Uncertainty quantification in application of linear lumped rainfall-runoff models

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Abstract. This study proposes a stochastic framework for a lumped rainfall-runoff problem at a catchment scale under the assumption of a linear relationship between the runoff discharge and the catchment storage. Both the rainfall and discharge are treated as random fields. An autoregressive (AR) model is adopted to account for the temporal variability of the rainfall process. For a stochastic description, solutions of the surface flow problem are derived in terms of first two statistical moments (namely, mean and variance) of the runoff discharge through the nonstationary Fourier-Stieltjes representation approach. The mean solution is an unbiased estimator of runoff discharge, and the variance can be used to characterize the uncertainty of mean model. The closed-form expression for the variance of runoff discharge may also be viewed as an index of temporal variability, allowing to assessing the impacts of the rainfall and catchment storage on the discharge variability. It is found that the temporal variability of the runoff discharge induced by a random rainfall process persists longer for smaller values of the storage or rainfall parameters.

1 Introduction

Rainfall-runoff models simulate the processes of converting rainfall to runoff. They are used for a variety of applications in hydrology (e.g., Beven, 2012; Falahi et al., 2012), for example, to predict the peak flow used in drainage design purposes, to estimate flows of ungauged catchments, to assess the effects of climate changes. The quantitation of rainfall-runoff processes is essential for providing a basis of water resources management and planning in river basins.

Rainstorm is the source to the generation of surface runoff and the production of runoff is, therefore, dependent on the characteristics of rainfall events. Rainfall processes are generally recognized as being affected by complex natural events. The details of the processes cannot be described precisely. Moreover, to carry out rainfall-runoff calculations detailed information about landscape properties and hydrologic states must be known in the whole catchment. The parameter values of the rainfall-runoff models may vary at different points of the catchment. It therefore requires a large quantity of measurements for accurate predictions of the hydrological response of the catchment. The number of measurement sites in most catchments, however, is likely to be small and therefore the amount of information is rather limited. Thus, it is very difficult to make an accurate prediction of catchment response based on insufficient measurements. As such, there is a great deal of uncertainty about the runoff prediction using a deterministic model. As such, the analysis of rainfall-runoff processes is often taken by means of a stochastic framework (e.g., Córdova and Rodríguez-Iturbe, 1985; Goel et al., 2000; Lee et al., 2001; Moore, 2007; Bartlett et al., 2016).

Much of stochastic research in rainfall-runoff modellings focused on development of the probability distribution of state variables (such as rainfall and flow discharge). In most cases, due to a complex non-linear behavior in general, the analytical solution for the probability distribution function does not exist. Alternatively, to take the advantage of closed-form expressions, the purpose of this study is to derive analytical solutions, namely the first two moments of runoff discharge, for a linear lumped rainfall-runoff problem. The first moment (ensemble mean) is used as an unbiased estimate of a system state, and the second moment (ensemble variance) is used as a measure of uncertainty by applying the mean model. Those expressions will be obtained using the nonstationary Fourier-Stieltjes representation approach along with the assumption of an AR rainfall model (e.g., Foufoula-Georgiou and Lettenmaier, 1987; Thyregod et al., 1999; Srikanthan, and McMahon, 2001; Rebora et al. 2006; Hannachi, 2014).

2 Stochastic Formulation

The physical-based equation in modeling the rainfall-runoff process is the equation of conservation of mass. If the control volume is extended to the scale of a catchment, the continuity equation for the free surface flow then takes on the lumped form of the storage equation as (e.g., Brutsaert, 2005; Beven, 2012)

$$\frac{dS}{dt} = R_t - E_t - Q \tag{1}$$

where S is catchment storage, R_t and E_t denote the rainfall and evapotranspiration at time t, respectively, and Q is the discharge from the catchment. The lumped model attempts to relate the forcing (rainfall input) to the model output (runoff) without considering the spatial variability. Therefore, S, Q, R_t and E_t in Eq. (1) represent spatial averages over the entire catchment area, and, as such, only their temporal variability is retained. That is, in a lumped system model, the flow is evaluated as a function of time alone at a particular location in large catchments.

Since there are two unknowns, namely Q and S, for only one equation, further knowledge of the relation of Q to S is needed in order to solve Eq. (1). In most practical applications, S in Eq. (1) is specified as an arbitrary function of Q (e.g., Lamb and Beven, 1997; Kirchner, 2009; Brauer et al. 2013). As such, the changes in S with time may be expressed as

$$\frac{dS}{dt} = \frac{dS}{dQ}\frac{dQ}{dt} \tag{2}$$

Given Eqs. (1) and (2), it follows that

$$\frac{dQ}{dt} + \frac{Q}{dS/dQ} = \frac{R_t - E_t}{dS/dQ} \tag{3}$$

This study will concentrate only on the case of S being a linear function of Q (e.g., Kaseke and Thompson, 1997; Botter et al., 2007; Suweis et al., 2010, Guinot et al., 2015):

$$S = KQ \tag{4}$$

where the constant K is termed as the storage parameter. Early work on a linear storage parameter of this type was reported by Nash (1959) and Dooge (1959). Consequently, Eq. (1) can be cast in the form

$$\frac{dQ}{dt} + \frac{Q}{K} = \frac{R_t - E_t}{K} \tag{5}$$

It is assumed in the following analysis that R_t is a temporal stochastic process (random field). We also assume that evapotranspiration has a negligible effect on Q as compared to that of rainfall ($R_t >> E_t$) (e.g., Jothityangkoon and Sivapalan, 2001; Dooge, 2005; Botter et al., 2009). Since the temporal random heterogeneity of R_t appears as a forcing term which generates the random variations in Q, the differential Eq. (5) is then viewed as a stochastic differential equation. The probabilistic structure of random Q is determined by its temporal statistical moments. In the present study, we are interested mainly in developing the first two moments of Q. The mean (unbiased estimate of) runoff discharge may also be interpreted as the solution predicted by the deterministic model. The second moment (variance) of catchment discharge derived below can then be used to characterize the uncertainty in applying the deterministic (or mean) model. The variance can be viewed as an index of large-scale discharge variability as well.

Due to its linearity, Eq. (5) may be split into two sub-equations: a mean equation governing the temporal behavior of mean catchment discharge,

$$\frac{d\overline{Q}}{dt} + \frac{\overline{Q}}{K} = \frac{\overline{R}}{K} \tag{6a}$$

and an equation for the perturbations describing the discharge perturbation produced as a result of the input rainfall perturbation,

$$\frac{dq}{dt} + \frac{q}{K} = \frac{r}{K} \tag{6b}$$

In Eq. (6), $\overline{Q}$ and $\overline{R}$ indicate the means of Q and R_t, respectively, and q (= Q-$\overline{Q}$) and r (= R_t - $\overline{R}$) are zero-mean perturbations.

Spectral representation theorem provides a very useful way of evaluating the variance of perturbations. To carry out the calculation, the perturbed-form Eq. (6b) must be solved in Fourier space. Since $r(t)$ in Eq. (6b) is a noise force contributing to the variations in q, the solution of Eq. (6b) requires knowledge of the temporal distribution of rainfall field. The section that follows attempts to develop the spectrum of $r(t)$ which will be achieved by solving an AR model for temporal rainfall processes through the nonstationary spectral approach.

3 Spectral Solution for the Rainfall field

The AR model specifies linear dependence of the output variable partly on its own previous values and partly on the random disturbance (or white noise) (e.g., Priestley, 1981; Vanmarcke, 1983). In other words, the AR model uses a linear equation with constant coefficients to define the relation between an output process and an input white noise process.

Throughout this study, it is assumed that the temporal distribution of rainfall field can be described by the AR model proposed by Vanmarcke (1983). Following Vanmarcke (1983), the random rainfall perturbation field $r(t)$ without directional preference may be expressed in the form

$$r(t) = a\left[r(t-1) + r(t+1)\right] + \xi(t) \tag{7a}$$

where a is a constant parameter and ξ is a stationary purely random (white noise) process. Subtracting $2ar(t)$ from both sides and rearranging terms yields (*Vanmarcke*,1983)

$$a\left[r(t-1) - 2r(t) + r(t+1)\right] - (1-2a)r(t) = -\xi(t) \tag{7b}$$

In continuous time, the natural analogue of the linear Eq. (7b) is a linear differential equation, of the form

$$\frac{d^2 r}{dt^2} - \alpha^2 r = \xi(t) \tag{8}$$

where α^2 = (1-2*a*)/*a*. In addition, the initial conditions are specified as

$$r(0) = 0 \tag{9a}$$

$$\frac{d}{dt} r(0) = 0 \tag{9b}$$

Eq. (8) along with Eq. (9) permits one to determine the spectrum of $r(t)$.

Whenever the random field is stationary, there always exists an unique representation of the process in terms of a Fourier-Stieltjes integral as (e.g., Lumley and Panofsky, 1964)

$$\xi(t) = \int_{-\infty}^{\infty} e^{i\omega t} dZ_{\xi}(\omega) \tag{10}$$

where $Z_{\xi}(\omega)$ is an orthogonal process (i.e., the random amplitudes dZ_{ξ} are uncorrelated) and ω denotes the frequency. Without the restriction that the $r(t)$ process must be stationary, the perturbed quantities $r(t)$ may be presented as (Priestley, 1965)

$$r(t) = \int_{-\infty}^{\infty} \Lambda_{r\xi}(t;\omega) e^{i\omega t} dZ_{\xi}(\omega) \tag{11}$$

In Eq. (11), $\Lambda_{r\xi}(\text{-})$ is referred to as the modulating function by Priestley (1965). Introducing Eqs. (10) and (11) into Eqs. (8) and (9), respectively, produces

$$\frac{d^2 \Lambda_{r\xi}}{dt^2} + i2\omega \frac{d\, \Lambda_{r\xi}}{dt} - (\omega^2 + \alpha^2) \Lambda_{r\xi} = 1 \tag{12}$$

with

$$\Lambda_{r\xi}(0;\omega) = 0 \tag{13a}$$

$$\frac{d\Lambda_{r\xi}(0;\omega)}{dt} = 0 \tag{13b}$$

The system of equations admits the solution as follows:

$$\Lambda_{r\xi}(t;\omega) = \frac{1}{\alpha^2+\omega^2}\left[-1+\frac{\alpha+i\omega}{2\alpha}e^{\eta-i\tau}+\frac{\alpha-i\omega}{2\alpha}e^{-\eta-i\tau}\right] \tag{14}$$

where $\eta = \alpha t$ and $\tau = \omega t$. Using Eq. (14), Eq. (11) implies

$$r(t) = \int_{-\infty}^{\infty}\frac{1}{\alpha^2+\omega^2}[-e^{i\tau}+\frac{\alpha+i\omega}{2\alpha}e^{\eta}+\frac{\alpha-i\omega}{2\alpha}e^{-\eta}]dZ_{\xi}(\omega) \tag{15}$$

It follows from using the representation theorem for $r(t)$ that the variance of $r(t)$, σ_r^2, admits a representation of the form

$$\sigma_r^2(t) = E[r(t)r^*(t)] = \int_{-\infty}^{\infty}\Lambda_{r\xi}(t;\omega)\,\Lambda_{r\xi}^*(t;\omega)\,S_{\xi\xi}(\omega)d\omega = \int_{-\infty}^{\infty}S_{rr}(\omega)d\omega \tag{16}$$

where E[-] indicates the ensemble average of the quantity, * denotes the complex conjugate, $S_{\xi\xi}(\omega)$ is the spectrum of $\xi(t)$, and $S_{rr}(t;\omega)$ is the evolutionary spectrum of $r(t)$, quantified corresponding to Eqs. (14) and (16) as

$$S_{rr}(t;\omega) = \frac{1}{\omega^4(1+\gamma^2)^2}\left[1-2\cos(\tau)\cosh(\eta)-\frac{2}{\gamma}\sin(\tau)\sinh(\eta)+\frac{1+\gamma^2}{2\gamma^2}\cosh(2\eta)+\frac{\gamma^2-1}{2\gamma^2}\right]S_{\xi\xi}(\omega) \tag{17}$$

In Eq. (17), $\gamma = \alpha/\omega$. The evolutionary spectrum referred by Priestley (1965) has the same physical interpretation as the spectrum of a stationary process that it describes the energy of a signal distributed with frequency. The latter is determined by the behavior of the process over all time, while the former represents specifically the spectral content of the process in the neighborhood of the time instant t.

As defined above, $\xi(t)$ represents a white noise process which consists of a sequence of uncorrelated random variables. The corresponding spectrum for such a process is

$$S_{\xi\xi}(\omega) = I_{\xi} \tag{18}$$

I_{ξ} in Eq. (18) is constant for all frequency. The variance of the rainfall field resulting from Eqs. (16)-(18) is now given by

$$\sigma_r^2(t) = \frac{\pi}{2\alpha^3}\Gamma_t I_{\xi} \tag{19}$$

where Γ_t = sinh(2η)-2η. It can be shown that the rainfall variance increases with time.

It follows from Eqs. (17)-(19) that for a given σ_r^2, the evolutionary spectrum of the rainfall response to white noise input can be rewritten as

$$S_{rr}(t;\omega) = \frac{2}{\pi}\frac{\gamma^3}{\omega(1+\gamma^2)^2}\Psi_t\sigma_r^2 \tag{20}$$

with

$$\Psi_t = \frac{1}{\Gamma_t}\left[1 - 2\cos(\tau)\cosh(\eta) - \frac{2}{\gamma}\sin(\tau)\sinh(\eta) + \frac{1+\gamma^2}{2\gamma^2}\cosh(2\eta) + \frac{\gamma^2-1}{2\gamma^2}\right] \tag{21}$$

The dependence of $S_{rr}(t;\omega)$ in Eq. (20) on rainfall parameter α is depicted in Fig. 1 at different times. The reduction of the temporal rainfall spectrum with α is clearly observed in the figure. This reflects that a larger α produces shorter persistence of rainfall perturbations, which, in turn, leads to less deviations of the rainfall perturbation from the mean rainfall profile and, consequently, less variability of the rainfall process. It can be shown that the variance of rainfall in Eq. (19) will decrease with a large α.

The results presented in this section will be employed in the derivation of solutions for the flow discharge problem in terms of its moments.

4 Moments of discharge

We consider the case where initially, there is no discharge from the catchment, implying that

$$\overline{Q}(0) = 0 \tag{22a}$$

$$q(0) = 0 \tag{22b}$$

The solution of Eqs. (6a) and (22a) for the mean runoff discharge is in the form

$$\overline{Q}(t) = \frac{\overline{R}}{K}\int_0^t e^{-(t-y)/K}\, dy = \overline{R}(1 - e^{-t/K}) \tag{23}$$

It is easy to see from Eq. (23) that the mean discharge decreases with a larger storage parameter.

We proceed to derive the variance of catchment discharge. A similar procedure to the above, applying the nonstationary spectral representation for the perturbed quantities $q(t)$

$$q(t) = \int_{-\infty}^{\infty} \Lambda_{q\xi}(t;\omega)\, e^{i\tau}\, dZ_{\xi}(\omega) \tag{24}$$

and Eq. (11) into Eqs. (6b) and (22b), leads to the following results

$$\frac{d\,\Lambda_{q\xi}}{dt} + (\frac{1}{K} + i\omega)\,\Lambda_{q\xi} = \frac{\Lambda_{r\xi}}{K} \tag{25a}$$

with

$$\Lambda_{q\xi}(0;\omega) = 0 \tag{25b}$$

The solution to this problem is

$$\Lambda_{q\xi}(t;\omega) = \frac{1}{K}\int_{0}^{t} \exp\left[-\frac{1+i\omega K}{K}(t-y)\right]\Lambda_{r\xi}(y;\omega)dy$$

$$= \frac{1}{2}\frac{e^{-i\tau}}{\alpha(\alpha^2+\omega^2)}\left[\frac{\alpha-i\omega}{\beta-1}\lambda_1 - \frac{\alpha+i\omega}{\beta+1}\lambda_2 + 2\frac{\alpha}{1+i\omega K}(e^{-\mu}-e^{-i\tau})\right] \tag{26}$$

where λ_1 = exp(-μ)-exp(-η), λ_2 = exp(-μ)-exp(η), $\beta = \alpha K$, and $\mu = t/K$. Eqs. (24) and (26) provide the framework required to express the discharge perturbation $q(t)$.

The variance of runoff discharge $\sigma_q^2(t)$ can now be obtained as follows:

$$\sigma_q^2(t) = E[q(t)\,q^*(t)] = \int_{-\infty}^{\infty}\left|\Lambda_{q\xi}(t;\omega)\right|^2 S_{\xi\xi}(\omega)d\omega = \int_{-\infty}^{\infty} S_{qq}(\omega)d\omega \tag{27}$$

where the evolutionary spectrum of $q(t)$ is given by

$$S_{qq}(t;\omega) = \frac{1}{4}\frac{1}{\alpha^2(\alpha^2+\omega^2)^2}\Big\{\frac{\alpha^2+\omega^2}{(1-\beta)^2}\lambda_1^2 + 2\frac{\alpha^2-\omega^2}{1-\beta^2}\left[e^{-\mu}(\lambda_1-e^{\eta})+1\right] - 4\frac{\alpha}{1-\beta}\Big[\frac{\alpha+K\omega^2}{1+K^2\omega^2}\lambda_1(e^{-\mu}-\cos(\tau))$$

$$+\frac{\omega(1-\beta)}{1+K^2\omega^2}\lambda_1\sin(\tau)\Big] + \frac{\alpha^2+\omega^2}{(1+\beta)^2}\lambda_2^2 - 4\frac{\alpha}{1+\beta}\Big[\frac{\alpha-K\omega^2}{1+K^2\omega^2}\lambda_2(e^{-\mu}-\cos(\tau)) - \frac{\omega(\beta+1)}{1+K^2\omega^2}\lambda_2\sin(\tau)\Big]$$

$$+4\frac{\alpha^2}{1+K^2\omega^2}\left[e^{-2\mu}-2\cos(\tau)e^{-\mu}+1\right]\Big\}S_{\xi\xi}(\omega) \tag{28}$$

The discharge variance follows from Eq. (27) through the application of Eqs. (18) and (28):

$$\sigma_q^2(t) = \frac{\pi}{2}I_{\xi}\frac{1}{\alpha^3}\Big[\frac{\lambda_1}{(1-\beta)^2}\left(\frac{\lambda_1}{2}-e^{-\eta}\right) + \frac{\phi_1}{(1+\beta)^2} - \frac{(1+3\beta)\,\lambda_1 e^{-\mu}}{(1-\beta)(1+\beta)^2} - \frac{e^{-\eta}\phi_2}{1-\beta^2}$$

$$+4\frac{\beta^2}{(1-\beta^2)^2}\left(\lambda_1 e^{-\eta} - \beta e^{-2\mu}\right)\Big] \tag{29}$$

with

$$\phi_1 = 1+2\beta+\frac{1+4\beta}{2}e^{-2\mu}+\frac{e^{2\eta}}{2}-e^{-2\eta}+e^{\eta-\mu} \tag{30a}$$

$$\phi_2 = \eta(\lambda_1 + \lambda_2) + \lambda_2 + 2(\eta+1)e^{-\mu} - \eta e^{\eta}(1-e^{-2\mu}) \tag{30b}$$

Finally, using the relation (19) leads to

$$\sigma_q^2(t) = \frac{\sigma_r^2}{\Gamma_t}\Big[\frac{\lambda_1}{(1-\beta)^2}\Big(\frac{\lambda_1}{2} - e^{-\eta}\Big) + \frac{\phi_1}{(1+\beta)^2} - \frac{(1+3\beta)\,\lambda_1 e^{-\mu}}{(1-\beta)(1+\beta)^2} - \frac{e^{-\eta}\phi_2}{1-\beta^2}$$

$$+ 4\frac{\beta^2}{(1-\beta^2)^2}(\lambda_1 e^{-\eta} - \beta e^{-2\mu})\Big] \tag{31}$$

The result of this type can be used directly to evaluate the uncertainty in the mean runoff discharge model when applying it to the field situations.

Figs. 3a and 3b display the runoff discharge variance in Eq. (31) as functions of the storage parameter *K* and rainfall parameter α, respectively, for various time scales. It is seen from Fig. 3a that the discharge variability increases with a decrease in *K* for a given α. This can be attributed to that persistence of random discharge fluctuations is reduced by a large *K*, which leads to smaller deviations of the discharge fluctuations. A similar conclusion has been made for the case of response of the Brownian particle motion to a stationary random noise forcing. Note that Eq. (6b) is in fact a generalized Langevin equation (e.g., van Kampen, 1981; Gardiner, 1985) arising in the analysis of Brownian motion, where *K* corresponds to a particle mass. It has been reported from the literature that the velocity variability of the Brownian particle is reduced by a large particle mass. That is, velocity fluctuations in stationary flow fields persist shorter with a larger particle mass.

In addition, Fig. 3b shows the reduction in the variability of the runoff discharge field with α for a fixed value of *K*. It is evident from Eq. (26) that in a linear system, the variability of output process correlates positively with that of input process. The larger the rainfall parameter, the smaller the variability of the rainfall field (Fig. 1), and, consequently, the smaller the variability of runoff discharge (Fig. 3b). In other words, the runoff processes in response to rainstorms characterized by a small rainfall parameter exhibit a relatively smoother data profile. The figure also indicates that the ratio of variabilities of runoff discharge to rainfall decreases with time for a fixed value of β. This refers to the fact that the growth rate of discharge variability with time is less than that of rainfall variability.

Quantitation of runoff discharge is the primary information for water resource management and planning in river basins. The quantitation generally involves a prediction over a relative large time scale, where direct measurements are not possible in many field cases. Under such conditions, there will be a great deal of uncertainty in applying the solution of the mean model (or the deterministic model). The temporal distributions of normalized mean runoff discharge ($\overline{Q}/\overline{R}$) predicted by Eq. (23) and its uncertainty (one standard deviation) estimated based on Eq. (29) shown in Figure 3 indicate that the uncertainty grows as the runoff discharge increases with time.

The consideration of a non-linear relationship between the discharge and the catchment storage will complicate the mathematical procedure. In general, an analytical solution to Eq. (25) doesn't exist. The runoff discharge variability of a non-linear reservoir modelling system will therefore be assessed numerically. It is expected that the discharge variability behavior of a non-linear reservoir modelling system will be qualitatively similar to that of a linear reservoir modelling system, although not quantitatively

5 Concluding remarks

In this work, the catchment-scale rainfall-runoff process is modeled by a linearized model and analyzed by means of a stochastic framework. In our derivation, the temporal distribution of the random rainfall process is described by an AR model. The closed-form solutions to the linear lumped rainfall-runoff model are expressed in terms of first two statistical moments through the nonstationary Fourier-Stieltjes representation. The first moment (mean) is used as an unbiased estimate of runoff discharge, while the second moment (variance) gives a quantitative measure of the uncertainty by applying the mean rainfall-runoff model to the field situations.

The analysis of the closed-form solutions clearly demonstrates that an introduction of a large rainfall parameter leads to the reduction in the variability of the rainfall process. The smaller the storage or rainfall parameters, the more persistence of the random fluctuations in runoff discharges and, in turn, the larger deviations from the mean, which results in larger variability of the runoff process.

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Figures

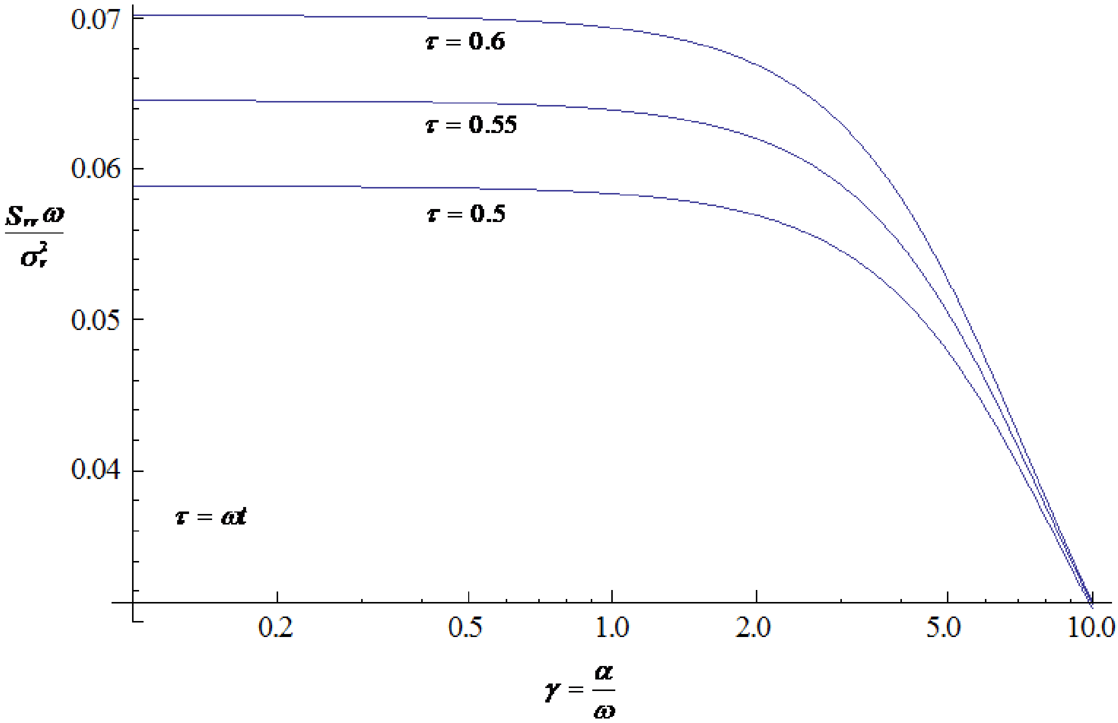

Figure 1. The dependence of $S_{rr}(t;\omega)$ in Eq. (20) on rainfall parameter α at different times.

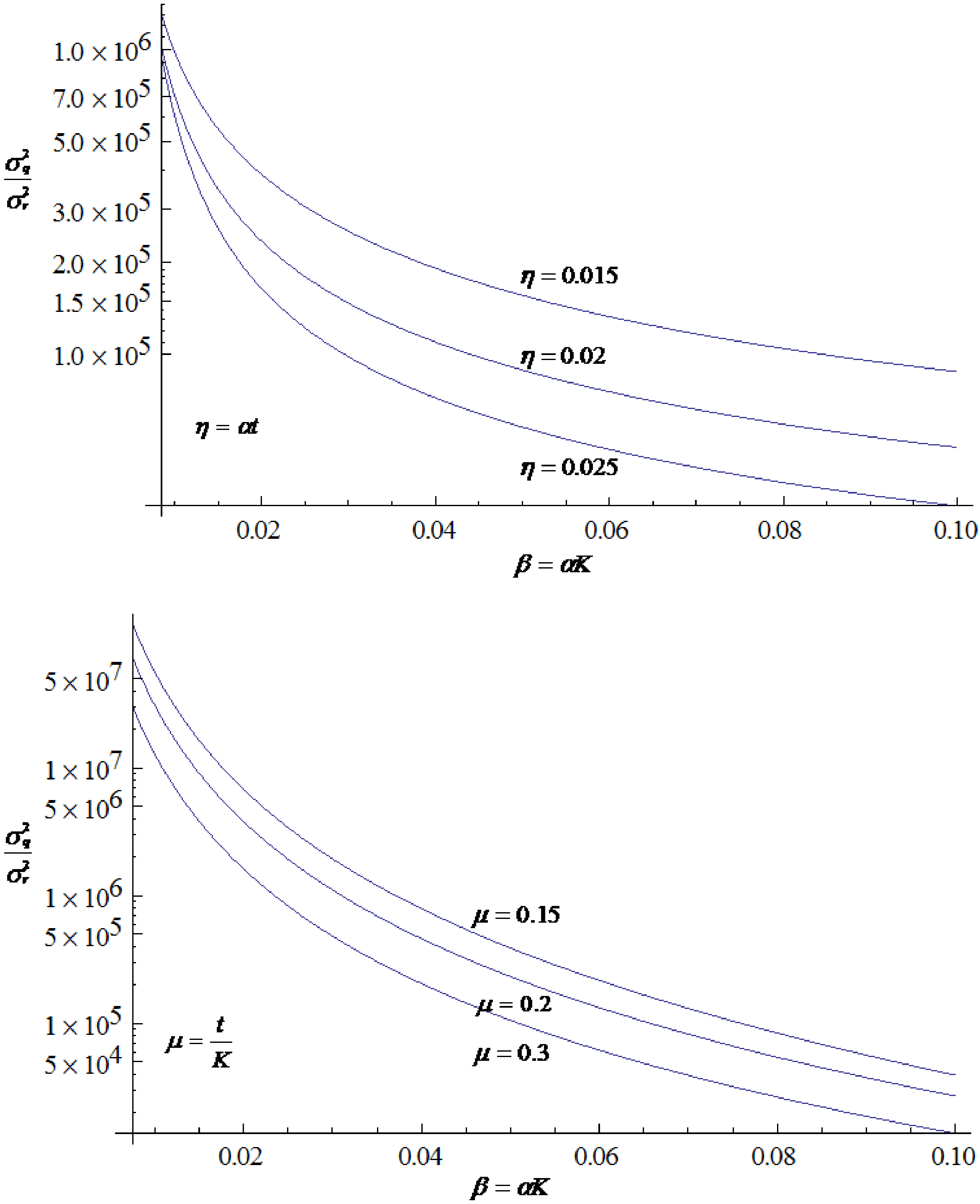

Figure 2. The dependence of σ_q^2 in Eq. (31) on (a) storage parameter K and (b) rainfall parameter α at different times.

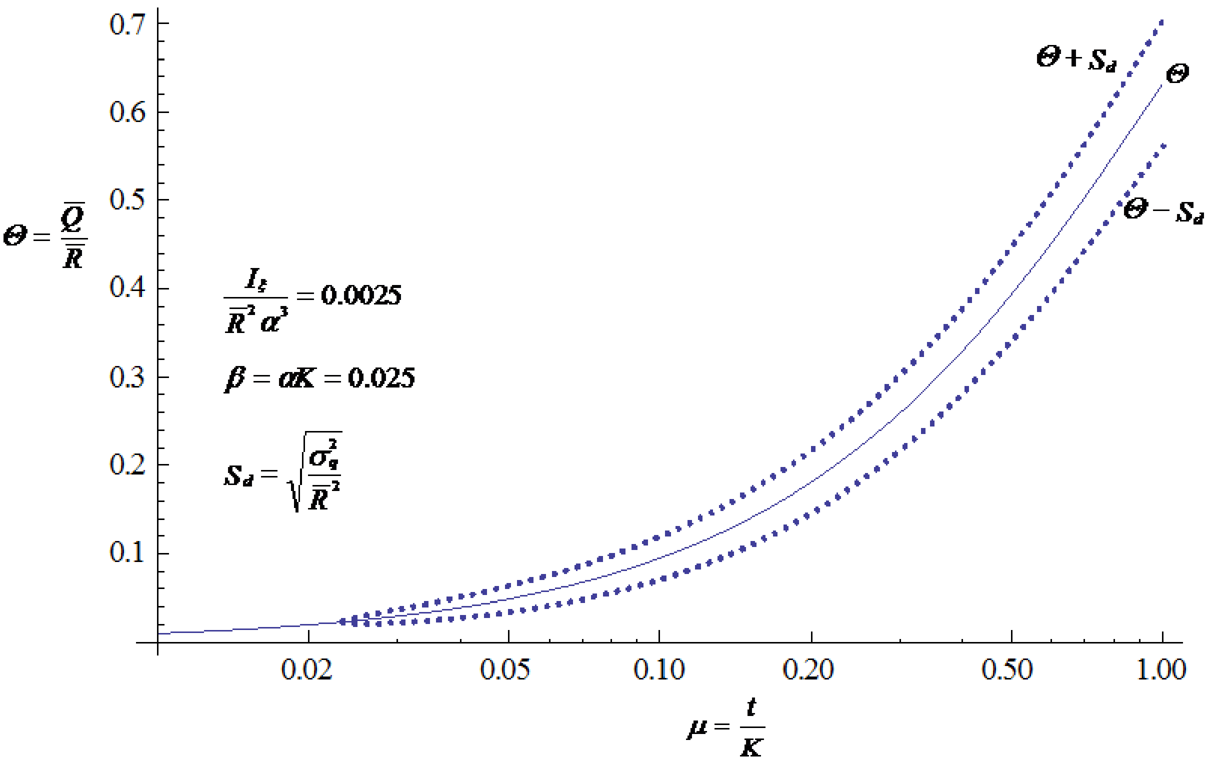

Figure 3. Normalized mean runoff discharge profiles along with one standard deviation intervals as a function of dimensionless time.