Response to comments of Anonymous Referee

We would like to thank Dr. Mylevaganam for the thoughtful comments and suggestions. The follows are our response in regard to the comments and suggestions.

Based on this review, the following comments are made:

1. As per the current version of the paper, the sentences in the abstract are scattered. From the reader's point of view, the abstract would be more concrete if the authors streamline the sentences to underscore the research carried out in this paper.

Response

As suggested, the abstract has been rewritten as

"This study proposes a stochastic framework for a lumped rainfall-runoff problem at a catchment scale under the assumption of a linear relationship between the runoff discharge and the catchment storage. Both rainfall and discharge are treated as random fields. An autoregressive model is adopted to account for the temporal variability of the rainfall process. For a stochastic description, solutions of the surface flow problem are derived in terms of first two statistical moments (namely, mean and variance) of the runoff discharge through the nonstationary Fourier-Stieltjes representation approach. The mean solution is an unbiased estimator of runoff discharge, and the variance can be used to characterize the uncertainty of mean model. The closed-form expression for the variance of runoff discharge may also be viewed as an index of temporal variability, allowing to assessing the impacts of the rainfall and catchment storage on the discharge variability. It is found that the temporal variability of the runoff discharge induced by a random rainfall process persists longer for smaller values of the storage or rainfall parameters."

2. In this paper, the evapotranspiration is the only abstraction from the catchment (line number 58 on page number 3). However, further simplification leads to drop the evapotranspiration from the continuity equation (line number 82 on page number four). In other words, the inflow (i.e., rainfall), outflow (i.e., runoff), and the storage are the only components of the system. The authors should clearly state the validity of this conceptualization. Otherwise, the title of the paper may mislead considering the fact that the conceptualized system does not account for all the inner details. Are

the authors formulating the system to derive a solution that is feasible in a mathematical environment?

Response

a. Since this work is an initial step in the stochastic quantification of the influence of random rainfall (R_t) on the runoff discharge variability, we assume $R_t >> E_t$ in order to isolate the stochastic effects in the clearest possible way. However, this assumption is not restrictive. To support that, three articles have been cited on page 4 at the end of the sentence in Line 90 as

"(e.g., Jothityangkoon and Sivapalan, 2001; Dooge, 2005; Botter et al., 2009)"

Botter, G., A. Porporato, A., I. Rodriguez-Iturbe, I., and A. Rinaldo, A.: (2009), Nonlinear storage-discharge relations and catchment streamflow regimes, Water Resour. Res., 45(10), W10427, 2009.

Dooge, J. C. I.: Bringing it all together, Hydrol. Earth Syst. Sci., 9(1/2), 3-14, 2005.

Jothityangkoon, C., Sivapalan, M.: Temporal scales of rainfall-runoff processes and spatial scaling of flood peaks: Space-time connection through catchment water balance, Adv. Water Resour., 24(9-10), 1015-1036, 2001.

b. The analysis of runoff process is based on a lumped rainfall-runoff model coupled with an autoregressive rainfall model. As indicated in the hydrology literature, those models have been widely applied to field situations. Therefore, we expected that the solution developed herein can be directly used for runoff predictions and uncertainty quantification under real field conditions. In addition, the stochastic methodology used here has been successfully employed in solving groundwater related problems (e.g., Gelhar, 1993; Rubin, 2003) under field conditions.

Gelhar, L. W.: Stochastic Subsurface Hydrology, Prentice Hall: Englewood Cliffs, N. J., 1993.

Rubin, Y.: Applied Stochastic Hydrogeology, Oxford University Press, New York, N. Y., 2003

3. Does the R.H.S of equation 7b need a sign? If the sign is absorbed within the function, the authors need to mention it in the manuscript. The equation 8 also needs to be checked.

Response

(a) Thanks for pointing out the typo. Eq. (7b) should read as

$$a[r(t-1)-2r(t)+r(t+1)]-(1-2a)r(t)=-\xi(t) \qquad (7b)$$

(b) As mentioned in the manuscript, Eqs. (7) - (8) were adopted from the book by Vanmarcke (1983). Yes, the sign in Eq. (8) is absorbed within the random function $\xi(t)$.

4. As per the authors, in most practical applications, S in Eq. (1) is specified as an arbitrary function of Q. To convince this statement, few journal papers need to be cited.

Response

As suggested, the following three articles have been cited on page 4 (Line 76) after the phrase "… an arbitrary function of Q" as "(e.g., Lamb and Beven, 1997; Kirchner, 2009; Brauer et al. 2013)".

Brauer, C. C., Teuling, A. J., Torfs, P. J. J. F., and Uijlenhoet, R.: Investigating storage-discharge relations in a lowland catchment using hydrograph fitting, recession analysis, and soil moisture data, Water Resour. Res., 49(7), 4257-4264, 2013.

Kirchner, J. W.: Catchments as simple dynamical systems: Catchment characterization, rainfall-runoff modeling, and doing hydrology backward, Water Resour. Res., 45(2), W02429, 2009.

Lamb, R. and Beven, K.: Using interactive recession curve analysis to specify a general catchment storage model, Hydrol. Earth Syst. Sci., 1(1), 101-113, 1997.

5. As per the current version of the paper, all variables and parameters in Eq. (1) represent spatial averages over the entire catchment area (line number 62 on page number three). What is meant by variables? What is meant by parameters?

Response

In this study, a variable is defined as a time-dependent field (a function of time). Its value varies depending on conditions, such as the initial condition and associated parameters representing the system characteristics. In other words, variables are regarded as the unknowns of the model, while parameters (commonly determined based on observed data a priori) are treated as knowns of the model. As such, the phrase "all variables and parameters" is replaced by "S, R_t, E_t, and Q" in Line 70 (page 4).

6. As per the authors, to carry out rainfall-runoff calculations detailed information about landscape properties and hydrologic states must be known in the whole catchment (line number 31 on page number two). Authors also state that such information is not available due to the heterogeneity in associated parameters (line number 33 on page number two).What is meant by heterogeneity in associated parameters? What are those associated parameters? Is it due to the heterogeneity that we do not have these information? Recently, it has been argued that the advances in data-intensive hydrologic science have laid the foundation for a data-driven hypothesis testing framework (http://www.hydrol-earth-syst-sci-discuss.net/hess-2016-695/). Therefore, the authors need to convince their statement.

Response

a. In "associated parameters", the parameters represent system characteristics or behavior in modeling the hydrological processes involved rainfall depth, surface runoff, catchment storage, water table, soil moisture profile, etc. The types of parameter may vary with the selected rainfall-runoff model.

b. To avoid confusion, that sentence in Line 37 (page 2) is replaced by following statements:

> "The parameter values of the rainfall-runoff models may vary at different points of the catchment. It therefore requires a large quantity of measurements for accurate predictions of the hydrological response of the catchment. The number of measurement sites in most catchments, however, is likely to be small and therefore the amount of information is rather limited. Thus, it is very difficult to make an accurate prediction of catchment response based on insufficient measurements."

c. Thanks for the information, "Scaling, Similarity, and the Fourth Paradigm for Hydrology" (hess-2016-695). However, the above-mentioned concept is different from that covered in hess-2016-695.

7. As per the authors, referring to line number 66 on page number 3, there are two unknowns, namely Q and S. What has motivated the authors to assert a statement of this nature? Is E_t known? If E_t is known, what has driven the authors to consider Q as unknown.

Response

The unknowns Q and S are evaluated based on given or assumed values of R_t and E_t.

8. From the reader's point of view, the title of section 2 is meaningless. What is meant by "problem"?

Response

The title of section 2 is replaced by "Stochastic Formulation".

9. As per the current version of the paper, "rainstorm" is the major input into the generation of surface runoff and the production of runoff (line number 28 on page number two). From the reader's point of view, this statement is not warranted. What is meant by "major input"?

Response

Thanks for the comment. We replace the phrase "the major input into the generation of …" (Line 32, page 2) by "the source to the generation of …".

10. As per the current version of the paper, the authors propose a stochastic framework for a linear lumped rainfall-runoff problem at the catchment scale (line number nine on page number one). The authors should clearly state this rainfall-runoff problem.

Response

As suggested, we modify that sentence as

"This study proposes a stochastic framework for a lumped rainfall-runoff problem at a catchment scale under the assumption of a linear relationship between the runoff discharge and the catchment storage."

11. As per the current version of the paper, the title of the paper is uncertainty quantification in application of linear lumped rainfall-runoff models. However, the abstract of the paper does not explicitly present about the uncertainty quantification.

Response

The following sentence has been added in the abstract

"The mean solution is an unbiased estimator of runoff discharge, and the variance can be used to characterize the uncertainty of mean model."

Please refer to response to comment 1 for the details.

12. How is the outcome of this research influenced if a non-linear relationship between the storage and the outflow is assumed? It would be more useful if the authors discuss on this.

Response

As suggested, a brief discussion on that is added on page 13 (Line 249) as

“The consideration of a non-linear relationship between the discharge and the catchment storage will complicate the mathematical procedure. In general, an analytical solution to Eq. (25) doesn’t exist. The runoff discharge variability of a non-linear reservoir modeling system will therefore be assessed numerically. It is expected that the discharge variability behavior of a non-linear reservoir modelling system will be qualitatively similar to that of a linear reservoir modelling system, although not quantitatively.”