|_{z=z_i} = \frac{\Delta z_{dn}}{l} = 1
\]

(7)

The above derivation of the consistent dual node approach from basic flow equations has
implications for how the dual node approach is conceptualized and how it should be implemented.
The idea that the coupling length must be directly related to the spatial discretization is an
important new insight. Namely, as the coupling length is related to grid topology, it does not
represent a non-physical parameter associated with a distinct interface separating the two domains.

It is also crucial to observe the difference between the consistent dual node approach and the common node approach regarding how the head continuity at the surface-subsurface interface is formulated. As explained in Section 2, the formulation in the common node approach is only correct if the topmost subsurface discrete volumes are very thin. In comparison, the formulation in the dual node approach is correct irrespective of the vertical discretization. Namely, irrespective of the vertical discretization the surface hydraulic heads equal the subsurface heads at the interface.

Since nodal values in cell-centered scheme are located at the centroids of the cells, the coupling length is simply given by \( l = z_s - z_{ss} \), where \( z_s \) and \( z_{ss} \) are the elevation heads \([L]\) associated with the surface node and the topmost subsurface node, respectively. This value has for the coupling length in cell-centered schemes has also been proposed suggested by others [Panday and Huyakorn, 2004]. However, in vertex-centered schemes the commonly used nodal configuration near the surface is such that \( z_s = z_{ss} \). If these elevation heads are used as the elevation heads at the dual nodes then \( \Delta z = z_s - z_{ss} = 0 \). Since the coupling length must be greater than zero, the coupling length cannot be defined as \( l = \Delta z \). Indeed, the coupling length in vertex-centered schemes is typically not related to grid structure [Liggett et al., 2013]. However, if \( \Delta z = 0 \) and the coupling length is some lumped parameter greater than zero, then the dual node approach is inconsistent. Namely, if \( \Delta z = 0 \) then the gradient in elevation head between the dual nodes equals zero. This may seem correct as the nodes are co-located. However, if \( z_{ss} = z_s \), then the physical principle of head continuity implies that \( p_{ss} = p_s \) must also hold. Moreover, however, in their work, the particular advantage of choosing this value (i.e. maintaining a unit gradient in elevation head) is not recognized. The coupling schemes as used by An and Yu [An
and Yu, 2014] and Kumar et al. [Kumar et al., 2009] are also in essence consistent dual node schemes. However, these schemes are not recognized as a dual node scheme. Instead, An and Yu [An and Yu, 2014] argue that their scheme is similar to the common node approach of Kollet and Maxwell [Kollet and Maxwell, 2006]. Kumar et al. [Kumar et al., 2009] argue that their scheme is similar to the dual node approach if the coupling length goes to zero which implies that their scheme would be similar to the common node approach. However, contrary to the common node approach the schemes of An and Yu [An and Yu, 2014] and Kumar et al. [Kumar et al., 2009] compute exchange fluxes between surface and topmost subsurface nodes and therefore these schemes are technically dual node schemes. As explained in this study, it is crucial to observe that the schemes of An and Yu [An and Yu, 2014] and Kumar et al. [Kumar et al., 2009] are actually quite different from the common node approach. As already mentioned, the consistent dual node scheme differs from the common node approach with respect to how the head continuity is formulated at the surface-subsurface interface. As discussed later on, this difference has crucial consequences in terms of accuracy as well as numerical efficiency.

In vertex-centered schemes the commonly used nodal configuration near the surface is such that \( z_s = z_{ss} \). However, even though the topmost subsurface node is located at the land surface in a vertex-centered scheme, the elevation head at this node should ideally correspond to the mean elevation head within the topmost subsurface discrete control volume such that \( z_{ss} < z_{ss, volume} \). This suggests that the topmost subsurface node should be moved to the centroid of the topmost subsurface discrete volume. Although this is a possible solution, the drawback of this solution is that the subsurface model ceases to be a purely vertex-centered scheme. Moreover, such an operation cannot be performed in finite element schemes since changing the nodal positions would redefine the geometry of the elements. Therefore, an alternative solution is proposed. To
enforce $l = z_s - z_{ss}$ without affecting the relative positions of nodes. Namely, in the subsurface grid, vertex-centered schemes the elevation of the surface nodes are changed according to

$$z_s = z_{ss} + l$$

where $l$ is equal to half the thickness of the topmost subsurface dual cell. This change does not affect the relative position of the nodes in the surface grid. The resulting nodal configuration is illustrated in Figure 1d. **When applying this solution, all the topmost subsurface cells must have the same thickness, such that the topography is increased with the same value everywhere.** In essence, the motivation behind this solution is that a more accurate approximation the hydraulic gradient (**i.e. enforcing a unit gradient in elevation head**) is more important than the actual elevation of the land surface. Indeed it can be argued that Similar to the change in land elevation will not drastically affect nodal configuration in ParFlow, the timing of runoff resulting nodal configuration may not seem ideal. Namely, once the topmost surface elevation does not coincide with the top of the subsurface node reaches fully saturated conditions, the amount of additional water needed to reach the elevated land surface is minor. Nonetheless, as long as the specific storage assigned to the topmost dual cell is relatively small.

It is crucial to observe that the proposed dual node implementation is not based illustrated later on assuming a distinct interface with a certain thickness between the subsurface and the surface. Instead, the coupling length is to be interpreted as a distance between dual nodes that accounts for the fact that the topmost subsurface nodal value ideally corresponds to a value below the land surface. This distance is related to the vertical discretization near the land surface and as such does not represent a non-physical parameter associated. Simulation results obtained with a distinct interface separating the two domains.

The common conceptualization of the dual node approach as a hydraulic separation by a interface with a given thickness [Kollet and Maxwell, 2006; Liggett et al., 2012; Liggett et al.,...
2013], may arise if dual node approach is interpreted as a second-order central finite difference
approximation evaluated at the centre of a saturated layer with a thickness equal to the coupling
length. If in addition the topmost subsurface head values are taken as values at the land surface,
then it follows that the dual node approach introduces a distinct interface between the two flow
domains. However, as explained the topmost subsurface head values should not be taken as values
at the land surface but as values at some distance from the land surface, such that the interface
defined by the coupling length occupies the upper half of the topmost subsurface discrete control
volumes.

It is also worthwhile to explain in further detail that the dual node approach does not
account for the relative hydraulic conductivity near the land surface. This does not imply that the
subsurface near the land surface is saturated. Namely, saturation in the topmost subsurface discrete
volume is computed with the pressure head at the topmost subsurface node which may well be
below zero. It may appear that the vertical hydraulic conductivity between the dual nodes should
be computed by weighting the vertical hydraulic conductivities at the dual nodes, which would
result in a dependency on the relative hydraulic conductivity as long as the topmost subsurface
node is not fully saturated. However, no weighting is needed if the dual node approach is
understood as a one-sided finite difference evaluated at the land surface. Namely, the vertical
hydraulic conductivity at the land surface is readily available. This is a difference with respect to
the approach of Morita and Yen [Morita and Yen, 2002] who do use a weighting scheme. Moreover, models typically apply upstream weighting to approximate the relative
hydraulic conductivities between nodes to avoid numerical instabilities [Forsyth and Kropinski,
1997]. Thus even if weighting is applied, then the dependency of the computations between the
dual nodes on the relative hydraulic conductivity will automatically disappear as the upstream node is always saturated—are reasonable.

To illustrate that the presented dual node approach exhibits consistent behaviour, the necessary conditions for ponding due to excess infiltration and exfiltration are considered. In general ponding starts when \( q_R > I \) [Hillel, 1982]. Setting \( q_R = I \), \( p_s = 0 \) and using \( h = p + z \), it follows from equation \( (6) \) and \( (7) \) that at the moment of ponding:

\[
p_{ss} = l \left( 1 \right) \left( \frac{q_R}{K_z} \right)
\]

\[
p_{ss} = l \left( 1 \right) \left( \frac{q_R}{K_z} \right)
\]

(8)

Ponding due to excess infiltration occurs if \( \frac{q_R}{K_z} > 1 \) and implies that saturation in the subsurface starts from the top down [Hillel, 1982]. Using \( \frac{q_R}{K_z} > 1 \) it follows from equation \( (8) \) that ponding due to excess infiltration occurs while \( p_{ss} < 0 \). This is reasonable since this value represents the pressure head at a certain depth below the land surface. Namely, if saturation occurs from the top-down then the saturation at a certain depth occurs later than saturation at the land surface. It is noted that if the ratio \( \frac{q_R}{K_z} \) is greater than but close to unity or if the coupling length is very small, then this condition becomes \( p_{ss} \approx 0 \). Once ponding starts the total flux rate between the dual nodes equals \( K_z \left( \left( p_s - p_{ss} \right) / l + 1 \right) \). Top-down saturation requires that this flux exceeds the vertical hydraulic conductivity. Reaching saturation at the topmost node \( (p_{ss} = 0) \) therefore requires \( p_s \geq 0 \). Thus, top-down saturation will occur after ponding is initiated. Ponding due to excess saturation occurs if \( \frac{q_R}{K_z} < 1 \) and implies that saturation in the subsurface starts
from the bottom up [Hillel, 1982]. Using $p_s = 0$, it follows from equation (8) that ponding due to excess saturation occurs while $0 < p_{ss} < l$. Thus ponding starts after reaching fully saturated conditions at the topmost subsurface node, which is again reasonable. Namely, the topmost subsurface node represents a value at a certain depth below the surface and thus bottom-up saturation implies that this node reaches saturation earlier than the surface. It is noted that if the ratio $q_R/K_z$ is smaller than but close to unity or if the coupling length is very small, then ponding occurs when $p_{ss} \approx 0$, $p_{ss} \approx 0$.

**4.14.3 Comparison to other dual node implementations**

To illustrate that it is crucial to account for the meaning of the values at the topmost subsurface nodes, it is instructive to consider what happens if these values are not taken as the mean values within discrete control volumes. As a first example, consider vertex-centered schemes where the dual nodes are defined such that $z_{ss} = z_s$ and $z_{ss} = z_s$ as illustrated in Figure 2c. As discussed in Section 41c. This is inconsistent because it defines a zero gradient in elevation head between the dual nodes. Nonetheless such schemes have been used in several models [Ebel et al., 2009; Liggett et al., 2012]. Since the vertical gradient in elevation head between the dual nodes is zero the total flux rate after ponding now equals $K_z(p_s - p_{ss})/l$. Top-down saturation requires that this flux exceeds the vertical hydraulic conductivity. Thus, reaching saturation at the topmost subsurface node $(p_{ss} = 0, p_{ss} = 0)$ requires $p_s > l$, $p_s > l$. Therefore, top-down saturation will not occur if runoff occurs and if the surface water depths remains smaller than the chosen coupling length. Indeed, it has been pointed out in other studies that the coupling length should be smaller than the rill storage height [Delfs et al., 2009; Liggett et al., 2012]. The zero vertical gradient in elevation head between the dual nodal also means that the required condition for ponding now
becomes \( p_{ss} = -\frac{q_r}{K} \). This implies that ponding due to excess saturation occurs while the topmost subsurface node is not yet saturated. This dual node approach has been compared to the common node approach in vertex-centered schemes [Liggett et al., 2012].

A second example is the dual node approach for cell-centered schemes as implemented in MODHMS which uses an adapted pressure-saturation relationship for the topmost subsurface nodes such that the topmost subsurface node only becomes fully saturated if hydraulic head at the node rises above the land surface [Liggett et al., 2013]. Since the topmost subsurface heads are associated with the cell centroid, this dual node scheme defines a unit gradient in elevation head at the land surface. However, the saturation value at the topmost node is associated with a location at the land surface and not with the centroid of a discrete control volume. This has undesirable consequences. Namely, saturating the topmost subsurface node \( (p_{ss} = l) \) due to excess infiltration requires that \( p_s > l \). Indeed, when simulating excess infiltration with MODHMS, a very small coupling length is needed to simulate top-down saturation due to excess infiltration. [Gaukroger and Werner, 2011; Liggett et al., 2013]. It can also be shown that ponding due to excess saturation occurs while \( 0 < p_{ss} < l \). But, because of the adapted pressure-saturation relationship this means that ponding starts while the topmost subsurface node is not yet saturated. Comparison of these results with the results for the consistent dual node implementation, it is clear that the adapted pressure-saturation relationship has undesirable consequences—been compared to the common node approach in cell-centered schemes [Liggett et al., 2013].

The above inconsistent implementations of the dual node approach have been used in several studies to compare the dual node approach with the common node approach. The two
comparison studies of Liggett et al. [Liggett et al., 2012; Liggett et al., 2013]. Such studies indicate that the dual node approach is typically only competitive with the common node approach in terms of accuracy once the coupling lengths are very small. However, the requirement for a very small coupling lengths, however, are a direct consequence of using inconsistent dual node approaches. Namely, if the topmost subsurface nodal values are not taken as the mean values within discrete volumes. In essence, by choosing a very small coupling lengths these inconsistencies are to some extent minimized. At best this minimization results in schemes that mimic the common node approach. However, as discussed, the common node approach is also inconsistent since it is not in agreement with the physical principle of a head continuity at the surface-subsurface interface. Since current views on how This contrasts with the consistent dual approach in which decreasing the coupling approaches compare are based on inconsistent dual node approaches, it is imperative to re-evaluate how the dual and common node approaches compare if the dual node approach is properly implemented.

Considering how the dual and the common node approach compare it is also crucial that the dual node approach is not to be conceptualized as a hydraulic separation between the flow domains in the form of a saturated interface. Namely, this conceptualization is often deemed a serious drawback of the dual node approach, since there is no evidence of such a distinct interface. Moreover, misconceptions about the coupling approaches can result in confusion. For example, in their paper An and Yu [An and Yu, 2014] reject the idea of using the dual node based on its classical conceptualization as a saturated interface and argue that their model is based on the approach proposed by Kollet and Maxwell [Kollet and Maxwell, 2006]. However, in their finite volume model the surface and subsurface nodes are not co-located. As such their coupling approach is, contrary to the claim of the authors, a dual node approach. This misunderstanding is probably also
related to aforementioned difficulties in inferring the nodal configuration as used in ParFlow. Nonetheless, their approach is actually a properly implemented dual node approach practically similar to the one proposed in this paper. Interestingly, the model of An and Yu [An and Yu, 2014] is less sensitive to the given vertical discretization near the land surface in comparison to ParFlow. However, since An and Yu were convinced that they followed the same coupling approach as ParFlow they hypothesized that the difference in performance was probably related to using irregular grids instead of orthogonal grids as in ParFlow [An and Yu, 2014]. However, if this difference is instead due to using a different coupling approach, then will result in more inaccurate simulation results as this would be an indication that a dual node approach is less sensitive to the vertical discretization near the land surface. This reinforces the idea that it is desirable to reconsider the comparison between the two coupling approaches numerically incorrect.

CATHY [Camporese et al., 2010] as well as the model of Morita and Yen [Morita and Yen, 2002] are examples of models which are neither based on the common node approach, nor a dual node approach. Both these models are conjunctive models in which the surface and subsurface flow are computed separately in a sequential fashion and in which coupling is established by matching the flow conditions along the surface-subsurface interface. A complete discussion is outside the scope of this paper, but it is worthwhile to mention that these models share some crucial characteristics with the consistent dual node approach. Although the two models are different, both models switch between appropriate boundary conditions along the surface-subsurface interface, such that infiltration fluxes are limited to the infiltrability. In both models the infiltration fluxes are computed while accounting for the unit vertical gradient in elevation head near the surface-subsurface interface. In addition, in both models ponding occurs when the infiltrability is exceeded.
5 Numerical experiments

5.1 Numerical model

To compare the consistent dual node approach with respect to the common node approach in terms of accuracy and computational efficiency numerical experiments are presented. These experiments are carried out with the model code DisCo which can simulate coupled surface-subsurface flow with the dual node approach using a fully implicit or monolithic scheme [de Rooij et al., 2013]. This means that subsurface flow is governed by the linearized Richards’ equation while surface flow is governed by the diffusive wave equation.

Starting from a dual node scheme, the implementation of a common node scheme is relatively straightforward. If the surface nodes are numbered last, a permutation vector can be constructed which gives the corresponding topmost subsurface node for each surface node. Then, the node numbering as used in the original dual node scheme can still be used to compute the surface and subsurface flow equations are terms. Subsequently, using the permutation vector the surface and subsurface flow terms associated with a common node can be combined into a single row of the global matrix system. In addition, when using the common node approach, there is no need to evaluate exchange flow terms between the two flow domains. It is noted that the surface flow and subsurface flow computations are exactly the same irrespective of the coupling approach. As such the model permits to compare the two approaches in terms of accuracy as well as numerical efficiency.

An adaptive error-controlled predictor-corrector one-step Newton scheme [Diersch and Perrochet, 1999] is used in which a single user-specified parameter controls the convergence as well the time stepping regime. It is assumed that by using the same error norms and the same model parameters that control the time-stepping, the simulations results as obtained by different
coupling approaches can be compared fairly in terms of accuracy and efficiency. Although, this scheme may not be necessary the most efficient scheme, it ensures that time discretization error is the same irrespective of the applied coupling approach. For brevity further details about the model are not discussed here and can be found elsewhere [de Rooij et al., 2013].

5.2 Hillslope scenarios

The model code is applied to a set of three hillslope scenarios. Table 1 lists the abbreviations used in the figures to distinguish between the coupling approaches, and to distinguish between cell-centered and vertex-centered schemes and to distinguish between models based on a. Each scenario is solved using different but uniform primary grid and grids that use a very thin primary top cell. The thickness of this top cell equals the thickness of the primary cells in the finest uniform grids. In models containing this thin layer of cells the vertical discretizations and $\Delta z$ specifies the discretization below the thin layer is based on the coarsest uniform grids. Further details about the discretizations are given in the figures.

The presented experiments focus mainly on the comparison between the consistent dual node approach and the common node approach. Inconsistent dual node implementations based on a zero hydraulic head gradient between the dual nodes are only considered for relatively coarse vertical discretizations to illustrate their short-comings vis-à-vis the consistent dual node approach. It is noted, that although these schemes are commonly used in vertex-centered schemes, for the purpose of this study they have also been implemented in the cell-centered schemes by using the nodal configuration depicted in Figure 1a. The scheme with an adapted pressure-saturation relationship is not considered.
5.1—Soil-column problems

These simulation scenarios consider infiltration into a vertical soil column and are inspired by scenarios as studied by Liggett et al. [Liggett et al., 2012; Liggett et al., 2013]. In the simulation scenarios rainfall is applied to a soil column with a height of 5 m. Initial conditions are defined by \( h = 0 \) m. The saturated conductivity is 1.0608 md\(^{-1}\). The porosity is 0.41 and the specific storage is \( 10^{-4} \) m\(^{-1}\). The van Genuchten parameters are given by \( s_r = 0.387, s_s = 1.0, \alpha = 7.5 \text{ m}^{-1} \) and \( n = 1.89 \).

For the first two scenarios a constant head boundary of \( h = 0 \) m is applied at the bottom of the column and the flux rate applied to the top of the soil column exceeds the saturated conductivity of the soil column, resulting in runoff due to excess infiltration. In the first scenario the applied flux rate is 1.1 md\(^{-1}\). Figure 2 and 3 illustrate the simulated runoff and the number of Newton steps for this scenario, respectively. Figure 4 illustrates the simulated runoff for the second scenario in which the flux rate is 10.608 md\(^{-1}\). It is noted that figure 4 does not display the results at later times when a steady-state is reached. However, to show the differences in results around the timing of ponding only a limited time period is displayed. Figure 5 illustrates the number of Newton steps for the second scenario. For the second scenario, Figure 6 compares the evolution in water depth between the common node approach and the dual node approach when using a relatively coarse vertical discretization and a cell-centered scheme.

To compare the different coupling approaches when simulating excess saturation, a third scenario is considered. The model setup is exactly the same as before, except that the effective rainfall rate is set to 0.5 md\(^{-1}\) and that the bottom boundary is changed into a no-flow boundary. The simulated runoff is depicted in Figure 7. Figure 8 shows the total number of Newton steps during the model runs. Figure 9 compares the evolution in water depth between the common node
approach and the dual node approach when using a relatively coarse vertical discretization and a cell-centered scheme.

5.2 Hillslope problems

In the following of the primary grid. The first two simulation scenarios consider hillslope problems as designed by Sulis et al. [Sulis et al., 2010]. For the purpose of this study, a third scenario is considered in which the initial and boundary conditions are different to create a flooding wave across an unsaturated hillslope. The problems consist of a land surface with a slope of 0.05 which is underlain by a porous medium. The domain is 400 m long and 80 m wide. The subsurface is 5 m thick. In the direction of the length and in the direction of the width the discretization is 80 m. Different vertical discretizations are considered. The van Genuchten parameters are given by $s_r = 0.2$, $s_s = 1.0$, $\alpha = 1 \text{ m}^{-1}$ and $n = 2$. The porosity is 0.4 and the specific storage is $10^{-4} \text{ m}^{-1}$. The manning's roughness coefficients are given by $3.3 \times 10^{-4} \text{ m}^{1/3} \text{ min}$. The surface flow domain has a zero-gradient outflow condition. For the first two simulation scenarios the domain is recharged with an effective rainfall rate of $3.3 \times 10^{-4} \text{ m/min}$ for a duration of 200 minutes and the initial water table depth is at a depth of 1.0 m below the land surface.

The first scenario considers excess infiltration and the saturated hydraulic conductivity equals $6.94 \times 10^{-6} \text{ m/min}$. Figure 10 and 11 show the simulated runoff and the number of Newton steps, respectively. For the second scenario which considers excess saturation, the saturated conductivity equals $6.94 \times 10^{-4} \text{ m/min}$. Figure 12 and 13 illustrate the simulated runoff and the number of Newton steps, respectively. Figure 4 and 5 illustrate the subsurface pressure heads at the topmost subsurface nodes and the water depths on the surface nodes. For the second scenario which considers excess infiltration the saturated hydraulic conductivity equals $6.94 \times 10^{-7} \text{ m/min}$. Figure 6 and 7 show the simulated runoff and the number of Newton steps, respectively.
and 9 illustrate the subsurface pressure heads at the topmost subsurface nodes and the water depths on the surface nodes for the finest and the coarsest vertical discretization, respectively. In the third scenario a surface water flood wave crossing the hillslope in the downhill direction is simulated by applying a Neumann boundary condition of 1.0 m$^3$/s for a duration of 200 minutes to the surface nodes with the highest elevation. The initial water table is located at a depth of 1.5 m. The vertical saturated hydraulic conductivity equals 6.94 x 10$^{-6}$ m/min. Figure 1410 illustrates the differences in simulated runoff and Figure 1511 illustrates the number of Newton steps of the model runs. Figure 16 compares and 13 illustrate the evolution in subsurface pressure heads at the topmost subsurface nodes and the water depths on the surface nodes as well as the time step sizes between the common node approach and for the dual node approach when using a relatively coarse and the coarsest vertical discretization and a cell-centered scheme, respectively.

**6 Discussion**

**6.1 Accuracy**

Considering the simulation of vertical flow through the unsaturated zone, a relatively fine vertical discretisation is needed to simulate sharp saturation fronts with the Richards’ equation [Pan and Wierenga, 1995; Ross, 1990]. A relatively fine vertical discretisation also implies that the common node approach will be in close agreement with the physical principle of head continuity along the surface-subsurface interface. Finally, if the vertical discretisation is relatively small then the coupling length for the consistent dual node approach is also small and this implies that the dual node approach mimics the common node approach. Therefore, it is expected that the coupling approaches will give similar and accurate results if the vertical discretization is sufficiently fine. Indeed, the simulations results indicate that a relatively fine and uniform vertical discretization
yields similar results for the common node approach as well as for the consistent dual node approach (Figure 2a, 4a, 5a, 7a, 10a, 12a and 14a). The simulation results based on the finest vertical discretization may thus be taken as reference solutions that enables a comparison of the coupling approaches when a coarser vertical discretization is used. This is an important issue, because using a relatively coarse vertical discretization is common practice in regional coupled surface-subsurface models [Jones et al., 2008; Kollet and Maxwell, 2008; Srivastava et al., 2014].

### 6.1.1 Excess saturation

As discussed by Ebel et al. [Ebel et al., 2009] and confirmed by others [Liggett et al., 2012] the dual node approach mimics the common node approach if the coupling length becomes sufficiently small. When comparing the consistent dual node approach and the common node approach a very similar observation applies. If the topmost subsurface cells are very thin, then the coupling length in the consistent dual node approach is very small. Also, if the topmost subsurface cells are sufficiently thin then the formulation of head continuity at the surface-subsurface interface in the common node approach is correct. Thus, the common node approach will mimic the consistent dual node approach. Indeed, the simulations results indicate that a relatively fine vertical discretization yields similar results for the common node approach as well as for the consistent dual node approach (Figure 2a, 4a, 6a, 8a, 10a and 12a).

A relatively fine uniform vertical discretisation also enables to simulate sharp saturation fronts with the Richards’ equation [Pan and Wierenga, 1995; Ross, 1990]. As such the simulation results based on the finest vertical discretization can be taken as reference solutions that enables comparisons of the coupling approaches when a coarser vertical discretization is used.
6.1.1 Excess saturation

The simulation results of runoff due to excess saturation as obtained by the common node approach and the consistent dual node approach as depicted in Figure 2 illustrate that simulating excess saturation runoff is not significantly affected by the vertical discretization (Figure 7 and 12). This is because the time needed to reach fully saturated conditions in the subsurface is a simple function of the flow boundary conditions and the initial water content. It is thus expected that the vertical discretization does not significantly affect the simulation of excess saturation. Although the vertical discretization may affect the computed initial water content, this effect is usually negligible. It has been found in other studies that the vertical discretization has little effect on simulated runoff due to excess saturation [Sulis et al., 2010].

As described in Section 4, when using the consistent dual node approach, ponding due to excess saturation occurs when \( 0 < \frac{p_{ss}}{l} < 1 \). Thus at the moment of ponding the hydraulic head at the topmost subsurface node is generally below the land surface. When using the common node approach, the hydraulic head at the topmost subsurface node is at the land surface at the moment of ponding. However, if the specific storage is relatively small, then the timing of runoff will be similar for both coupling approaches. Both approaches are thus expected to yield similar and reasonably accurate results even when the vertical discretization is relatively coarse. Indeed, the simulation results indicate that there is little difference between the common node approach and the consistent dual node approach (Figure 7 and 12).

As indicated in Figure 7d, when using an inconsistent dual node approach, the timing of runoff may be underestimated unless a very small coupling length is being used. As discussed in section 5 this is expected.
6.1.2 Excess infiltration

When simulating excess infiltration the common node approach requires fully saturated conditions at the topmost subsurface node for ponding to occur. This is a direct consequence of the head continuity between the surface nodes and the topmost subsurface nodes. However, top-down saturation associated with excess infiltration implies that reaching fully saturated conditions in the topmost subsurface discrete volumes should requires more time than reaching fully saturated conditions in the very nearland surface, especially if the vertical discretization is relatively coarse. It is thus expected that the common node approach delays runoff and that this delay increases for a coarser vertical discretization. In addition, if the saturation fronts are less sharp due to a relatively coarse vertical discretization, it takes more time to reach saturated conditions at the common node. This will further delay runoff. Indeed, the simulation results indicate clearly that runoff is delayed when using the common node approach, particularly if the vertical discretization is relatively coarse (Figure 2, 46, 9a, 10 and 1413a). It has also been found in other studies that the common node approach delays runoff due to excess infiltration if the vertical discretization is relatively coarse [Sulis et al., 2010]. The overestimation of the infiltration associated with the delay in runoff may result in runoff due to excess saturation even if the applied flux rate should result in runoff due to excess infiltration. This is illustrated in Figure 10c for the model run based on a cell-centered scheme and the common node approach. This Figure illustrates that overestimating the infiltration can yield a distinctive higher peak in runoff. Comparing this peak with the runoff responses in Figure 12, it is clear that this model run simulates runoff due to excess saturation.

In comparison, as explained in Section 4.2, when using the consistent dual node approach, ponding due to excess infiltration displays more desirable behaviour. Namely, as explained in Section 4 approach, ponding due to excess
infiltration occurs before reaching fully saturated conditions at the topmost subsurface node—which is arguably more correct if saturation occurs from the top-down, particularly if the vertical discretization is relatively coarse. When using the consistent dual node approach, the moment of ponding depends on the computation of the infiltrability. More specifically, ponding occurs when the infiltrability is exceeded. Compared to the condition for ponding in the common node approach this is arguably more correct. Namely, if saturation occurs from the top-down then the saturation at a certain depth occurs later than saturation at the land surface. Indeed, simulation results indicate that when simulating excess infiltration the consistent dual node approach is less sensitive to the vertical discretization in comparison to the common node approach. This is clearly indicated in Figure 6b-d, 9a, 10b-d and 13a. To further explain this difference in accuracy, it is emphasized that the spatial resolution only affects the accuracy of the flow computations when using the consistent dual node approach and that the formulation of head continuity at the interface remains correct. In contrast, when using the common node approach, if the spatial resolution is too coarse then this does not only affect the accuracy of the flow computations but in addition the formulation of head continuity becomes incorrect. It must be emphasized, however, that regardless of the applied coupling approach, the vertical discretization must be relatively fine. As indicated by Figure 6b-d, 9a, 10b-d and 13a the difference between the simulated results and the reference solution increase for a coarser discretization. Eventually such differences will lead to unreasonable results regardless of the coupling approach.

It is interesting to note that An and Yu [An and Yu, 2014] also found that their model was less sensitive to the vertical discretization in comparison to ParFlow when simulating runoff due to excess infiltration. Whereas An and Yu [An and Yu, 2014] hypothesized that this difference in performance was related to using irregular grids instead of orthogonal grids as in ParFlow, it is
argued here that this difference can be explained by the fact that both models use a different coupling approach.

Although the consistent dual node approach is less sensitive to the vertical discretization in comparison to the common node approach, it is useful to explain in detail how the vertical discretization affects the accuracy of the consistent dual node approach to the vertical discretization. A relatively coarse vertical discretization may result in an underestimation of the vertical pressure gradient at the land surface. This is because in a soil close to hydrostatic conditions the pressure heads increase with depth. Therefore, the infiltrability during the early stages of infiltration may be underestimated. If the applied flux rate is sufficiently large such that the underestimated infiltrability is exceeded, then this underestimation will result in an underestimation of the timing of runoff. It may be observed from equation (8) that if the ratio $q_r/K_z$ or the coupling length is sufficiently large, then ponding is initiated immediately overestimated. Figure 10c and 14c illustrate that the timing of runoff can indeed be underestimated due to a relatively coarse vertical discretization when using the consistent dual node approach. However, is indeed overestimated at early times. During the later stages of infiltration the pressure head at the topmost subsurface node will be underestimated due to the combined effect of an underestimated infiltration rate and the overly diffused saturation fronts. This results in an overestimation of the infiltration rate in the later stages. Thus at some time after ponding has started, it is expected that the amount of runoff is underestimated. Contrary to the common node approach, however, there will be a time at which runoff is simulated correctly (Figure 10c and 14c).

If the applied flux rate is not sufficiently large, then the underestimated infiltrability in the early stages of infiltration will not be exceeded. In that case, the overly diffused
saturation fronts resulting from a relatively coarse vertical discretization will eventually lead to an underestimation of pressure head at the topmost subsurface node and as such the infiltrability may be overestimated at later times. Consequently, when using the consistent dual node approach the timing of runoff due to excess infiltration may also be underestimated. As discussed in section 4 if the ratio \( q_R/K_z \) goes to unity, then the consistent dual node approach behaves practically similar to the common node approach. Indeed, Figure 2b which depicts a simulation with a relatively small ratio \( q_R/K_z \) clearly illustrates that the timing of runoff may be underestimated when using the consistent dual node approach. Runoff due to excess infiltration may be delayed. However, the delay in runoff as simulated by the consistent dual node approach will only equal the delay in runoff as simulated by the common node approach in the limit when \( q_R/K_z \) goes to unity. Namely, as explained in Section 4.2 if \( q_R/K_z \) goes to unity, then the consistent dual node approach behaves similar as a common node approach. However, in general, if the consistent dual node approach delays runoff, this delay will be smaller than the delay in runoff as simulated by the common node approach (Figure 2b). Overall, regardless if the consistent dual node approach underestimates or overestimates the timing of runoff, the simulation results indicate that the consistent dual node approach is generally less inaccurate than the common node approach for simulating excess infiltration when using a relatively coarse uniform vertical discretization.

Comparing Figure 12a and 13a it can be observed that if the vertical discretization is relatively coarse then a common node can act as an artificial barrier for a surface water wave advancing across an initially unsaturated subsurface domain. Namely, as the wave travels downstream the wave can only advance to the next common node once it is fully saturated. The effect of this artificial barrier is that the front of the surface water wave is steepened. In contrast,
the consistent dual approach simulates a wave that becomes less steep as it advances downstream for relatively fine as well as relatively coarse vertical discretizations as depicted in Figure 13a.

As illustrated in Figure 2b, 4b, 6b-d, and 10b and 14b-d, if the coupling approach and the vertical discretization are identical and if the thin layer is absent, then the vertex-centered schemes are more accurate, closer to the reference solution with respect to the cell-centered schemes. This difference in accuracy results solely from the fact the primary mesh is the same for both schemes. As such the vertical extent of the topmost subsurface volumes is twice as small when using the vertex-centered scheme. This difference in vertical grid resolution near the land surface explains the differences in accuracy between the schemes.

When using a thin layer at the top of the model the common node approach and consistent dual node approach provide similar simulation results as shown in Figure 2c, 4c, 10d and 14d. This is expected, because the thin layer implies a small coupling length and as such the consistent dual node approach mimics the common node approach. In essence, in schemes using the consistent dual node approach the thin layer establishes a near head continuity between the dual nodes. If the simulation results are compared to the models based on the coarsest uniform discretization (Figure 2b, 4b, 10c and 14c), it is observed that adding a thin layer has only a positive effect on the cell-centered schemes based on the common node approach. This positive effect is explained by the fact that due to the thin layer the common node approach is in almost full agreement with the principle of head continuity at the land surface. Vis-à-vis the corresponding model without a thin layer, the thin layer has a negligible effect on the cell-centered scheme based on the consistent dual node approach. This is because the thin layer establishes a head continuity between the dual nodes and the topmost subsurface node and the adjacent subsurface node below act like the dual nodes in the model without the thin layer. The thin layer has also a negligible effect on the vertex-centered
scheme based on the common approach. In this case the thin layer establishes a near head continuity between the topmost subsurface node and the adjacent node below and ponding due to excess infiltration will require almost fully saturated conditions in the two topmost subsurface volumes. The sum of these two volumes is equal to the topmost volume in the model without the thin layer and therefore the effect of the thin layer is minimal. In a vertex-centered scheme based on the consistent dual node approach, the thin layer has a clear negative effect. In essence the head continuity between the dual nodes removes the benefits of using the consistent dual node approach and contrary to the cell-centered scheme based on the consistent dual node approach the topmost subsurface node and the adjacent subsurface node below do not act like the dual nodes in the model without the thin layer. This is because the thin layer creates a non-uniform primary mesh in which the subsurface node directly below the topmost subsurface node is not located at the centroid of its associated dual cell.

As indicated in figure 2d and 4d, when using an inconsistent dual node approach, the runoff is overestimated unless a very small coupling length is being used. As discussed in section 5, this is expected.

6.2 Computational efficiency

During the early stages of ponding the rates at which the water depths are changing can be relatively fast as the applied flux rates on the land surface are possibly quite large. Typically, a numerical model with adaptive time-stepping will decrease the time step size at the moment of ponding to handle the non-linear flow terms and the high rates of change in water depth. Since a higher infiltration rate at the moment of ponding results in lower initial rates of change in water depth, it is expected that the most efficient coupling approach is characterized by a higher infiltration rate at the moment of ponding.
The computational efficiency of the schemes is measured in terms of the number of Newton steps. The number of Newton steps equals the number of times that the linearized system of equations is solved and this number depends on the time step sizes as well as the number of failed Newton steps. It is emphasized that the measured efficiency depends crucially on the applied model code. Nonetheless, as shown in the following, the measured differences in efficiencies can be explained in terms of abrupt changes in how fast pressure heads near the surface-subsurface interface are evolving with time. Regardless of the type of scheme used to solve the non-linear flow equations, such abrupt changes are difficult to solve.

Once ponding occurs a surface-subsurface flow model will encounter significant numerical difficulties as surface flow terms are activated. In essence, the activation of these terms represents a discontinuity in flow behaviour which is challenging to resolve [Osei-Kuffuor et al., 2014]. Indeed, the Newton steps as depicted in Figure 3 and 7 indicate that simulations encounter difficulties at the moment of ponding. These figures also indicate that the consistent dual node approach can be more efficient in comparison to the common node approach.

### 6.2.1 Excess saturation

When simulating excess saturation the subsurface is fully pressurized at just before the moment of ponding due to excess saturation, the rate of change in pressure heads at the topmost subsurface nodes is relatively high for both coupling approaches. This high rate is related to the shape of the water retention curve. Typically, the derivative of the saturation with respect to the pressure head goes to zero when approaching fully saturated conditions. Once ponding starts, the surface flow terms are activated and can only accommodate therefore the rate of changes in pressure heads at the topmost subsurface nodes decreases drastically. Both approaches must handle this drastic
change. However, from Figure 4b and 5b it can be observed that the rate of change decreases more
abruptly when using the common node approach.

When using the common node approach the vertical hydraulic gradients in the subsurface
are close to zero at the moment of ponding, since additional water volumes can only be
accommodated by means of the specific storage. As such the column will be close to hydrostatic
conditions at the moment of ponding. When using the common node approach this implies
that the hydraulic gradient between the common node and the adjacent subsurface node below is
very close to zero. When using the consistent infiltration rate drops instantaneously at the moment
of ponding. In contrast, in the dual node approach ponding due to excess saturation occurs when
0 < p_{so} < l, the infiltrability is exceeded. Thus, at the moment of ponding, the infiltration rate
is higher in comparison to the common node approach. After ponding this infiltration rate will
decrease quickly as the hydraulic heads at the dual nodes equilibrate. This difference in the
infiltration rate at the moment of ponding explains why the topmost subsurface hydraulic heads
change more smoothly when using the dual node approach. If the vertical discretization is
generally still below the land surface. This means that the infiltration rate at the
moment of ponding as computed by the consistent dual node approach is higher in comparison to
the rate as computed by the common node approach. It is thus expected that the consistent
dual node approach is more efficient when simulating excess saturation. Indeed, Figure 8 and 13
illustrate that, when simulating excess saturation, the consistent dual node approach is more
efficient than the even higher rate initial rate of change in water
depth as depicted in Figure 5a.

The more abrupt changes in pressure heads at the common node in comparison to the
changes in pressure heads at the dual nodes mean that solving the activation of ponding with the
common node approach. Figure 10 illustrates the pressure heads on the nodes near the land surface as is more difficult. It is noted that the differences in the infiltration rates between the two coupling approaches only occur at the moment of ponding and directly thereafter when water depths are relatively small. Namely, quickly after ponding, the hydraulic heads at the dual nodes will equilibrate and after that the two coupling approaches will behave similar. This explains why these differences in infiltration rates do not significantly affect the accuracy of simulated by the models based on the cell-centered scheme and the coarsest vertical discretization. As illustrated, the pressure head gradient governing the infiltration rate at the moment of ponding is larger when using the consistent dual node approach and consequently the rate of change in water depth is smaller—runoff.

6.2.2 Excess infiltration

As discussed in section 7.1.2, in comparison to the consistent dual node approach, the common node approach yields a later time of ponding due to excess infiltration. Since saturation fronts in a homogeneous medium become more diffused with time, it follows that the common node approach yields a smaller infiltration rate at the moment of ponding. Namely, if the saturation fronts are more diffused, then the pressure head gradient governing the infiltration rate is less sharp. Therefore, it is expected that the common node approach is computationally less efficient than the consistent dual node approach, particularly if ponding is significantly delayed. Figure 5 illustrates clearly, that the consistent dual node approach can be more computationally efficient. For the simulation scenario depicted in Figure 5, the consistent dual node approach is also more accurate. Figure 4b illustrates that, compared to the consistent dual node approach, the common node approach can result in a relatively high rate of change in runoff at the moment of ponding. This is indicative of a relatively high initial rate of change in water depth at the moment of ponding. Figure
7 illustrates the pressure heads at the nodes near the land surface as simulated by the cell-centered schemes based on the coarsest vertical discretization. It can be observed that the pressure head gradient at the moment of ponding is larger when using the consistent dual node approach. This implies a higher infiltration rate and a lower rate of change in water depth. Figure 11 also illustrates that the consistent dual node approach is more efficient when handling the activation of ponding. However, considering the entire simulation period, the dual node approach is not always more efficient. As illustrated by Figure 11b and 11c, when the discretization is relatively coarse the common node approach is sometimes more efficient during the later stages of the simulation. However, in these cases the common node approach is only more efficient, because its inaccuracy leads to an easier flow problem to be solved. Namely, the underestimation of runoff results in more diffused saturation fronts in the subsurface.

Figure 2 shows that if the ratio $q_R/K_z$ is relatively small, then the differences in computational efficiency are relatively small. As discussed in section 4 this is because the consistent dual node approach behaves very similar to the common node approach if the ratio $q_R/K_z$ is relatively small.

Another factor that affects the efficiency of the common node approach is that the delay in ponding can act as an artificial barrier for a surface water wave advancing across an initially unsaturated subsurface domain. The effect of this artificial barrier is that the front of the surface water wave is steepened. This steepening of the surface wave front results in higher rates at which the water depth is changing and is undesirable because it decreases the computational efficiency. This is clearly illustrated in Figure 15. Figure 16 illustrates the evolution of water depth at the land surface for the cell-centered schemes using the coarsest vertical discretization. As shown, the common node approach delays and steepens the surface water front. This results in relatively high
rates of change in water depth at the moment of ponding. Consequently, the common node approach is less efficient than the dual node approach. It is noted that for this scenario the consistent dual node approach is more efficient as well as more accurate.

Figure 8, 9, 12 and 13 illustrate the evolution of pressure heads at dual nodes and common nodes when simulating excess infiltration. When applying the consistent dual approach, the net flux into a topmost subsurface cell will decrease once ponding occurs, because the applied flux rate will be partitioned between dual nodes (i.e. between the surface flow and subsurface flow domain). This occurs while the topmost subsurface node is not yet fully saturated. After ponding the infiltration rate decreases such that if the topmost subsurface node reaches fully saturated conditions the net flux into the topmost subsurface node is relatively small. In contrast, partitioning of the applied flux rate on a common node between the surface flow and subsurface domain starts when the common node reaches fully saturated conditions at this node. This means that just before ponding the rate of change in pressure head is relatively high as the common node is driven towards fully saturated conditions while the infiltration rate is relatively high. This means that similar to the excess saturation scenario the rate of change in pressure head at the common node is high just before ponding. At the moment of ponding, this rate must drop abruptly as surface flow terms are activated. This abrupt change explains why the common node approach is less efficient.

Figures 7 and 11 also indicate that a coarser vertical discretization only provides a significant gain in efficiency in terms of Newton steps when using the consistent dual node approach. When using the common node approach, a coarser discretization does not change the fact that the topmost subsurface node must reach fully saturated conditions for ponding to occur and that the infiltration rate is relatively high just before ponding. When using the consistent dual
node approach, a coarser vertical discretization means that the saturation fronts are more diffused such that the flow problem becomes easier to solve.

Figure 8a and 9a illustrate that for the second simulation scenario, ponding occurs almost simultaneously at all the surface nodes. Figure 12a and 13a show that this is different for the third scenario where ponding occurs at different times as the flooding wave travels downstream. When Figure 11a is compared with Figure 12a and when Figure 11d is compared with Figure 13a, it is clear that the common node approach encounters difficulties around each time ponding starts at a surface node. Figure 11 shows that these difficulties are encountered for all discretizations. In contrast the consistent dual node approach has much less difficulties solving these difficulties. As discussed in Section 6.1.2. the common node approach may result in steepening the advancing wave. This implies that water depths will be changing more quickly. This presents an additional difficulty for solving this flow problem with the common node approach.

7 Conclusions

In this study it is shown that contrary to the common held view, the dual node approach if properly implemented is actually the more general, the more elegant as well as the more accurate coupling approach in comparison to the common node approach. This consistent dual node approach is implemented in cell-centered as well as vertex-centered finite difference schemes. The consistent dual node approach is derived from basic equations that govern infiltration and infiltrability at the land surface using the dual node approach should be conceptualized and implemented as a one-sided finite differences approximation of the vertical hydraulic gradient at the land surface. In both cell-centered as vertex-centered schemes this provides an important new insight into the coupling length. Namely, if the dual node approach is properly implemented then
The coupling length is related to the grid geometry. As discussed, the dual node approach should not be conceptualized as a distinct interface between the surface and the subsurface. Moreover, this approach is in agreement with principle of head continuity along the land surface whereas the common node approach is not, unless the vertical discretization is grid resolution. Thus, the coupling length does not represent an additional non-physical model parameter and therefore the dual node approach is not automatically a less physically based approach in comparison to the common node approach. Actually, this study shows if the vertical discretization is not sufficiently fine then the head continuity at the surface-subsurface interface is formulated more correctly in the consistent dual node scheme. This difference in formulation has consequences for how both approaches compare in terms of accuracy and efficiency.

Numerical experiments indicate that if the vertical discretization is relatively coarse, then the consistent dual node approach is often less inaccurate as well as more computationally efficient—equally accurate or more accurate than the common node approach. It has been shown that in comparison to the common node approach for the consistent dual node approach is less sensitive to the vertical discretization when simulating excess infiltration. For simulating excess saturation both coupling approaches are more or less equally accurate, but the consistent dual node approach was found to be more computationally efficient. Therefore, overall it can be argued that the consistent dual node approach is to be preferred to the common node approach unless the practical advantage of the consistent dual node approach in terms of accuracy is limited. Namely, if the vertical discretization is sufficiently fine—such that refined, both approaches will converge to more accurate and eventually similar results when simulating excess infiltration. When simulating excess saturation both approaches yield similar results even if the vertical discretization is relatively coarse.
Nonetheless, even though the advantage of the consistent dual node approach in terms of accuracy is limited, the fact that the consistent dual node approach is equally or more accurate than the common node approach is a significant finding. Namely, this finding is different from the commonly held view that a dual node approach is most accurate if it mimics the common node approach. Moreover, it also illustrates clearly that the consistent dual node approach is not similar to a common node approach.

Numerical experiment indicate that the consistent dual node approach can be more efficient than the common node approach while being equally or more accurate than the common node approach. It has been shown that this difference in efficiency is related to abrupt changes in the evolution of pressure heads around the moment that ponding is initiated.

Based on the findings in this study the models of An and Yu [An and Yu, 2014] and Kumar et al. [Kumar et al., 2009] are expected to have some advantages with respect to models that are based on the common node approach. This is because these models are based on a consistent dual node approach. Moreover, given a model that uses an alternative dual node approach, it is relatively straightforward to implement the numerically more correct consistent dual node approach.

Acknowledgements
This research was funded by the Carl. S. Swisher Foundation.

References


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Therrien, R., R. G. McLaren, E. A. Sudicky, and S. M. Panday (2010), HydroGeoSphere—a three-dimensional numerical model describing fully-integrated subsurface and surface flow and solute transport (draft), Groundwater Simulations Group, University of Waterloo.


Table 1: Abbreviations as used in the figures.

<table>
<thead>
<tr>
<th>abbreviation</th>
<th>meaning</th>
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<tbody>
<tr>
<td>cc</td>
<td>cell-centered</td>
</tr>
<tr>
<td>vc</td>
<td>vertex-centered</td>
</tr>
<tr>
<td>dn</td>
<td>dual node</td>
</tr>
<tr>
<td>cn</td>
<td>common node</td>
</tr>
<tr>
<td>TL</td>
<td>tiny-layer</td>
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Figure 1: a) Common nodes and co-located dual nodes in cell-centered schemes. bb) Dual nodes in cell-centered-centered schemes. c) Common nodes and co-located dual nodes in vertex-centered schemes. c) Dual nodes in cell-centered-centered schemes (not co-located). d) Dual nodes in vertex-centered schemes (not co-located). The white squares and white circles represent surface and subsurface nodes, respectively. The solid and dashed lines represent the primary mesh and the dual mesh, respectively. The grey-shaded area is a topmost discrete volume as associated with a topmost subsurface node. The black dot represents the centroid of this volume. The coupling length \( l \) as depicted in this figure applies to the consistent dual node approach.
Figure 2: Simulated runoff outflow response for excess infiltration-insaturation on a vertical soil column hillslope (first scenario) using different vertical discretizations ($q_R = 1.1 \text{ m d}^{-1}$).
Figure 3: Number of Newton steps for excess infiltration in a vertical soil column saturation on a hillslope (first scenario) using different vertical discretizations ($q_R = 1.1 \, \text{md}^{-1}$).
Figure 4: Simulated runoff values at the common nodes for excess saturation on a hillslope (first scenario) with a cell-centered scheme and Δz = 0.0125 m. a) Water depths. b) Pressure heads. Nodes are numbered 1-5 in the down-slope direction.
Figure 5: Simulated values for excess saturation on a hillslope (first scenario) with a cell-centered scheme and Δz = 0.2 m. a) Water depths at the surface nodes. b) Pressure heads at the topmost subsurface nodes. Nodes are numbered 1-5 in the down-slope direction.
Figure 6: Outflow response for excess infiltration in a vertical soil column on a hillslope (second scenario) using different vertical discretizations ($\theta_R = 10.608 \text{ md}^{-1}$).
\[ \Delta z = 0.05 \text{ m} \]

\[ \Delta z = 0.05 \text{ m} \]

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\[ \Delta z = 0.05 \text{ m} \]
Figure 5.7: The total number of Newton steps for excess infiltration in (second scenario) on a vertical soil column on hillslope using different vertical discretizations \((q_R = 10.608 \text{ md}^{-1})\).
Figure 6: Changes in pressure heads near the surface-subsurface interface:
Simulated values at the common nodes for excess infiltration on a vertical soil column ($q_R = 10.608 \text{ md}^{-1}$). Left: dn(cc) slope (second scenario) with a cell-centered scheme and $\Delta z = 0.5125 \text{ m}$. Right: cn(cc) $\Delta z = 0.0125 \text{ m}$. Water depths. b) Pressure heads. Nodes are numbered 1-5 in the down-slope direction.
Figure 7: Simulated runoff for excess saturation in a vertical soil column.

Figure 9: Simulated values for excess infiltration on a hillslope with a cell-centered scheme (second scenario) and $\Delta z = 0.2$ m. a) Water depths at the surface nodes, b) Pressure heads at the topmost subsurface nodes. Nodes are numbered 1-5 in the down-slope direction.
Figure 10: Outflow response for flooding an unsaturated hillslope using different vertical discretizations.
Figure 8: The total number of Newton steps for excess saturation in a vertical soil column flooding an unsaturated hillslope using different vertical discretizations.
Figure 9: Changes in pressure heads near the surface–subsurface interface for excess saturation in a vertical soil column. Left: $dn(cc) \Delta z = 0.5 \text{ m}$. Right: $cn(cc) \Delta z = 0.5 \text{ m}$.
Figure 10: Outflow response for excess infiltration (third scenario) on a hillslope with a cell-centered scheme and $\Delta z = 0.0125$ m. a) Water depths at the surface nodes. b) Pressure heads at the topmost subsurface nodes. Nodes are numbered 1-5 in the downslope direction.
Figure 11: The total number of Newton steps for excess infiltration on a hill slope using different vertical discretizations.
Figure 12: Outflow response for excess saturation (third scenario) on a hill slope using different vertical discretizations.
Figure 13: Number of Newton steps for excess saturation on hillslope with a hill slope using different vertical discretizations.
Figure 14: Outflow response for flooding an unsaturated hill slope using different vertical discretizations.
Figure 15: Number of Newton steps for flooding an unsaturated hill slope using different vertical discretizations.
Figure 16: Response in water depth at the fivecell-centered scheme and $\Delta z = 0.2$ m. a) Water depths at the surface nodes (numbered from upstream to downstream) for flooding an unsaturated hill-slope. Left: $dn(ce) \Delta z = 0.5$ m. Right: $cn(ce) \Delta z = 0.5$ m. b) Pressure heads at the topmost subsurface nodes. Nodes are numbered 1-5 in the down-slope direction.)