Anonyms Referee # 1

We thank the referee (#1) for reviewing our manuscript entitled ‘Modeling the Changes in Water Balance Components of Highly Irrigated Western Part of Bangladesh’ and for his/her valuable comments to improve our manuscript. We have responded to referee (#1) comments below:

Overall Comments:

This paper by A.T.M. Sakiur Rahman et al (the authors) describes the development of wavelet autoregressive moving average models to forecast changes in water balance components. The approach is applied to a highly-irrigated area in Western Bangladesh using data collected between 1982 and 2013. The authors show that the approach can be used to forecast short term changes in water balance components but suggest that models could be further improved using different combinations of wavelet analysis.

Reply to overall comments:

We are very much grateful to you for your valuable comments about our study ‘Modeling the Changes in Water Balance Components of Highly Irrigated Western Part of Bangladesh’. We have gone through your comments and we will incorporate the necessary corrections in the relevant sections. We are also doing necessary corrections in language which is your main concern regarding the manuscript. Thank you very much for your suggestions that will help us to prepare a good paper. Yes, the paper is methodological in nature; we have tried to forecast water balance components (WBCs) more precisely after denoising the time series by discrete wavelet transformation. We also expect that there is a scope for further improvement of the methodology by different combinations of wavelet techniques. We will add a discussion section (3.4) where we discuss the advantages and limitations of our study. Please go through reply 11 of anonym’s referee-

Action: We are doing necessary corrections in language. We have also written a discussion section. Please go through reply 11 of anonymous referee-

The responses to the specific comments are also presented as follows:

Reply to the Specific comments

Comment 1: This is a technical paper with a strong methodological focus. While the approaches used are well described, I would recommend the authors more clearly define the importance of the work in a broader hydrological context; for example, the relevance to hydrology and water resource management regionally and globally as this is only mentioned briefly.

Reply 1: We have rewritten the introduction section and discussed about the time series analysis in a broader hydrological context following your suggestions (please go through the reply 1 of referee #2). Thank you very much for your constructive comments.

Action: We have rewritten the introduction section following the reviewer’s suggestions. Please go through reply 1 of anonym referee#2.

Comment 2: This study is applied to a region in Bangladesh, however, both the approach and the paper would benefit if the transferability of the methodology could be highlighted by the authors; i.e. in what environments and under what conditions would the methods described work well.

Reply 2: We are very pleased after going through your comments. ARIMA models (Box and Jenkins, 1976) are very much useful for forecasting hydrological variables such as rainfall (e.g., rainfall of the USA by Burlando et al., 1993), temperature (e.g., temperature of Bangladesh, Nary et al., 2017), PET (e.g., PET of Iran, Valipour, 2012), groundwater level (e.g., groundwater level of Canada by Adamowski and Chan), runoff (e.g. runoff of Russia by Nigam et al. 2014), water quality (water quality of Turkey by Faruk 2010) etc. These are the few examples of the application of ARIMA models in hydrology. However, ARIMA models have a limitation; these models cannot appropriately handle non-stationary hydrological data. Wavelet analysis is a suitable technique to overcome this problem. Several studies have already demonstrated the advantages of wavelet analysis (Sang, 2013). Wavelet denoising has not attracted much attention in hydrologic science, though it has been used in the other science and engineering fields (Sang, 2013). We have discussed the advantages of denoising for forecasting the hydrological data in our article. Water balance components are related with a range of hydro-
meteorological variables. As we have given here few examples of worldwide applications of ARIMA models for forecasting the hydrological variables. Therefore, we may assume that our developed wavelet denoised ARIMA models can be applied for forecasting the hydrological variables worldwide. We will briefly discuss the matter in our new discussion section. Thank you very much for reminding us to write something about this important issue.

**Action:** As mentioned earlier, we have added a discussion section following the reviewer’s suggestions. Please go through reply 11 of anonymous refree#2.

**Comment 3:** The paper would also benefit from a separate limitations section; for example, as an additional section 3.4. Limitations, in addition to those found within the methods, should also be highlighted. This may include, for example, scarcity of data. Also, data from 11 stations are used to represent over 61,000 km². A comment on how representative this data is would be welcome.

**Reply 3:** As mentioned earlier, we will incorporate a discussion section where we highlight the limitations of our present study. We are doing another research work, and we hope that we will mention the scarcity of data and representativeness of the data for a large area in our next study.

**Action:** As mentioned earlier, we have added a discussion section. Please go through reply 11 of anonymous refree#2.

**Comment 4:** Some of the text in Figure 6 is difficult to read. Also, please be consistent with the font.

**Reply 4:** Thank you very much again for your valuable suggestions. We have already prepared new figures following your suggestions. We will add this figure in our final manuscript.

**Action:** We have prepared a new figure.
Figure 6: Plot of best WD-ARIMA model first panel represents actual versus fitted values for the period of 1981-82 to 2012-2013, the second panel is normal Q-Q plot of residuals of the model, and the third panel shows actual, fitted and forecasted values for 2009-2010 to 2012-13 (a) PET of Rangpur station located in north; (b) AET of Ishurdi station located in the central part, (c) deficit of Rajshahi station located in NW Bangladesh and (d) surplus of Bholo station located in south of the study area.

Comment 5: Some examples of typos and where sentence rephrasing would be beneficial:……

Reply 5: We are going through the manuscript carefully and doing necessary corrections to improve the language. Thank you very much for your time and comments that help us a lot to improve the article.

Action: As mentioned earlier, we are doing necessary language corrections with the help of two professors of English department.

References


Anonyms Referee # 2

Responses to interactive comments of anonymous referee (#2) about our study “Modeling the Changes in Water Balance Components of Highly Irrigated Western Part of Bangladesh”

General Comments:

This paper investigates changes in water balance components between 1981-82 to 2012-13 in an area of Bangladesh that is intensively irrigated. First, historical trends are examined using the Mann-Kendal test and discrete wavelet transformation. Then, ARIMA models are developed in order to forecast changes in water balance components. The paper produces some interesting results; particularly around the use of ARIMA models that are fitted to wavelet denoised time series data.

The paper is well organised and about the right length for a study of this kind. Generally speaking the equations are well laid out and easy to follow. However, as it stands the level of English used in the paper is poor, which makes some sections very difficult to follow. I would strongly advise the authors to consult a proofreader who has full professional proficiency in written English. Nevertheless in my judgement the scientific content is sound and represents an interesting approach to analysing and forecasting changes in the water balance. Thus, I would reconsider this paper for publication following a major revision to improve the quality of English as well as addressing the specific points mentioned below.

Reply to general comments:

We are very much grateful to you for your valuable comments about our study ‘Modeling the Changes in Water Balance Components of Highly Irrigated Western Part of Bangladesh’. We have already gone through your comments and we will incorporate the necessary corrections in the relevant sections. We are also doing necessary corrections in language which is your main concern regarding the manuscript. Actually, we will receive help from two professors of English for doing the corrections in our manuscript. Thank you very much for your suggestions that will help us to prepare a well-organized paper. Yes, the paper is methodological in nature; we have tried to forecast water balance components (WBCs) more precisely after denoising the time series by discrete wavelet transformation. We also expect that there is a scope for further improvement of the methodology by different combinations of wavelet techniques. We will add a discussion section following your suggestions.

Action: We have written a discussion section (3.4). Please go to reply 11.

The responses to the specific comments are also presented as follows:

Reply to the Specific comments

Comment 1: The Introduction lacks focus, does not provide much critical analysis and does not place the work in the broader context of water resources management. I would like to see the aim of the paper clearly stated in the first paragraph of the paper, so that readers know what the paper is setting out to achieve. The paper is methodological in nature, so the Introduction must make clear to the reader the state-of-the-art in time series analysis for water resources management. Line 82 onwards does include some critical analysis but, in my opinion, it is insufficient to persuade the reader of the approach and its relevance to hydrology more generally. Focus less on the results of studies and instead examine and compare the different ways that previous researchers have tackled the problem.

Reply 1: We also agree with your comments that we need to give more emphasis on the state-of-the-art in time series analysis for water resources management in the introduction section. Thus, we will revise the introduction section following your suggestions.

Action: We have rewritten the introduction section (1) following the reviewers suggestions.

I. Introduction

After introducing the monthly water balance model by Thronthwaite (1948) and afterward followed by Thronthwaite and Mather (1957), this model is going through modifications for adaptation in the different areas
of the world. The development of the new model is still ongoing (Xu and Singh, 1998) as the water balance model is significantly important in water resources management, irrigation scheduling and crop pattern designing (Kang et al., 2003; Valipour, 2012). Moreover, it can be used for the reconstruction of catchment hydrology, climate change impact assessment and streamflow forecasting (e.g. Alley, 1985; Arnall, 1992, Xu and Halldin, 1996; Molden and Sakhivadivel, 1999; Boughton, 2004; Anderson et al., 2006; Healy et al., 2007; Moriarty et al., 2007; Karimi et al., 2013). Therefore, detecting the changes in WBCs and more accurate forecasting of WBCs are important for achieving the sustainability of water resources management. However, hydro-meteorological time series are contaminated by noises from hydro-physical processes that affect the accuracy of analysis, simulation and forecasting (Sang et al., 2013 and Wang et al., 2014). Hence, it is necessary to denoise the time series for improving the accuracy of the obtained results. In the present study, wavelet denoising technique has been coupled with ARIMA models for forecasting the WBCs after detecting the changes in WBCs by different forms of MK tests and identifying the time period responsible for trends in WBCs time series using DWT time series data.

Generally, physics based numerical models are used for understanding a particular hydrological system and forecasting the water balance or budget (e.g. Fulton et al., 2015, Leta et al., 2016) components. In this method, for reliable forecasting, a large amount of hydrological data is required to assign physical properties of the grid and model parameters and to calibrate the model simulation. However, they have a number of limitations in practice including the cost, time and availability of the data (Yoon et al., 2011; Adamowski and Chan, 2011). Data based forecasting models, statistical models, are suitable alternatives to overcome these problems. The most common statistical methods for hydrological forecasting are ARIMA models and multiple linear regression (Young, 1999; Adamowski, 2007). Many studies use ARIMA model to predict water balance input parameters like rainfall (e.g., Rahman et al., 2015; Rahman et al., 2016), temperature (e.g. Nury et al., 2016) and $P_{ET}$ (e.g., Valipour, 2012). However, ARIMA model cannot handle non-stationary hydrological data without pre-processing of the input time series data (Tiwari and Chatterjee, 2010; Adamowski and Chan, 2011). Wavelet analysis, a new method in the area of hydrological research, is such a method that is able to handle non-stationary data effectively (Adamowski and Chan, 2011). However, over the course of time some research works have already been done. For example, Adamowski and Chan (2011) coupled wavelet analysis with Artificial Neural Network (ANN) models for forecasting the hydrological variables like groundwater level in Quebec, Canada. Kisi (2008) and Partla (2009) and Santos and da Silva (2014) develop a hybrid wavelet ANN models for monthly and daily streamflow forecasting respectively. A study conducted by Rahman and Hasan (2014) also finds that the performance of the wavelet-based ARIMA models is better than the classical ARIMA model for forecasting the humidity of Rajshahi meteorological station in Bangladesh. A comparative study of wavelet ARIMA models and wavelet ANN models has been conducted by Nury et al. (2017). The study shows that the wavelet ARIMA models are more effective than the wavelet ANN for temperature forecasting. Khalek and Ali (2016) developed wavelet seasonal ARIMA (W-SARIMA) and neural network autoregressive (W-NNAR) model for forecasting the groundwater level. The study also finds that the performance of W-SARIMA model is better than the performance of W-NNAR models. All of these studies mentioned above find that the performance of wavelet aided model is better than classical ARIMA models and ANN models. Moreover, the analysis of periodicity using wavelet transformed details, and approximation components of hydro-meteorological time series data can better provide insight into trends and effects of time period on trend (e.g. Nalley et al., 2013;
Araghi et al., 2014; Pathak et al., 2016). As a result, wavelet transformation of hydro-meteorological time series is gaining popularity in recent years to detect periodicity (e.g. Partal and Kılıç, 2006; Partal, 2009; Nalley et al., 2013; Araghi et al., 2014; Pathak et al., 2016). Some studies have been conducted on spatio-temporal characteristics of hydro-meteorological variables such as rainfall (e.g. Shahid and Khairulmaini, 2009; McSweeney et al., 2010; Ahasan et al., 2010; Kamruzzaman et al., 2016a, Rahman and Lateh, 2016; Rahman et al., 2016; Syed and Al Amin, 2016), temperature (e.g. Shahid, 2010; Nasher and Uddin, 2013; Rahman, 2016; Syed and Al Amin, 2016; Kamruzzaman et al., 2016a), P\textsubscript{ET} (Hasan et al., 2014; Acharjee, 2017) in Bangladesh. Karim et al. (2012) study the WBCs like P\textsubscript{ET}, AET, deficit and surplus of water of 12 districts in Bangladesh and Kanoua and Merkel (2015) study the water balance of Titus Upazila (Sub-district) in Bangladesh. So far, all studies carried out on hydrological variables in Bangladesh have the following limitations: most of the studies were limited to detect trends or forecasting of rainfall and temperature and a few studies on P\textsubscript{ET} and water balance. Therefore, the present study has been conducted to detect trends and to identify periodicities in WBCs such as potential evapotranspiration (P\textsubscript{ET}), actual evapotranspiration (A\textsubscript{ET}), annual deficit and surplus of water by co-utilizing DWT and different forms of Mann-Kendal (MK) test in the western part of Bangladesh; and to develop WD-ARIMA models for forecasting the WBCs. To date, there is no comprehensive study that couples wavelet denoising methods with ARIMA models for forecasting WBCs. Wavelet denoising methods are widely used in many other engineering and scientific fields; however, they have been little used in hydrology (Sang, 2013). Hence, it is expected that the new combinations will better explore insight the water balance components which will ultimately help policymakers to prepare sustainable water resources management plans.

Comments 2 to 6:

Comment 2: The first half of Section 2.3.1 is probably superfluous: it is well known that Penman-Monteith is the most appropriate method to use to calculate PET, data permitting.

Comment 3: Line 137: I’m not exactly sure what ‘Deficit’ and ‘Surplus’ mean in this context (nor is it clear why they are capitalised) – provide additional explanation.

Comment 4: Line 137-139: It is presented as a fact that ‘the concept of water balance in the unsaturated zone…give the best estimation for the real world’ - this is quite a statement and surely unjustified. I note that Bakundukize et al 2011 were investigating hydrology in Burundi – are there similarities to Bangladesh? Provide some additional arguments for using the Thornthwaite and Mather model.

Comment 5: Line 143: Wolock and McCabe (1999) examined hydrology in the United States – is it reasonable to assume a 5% runoff in Bangladesh, given its tropical climate?

Comment 6: Line 145-151: Express the water balance model as equations. The calculation of the water balance is fundamental to the subsequent analysis, so it should be clear what you have done.

Reply to comments 2-6:

Thank you very much for your valuable comments. These comments are related to the section ‘Calculation of P\textsubscript{ET} and WBC (2.3.1)’. We will rewrite this section following your suggestions. Line 137.-we will also add a brief description of A\textsubscript{ET}, deficit and surplus of water. In line 143, firstly, direct runoff (DRO) is not the total runoff. It is the fraction of rainfall that immediately enters low-lying areas and/or stream channels because of infiltration-excess flow is known as DRO. “The fraction of P\textsubscript{rain} that becomes DRO is specified; based on previous water-balance analyses, 5 percent is a typical value to use (Wolock and McCabe, 1999)”. This concept has also been applied to estimate the direct runoff in Bangladesh and yields good results (Karim et al., 2012; Kanoua and Merkel, 2015). About line 145-151, we also agree with you and grateful to you for your critical
findings. We will add the equations of water balance components in the main manuscript. However, we may not add the Penman-Monteith equation (Allen et al., 1998) as it is a well-established method.

**Action:** We have rewritten the section 2.3.1 following the reviewer’s suggestions.

### 2.3.1 Calculation of Potential evapotranspiration and Water Balance Components

Potential evapotranspiration ($P_{ET}$) is the key parameter to estimate WBCs. It has been calculated by Penman-Monteith equation (Allen et al., 1998) in the present study. The soil-water balance concept proposed by Thornthwaite and Mather (1955) is one of the most widely used methods for estimating the WBCs. It is suitable for assessing the effectiveness of agricultural water resources management practices and regional water balance studies as it allows estimating the actual evapotranspiration ($A_{ET}$), water deficit and surplus (e.g., Chapman and Brown 1966, Bakundukize et al., 2011, Karim et al., 2012, Viaroli et al., 2017). $A_{ET}$ is the amount of water which is removed from the surface due to the process of evaporation and transpiration. The amount by which $P_{ET}$ exceeds $A_{ET}$ is termed as deficit and surplus is the excess rainfall after the soil has reached its water holding capacity (de Jong and Bootsma, 1997). It is necessary to calculate the field capacity of the soil for estimating the WBCs. Field capacity of soil in the study area has been calculated using the soil texture map of Bangladesh prepared by Soil Resource Development Institute Bangladesh (SRDI, 1998) where the description of soils has been presented by Huq and Shoaib (2013). The values for water holding capacity of soil and rooting depth of the plants suggested by Thornthwaite and Mather (1957) have been used for WBCs estimation in the present study. The first step of the calculation is the subtraction of 5% rainfall from the monthly rainfall data as this amount of water has been lost due to direct runoff (Wolock and McCabe, 1999; Karim et al., 2012; Kanoua and Merkel, 2015). The remaining amount of rainfall has been included in the calculation. The WBCs like $A_{ET}$, surplus and deficit have been estimated based on the formulas presented in Table 1 and details of WBCs calculation can be found in Electronically Supplementary Martial (EMS).

### Table 1: Calculations of water balance components (Thornthwaite and Mather, 1957)

<table>
<thead>
<tr>
<th></th>
<th>Wet months ($P - R_0 &gt; P_{ET}$)</th>
<th>Dry months ($P - R_0 &lt; P_{ET}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{ET}$</td>
<td>$P_{ET}$</td>
<td>$(P - R_0) + \Delta S_B$</td>
</tr>
<tr>
<td>Deficit</td>
<td>0</td>
<td>$P_{ET} - A_{ET}$</td>
</tr>
<tr>
<td>Surplus</td>
<td>$(P - R_0) - P_{ET}$</td>
<td>0</td>
</tr>
</tbody>
</table>

Where $P$ is the rainfall (mm), $R_0$ is the direct runoff (mm), $P_{ET}$ is the potential evapotranspiration (mm), $A_{ET}$ is the actual evapotranspiration (mm) and $\Delta S_B$ is the changes in soil moisture storage (mm).

**Comment 7.** In my view Section 2 would benefit from a short overview describing the reason for carrying out the various steps (water balance, Mann-Kendal, wavelet analysis, ARIMA) and how they relate to one another. At the moment this is not clear.

**Reply 7:** We will incorporate a short overview in the section 2.3. However, there are descriptions on water balance, wavelet analysis and ARIMA model in sections 2.3.1, 2.3.3 and 2.3.4 respectively. Therefore, we will only revise the trend test section (2.3.2).

**Action:** We have written a short overview of the methods in section 2.3 and rewritten the section 2.3.2 following the reviewer’s suggestions.
2.3 Methods

In the present study, WBCs have been calculated and trends in WBCs have been identified by MK/MMK test for evaluating the long-term water balance of the highly irrigated western part of Bangladesh. DWT data of WBCs time series has been analyzed for identifying the time period responsible for the trend in the data. WBCs have been forecasted by ARIMA models and the model performance has been evaluated statistically. If the performance of the model is not satisfactory for forecasting the WBCs, the denoising of original time series has been done using discrete wavelet transformation techniques to improve the performance of the model. The descriptions of the methods have been presented in the following sections.

2.3.2 Trend Test

In the present study, the trends in WBCs have been detected by non-parametric Mann–Kendall (MK) (Mann, 1945; Kendall, 1975) test as it shows better performance to identify trends in hydrological variables like rainfall (e.g. Shahid, 2010), temperature (e.g. Kamruzzaman et al., 2016a), P<sub>ET</sub> (e.g. Kumar et al., 2016), soil moisture (e.g. Tabari and Talaei, 2013), runoff (e.g. Pathak et al., 2016), groundwater level (e.g. Rahman et al., 2016), water quality (e.g. Lutz et al., 2016) in comparison to the parametric test (Nalley et al., 2012). MK test cannot appropriately calculate the test statistic (Z) due to underestimating the variance (Hamed and Rao, 1998) if there is a significant serial correlation at lag-1 in the time series data (Yue et al., 2002). The lag-1 auto-correlation has been checked before analyzing the time series data if there is a significant lag-1 auto-correlation at 5% level, the Modified MK test (Hamed and Rao, 1998) has been applied instead of MK test. The estimated Z statistic of MK/MMK test has been evaluated for the direction of the trend such as positive Z statistic to indicate increasing trend and vice versa. Moreover, it also indicates the level of significance of the obtained trend, for example, if the calculated Z statistic is equal to or greater than the tabulated value of Z statistic +1.96 that indicates a significant positive trend at 95% confidence level or if it is equal to or less than -1.96 that indicates a significant decreasing trend. Moreover, the sequential values of u(t) statistic of MK test derived from the progressive analysis of MK test (Sneyers, 1990), u(t) is similar to the Z statistic (Partal and Küçük, 2006), have been used for investigating the change point detection. The magnitude of the change has been calculated by Sen’s slope estimator (Sen, 1968). There are many good explanations (notably Nalley et al., 2012) of these methods mentioned in this section and details regarding these, furthermore, can be referred to Mann (1945); Sen (1968); Kendall (1971); Hamed and Rao (1998); Sneyers (1990); Yue et al. (2002).

Comment 8. Line 154: Which hydrological variables were investigated?

Reply 8: Hydrological variables like rainfall, temperature, P<sub>ET</sub>, runoff, groundwater level and water quality have been investigated by MK test to detect trends in time series data. We have mentioned about these in reply 7 (revised section 2.3.2).

Action: As we mentioned in reply 7, we have rewritten the section 2.3.2.

Comment 9. Line 159: What are these ‘Z’ values, and what is their importance? This is the first time they have been mentioned.

Reply 9: Thank you very much for noticing the Z statistic. We have incorporated text about Z statistic in reply 7 (revised section 2.3.2).

Action: As mentioned earlier, we have rewritten the section 2.3.2 and added necessary text on Z statistic.
Comment 10. Section 2.3.7 is unnecessary here unless it specifically influences the scientific results. Instead put this information in an Appendix or similar (however, I congratulate the authors for putting their computer code alongside the paper – this is not done often enough).

Reply 10: This section will be moved to electronical supplementary material (ESM).

Action: This section will be moved to electronical supplementary material.

Comment 11: I think Section 3 should simply describe the results, with an additional ‘Discussion’ section for placing the results in the context of other studies (e.g. Line 281, 288, 321 etc. should be put in a Discussion section). The discussion should include additional analysis discussing the various limitations and weaknesses of the present study as well as suggesting improvements.

Reply 11: We are grateful to you for your valuable comments. We will incorporate a discussion section (3.4) after the results of analysis following your suggestions. We also hope that this section will help readers about the results described in the manuscript and how can we improve the performance of the model.

Action: We have written a discussion section (3.4) following the suggestions of the reviewers.

3.4 Discussion

The present study reveals that a decreasing trend in $P_{ET}$ dominates over the study area. However, positive trends in rainfall and temperature dominate in the western part of Bangladesh (e.g. Shahid and Khairelmian, 2009; Kamruzzaman et al., 2016a). Moreover, a recent study has also found a negative trend in evapotranspiration in four stations located in northwest Bangladesh (Acharjee et al., 2017). Though annual rainfall and temperature of Satkhira station show positive trends (Kamruzzaman et al., 2016a), $P_{ET}$ shows a significant downward trend. Increasing trends in temperature have been found in Yunnan Province of South China, but $P_{ET}$ shows decreasing trend (Fan and Thomas, 2012). McVicar et al. (2012) have also found decreasing trends in $P_{ET}$ in the different parts of the world. Therefore, temperature-based models for the estimation of $P_{ET}$ cannot well explain the causes of changes in $P_{ET}$, though the temperature is the primary driver of changes in $P_{ET}$ (IPCC, 2007). To get a detailed idea about the underlying mechanisms of changes in $P_{ET}$, it is necessary to do a detailed analysis of all climatic variables such as rainfall, temperature, sunshine hours, wind speed, humidity and climate controlling phenomena like El Niño Southern Oscillations (ENSO).

The study has also developed WD-ARIMA models for forecasting the WBCs. The performance of the model shows the benefit of denoising of hydrological time series data like $P_{ET}$, $A_{ET}$, surplus and deficit. However, the model performance analysis criterion like NSE indicates that the performance of the model for $P_{ET}$ forecasting is acceptable ($NSE \geq 0.65$). To have a closer look at the forecasted values and actual values, the deviation between forecast values and actual values increases with increasing time steps. Therefore, WD-ARIMA models are not suitable for long-term forecasting. The present study has developed the WD-ARIMA model by coupling the discrete wavelet denoise time series data and ARIMA model. The soft threshold method has been selected for denoising the time series data and universal threshold (UT) method which has been used for the determination of the threshold value. However, there are some approaches for threshold value determination such as SURE (Stein, 1981), MINMAX (Donoho and Johnstone, 1998) and so on. Moreover, Wang et al. (2014) develop a hybrid approach for denoising the hydro-meteorological time series such as rainfall and streamflow called adaptive wavelet de-noising approach using sample entropy (AWDA-SE). The study has shown that the performance of the developed denoising method is better than conventional de-noising methods for denoising rainfall and streamflow. These approaches may apply to increase the performance of ARIMA models for forecasting hydrological variables like $P_{ET}$. Moreover, there are several mother wavelet families such as
Daubechies, Harr, Coiflets, Morlet, Mexican Hat and so on (Sang, 2013). In the present study, only Daubechies-6 from Daubechies wavelet family has been applied as mother wavelet of discrete wavelet transformation. WD-ARIMA models for forecasting the A\textsubscript{ET}, surplus and deficit show very good performance, whereas the classical ARIMA model shows poor performance or unable to forecast the WBCs. Moreover, studies (e.g. Chou, 2011; Kisi, 2008; Partla, 2009; Santos and da Silva, 2014; Rahman and Hasan, 2014; Nury et al., 2016; Adamowski and Chan, 2011; Khalek and Ali, 2016) have also mentioned that the performance of wavelet aided models for forecasting non-stationary hydro-meteorological variables is better than classical ARIMA and ANN models. As the traditional methods such as Wiener filtering, Kalman filtering, Fourier transform are not suitable for non-stationary hydrological time series data (Adamowski and Chan, 2011; Sang, 2013), wavelet denoising can be used to improve the performance of the classical ARIMA models for forecasting hydrological variables.


Reply 12: We will incorporate your suggestion. We will replace \( P_{ET} \) by Potential Evapotranspiration (3.2.1) and \( A_{ET} \) by Actual Evapotranspiration (3.2.2).

Action: We will replace \( P_{ET} \) by Potential Evapotranspiration (3.2.1) and \( A_{ET} \) by Actual Evapotranspiration (3.2.2) in the heading.

Comment 13. Line 359: This is just a piece of computer code – what does it do, and what insight does it provide that you cannot gain from manual interpretation of ACF, PACF, AIC, BIC?

Reply to Comment 13: ACF, PACF, AIC, BIC are important parameters for selection of an accurate ARIMA model for forecasting. For manual model sections, we need to find out the best combinations of these parameters with acceptable error. Besides manual model selections, automatic model selection option of the forecast package of R (R-language software) has been used in the present study. This option helps us find out the best model, especially when we could not find a satisfactory model (model with acceptable error) by manual interpretation of ACF, PACF, AIC and BIC.

Action: We have added the answer here for the reviewer.

Comment 14. Line 362: What is a Q-Q plot?

Reply 14: The quantile-quantile (Q-Q) plot is a probability plot to check the hypothesis of normality for a certain samples. It is graphical method which compares probability distributions based on the quantile values (Filliben, 1975). In our study, we have prepared Q-Q plot to check the normality of residuals.

Action: We have added the answer here for the reviewer.

Comment 15. Line 386-416: To my mind this passage is the strongest part of the paper -the discussion should emphasise this result and its relevance to water resources management more generally.

Reply to Comment 15: Thank you very much again for your observations and comments. We will add a discussion section as we have mentioned and added in reply 11.

Action: As mentioned earlier, we have added a discussion section (3.4). Please go to the reply 11.

Comment 16: As I mentioned earlier, I would strongly suggest creating an additional Discussion section in which to discuss the results in the context of other studies, highlight limitations and propose future research directions.

Reply to Comment 16: We have mentioned the matter earlier. We are grateful to you for your comments that help us improve the quality of our present research work. Thank you very much again.

Action: As mentioned earlier, we have added a discussion section (3.4). Please go to the reply 11.


Bakundukize, C., CAMP, M. V. and Walraevens, K. Estimation of groundwater recharge in Bugesera region (Burundi) using soil moisture budget approach, Geologia Belgica, 14/1–2, 85–102, 2011.


IPCC (Inter-governmental Panel on Climate Change). In: Solomon, S. et al. (eds.) Technical summary of climate change 2007: the physical science basis. Contribution of working group I to the fourth assessment report of the intergovernmental panel on climate change. Cambridge University Press, Cambridge, 2007


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Modeling the Changes in Water Balance Components of Highly Irrigated Western Part of Bangladesh


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Abstract. The objectives of the present study are to explore the changes in water balance components (WBCs) by co-utilizing Discrete Wavelet Transformation (DWT) and different forms of Mann–Kendal (MK) test and to develop Wavelet Denoise Autoregressive Integrated Moving Average (WD-ARIMA) models for forecasting the WBCs. The results reveal that most of the trends (about 73%) identified in potential evapotranspiration (PET) have decreasing tendency during the hydrological years 1981-82 to 2012-13 in the western part of Bangladesh. However, most of the trends (about 82%) are not statistically significant at 5% level of significance. Actual evapotranspiration (AET), annual deficit and annual surplus also show almost similar tendency. Rainfall and temperature show increasing trends, but WBCs show inverse tendency suggesting traditional concept of change in PET associated with changes in temperature, those cannot explain the change in WBCs. Moreover, it is found that generally 8-year (D3) to 16-year (D4) periodic components are effective components and are responsible for trends found in original data of WBCs in the area. The wavelet denoising of WBCs time series has been done to improve the performance of ARIMA model as actual data affected by noise and show unsatisfactory model performance. The quality of denoising time series data has been ensured by relevant statistical analysis. The performance of WD-ARIMA model has been assessed by Nash–Sutcliffe Efficiency (NSE) coefficient and coefficient of determination (R²). The WD-ARIMA model shows acceptability with very good performance that clearly demonstrates the advantages of denoising of the time series data for forecasting WBCs. The validation results of models reveal that the forecasted values are very close to actual ones with acceptable mean percentage error, and residuals also follow the normal distribution. The performance and validation results indicate that models can be used for short-term forecasting of WBCs. Further studies on different combinations of wavelet analysis would be facilitated to develop better model for hydrological forecasting in context of climate change, and findings of the study can be used to improve the water resources management in highly irrigated western part of Bangladesh.

Keywords: Discrete Wavelet Transformation, Wavelet Denoising, Water Balance, ARIMA Model
1. Introduction

After introducing the monthly water balance model by Thronthwaite (1948) and afterward followed by Thorntwaite and Mather (1957), this model is going through modifications for adaptation in the different areas of the world. The development of the new model is still ongoing (Xu and Singh, 1998) as the water balance model is significantly important in water resources management, irrigation scheduling and crop pattern designing (Kang et al., 2003; Valipour, 2012). Moreover, it can be used for the reconstruction of catchment hydrology, climate change impact assessment and streamflow forecasting (e.g. Alley, 1985; Arnall, 1992, Xu and Halldin, 1996; Molden and Sakthivadivel, 1999; Boughton, 2004; Anderson et al., 2006; Healy et al., 2007; Moriarty et al., 2007; Karimi et al., 2013). Therefore, detecting the changes in WBCs and more accurate forecasting of WBCs are important for achieving the sustainability of water resources management. However, hydro-meteorological time series are contaminated by noises from hydro-physical processes that affect the accuracy of analysis, simulation and forecasting (Sang et al., 2013 and Wang et al., 2014). Hence, it is necessary to denoise the time series for improving the accuracy of the obtained results. In the present study, wavelet denoising technique has been coupled with ARIMA models for forecasting the WBCs after detecting the changes in WBCs by different forms of MK tests and identifying the time period responsible for trends in WBCs time series using DWT time series data.

Generally, physics based numerical models are used for understanding a particular hydrological system and forecasting the water balance or budget (e.g. Fulton et al., 2015, Leta et al., 2016) components. In this method, for reliable forecasting, a large amount of hydrological data is required to assign physical properties of the grid and model parameters and to calibrate the model simulation. However, they have a number of limitations in practice including the cost, time and availability of the data (Yoon et al., 2011; Adamowski and Chan, 2011). Data based forecasting models, statistical models, are suitable alternatives to overcome these problems. The most common statistical methods for hydrological forecasting are ARIMA models and multiple linear regression (Young, 1999; Adamowski, 2007). Many studies use ARIMA model to predict water balance input parameters like rainfall (e.g., Rahman et al., 2015; Rahman et al., 2016), temperature (e.g. Nury et al., 2016) and $P_{ET}$ (e.g., Valipour, 2012). However, ARIMA model cannot handle non-stationary hydrological data without preprocessing of the input time series data (Tiwari and Chatterjee, 2010; Adamowski and Chan, 2011). Wavelet analysis, a new method in the area of hydrological research, is such a method that is able to handle non-stationary data effectively (Adamowski and Chan, 2011). However, over the course of time some research works have already been done. For example, Adamowski and Chan (2011) coupled wavelet analysis with Artificial Neural Network (ANN) models for forecasting the hydrological variables like groundwater level in Quebec, Canada. Kisi (2008) and Partla (2009) and Santos and da Silva (2014) develop a hybrid wavelet ANN models for monthly and daily streamflow forecasting respectively. A study conducted by Rahman and Hasan (2014) also finds that the performance of the wavelet-based ARIMA models is better than the classical ARIMA model for forecasting the humidity of Rajshahi meteorological station in Bangladesh. A comparative study of wavelet ARIMA models and wavelet ANN models has been conducted by Nury et al. (2017). The study shows that the wavelet ARIMA models are more effective than the wavelet ANN for temperature forecasting. Khalek and Ali (2016) developed wavelet seasonal ARIMA (W-SARIMA) and neural network autoregressive (W-NNAR) model for forecasting the groundwater level. The study also finds that the performance of W-SARIMA
model is better than the performance of W-NNAR models. All of these studies mentioned above find that the
performance of wavelet aided model is better than classical ARIMA models and ANN models. Moreover, the
analysis of periodicity using wavelet transformed details, and approximation components of hydro-
meteorological time series data can better provide insight into trends and effects of time period on trend (e.g.
Nalley et al., 2013; Araghi et al., 2014; Pathak et al., 2016). As a result, wavelet transformation of hydro-
meteorological time series is gaining popularity in recent years to detect periodicity (e.g. Partal and Küçük,
2006; Partal, 2009; Nalley et al., 2013; Araghi et al., 2014; Pathak et al., 2016). Some studies have been
conducted on spatio-temporal characteristics of hydro-meteorological variables such as rainfall (e.g. Shahid and
Khairulmaini, 2009; McSweeney et al., 2010; Ahasan et al., 2010; Kamruzzaman et al., 2016a, Rahman and
Lateh, 2016; Rahman et al., 2016; Syed and Al Amin, 2016), temperature (e.g. Shahid, 2010; Nasher and Uddin,
2013; Rahman, 2016; Syed and Al Amin, 2016; Kamruzzaman et al., 2016a). $P_{ET}$ (Hasan et al., 2014; Acharjee,
2017) in Bangladesh. Karim et al. (2012) study the WBCs like $P_{ET}$, AET, deficit and surplus of water of 12
districts in Bangladesh and Kanoua and Merkel (2015) study the water balance of Titas Upazila (Sub-district) in
Bangladesh. So far, all studies carried out on hydrological variables in Bangladesh have the following
limitations: most of the studies were limited to detect trends or forecasting of rainfall and temperature and a few
studies on $P_{ET}$ and water balance. Therefore, the present study has been conducted to detect trends and to
identify periodicities in WBCs such as potential evapotranspiration ($P_{ET}$), actual evapotranspiration ($A_{ET}$),
annual deficit and surplus of water by co-utilizing DWT and different forms of Mann-Kendal (MK) test in the
western part of Bangladesh; and to develop WD-ARIMA models for forecasting the WBCs. To date, there is no
comprehensive study that couples wavelet denoising methods with ARIMA models for forecasting WBCs.
Wavelet denoising methods are widely used in many other engineering and scientific fields; however, they have
been little used in hydrology (Sang, 2013). Hence, it is expected that the new combinations will better explore
insight the water balance components which will ultimately help policymakers to prepare sustainable water
resources management plans.

2. Study Area, Data and Methods

2.1 Study area

Bangladesh enjoys a humid, warm and tropical climate. The western part of Bangladesh covers about 41% or
60,165 km$^2$ of the country. The geographic coordinates of the study area extends between 21°36’-26°38’N
latitude and 88°19’-91°01’E longitude. Annual rainfall and average temperature in the area vary from 1492 to
2766 mm with an average of 1925 mm and 24.18 to 26.17°C with an average of 25.44°C respectively
(Kamruzzaman et al., 2016a). Bangladesh is the fourth biggest rice producing country in the world (Scott and
Sharma, 2009) and the livelihoods of the majority of the people (about 75%, Shahid and Behrawan, 2008;
Kamruzzaman et al., 2016b) are related to agricultural practices. Crop calendar of Bangladesh is related to the
climatic seasons. Rice grows in three seasons (Aus, Aman and Boro seasons) in Bangladesh. Almost 73.94%
cultivable area is used for Boro rice cultivation in the country (Banglapedia, 2003). Aus and Aman rice are
mainly rain-fed crops; however, Boro rice is almost groundwater-fed (Ravenscroft et al., 2005) and requires
about 1m of water per square meter in Bangladesh (Harvey et al., 2006; Michael and Voss, 2009).

2.2 Data
National climate database of Bangladesh prepared by Bangladesh Agricultural Research Council (BARC) has been used for the study. The database is available for research and can be found in BARC website (http://climate.barcapps.gov.bd/). The database has been prepared from the data recorded by Bangladesh Meteorological Division and contains long-term monthly climate data such as rainfall, minimum, maximum and average temperatures, humidity, sunshine hours, wind speed and cloud cover. The locations of the meteorological stations in the study area are shown in Figure 1. The data has been rearranged following the hydrological year for the period 1981-82 to 2012-13. The hydrological year in Bangladesh starts in April and ends in March.

2.3 Methods

In the present study, WBCs have been calculated and trends in WBCs have been identified by MK/MMK test for evaluating the long-term water balance of the highly irrigated western part of Bangladesh. DWT data of WBCs time series has been analyzed for identifying the time period responsible for the trend in the data. WBCs have been forecasted by ARIMA models and the model performance has been evaluated statistically. If the performance of the model is not satisfactory for forecasting the WBCs, the denoising of original time series has been done using discrete wavelet transformation techniques to improve the performance of the model. The descriptions of the methods have been presented in the following sections.

2.3.1 Calculation of Potential Evapotranspiration and Water Balance Components

Potential evapotranspiration ($P_{ET}$) is the key parameter to estimate WBCs. It has been calculated by Penman-Monteith equation (Allen et al., 1998) in the present study. The soil-water balance concept proposed by Thornthwaite and Mather (1955) is one of the most widely used methods for estimating the WBCs. It is suitable for assessing the effectiveness of agricultural water resources management practices and regional water balance studies as it allows estimating the actual evapotranspiration ($A_{ET}$), water deficit and surplus (e.g., Chapman and Brown 1966, Bakundukize et al., 2011, Karim et al., 2012, Viaroli et al., 2017). $A_{ET}$ is the amount of water which is removed from the surface due to the process of evaporation and transpiration. The amount by which $P_{ET}$ exceeds $A_{ET}$ is termed as deficit and surplus is the excess rainfall after the soil has reached its water holding capacity (de Jong and Bootsma, 1997). It is necessary to calculate the field capacity of the soil for estimating the WBCs. Field capacity of soil in the study area has been calculated using the soil texture map of Bangladesh prepared by Soil Resource Development Institute Bangladesh (SRDI, 1998) where the description of soils has been presented by Huq and Shoaib (2013). The values for water holding capacity of soil and rooting depth of the plants suggested by Thornthwaite and Mather (1957) have been used for WBCs estimation in the present study. The first step of the calculation is the subtraction of 5% rainfall from the monthly rainfall data as this amount of water has been lost due to direct runoff (Wolock and McCabe, 1999; Karim et al., 2012; Kanoua and Merkel, 2015). The remaining amount of rainfall has been included in the calculation. The WBCs like $A_{ET}$, surplus and deficit have been estimated based on the formulas presented in Table 1 and details of WBCs calculation can be found in Electronically Supplementary Martial (EMS).

2.3.2 Trend Test

In the present study, the trends in WBCs have been detected by non-parametric Mann–Kendall (MK) (Mann, 1945; Kendall, 1975) test as it shows better performance to identify trends in hydrological variables like rainfall.
(e.g. Shahid, 2010), temperature (e.g. Kamruzzaman et al., 2016), soil moisture (e.g. Tabari and Talaee, 2013), runoff (e.g. Pathak et al., 2016), groundwater level (e.g. Rahman et al., 2016), water quality (e.g. Lutz et al., 2016) in comparison to the parametric test (Nalley et al., 2012). MK test cannot appropriately calculate the test statistic (Z) due to underestimating the variance (Hamed and Rao, 1998) if there is a significant serial correlation at lag-1 in the time series data (Yue et al., 2002). The lag-1 auto-correlation has been checked before analyzing the time series data if there is a significant lag-1 auto-correlation at 5% level, the Modified MK test (Hamed and Rao, 1998) has been applied instead of MK test. The estimated Z statistic of MK/MMK test has been evaluated for the direction of the trend such as positive Z statistic to indicate increasing trend and vice versa. Moreover, it also indicates the level of significance of the obtained trend, for example, if the calculated Z statistic is equal to or greater than the tabulated value of Z statistic +1.96 that indicates a significant positive trend at 95% confidence level or if it is equal to or less than -1.96 that indicates a significant decreasing trend. Moreover, the sequential values of $u(t)$ statistic of MK test derived from the progressive analysis of MK test (Sneyers, 1990), $u(t)$ is similar to the Z statistic (Partal and Küçük, 2006), have been used for investigating the change point detection. The magnitude of the change has been calculated by Sen’s slope estimator (Sen, 1968). There are many good explanations (notably Nalley et al., 2012) of these methods mentioned in this section and details regarding these, furthermore, can be referred to Mann (1945); Sen (1968); Kendall (1971); Hamed and Rao (1998); Sneyers (1990); Yue et al. (2002).

### 2.3.3 Wavelet Transform and Periodicity

The wavelet analysis has been used to identify periodicity in hydro-climatic time series data (e.g., Smith et al., 1998; Azad et al., 2015; Nalley et al., 2012; Araghi et al., 2014; Pathak et al., 2016) for different parts of the world. Wavelet transform (WT), a multi-resolution analytical approach, can be applied to analyze time series data as it offers flexible window functions that can be changed over time (Nievergelt, 2001; Percival and Walden, 2000). It can be applied to detect the periodicity in hydro-climatic time series data (Smith et al., 1998; Pišoft et al., 2014; Torrence and Compo, 1998; Araghi et al., 2014; Pathak et al., 2016) and produces better performances in comparison to traditional approaches (Sang, 2013). There are two main kinds of wavelet transform such as continuous wavelet transform (CWT) and discrete wavelet transform (DWT). The application CWT is complex, as it produces a lot of coefficients (Torrence and Compo, 1998; Araghi et al., 2014), whereas DWT is simple and useful for hydro-climatic analysis (Partal and Küçük, 2006; Nalley et al., 2012). The wavelet coefficients following the DTW with dyadic format can be calculated as (Mallat, 1989):

$$
\psi_{m,n}(t) = s_{0}^{-1/2} \psi \left( \frac{t - n \tau_{0} s_{0}^{m}}{s_{0}^{m}} \right) \quad \ldots \quad \ldots \quad \ldots \quad (1)
$$

Where $\psi$ is the mother wavelet, the integers $m$ and $n$ are wavelet dilation and translation respectively. Specified fixed dilation step ($s_{0}$) is greater than 1 and $\tau_{0}$ is location parameter. For the practical application, the values of parameters $s_{0}$ and $\tau_{0}$ are considered as 2 and 1 respectively (Partal and Küçük, 2006; Pathak 2016). After substituting these values in equation (1), the DWT for a time series $x_{t}$ becomes:

$$
W_{m,n} = 2^{-m/2} \sum_{i=0}^{N-1} x_{i} \psi(2^{-m} i - n) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2)
$$

Where $W$ indicates wavelet coefficient at scale $s = 2^{m}$ and location $\tau = 2^{m} n$. 

In the DWT, details (D) and approximations (A) time series can emerge from the original time series after passing through low-pass and high-pass filters respectively. While approximations are the high scale and low-frequency components, details are the low scale and high-frequency components. Successive, iterations have been performed to decompose the time series into their several lower resolution components (Mallat, 1989; Misiti et al., 1997). In the present study, four levels (D1-D4) of decompositions have been performed following the dyadic scales and referred as D1, D2, D3 and D4 which are corresponds to 2, 4, 8 and 16 year periodicity. Daubechies wavelet has been used in the present study as it performs better in hydro-meteorological studies (Nalley et al., 2012, 2013; Ramana et al., 2013; Araghi et al., 2014). To confirm about the periodicity present in the time series, correlation coefficient (C0) between u(t) of original data, u(t) of decomposition (D) time series data and different models (D1+A……D4+D3+A) time series data have been calculated and the obtained results have been compared accordingly (Partal and Küçük, 2006; Partal, 2009).

2.3.4 ARIMA Models

To identify the complex pattern in data and to project the future scenario, ARIMA model (Box and Jenkins, 1976) has been used in hydrological science (e.g. Adamowski and Chan, 2011; Valipour et al., 2013; Nury et al., 2017; Khalek and Ali, 2016). The method includes three terms: (1) an autoregressive process (AR) represented by order-p, (2) nonseasonal differences for non-stationary data termed as order-d and (3) moving average process (MA) represented by order-q. ARIMA model of order (p, d, q) can be written as:

\[ \phi_p(L) (1 - L)^d Y_t = \theta_0 + \theta_q(L) U_t \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3) \]

Where, \( \theta_0 \) and \( U_t \) are the intercept and white process with zero mean and constant variance respectively. \( \phi_p(L) \) stands for AR term \((1 - \phi_1 L - \cdots - \phi_p L^p)\) and \( \theta_q(L) \) represents MA term \((1 - \theta_1 L - \cdots - \theta_p L^p)\).

2.3.5 Wavelet Denoising

Wavelet de-noising based on thresholds introduced by Donoho et al. (1995) has been applied to the hydro-meteorological analysis (Wang et al., 2005 and 2014; Chou, 2011). In the present study, three-steps of analysis has been done for denoising the time series data as follows:

1. Decomposing the time series data \( x(t) \) into \( M \) resolution level for obtaining the detail coefficients \( (W_{f,k}) \) and approximation coefficients using DWT.
2. The detail coefficients obtained from DWT (1 to \( M \) levels) have been treated with threshold \((T_j)\) selection. There are soft threshold and hard threshold to deal with detail coefficients and to get decomposed coefficient. In the present study, soft threshold has been selected as it’s performs better than hard (Wang et al., 2014; Chou, 2011):

   Soft threshold processing:
   \[ W'_{f,k} = \begin{cases} 
   sgn(W_{f,k}) \left( |W_{f,k}| - T_j \right) & \text{if} \ |W_{f,k}| > T_j \\
   0 & \text{if} \ |W_{f,k}| < T_j 
   \end{cases} \]

3. Details coefficients from 1 to \( M \) level and approximate coefficients at level \( M \) have been reconstructed to get denoising time series data.
It is also necessary to select the threshold value for denoising the data. In the present study, Universal threshold
(UT) method (Donoho and Johnstone, 1994) has been used for estimating the threshold value as it shows good
performance in analyzing hydro-meteorological data (Wang et al., 2005; Chou, 2011).

2.3.6 Assessment of Model Performance

There are several indicators to assess the performance of the models. Nash–Sutcliffe Efficiency (NSE) (Nash
and Sutcliffe, 1970) coefficient, a normalized goodness-of-fit statistic, is the most powerful and popular method
for measuring the performance of hydrological models (McCuen et al., 2006; Moussa, 2010; Ritter and Muñoz-
Carpena, 2013). To evaluate and make a comparison between ARIMA and WD-ARIMA model, NSE has been
used in the study. NSE can be calculated as (Nash and Sutcliffe, 1970):

\[
NSE = 1 - \frac{\sum_{i=1}^{N}(O_i - P_i)^2}{\sum_{i=1}^{N}(O_i - \bar{O})^2} = 1 - \left( \frac{RMSE}{SD} \right)^2
\]

Where, \(N\), \(O_i\) and \(P_i\) are the sample size, number of observation and model estimates respectively and
\(\bar{O}\) and \(SD\) are the mean and standard deviation of the observed values. The performance of a model can be
evaluated based on NSE value as: very good (NSE ≥ 0.90); good (NSE = 0.80-0.90); acceptable (NSE ≥ 0.65);
and unsatisfactory (NSE<0.65) (Ritter and Muñoz-Carpena, 2013). \(E_{RMS}\) is the root mean square error that can
be calculated as:

\[
E_{RMS} = \sqrt{\frac{\sum_{i=1}^{N}(O_i - P_i)^2}{N}}
\]

The coefficient of determination \((R^2)\) is another goodness of fit test to measure the performance of the models.
The perfect fit of the model draws a line between the actual values and fitted values, where \(R^2\) value is 1. If \(y_i\) is
the observation data, \(\hat{y}_i\) is the model forecasted values of \(y_i\) and \(N\) is the number of data point used, \(R^2\) can be
given as (Sreekanth et al., 2009):

\[
R^2 = 1 - \frac{\sum_{i=1}^{N}(y_i - \hat{y}_i)^2}{\sum_{i=1}^{N}(y_i - \bar{y})^2}
\]

Moreover, mean percentage error (\(E_{MP}\)) and mean error (\(E_M\)) have also been calculated to evaluate the validation
of the model for forecasting. \(E_{MP}\) reveals the percentage of bias (larger or smaller) of forecasted data over the
actual counterparts (Khalek and Ali, 2016). \(E_{MP}\) and \(E_M\) can be calculated as follows:

\[
E_{MP} = \left( \frac{1}{n} \sum_{t=1}^{n} \frac{y_t(actual) - y_t(forecasted)}{y_t(actual)} \right) \times 100\% \quad (7)
\]

\[
E_M = \frac{1}{n} \sum_{t=1}^{n} [y_t(actual) - y_t(forecasted)]^2 \quad (8)
\]

3. Results of Analysis

3.1 Exploratory Statistics of Water Balance Components
Mean annual $P_{ET}$ during the period of 1981-82 to 2012-2013 in the study area varies from 1228 to 1460 mm (Figure 2) with an average of 1338 mm. The higher $P_{ET}$ values are found in the central part of the area where the annual rainfall is lower, but the temperature is higher (Kamruzzaman et al., 2016a). The standard deviations of $P_{ET}$ vary from 205 mm (in Jessore station) to 41 mm (in Bhola station). The $A_{ET}$ value (average = 925 mm) is almost 31% less than the $P_{ET}$ value as during the dry months (Dec-May), soil moisture condition reaches in a critical stage and $A_{ET}$ value is much lower than $P_{ET}$. The annual surplus of water varies from 515 to 1277 mm with an average of 838 mm. According to Wolock and McCabe (1999), 50% of surplus water can be considered as runoff for the major parts of the world. The higher surplus amount of water has been found in the northern part of the area and along the coastal area. The annual deficit of water that mainly occurs during the dry season (Dec to May) varies from 329 to 556 mm with an average of 416 mm (Figure 2). The highest annual deficit of water found in Rajshahi which is located in the central western part of the area where the depth of groundwater below the ground surface increases rapidly (Shamsudduha et al., 2009; Rahman et al., 2016).

### 3.2 Trend and Periodicity in Water Balance Components

#### 3.2.1 Potential Evapotranspiration

The MK test or MMK test based on the lag-1 auto-correlation has been applied to detect the trend in $P_{ET}$. Table 2 shows the Z statistic of MK or MMK test of original time series data of $P_{ET}$ and Z statistic of the decomposition time series (D1-D4), approximation (A) and model (D1+A.....D3+D4+A) time series. The estimated Z statistic of original data ranges from -2.07 (Satkhira station) to 2.37 (Bhola station). These two stations out of total eleven show significant trends in $P_{ET}$. The plots of sequential $u(t)$ statistic of SMK test of these two stations are shown in Figure 3 where the dashed lines correspond to 5% significance level (±1.96).

The decreasing trend in $P_{ET}$ in Satkhira station started in the year 1985-86 and a significant decreasing trend started in 1993-94 hydrological year, and the trend become reverse after 2007-08. However, the significant increasing trend in $P_{ET}$ of Bhola station has been started very recently after some fluctuation.

Most of the trends (73%) in $P_{ET}$ in the study are negative and statistically insignificant at 95% confidence level or 5% significance level. Moreover, Z statistic of approximation (A) time series obtained by DWT indicates decreasing trends in $P_{ET}$ in all stations. The calculated Z statistic of approximation (A) time series is about -1.80 after rounding the figure for all stations as A time series data of all stations show a similar pattern (Electronic Supplementary Material (ESM) Fig. S1) over the time. The magnitude of change in $P_{ET}$ ranges from -10.89 mm/year in Satkhira station to 1.67 mm/year in Bhola station (Figure 4). The MK or MMK test has also been applied to the decomposition time series and model time series generates from the combination of approximation and decomposition time series data (Table 2 represents results of four stations based on alphabetic order and the full Table can be found in ESM Table S1). To find out the dominant periodicity affecting the trends in $P_{ET}$, two steps of analysis have been done. Firstly, the Z statistic which is the closest to the Z statistic of original time series data has been found out from the values of Z statistic of different models and decomposition (D) time series data. Secondly, the correlation coefficients ($C_o$) of pairs of data (such as $C_o$ between $u(t)$ statistics of SMK of the original time series data and $u(t)$ statistics of SMK of D time series data) have been estimated and found out the highest $C_o$ from the estimated $C_o$ values for different pairs (Table 2). For example, the Z statistic of D4 time series data of Barisal station is 0.76 which is the nearest to Z statistic (0.72)
of the original time series data among the different models (Table 2). Moreover, Z statistic of model (D3+D4+A) time series data is 0.56 which is the second nearest value to original time series with the highest correlation coefficient ($C_o = 0.85$). Again D4 is present, hence D4 (16-year) is the dominant periodic component on the trend in original data. However, D3 has also effect on the trend in the data. Therefore, D4 (16-year) is the basic periodic component, but 8-year (D3) periodicity has also effect on the trend. An additional example, Z (2.47) statistic of the original time series of Bhola station is the closest to Z (2.36) statistic of the model (D2+D4+A) time series data. However, the values of the Z statistic of D2, D4, D2+A and D4+A time series are 0.61, 1.20, 0.48 and 0.90 respectively, which are not close to the Z statistic of the original time series data. Hence, it is not clear from the Z statistic which periodic component (D2/D4) is the basic periodic component for the significant trend in the original data. To get a clear idea about the dominant periodic component, $C_o$ coefficient values have been analyzed. It is seen that the $C_o$ between $u(t)$ statistic of SMK of original time series data and $u(t)$ statistic of SMK of D4 time series data is higher than the $C_o$ between $u(t)$ statistic of SMK of original time series data and $u(t)$ statistic of SMK of D2 time series data (Table 2). Moreover, Moreover, values of Z statistic of time series with D4 components like D4 and (D4+A) model time series are higher than time series with D2 component (D2 and D2+A) (Table 2). It is, therefore, clear that D4 is the main periodic component responsible for the trend in $P_{\text{ET}}$ data of Bhola station. However, Z statistic of D4 or D4+A is not close to the Z statistic of original data (Table 2). Moreover, there is a statistically significant positive trend in original data of $P_{\text{ET}}$ of Bhola station, but the trends of D4 and (D4+A) model time series data are not statistically significant. When D2 time series add with (D4+A) model time series data, the Z statistic of the resultant (D2+D4+A) model time series data becomes very close to original time series data. The trend of (D2+D4+A) model time series is also statistically significant like the trend in original time series data (Table 2). Hence, D2 has also effect on the trend in the original time series data. Station-wise analysis indicates that almost half of the stations show the harmoniousness between the Z statistic of (D3+D4+A) model and original time series data. When D3 and D4 time series have been analyzed separately, it is found that the higher relationship exists between D4 and original time series data. Again, three stations (Dinajpur, Ishurdi and Jessore) show the similarity in estimated Z statistic of original and (D1+D4+A) model time series data with higher $C_o$ values of $u(t)$ statistic of SMK between D4 time series and original data except for the Ishurdi station. Moreover, two stations (Bhola and Satkhira) show significant trends in original data. The closest Z statistic is found between original and model (D2+D4+A) time series data for both stations. Again, D4 (16-year periodicity) is the dominant periodic component based on $C_o$ for both of these stations. Therefore, 16-year periodicity is the main periodic component which is responsible for trends in $P_{\text{ET}}$ data over the study area. Moreover, D3 (8-year) periodicity also has some effect on the trends and present in some stations (Table 2 and also see ESM Table S1). D4 (16-year) periodicity dominates in annual rainfall in Marmara region in Turkey (Partal and Küçük, 2006). Araghi et al. (2016) found that 8 to 16 year (D3 to D4) periodicity is responsible for trends in annual temperature in Iran.

### 3.2.2 Actual Evapotranspiration

All of the stations except Bogra show decreasing trends in $A_{\text{ET}}$ and the calculated Z statistic ranges from -2.90 in Bogra station to 0.31 in Ishurdi station. Similar to the trends found in $P_{\text{ET}}$, trends in $A_{\text{ET}}$ are also insignificant at 5% significance level except Ishurdi station which shows significant (at 5% significant level) decreasing trend.
The magnitudes of the trends of \textit{original} $A_{ET}$ data vary from -5 mm/year in Faridpur station to 0.75 mm/year in Bogra station. The distribution of the magnitude of the trend is shown in Figure 4b. The periodicity in $A_{ET}$ is slightly different from $P_{ET}$ (see ESM Table S2). Almost half of the (five) stations show that D2 (4-year) is the main periodic component and D4 (16-year) has also effects on trend as Z statistic of (D2+D4+A) model time series is the nearest to original series for Khulna and Ishurdi stations. Moreover, D4 (16-year) is the main periodicity for Rangpur and Rajshahi stations. In addition, D1 (2-year) is the dominant periodicity in Barisal, Bhola and Bogra stations. $A_{ET}$ depends on climatic factors such as $P_{ET}$ and rainfall as well as on soil moisture conditions. The variations in periodicity in $A_{ET}$ from $P_{ET}$, hence, are mainly related to soil moisture conditions of the area.

### 3.2.3 Surplus

Almost 82% stations show insignificant decreasing trends in annual surplus of water. The magnitude of trends of original \textit{annual surplus data} ranges from -11.63 mm/year to 6.71 mm/year (Figure 4c). There is a similarity in periodicity characteristics of $P_{ET}$ and surplus (See EMS Table S3). D4 (16-year) is the main periodic component present in seven stations and in most of the cases D2 is also present (D2+D4+A) except in Rajshahi. D3 (8-year) is mainly responsible for trend in surplus in three stations. Surplus mainly occurs during the rainy season (Jun-Oct) in the study area when soil moisture is almost full and $A_{ET}$ is equal to $P_{ET}$. Surplus mainly depends on rainfall. Therefore, it also provides an idea about the periodicity in rainfall.

### 3.2.4 Deficit

Approximately 73% stations show increasing trends in the annual deficit of water. The increasing trends are significant in two stations at 95% confidence level (see ESM Table S4). However, Satkhira station shows a significant decreasing trend ($Z = -2.08$) in deficit. The magnitude of trends of original \textit{annual deficit data} ranges from -8.1 to 7.7 mm/year (Figure 4b). The periodicity analysis reveals that D4 (16-year periodicity) is the main responsible factor for the trends in the deficit. The Z statistic of (D2+D4+A) model time series data is close to the Z statistic of original time series data (ESM Table S4). D3 (8-years periodicity) is also responsible for trends in data of two stations.

### 3.3 Model Selection and Forecasting Ability

Firstly, ARIMA model has been selected for forecasting the WBCs time series. \textit{Four-step analysis has been done} during the time series modeling: (1) stationarity in the data has been checked by Augmented (ADF) test, (2) auto-correlation function (ACF) has been used for selecting the order of MA process (see ESM Fig. S2-S5), (3) partial auto-correlation function (PACF) has been used for selecting the order of AR process (see ESM Fig. S2-S5) and (4) finally, the appropriate model has been selected based on several trials, values of model selection criteria like Akaike information criterion (AIC) and Bayesian information criterion (BIC). During the trails for selecting the model, besides the manual model selection based on ACF, PACF, AIC and BIC, the \textit{auto ARIMA function of the ‘forecast’ package} (Hyndman et al., 2017) of R (R 3.4.0 language developed by R Development Core Team, 2016) has been used to get reasonable information about the nature of the data for modeling. The best model has been selected based on lower values of AIC, BIC, and higher value of $R^2$. The Q-Q plot has been prepared to check the normality of residuals. The performance of ARIMA model (parameters can be found in ESM Table S5) has been evaluated by $NSE$ and $R^2$ (Table 3). The estimated values of $NSE$ of ARIMA model of
$P_{ET}$ time series vary from -0.60 for Bhola station to 0.81 for Jessore station (Table 3). ARIMA models for almost all stations show unsatisfactory performance as the average $NSE$ value of eleven stations is 0.38 and $R^2$ values range from 0.10 to 0.81 with an average of 0.38. Moreover, the $NSE$ value of Bhola station indicates that ARIMA model is not suitable for forecasting the $P_{ET}$. ARIMA model has also been applied to $A_{ET}$, surplus and deficit time series data. After carefully checking the ACF and PACF (see ESM Figure S2–S5) of $A_{ET}$, it is found that there are no significant spikes in ACF and PACF. Moreover, the results obtained from auto ARIMA functions also show similar results. Therefore, ARIMA model is not satisfactory for forecasting the variability or changing pattern of $A_{ET}$. For WBCs like surplus and deficit, the performance of ARIMA model is almost similar to $A_{ET}$ except for few cases. As the hydro-meteorological data are affected by noises from different hydro-physical processes (Wang et al., 2014), results obtained from ARIMA models show the unsatisfactory performance. To improve the model performance, it is necessary to remove the noise from the data. DWT denoising has been applied to the WBCs data in the present study and the quality of the denoising time series data has been checked before further processing. The important criteria to select a method for denoising the time series using wavelet transformation are the mean of the original series and denoising time series data should be close and standard deviation of denoising time series should be less than the original series (Wang et al., 2014). Figure 5(a) displays mean of the actual time series of $P_{ET}$ and mean of wavelet denoising time series of $P_{ET}$. It is seen that there are no visible differences between the mean of the original time series data and DWT wavelet denoise time series data. Moreover, the standard deviation of $P_{ET}$ of wavelet denoising time series is lower than the original time series (Figure 5b). $A_{ET}$, surplus and deficit time series also show the similar results (see ESM Figure S4–S5). Furthermore, lag-1 auto-correlation of wavelet denoise time series data must be higher than the original time series (Wang et al., 2014). For this consideration, wavelet denoise time series also shows that lag-1 absolute value of auto-correlation is higher than that of original series value [see ESM Figure S2 (b), S3 (b), S4 (b) and S5 (b)]. The performance of WD-ARIMA model is shown in Table 3. After denoising the data, the performance of ARIMA model is satisfactory for all WBCs time series data (Table 3). The average $NSE$ value of WD-ARIMA models for $P_{ET}$ time series of eleven stations located in the western part of Bangladesh is 0.76 and an average $R^2$ value is 0.67. Both performance indicators reveal that the performance of the WD-ARIMA model is better than the classical ARIMA model (Table 3). Moreover, the average $NSE$ value of WD-ARIMA models of $P_{ET}$ time series of these stations is 0.92 which indicates that the performance of the model is very good and the average $R^2$ value is 0.89 which indicates the model can explain almost 89% variance of the data (Table 3). Results obtained from WD-ARIMA models of annual surplus and annual deficit also indicate very good performance for forecasting these variables (Table 3). The average $NSE$ value of eleven stations of WD-ARIMA models for the annual surplus is about 0.92 and average $R^2$ value is 0.90. WD-ARIMA models for forecasting the annual deficit (average $NSE = 0.88$) also show good performance. The comparative study of the performance of the WD-ARIMA models of WBCs reveals that model performance is very good or good for $A_{ET}$, annual surplus and deficit. However, the performance is acceptable for $P_{ET}$. This deviation may arise from the variability of the $P_{ET}$ is higher than others WBCs or may relate to the variability of climatic variables.

Moreover, validations of the models have been done to explore the forecasting ability of the fitted models. The mean percentage error ($E_{MP}$) of the forecasted values for the four year period from 2008-09 to 2012-13 has been calculated to know the percentage bias of the forecasted data (Table 4). The average $E_{MP}$ of eleven stations of WD-ARIMA models for $P_{ET}$ is -0.6 (with ranges from 0.75 to -3.34) that indicates the forecasted values are
slietically better than conventional de-

transpiration in owever, the differences are generally increasing after these periods

Figure 6. The plot of actual versus forecasted values (Figure 6)

Moreover, the actual versus the model calculated fitted values are very close to each other. The

normal Q-Q plots reveal that the residuals of the models are near normal. The $E_{MP}$ values of WD- ARIMA

models for $A_{ET}$ range from -0.7 to 0.2 with an average of -0.09 which also indicates that forecasted $A_{ET}$ values

are slightly lower than actual $A_{ET}$ values. The $E_{MP}$ values for annual surplus (average = -0.75) and annual deficit

(average = -0.12) are almost similar to the $A_{ET}$ and $P_{ET}$. It is also notable that the average $E_{MP}$ values for all

WBCs are negative, which indicate the forecasted values of WBCs are slightly lower than the actual values for

most of the stations.

3.4 Discussion

The present study reveals that a decreasing trend in $P_{ET}$ dominates over the study area. However, positive trends

in rainfall and temperature dominate in the western part of Bangladesh (e.g. Shahid and Khairulmaini, 2009;

Kamruzzaman et al., 2016a). Moreover, a recent study has also found a negative trend in evapotranspiration in

four stations located in northwest Bangladesh (Acharjee et al., 2017). Though annual rainfall and temperature of

Satkhira station show positive trends (Kamruzzaman et al., 2016a), $P_{ET}$ shows a significant downward trend.

Increasing trends in temperature have been found in Yunnan Province of South China, but $P_{ET}$ shows decreasing
trend (Fan and Thomas, 2012). McVicar et al. (2012) have also found decreasing trends in $P_{ET}$ in the different

parts of the world. Therefore, temperature-based models for the estimation of $P_{ET}$ cannot well explain the causes

of changes in $P_{ET}$, though the temperature is the primary driver of changes in $P_{ET}$ (IPCC, 2007). To get a
detailed idea about the underlying mechanisms of changes in $P_{ET}$, it is necessary to do a detailed analysis of all

climatic variables such as rainfall, temperature, sunshine hours, wind speed, humidity and climate controlling

phenomena like El Niño Southern Oscillations (ENSO).

The study has also developed WD-ARIMA models for forecasting the WBCs. The performance of the model

shows the benefit of denoising of hydrological time series data like $P_{ET}$, $A_{ET}$, surplus and deficit. However, the

model performance analysis criterion like NSE indicates that the performance of the model for $P_{ET}$ forecasting is

acceptable ($NSE \geq 0.65$). To have a closer look at the forecasted values and actual values, the deviation between

forecast values and actual values increases with increasing time steps. Therefore, WD-ARIMA models are not

suitable for long-term forecasting. The present study has developed the WD-ARIMA model by coupling the

discrete wavelet denoise time series data and ARIMA model. The soft threshold method has been selected for

denoising the time series data and universal threshold (UT) method which has been used for the determination

of the threshold value. However, there are some approaches for threshold value determination such as SURE

(Stein, 1981), MINMAX (Donoho and Johnstone, 1998) and so on. Moreover, Wang et al. (2014) develop a

hybrid approach for denoising the hydro-meteorological time series such as rainfall and streamflow called

adaptive wavelet de-noising approach using sample entropy (AWDA-SE). The study has shown that the

performance of the developed denoising method is better than conventional de-noising methods for denoising

rainfall and streamflow. These approaches may apply to increase the performance of ARIMA models for
forecasting hydrological variables like $P_{ET}$. Moreover, there are several mother wavelet families such as Daubechies, Harr, Coiflets, Morlet, Mexican Hat and so on (Sang, 2013). In the present study, only Daubechies-6 from Daubechies wavelet family has been applied as mother wavelet of discrete wavelet transformation. WD-ARIMA models for forecasting the $A_{ET}$, surplus and deficit show very good performance, whereas the classical ARIMA model shows poor performance or unable to forecast the WBCs. Moreover, studies (e.g. Chou, 2011; Kisi, 2008; Partla, 2009; Santos and da Silva, 2014; Rahman and Hasan, 2014; Nury et al., 2016; Adamowski and Chan, 2011; Khalek and Ali, 2016) have also mentioned that the performance of wavelet aided models for forecasting non-stationary hydro-meteorological variables is better than classical ARIMA and ANN models. As the traditional methods such as Wiener filtering, Kalman filtering, Fourier transform are not suitable for non-stationary hydrological time series data (Adamowski and Chan, 2011; Sang, 2013), wavelet denoising can be used to improve the performance of the classical ARIMA models for forecasting hydrological variables.

5. Summary and Conclusions

The study explores the changes in WBCs using wavelet aided various forms of MK test and develops wavelet aided ARIMA models for forecasting the WBCs. The results obtained from trends analysis indicate that decreasing trends are dominant in all WBCs in the western part of Bangladesh during the period of 1982-83 to 2012-13. However, most of the trends are insignificant at 95% confidence level. One positive and one negative significant trend in $P_{ET}$ have been found in Satkhira and Bhola stations respectively. The study analyzed different combinations of D and A (i.e. D+A and D+A+A) components of DWT with Co of $a(t)$ statistic of SMK test that provides details information about the dominant periodicity that clearly affects the trend in original data and the time period which has also effect on trend in data (see section trend and periodicity or for example of Bhola station). The findings of the study reveal that to get details about the time period responsible for trends in data, it is necessary to analyze different combinations of D+A and D+A+A components rather than only details component (D) or approximation of wavelet transform data. Moreover, the study explored that changes in temperature or rainfall or both of these are not only associated with changes in $P_{ET}$. Before concluding the attribute of changes in $P_{ET}$, it is necessary to do details analysis of all the relevant climatic variables. In the western part of Bangladesh, D3 (8-year) and D4 (16-year) components have dominant effects on trends in original WBCs time series data. D2 (4-year) periodicity are also present in some cases, especially for $A_{ET}$. As surplus occurs during the rainy season and most of the rainfall occurs during this season, it may point out that rainfall pattern may have a similar periodicity (D3 to D4).

Modeling of the study reveals that WBCs time series data is affected by noises from different hydro-physical interactions. As a result, classic ARIMA models show unsatisfactory performance for most of the cases (for example $P_{ET}$) or unable to model the variability and changes in $A_{ET}$, surplus and deficit. The study has showed that ARIMA model can be used to model the WBCs time series after the denoising the WBCs time series using DWT with a universal threshold. The quality of wavelet denoise time series data has been evaluated and found satisfactory results for WBCs denoising. The fitted WD-ARIMA model performance has been evaluated by $NSE$ and $R^2$ (average $NSE$ and $R^2$ values of eleven stations located in western part of Bangladesh are 0.76 and 0.67 for $P_{ET}$: 0.92 and 0.89 for $A_{ET}$; 0.92 and 0.90 for annual surplus, and 0.88 and 0.88 for annual deficit respectively). The validation of WD-ARIMA models shows acceptable to very good performance for the short-term forecasting of WBCs as the validation for the period of 2009-10 to 2012-13 shows the acceptable $E_{MAE}$
value. However, the gap between the actual data and forecasted data increases with increasing time period. The obtained results are encouraging for further studies to find out a realistic model for real-world application under the changing climate. The results of the study can be incorporated into water resources management plans for the highly irrigated western part of Bangladesh where groundwater resource is at a critical stage. Further studies, therefore, denoising of hydrological time series data using different mother wavelets such as Haar, Coiflet and determination of thresholds using MINMAX, SURE or entropy based adaptive denoising approaches would be helpful for developing the better models for hydro-climatic time series in the context of climate change and would be beneficial for managing water resources in a sustainable manner.

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Figure 1: Study area western part of Bangladesh with locations of meteorological stations.

Figure 2: Distribution of mean annual (a) $P_{ET}$, (b) $A_{ET}$, (c) surplus and (d) deficit of water in the study area during the hydrologic year 1981-82 to 2012-13.
Figure 3: Sequential values of the statistics $u(t)$ of (a) Satkhira station and (b) Bhola station.

Figure 4: Distribution of rate of changes of WBCs during the period of 1981-82 to 2012-13.

Figure 5: Comparison between actual and wavelet denoised $P_{ET}$ time series (a) mean and (b) standard deviation.
Figure 6: Plot of best WD-ARIMA model first panel represents actual versus fitted values for the period of 1981-82 to 2012-2013, the second panel is normal Q-Q plot of residuals of the model, and the third panel shows actual, fitted and forecasted values for 2009-2010 to 2012-13 (a) $P_{ET}$ of Rangpur station located in north; (b) $A_{ET}$ of Ishurdi station located in the central part, (c) deficit of Rajshahi station located in NW Bangladesh and (d) surplus of Bhola station located in south of the study area.
Table 1: Calculations of water balance components (Thornthwaite and Mather, 1957)

<table>
<thead>
<tr>
<th></th>
<th>Wet months ($P - R_d &gt; P_{ET}$)</th>
<th>Dry months ($P - R_d &lt; P_{ET}$)</th>
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</thead>
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<tr>
<td>$A_{ET}$</td>
<td></td>
<td>$P_{ET}$</td>
</tr>
<tr>
<td>Deficit</td>
<td>0</td>
<td>$(P - R_d) + \Delta S_d$</td>
</tr>
<tr>
<td>Surplus</td>
<td>$P_{ET} - A_{ET}$</td>
<td>0</td>
</tr>
</tbody>
</table>

Where $P$ is the rainfall (mm), $R_d$ is the direct runoff (mm), $P_{ET}$ is the potential evapotranspiration (mm), $A_{ET}$ is the actual evapotranspiration (mm) and $\Delta S_d$ is the changes in soil moisture storage (mm).

Table 2: Z statistic of MK or MMK of original time series, approximation and different models $P_{ET}$ of DWT

<table>
<thead>
<tr>
<th>Stations Models</th>
<th>Barisal</th>
<th>Bhola</th>
<th>Bogra</th>
<th>Dinajpur</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Z$</td>
<td>$Co$</td>
<td>$MSE$</td>
<td>$Z$</td>
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<tr>
<td>Original</td>
<td>0.72</td>
<td>2.37*</td>
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<td>-0.98</td>
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<tr>
<td>A</td>
<td>-1.80</td>
<td>0.24</td>
<td>11.56</td>
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</tr>
<tr>
<td>D1</td>
<td>0.91</td>
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<td>0.50</td>
<td>2.02*</td>
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<td>0.17</td>
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<td>0.61</td>
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<tr>
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<td>0.17</td>
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<td>D4</td>
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<td>0.50</td>
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<tr>
<td>D3+D4+A</td>
<td>0.56</td>
<td>0.85</td>
<td>2.04</td>
<td>1.83</td>
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</tbody>
</table>

$MSE$, total mean square error; $Co$, correlation between original data and DWT models.
### Table 3: Comparison of performance of ARIMA model and WD-ARIMA model

<table>
<thead>
<tr>
<th>Stations</th>
<th>$P_{ET}$</th>
<th>$A_{ET}$</th>
<th>Surplus</th>
<th>Deficit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td>WD-ARIMA</td>
<td>ARIMA</td>
<td>WD-ARIMA</td>
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<td>$R^2$</td>
<td>NSE</td>
<td>$R^2$</td>
<td>NSE</td>
</tr>
<tr>
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<td>0.43</td>
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<tr>
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<tr>
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<tr>
<td>Avg.</td>
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<td>0.38</td>
<td>0.76</td>
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### Table 4: Accuracy of WD-ARIMA models of WBCs for validation of the model’s predictive ability for the period of 2009-10 to 2012-2013

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<th>$A_{ET}$</th>
<th>Surplus</th>
<th>Deficit</th>
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