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# Development and Evaluation of a Stochastic Daily Rainfall Model with Long Term Variability

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AFM Kamal Chowdhury[1], Natalie Lockart[1], Garry Willgoose[1], George Kuczera[1], Anthony S. Kiem[2], and Nadeeka Parana Manage[1]

[1] School of Engineering, The University of Newcastle, Callaghan 2308, New South Wales, Australia
[2] School of Environmental and Life Sciences, The University of Newcastle, Callaghan 2308, New South Wales, Australia

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*Correspondence to:* Professor Garry Willgoose (garry.willgoose@newcastle.edu.au*)*

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**Abstract.** The primary objective of this study is to develop a stochastic rainfall generation model that can match not only the short resolution (daily) variability, but also the longer resolution (monthly to multiyear) variability of observed rainfall. This study has developed a Markov Chain (MC) model, which uses a two-state MC process with two parameters (wet-to-wet and dry-to-dry transition probabilities) to simulate rainfall occurrence and a Gamma distribution with two parameters (mean and standard deviation of wet day rainfall) to simulate wet day rainfall depths. Starting with the traditional MC-Gamma model with deterministic parameters, this study has developed and assessed four other variants of the MC-Gamma model with different parameterisations. The key finding is that if the parameters of the Gamma distribution are randomly sampled each year from fitted distributions, rather than fixed parameters with time, the variability of rainfall depths at both short and longer resolutions can be preserved, while the variability of wet periods (i.e. number of wet days and mean length of wet spell) can be preserved by decade-varied MC parameters. This is a straightforward enhancement to the traditional simplest MC model and is both objective and parsimonious.

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# 1. Introduction

Observed rainfall data generally provides a single realisation of a short record, often not more than a few decades. The direct application of these data in hydrological and agricultural systems may not provide the necessary robustness in identification and implication of extreme climate conditions (e.g. droughts, floods). In particular, for urban water security analysis of reservoirs, long-term hydrologic records are required to sample extreme droughts that drive the security of the urban system (Mortazavi et al., 2013). However, the observed data may still be suitable to calibrate stochastic rainfall models that can, in turn, be used to generate long stochastic streamflow sequences for use in reservoir reliability modelling. In addition to historical and current scenarios, the stochastic models are useful to evaluate the climate and hydrological characteristics of future climate change scenarios (Glenis et al., 2015).

There is a major issue in the use of stochastic daily rainfall models. The daily models generally preserve the short-term daily rainfall variability (since they are calibrated to the daily resolution data) but tend to underestimate the longer-term rainfall variability of monthly and multiyear resolutions (Wang and Nathan, 2007). Such underestimation is critically important for the application of these models in hydrological planning and design. Preserving the long-term variability is important for drought security analysis of reservoirs because the reservoir water levels usually vary at monthly to multiyear resolutions. The underestimation of longer-term variability of rainfall may cause an overestimation of reservoir reliability in urban water planning (Frost et al., 2007). Therefore, preserving key statistics of wet and dry spells, and rainfall depths in daily to multiyear resolutions is important in stochastic rainfall simulation.

Markov Chain (MC) models are very common for stochastic rainfall generation. A typical MC rainfall model is composed of two parts: a rainfall occurrence model that uses a transition probability between wet and dry days, and a rainfall magnitude model that uses a probability distribution of wet day rainfall depths (commonly a Gamma distribution) fitted to the observed data. The two-part MC-Gamma model is one of the most popular parametric models for daily rainfall simulation, primarily proposed by Richardson (1981) and known as Weather Generator (WGEN). In addition to rainfall, the WGEN also simulates temperature and solar radiation. While other models such as point process models (Cowpertwait et al., 1996) are also used for stochastic rainfall generation, this study has focused on MC type models.

The first component of the MC model defines wet and dry days. This is determined by the state and order of the Markov process. Most MC models (Richardson, 1981; Dubrovský et al., 2004) use a simple two-state, first-order approach, that is, a day can be either 'wet' or 'dry' (two-state) and the state of the current day is only dependent on the state of the preceding day (first-order). Other models use higher states and orders – examples include, the four-state model (Jothityangkoon et al., 2000), alternating renewal process model with negative binomial distribution of wet and dry spell lengths (Wilby et al., 1998), bivariate mixed distribution model (Li et al., 2013), and multi-order model (Lennartsson et al., 2008). These models are more complex as the number of parameters required in the model increases with the number of states and orders of the Markov process. However, the two-state, first-order MC model can often reproduce the statistics of wet and dry periods as well as these higher state/order models (Chen and Brissette, 2014). Dubrovský et al. (2004) recommended that, rather than trying an increased order MC, one should consider other approaches for better reproduction of wet and dry days. Mehrotra and Sharma (2007) proposed a modified MC process using memory of past wet periods, which has been found to reproduce the wet and dry spell statistics reasonably well. They also tested a first-order and a second-order process in their modified MC model and found that the second-order process provided only marginal improvements over the first-order process. Another important finding of Dubrovský et al. (2004) was that the order of MC generally had no effect on the variability of monthly rainfall depths.

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83  The second component of the MC model is the probability distribution for the wet day rainfall. As the distribution of wet day
84  rainfall is generally right-skewed (Hundecha et al., 2009), it is common practice to use right-skewed exponential type
85  distributions. Common distributions include the Gamma distribution (Wang and Nathan, 2007; Chen et al., 2010), Weibull
86  distribution (Sharda and Das, 2005), truncated normal distribution (Hundecha et al., 2009), and kernel-density estimation
87  techniques (Harrold et al., 2003). A number of other studies fitted a mixture of two or more distributions, for example mixed
88  exponential distribution (Wilks, 1999a; Liu et al., 2011), Gamma and generalised Pareto distribution (Furrer and Katz,
89  2008), and transformed normal and generalised Pareto distribution (Lennartsson et al., 2008). However, the Gamma
90  distribution is the most commonly used distribution, because it has only two parameters, that can be calculated calculated
91  from the mean and standard deviation (SD) of wet day rainfall, and which adequately represent the rainfall probability
92  distribution functions. The parameterisation and application of the distribution in the model is straightforward. Although the
93  Gamma distribution has been found to be appropriate for simulating most of the variability of rainfall depth (Bellone et al.,
94  2000), the major drawback of using a Gamma distribution is that its tail is too light to capture heavy rainfall intensities (Vrac
95  and Naveau, 2007). Therefore, direct use of a Gamma distribution usually causes an underestimation of SD and extreme
96  rainfall depths at monthly to multiyear resolutions.

97  A number of methods have been developed in an attempt to resolve the underestimation of long-term variability. The major
98  approaches for resolving this issue include (i) Models with mixed distributions, (ii) Nesting-type models, (iii) Models with
99  rainfall-climate index correlation, and (iv) Models with modified Markov Chains.

100 The models with mixed distributions concentrate on the upper tail behaviour of the probability distribution of wet day
101 rainfall depths. Since a single component distribution cannot incorporate the extreme rainfall depths well, a mixture of
102 distributions is introduced. In these models, rainfall above a threshold depth is defined as 'extreme' and two separate
103 distributions are used to simulate the 'extreme' and 'small' rainfall amounts. Wilks (1999a) proposed a mixture of two
104 exponential distributions with one shape parameter, but two scale parameters which are used to incorporate the extreme and
105 small rainfall depths respectively. In other models, the 'extreme' rainfall depths are modelled by a generalised Pareto
106 distribution (Vrac and Naveau, 2007) or stretched exponential distribution (Wilson and Toumi, 2005), while small rainfall
107 depths are modelled by a Gamma distribution. Nonetheless, these models have difficulty in objectively defining the
108 threshold corresponding to the 'extreme value'. Wilson and Toumi (2005) defined extreme rainfall as daily totals with
109 exceedance probability less than 5%. Although Vrac and Naveau (2007) used a dynamic mixture to avoid choosing a
110 threshold for 'extreme', Furrer and Katz (2007) described the method as over-parameterised. Recently, Naveau et al. (2016)
111 proposed a new model with smooth transition between the 'small rainfall' and 'extreme rainfall' simulation process to
112 generate low, moderate and heavy rainfall depths without selecting a threshold.

113 Nesting models adjust the daily rainfall series at different temporal resolutions to obtain statistics that are optimal for all
114 resolutions. These models initially generate a daily rainfall series, which is then modified to adjust the monthly and yearly
115 statistics. Several models (Dubrovský et al., 2004; Wang and Nathan, 2007; Srikanthan and Pegram, 2009; Chen et al., 2010)
116 use the nesting method. They generally generate a daily rainfall series, then the generated daily rainfall data are aggregated
117 to monthly rainfall values, and these monthly values are modified by a lag–1 autoregressive monthly rainfall model. The
118 modified monthly rainfall values are aggregated to annual rainfall values and these values are then modified by another lag–
119 1 autoregressive annual model (Srikanthan and Pegram, 2009). The nesting-type models generally perform well to reproduce
120 the rainfall variability at all resolutions. Dubrovský et al. (2004) also showed satisfactory performance of their nesting-type
121 model to reproduce the variability of monthly streamflow characteristics and the frequency of extreme streamflow. Although

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the nesting-type models preserve the daily, monthly and yearly statistics, they are generally based on subjective statistical adjustments and thus have a weak physical basis.

Some parametric models introduced the influence of large-scale climate mechanisms such as the El Niño/Southern Oscillation (ENSO) in parameterisation (Hansen and Mavromatis, 2001; Furrer and Katz, 2007). Bardossy and Plate (1992) used the correlation between atmospheric circulation patterns and rainfall in a transformed conditional multivariate autoregressive AR (1) model for daily rainfall simulation. Katz and Parlange (1993) developed a model with parameters conditioned on the ENSO indices. Yunus et al. (2016) developed a generalised linear model for daily rainfall by using ENSO indices as predictors. Although the climate indices were often not strongly correlated to the rainfall, Katz and Zheng (1999) described it as a diagnostic element to detect a 'hidden' (i.e. unobserved) index which could be used to obtain long-term variability. Thyer and Kuczera (2000) developed a hidden state MC model for annual data, while Ramesh and Onof (2014) developed a hidden state MC model for daily data. The major drawback of this model approach is that the 'hidden' index is unobserved and its origin is unknown.

Modified MC models concentrate not only on the order of MC, but also introduce modifications to the parameterisation of the MC process to better reproduce the rainfall variability. The transition probabilities are generally modified by considering their long-term variability (i.e. memory of past wet and dry periods), and the wet day rainfall depth is modelled using a nonparametric kernel-density simulator conditional on previous day rainfall (Lall et al., 1996; Harrold et al., 2003). The nonparametric kernel-density techniques usually used resampling of observed data (Rajagopalan and Lall, 1999). While these models perform reasonably well, they usually cannot generate extreme values higher than the observed extremes, because only the original observations are resampled in the model (Sharif and Burn, 2006). Mehrotra and Sharma (2007) proposed a semi-parametric Markov model, which was further evaluated by Mehrotra et al. (2015). To incorporate the long-term variability, they modified the transition probabilities of the MC process by taking the memory of past wet periods (i.e. beyond lag-1) into account, while the wet day rainfall depths were simulated by a nonparametric kernel-density process. For raingauge data around Sydney, the semi-parametric model preserved the rainfall variability at daily to multiyear resolutions (Mehrotra et al., 2015).

The MC models that focus specifically on resolving the underestimation of long-term variability involve subjective assumptions and limitations. In the models with mixed distributions, defining a certain rainfall depth as an extreme value is subjective. The nesting type models used empirical adjustment factors, generally without physical foundation. The hidden indices of hidden state MC models are unobserved. The models with modified MC parameters modified the transition probabilities of wet and dry periods to obtain long term variability, but used the kernel density technique to resample wet day rainfall depths from observed records. Therefore, they usually cannot generate more extreme values than the observed extremes.

The overarching objectives of the research, that this paper forms part of, is to develop a stochastic rainfall generator that can be calibrated to daily rainfall data derived from dynamically downscaled global climate simulations and which also preserves long-term variability (Evans et al., 2014). A common problem with these simulations is that typical computational CPU limits mean that the length of the simulation is rarely more than a few decades, not long enough to facilitate stochastic assessment of the reliability of water supply reservoirs (e.g. Lockart et al., 2016). Accordingly, we need a rainfall simulator that can be calibrated and run at the daily timescale (to be used as input into a hydrology model at the daily resolution), but which has the right statistical properties (specifically variability about the mean) when averaged over periods out to a decade. In this paper, we develop and test five models against observed rainfall at two sites in Australia with contrasting climate.

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Accordingly, this study details the development of a MC model for stochastic generation of daily rainfall. This MC model uses a two-state MC process with two parameters (wet-to-wet and dry-to-dry transition probabilities) for simulating rainfall occurrence and a two parameter Gamma distribution (mean and SD of wet day rainfall) for simulating wet day rainfall depths. Five variants of the MC model, with gradually increasing complexity of parameterisation, are developed and assessed. Starting with a very simple model against which the performances of the other models will be compared, each of the successive models provides better performance in reproducing the variability and dependence of observed rainfall over the range of resolutions from day to decade, and we assess the incremental improvements in performance against the incremental increases in model complexity.

## 2. Data and Study Sites

This study has used daily raingauge data from Sydney Observatory Hill and Adelaide Airport stations (station number 66062 and 023034 respectively) obtained from the Bureau of Meteorology (BoM), Australia (Figure 1) for 1979-2008 (BoM, 2013). These two stations have been used because they provide a contrast between a highly seasonal Mediterranean climate with low inter-decadal variability in Adelaide and a relatively non-seasonal climate with high inter-decadal variability in Sydney (see Figure 2). Risbey et al. (2009) also showed that the major climate drivers of rainfall (e.g. ENSO) in Sydney and Adelaide are different for all seasons. This paper also used the Oceanic Nino Index (ONI) and Inter-Decadal Pacific Oscillation (IPO) index at monthly resolution for 1979–2008 period (Folland, 2008; NOAA, 2014). These climate indices are used to develop two variants of the MC models discussed in section 4.2.2.

## 3. Model Assessment Procedures

### 3.1. Statistics for Assessment of Model Performance

Each model developed in this study has been assessed to understand its ability to reproduce the distribution and autocorrelation of observed rainfall. Assessment of the distribution and autocorrelation are generally used to inform the suitability of the model for urban drought security assessment. The assessment criteria of each model consider its ability to reproduce (i) mean, SD and 95th percentile of rainfall depths at daily to multiyear resolutions, (ii) mean and SD of the number of wet days and mean length of wet spells at monthly to multiyear resolutions, and (iii) month-to-month autocorrelations of monthly rainfall depths and monthly number of wet days. The performances of the MC models for dry period statistics are found to be similar to the wet period statistics (the term 'wet period statistics' will hereafter refer the number of wet days and mean length of wet spells), and hence, only representative results for annual mean length of dry spells are shown.

At daily and monthly resolutions, the distribution statistics are assessed for each month starting from January, while at multiyear resolutions, the distribution statistics are assessed for 1 to 10 overlapping years. Mean length (in days) of wet spells are calculated at monthly and annual resolution by extracting wet spells of one or more consecutive wet days (two successive wet spells are separated by at-least one dry day) and using equation 1:

$$\text{mean length of wet spell} = \frac{\sum(\text{length of wet spells})}{\sum(\text{occurrences of wet spells})} \qquad (1)$$

Similar to wet spells, the mean length of dry spells are also calculated at monthly and annual resolution by extracting dry spells of one or more consecutive dry days.

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## 3.2. Calculation of Z Scores

For the distribution statistics (i.e. mean and SD) of rainfall depths and wet periods (number of wet days and mean length of wet spells), this study has calculated the expected value and error limit (SD) to calculate the Z score of a model simulation. The calculation of the Z score is as follows:

1. Run the model 1000 times using the probability distribution of the parameters calibrated to the observed data, with each run being the same length as the observed data.
2. Calculate the desired statistics (e.g. mean and SD of the daily rainfall depths) in each run, which gives 1000 realisations of each statistic.
3. For each statistic, calculate the mean (expected value) and SD (error limit) of the 1000 realisations.
4. Calculate the Z Score of each statistic by comparing the expected value with the respective observed value (calculated from the observed data), as follows:

$$Z \; Score = \frac{Observed \; value - Expected \; value}{SD} \qquad (2)$$

A Z Score between –2 and +2 for a statistic indicates that the observed value falls within the 95% confidence limits of the simulated rainfall assuming a normal distribution approximates the sampling distribution of Z. A Z Score less than –2 or greater than +2 suggests that the statistic is over- or under-estimated respectively in the model simulation.

## 4. Markov Chain (MC) Models

This study has developed and assessed the following five variants of a Markov Chain (MC) model:

- Model 1: Average Parameter Markov Chain (APMC) model,
- Model 2: Decadal Parameter Markov Chain (DPMC) model,
- Model 3: Compound Distribution Markov Chain (CDMC) model,
- Model 4: Hierarchical Markov Chain (HMC) model,
- Model 5: Decadal and Hierarchical Markov Chain (DHMC) model.

### 4.1. Model 1: Average Parameter Markov Chain (APMC) model

The first MC model – the APMC – is a traditional two-part MC-Gamma distribution model. This is similar to the rainfall generator proposed by Richardson (1981), widely known as the Weather Generator (WGEN) model. The exception is that the parameters in WGEN were smoothed with Fourier harmonics, which has not been done in the case of APMC parameters. Although APMC is not the final model of this study, it is the baseline modelling approach against which the more sophisticated models developed in this study are compared.

The APMC simulates the daily rainfall in two steps: daily rainfall occurrence (i.e. wet and dry day) simulation by first-order Markov Chain, and wet day rainfall depth simulation by Gamma distribution. To incorporate the seasonal variability in the model, the APMC uses a separate set of parameters for each month, where the first month of the simulation is January.

### 4.1.1. Rainfall occurrence simulation

The APMC uses 24 (2 parameters × 12 months) MC parameters, transition probabilities of dry-to-dry day ($P_{00}$) and wet-to-wet day ($P_{11}$), for wet and dry day occurrence simulation. In addition, the unconditional probability of a dry day ($\pi_0$) in

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January is used to simulate rainfall occurrence for the first day of the series. In the model calibration, these deterministic MC parameters are calculated from the observed daily rainfall data. To calculate these parameters, a day with rainfall depth of 0.3 mm and above has been considered a wet day, otherwise it was considered a dry day (similar to Mehrotra et al, 2015). In simulation, the MC parameters are used in a Monte-Carlo process to simulate the occurrences of wet and dry days.

### 4.1.2. Rainfall depth simulation

After simulation of the rainfall occurrence using MC parameters, the next step is to generate rainfall depths for the wet days. The rainfall depth for dry days is zero. The APMC rainfall depth simulation process assumes that (i) daily rainfall depth for wet days follows a Gamma distribution, and (ii) the rainfall depth for a wet day is independent of the rainfall depth of the preceding day.

The Gamma distribution has two parameters $\alpha$ (shape parameter) and $\beta$ (scale parameter) with mean $\mu = \alpha\beta$ and variance $\sigma^2 = \alpha\beta^2$. Since both $\alpha_i$ and $\beta_i$ are directly proportional to and can be derived from $\mu_i$ and $\sigma_i$ of wet day rainfall of the month $i$, during calibration of the model it is convenient to calculate $\mu_i$ and $\sigma_i$ values from the daily rainfall observed data. The appropriate ratios of $\mu_i$ and $\sigma_i$ can then be used in the rainfall depth generation process using the Gamma distribution. Therefore, $\mu_i$ and $\sigma_i$ will be referred to as the Gamma distribution parameters in further discussions of this paper.

In calibration of APMC, deterministic average values of $\mu_i$ and $\sigma_i$ are calculated from the entire period of data record for each month. This gives 12 values of $\mu$ and $\sigma$ each. In simulations, the rainfall depth for each wet day of a month $i$ is generated using the $\mu_i$ and $\sigma_i$ values of the respective month using the Gamma distribution. In generating the rainfall depth for a wet day, if a random sample from the Gamma distribution gives a rainfall depth less than 0.3 mm then the rainfall for that day is set to 0.3 mm (i.e. the threshold rainfall depth), while the rainfall depths for dry days are set to 0.0 mm. Chowdhury (2016) showed that setting rainfall below 0.3 mm to 0.3 mm for the lowest rainfall depth does not significantly affect the overall distribution of modelled rainfall depths.

### 4.1.3. Independence of rainfall depths in successive wet days

The APMC assumes that the rainfall depth for a particular day is independent of the rainfall depth of the preceding day. To validate this assumption, this study examined the autocorrelation of wet day rainfall depths, and only found very weak lag-1 autocorrelations ($r^2 < 0.1$) for both Sydney and Adelaide. This finding is consistent irrespective of seasonal variations. The conclusion is that the underlying assumption of daily independence of the APMC is consistent with the respective characteristic of the observed data.

### 4.2. Model 2: Decadal Parameter Markov Chain (DPMC) Model

Section 6 will show that the APMC significantly underestimates the rainfall variability at monthly to multiyear resolutions. The DPMC assumes that the inter-annual rainfall variability can be captured by the decade-to-decade variability of the parameters that APMC failed to capture. The idea is to divide the observed rainfall sample into sub-samples of 10-year lengths (similar models with climate-based sub-samples are discussed in section 4.2.2). For example, a 30-year rainfall sample is divided into three sub-samples of 10-years in length. Then, $4 \times 12$ parameters of $P_{00}$, $P_{11}$, $\mu$, and $\sigma$ (one set of four parameters for each of the 12 months) are calculated from each of the sub-samples. The simulation proceeds in a way similar to the APMC, except that the deterministic, decadal average, parameters of DPMC are varied from decade to decade.

### 4.2.1. Decadal variability of DPMC parameters

Figure 2 shows the DPMC values of $P_{11}$ and $\mu$ for each decade along with APMC values (i.e. the 30-year averages) for Sydney and Adelaide. For Sydney, DPMC values of $P_{11}$ and $\mu$ show clear variability between the three decadal samples and deviations from the APMC values. However, DPMC values of $P_{11}$ and $\mu$ for Adelaide show less variability between the decadal samples.

The use of decade-varied parameters in DPMC is subject to the question of how significant the decadal variability of these parameters is – is the decadal variability statistically significant or just sampling variability? Therefore, the statistical significance of the decadal variability of DPMC parameters were examined by Monte-Carlo simulations as per section 3.2. This examination suggested that the sampling variability of DPMC parameters in decadal samples is mostly within the sampling variability of their corresponding APMC values (not shown). This suggests that the decadal variability of DPMC parameters is not statistically significant.

### 4.2.2. Potential impact of climate modes

This study has also investigated other sub-sampling approaches of the MC-Gamma parameters similar to the DPMC. In these models, this study has calibrated the MC-Gamma parameters to sub-samples of rainfall timeseries divided according to the phases of IPO (e.g. positive and negative) and ENSO (La Niña, Neutral and El Niño). Since previous studies (Verdon-Kidd et al., 2004) found that the inter-annual variabilities of East-Australian rainfall are influenced by these large-scale climate drivers, the idea behind these models was to introduce more inter-annual variability to the model by simulating rainfall for different phases of climate drivers with parameters calibrated to respective phases. These climate-based models are very similar to DPMC, except that the sub-samples are different. The following two types of climate-based models have been tested:

- IPO based model: The observed data for every month was divided into two sub-samples according to the positive and negative phases of the monthly IPO index (e.g. for January, data of the years with positive IPO index and data of the years with negative IPO index are separated). Then, for each month, the MC-Gamma parameters ($P_{00}$, $P_{11}$, $\mu$, and $\sigma$) are calibrated to each sub-sample. In simulation, the rainfall for the months of each IPO phase were modelled by using parameters of the respective phase.
- ENSO based model: The observed data for every month was divided into three sub-samples according to monthly ONI index: La Niña (ONI $\leq$ –0.5), Neutral (–0.5 < ONI < 0.5), and El Niño (ONI $\geq$ 0.5). Then, the MC-Gamma parameters are calibrated to each sub-sample and the rainfall for the months of each ENSO phase were modelled by using parameters of the respective phase.

### 4.3. Model 3: Compound Distribution Markov Chain (CDMC) Model

The results in section 6 will show that neither APMC nor DPMC can satisfactorily reproduce the SD of rainfall depths for monthly to multiyear resolutions. Therefore, in the third MC model – the CDMC – this study has incorporated the long-term variability of rainfall depths by introducing random variability in $\mu$ and $\sigma$. However, for wet and dry period simulation, the CDMC still uses the deterministic parameters of $P_{00}$ and $P_{11}$, as in the APMC. Thus this model stochastically varies the rainfall depth model, but not the rainfall occurrence model.

In the CDMC, $\mu_i$ and $\sigma_i$ are randomly sampled for each month of each year. The random sampling was done independently of the sampling for the preceding month/s. To estimate the distribution of $\mu_i$ and $\sigma_i$, this study has calculated $\mu_i$ and $\sigma_i$ for every month of every year from the observed data. For example, from the 30-year observed data, for January ($i = 1$), this study has calculated 30 samples of $\mu_1$ and $\sigma_1$ values each.

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By testing the probability distributions of $\mu_i$ and $\sigma_i$ values for each month, this study has found that both $\mu_i$ and $\sigma_i$ values for each month are lognormally distributed. Figure 3 shows the lognormal probability plots of $\mu_i$ and $\sigma_i$ values for July ($i = 7$) which is representative of the other months. The $r^2$ for log $\mu_i$ and log $\sigma_i$ are generally above 0.90, indicating a very good fit of the lognormal distributions. Additionally, the hypothesis that log $\mu_i$ and log $\sigma_i$ are normally distributed is supported by the Kolmogorov-Smirnov test at 5% significance level. In addition to the lognormally distributed $\mu_i$ and $\sigma_i$ values, this study has also found that the log $\mu_i$ and log $\sigma_i$ values for each month are strongly correlated with each other with correlation coefficient $r_{c,i}$ around 0.90 (Figure 4). Therefore, for each month $i$, this study has fitted a bivariate-normal distribution to the log $\mu_i$ and log $\sigma_i$ values with parameters $(\lambda_{\mu,i}, \zeta_{\mu,i})$, $(\lambda_{\sigma,i}, \zeta_{\sigma,i})$ and $r_{c,i}$. The $\lambda$ and $\zeta$ parameters denote the mean and SD of the log variate, while $r_c$ is the correlation coefficient between log $\mu$ and log $\sigma$.

At the start of each month of each year of the simulation, the log $\mu_i$ is sampled from its fitted normal distribution log $\mu_i \sim N(\lambda_{\mu_i}, \zeta_{\mu_i}^2)$ for month $i$. Then, the log $\sigma_i$ is sampled from the fitted conditional normal distribution:

$$\log \sigma_i \mid \log \mu_i \ \sim \ N\left(\lambda_{\sigma_i} + \frac{\zeta_{\sigma_i}}{\zeta_{\mu_i}} r_{c,i}\left(log\mu_i - \lambda_{\mu_i}\right), \quad \left(1 - r_{c,i}^2\right)\left(\zeta_{\sigma_i}\right)^2\right) \tag{3}$$

These stochastically sampled $\mu_i$ and $\sigma_i$ values are then used to generate rainfall in the wet days for the month in question, while the sequence of wet and dry days is determined using the deterministic APMC values of $P_{00,i}$ and $P_{11,i}$. However, the sampled $\mu_i$ and $\sigma_i$ values of a month ($i$) are not correlated to the $\mu_{i-1}$ and $\sigma_{i-1}$ of the preceding month ($i$–1) as this study has found that the month-to-month autocorrelations of $\mu$ and $\sigma$ values are not strong (Figure 5).

### 4.4. Model 4: Hierarchical Markov Chain (HMC) Model

The results in section 6 will show that the CDMC cannot satisfactorily reproduce the SD of wet periods for monthly to multiyear resolutions. Therefore, in the fourth MC model – the HMC – we introduce stochastic variation in both MC and the Gamma distribution models to incorporate long-term variability of rainfall depths as well as wet and dry periods. In calibration, for month $i$, the $P_{00,i}$ and $P_{11,i}$ are calculated for each month of each year from the observed data. For month $i$, these $P_{00,i}$ and $P_{11,i}$ values (e.g. 30 $P_{11,7}$ values for July from the 30-year observed data) are found to be normally distributed with values between 0 and 1 (Figure 6). Therefore, this study has fitted a truncated normal distribution, bounded by 0 and 1 to the calculated $P_{00}$ and $P_{11}$ values. In simulation, for each year, the $P_{00,i}$ and $P_{11,i}$ are sampled from their truncated normal distributions. This procedure is similar to what was done for $\mu_i$ and $\sigma_i$ to sample from bivariate-lognormal distribution. However, it does not include a bivariate distribution because the correlation between $P_{00,i}$ and $P_{11,i}$ was weak.

#### 4.4.1. Impact of autocorrelations on stochasticity of MC parameters

In the HMC, the sampled MC parameters of each month are independent of the parameters of preceding month. However, this study has found strong month-to-month autocorrelations of the $P_{00}$ and $P_{11}$ for Adelaide (Figure 5a), although the autocorrelations are weak for Sydney (Figure 5b). Therefore, this study has tested an alternative to the HMC (referred to as HMC2), which uses a lag–1 autocorrelation equation (a similar equation was used by Wang and Nathan (2007) in their rainfall depth model) in the stochastic sampling of $P_{00,i}$ and $P_{11,i}$ from the truncated normal distribution. The following lag–1 autocorrelation equation has been used to modify the randomly sampled $P_{00,i}$ (same method used for $P_{11,i}$) for month $i$ by correlating with the $P_{00,i-1}$ of month $i$–1 (preceding month):

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$$\frac{\overline{P_{00,t}} - mean(P_{00,i})}{sd(P_{00,i})} = r \times \frac{\overline{P_{00,t-1}} - mean(P_{00,i-1})}{sd(P_{00,i-1})} + (1-r^2)^{1/2}\,\frac{\overline{P_{00,t}} - mean(P_{00,i})}{sd(P_{00,i})} \qquad (4)$$

where, for a month $i$ (e.g. January),

- $\overline{P_{00,t}}$
- $mean(P_{00,i})$ is mean of the yearly-varied parameter values calculated from observed data for month $i$ (e.g. mean of 30 $P_{00,7}$ values for July),
- $sd(P_{00,i})$ is SD of the yearly-varied parameter values calculated from observed data for month $i$,
- $\overline{P_{00,t-1}}$
- $mean(P_{00,i-1})$ is mean of the parameter values calculated from observed data for month $i$–1,
- $sd(P_{00,i-1})$ is SD of the parameter values calculated from observed data for month $i$–1,
- $r$ is lag–1 autocorrelation coefficient for observed month-to-month autocorrelation of $P_{00}$ (constant for all month),
- $\overline{P_{00,t}}$ is stochastic parameter value sampled from a truncated normal distribution fitted to the yearly-varied observed parameter values for month $i$,
- $\overline{P_{00,t-1}}$ is auto-correlated parameter value for month $i$–1 (used to simulate the dry days of the preceding month),
- $\overline{P_{00,t}}$ is the final auto-correlated parameter value which was used in simulation of dry days for month $i$.

$\overline{P_{11,t}}$ for month $i$ is sampled using a similar process.

### 4.4.2. Impact of cross-correlations on stochasticity of MC parameters

We observed a strong positive correlation between $P_{11,i}$ and log $\mu_i$ and log $\sigma_i$, although the correlations between $P_{00,i}$ and log $\mu_i$ and log $\sigma_i$ are weak. Therefore, another alternative to HMC (refer as HMC3) was tested by using a multivariate sampling system for the $P_{11,i}$, $\mu_i$ and $\sigma_i$, while $P_{00,i}$ remains independent.

### 4.5. Model 5: Decadal and Hierarchical Markov Chain (DHMC) Model

Section 6 will show that the CDMC, with APMC values of MC parameters, significantly underestimates the wet period variability at multiyear resolutions, while the HMC (including the two alternatives HMC2 and HMC3) with stochastic MC parameters, significantly overestimates the wet period variability at monthly resolution. However, we found that the DPMC can satisfactorily preserve the wet period variability at both monthly and multiyear resolutions, although it underestimates the rainfall depths variability. Therefore, in the DHMC model, this study has used the DPMC values of MC parameters (the parameter values vary for each decade of simulation) for simulation of wet and dry days, while the stochastic parameters of the Gamma distribution (same as CDMC) are used for simulation of wet day rainfall depths.

### 5. Methodological Comparison of Five MC Models

The following points discuss the key common features in the five MC models of this study, while other key methodological comparisons are shown in Table 1.

- All models use first-order MC parameters to simulate the rainfall occurrences and Gamma distribution to simulate rainfall depths in wet days.
- Simulation of rainfall depth for each wet day is independent of the rainfall depth of the preceding day.

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- Separate sets of parameters are used for each month (e.g. 12 sets of MC and Gamma parameters) to incorporate seasonal variability.

## 6. Model Comparison for Distribution Statistics

This section compares the performances of the five MC models for the mean and SD of rainfall depths and wet period statistics (i.e. number of wet days and mean length of wet spells).

### 6.1. Mean and SD of Rainfall Depths

Figures 7, 8 and 9 compare the five MC models for the mean and SD of rainfall depths at daily, monthly and multiyear resolutions respectively. Figure 9 also compares the 95th percentile of multiyear rainfall depths. For mean and SD of rainfall depths, the performances of APMC and DPMC are similar. The performances of CDMC, HMC and DHMC are also similar, but different from APMC and DPMC. All five models preserve the mean (i.e. satisfactorily reproduce the observed mean) rainfall depths at all resolutions with Z scores between -2 and +2. However, the CDMC, HMC, and DHMC show a tendency to underestimate the mean with mostly positive Z scores (between 0 and +2). The APMC and DPMC preserve the SD of rainfall depths only at daily resolution and significantly underestimate the SD at monthly and multiyear resolutions for Sydney but preserve the SDs at all resolutions for Adelaide (Figure 7, 8 and 9). The CDMC, HMC and DHMC preserve the SD of rainfall depths at all resolutions for both stations except a slight tendency to underestimate the SD for February and November at daily resolution in Sydney. We conclude that those models with stochastic parameters for the Gamma distribution (i.e. CDMC, HMC, and DHMC) best preserve SDs at all resolutions for both stations. For the 95th percentile of rainfall depths, we found that models which can preserve the SD at a given resolution can also preserve the 95th percentile at that resolution and vice-versa. In Figure 9, the representative results at multiyear resolution (average of the absolute values of Z scores for daily and monthly resolutions are shown in Table 2) show that the CDMC, HMC and DHMC preserve the 95th percentile for both stations but the APMC and DPMC underestimate the statistic for Sydney.

### 6.2. Mean and SD of Number of Wet Days

Figure 10 and 11 compare the five MC models for the mean and SD of number of wet days at monthly and multiyear resolutions respectively. All five models preserve the mean of number of wet days for both monthly and multiyear resolutions. For the SD of the monthly number of wet days, all models except HMC can satisfactorily reproduce the SD with Z scores between -2 and +2 for almost all months of both stations, while the HMC tends to overestimate the SD (Figure 10). For SD of multiyear number of wet days, the APMC and CDMC significantly underestimate the SD for Sydney but preserve the statistic for Adelaide. The DPMC and DHMC perform similarly and satisfactorily to preserve the SD of multiyear number of wet days for both Sydney and Adelaide, while HMC also preserves the statistic for both stations. We conclude that the models with decade-varied MC parameters (i.e. DPMC and DHMC) perform satisfactorily at reproducing the variability of the number of wet days at monthly and multiyear resolutions for both stations.

### 6.3. Mean and SD of Mean Length of Wet and Dry Spells

Figure 12 compares the five MC models for the mean and SD of mean length of wet and dry spells at annual resolution. The average of the absolute values of the Z scores for monthly resolution are shown in Table 2. The comparative performances of the five MC models for the mean and SD of mean length of wet spells at monthly (Table 2) and annual (Figure 12) resolutions are mostly consistent with their respective performances for mean and SD of number of wet days. All models except HMC preserve the mean and SD of mean length of wet spells, while the HMC tends to overestimate the SD (Figure 12).

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For the mean and SD of mean length of dry spells, we found that models that can preserve the wet spells distributions also preserve the dry spells distributions and vice-versa. As a representative result, the Z scores for the mean and SD of annual mean length of dry spells shown in Figure 12 indicate that all models except HMC preserve both mean and SD, while HMC overestimates the SD. Figure 12 also indicates that the DPMC and DHMC perform better (Z scores closer to zero) than the APMC and CDMC to reproduce the SD of annual mean length of dry spells.

We conclude that models with decade-varied MC parameters (i.e. DPMC, DHMC) perform relatively better and more satisfactorily at reproducing the variability of the length of wet and dry spells. The HMC introduces too much variability in the length of wet and dry spells, while the APMC and CDMC tend to underestimate the variability.

### 6.4. Potential Impact of Climate Modes

Since the DPMC significantly underestimates the SD of rainfall depths at monthly and multiyear resolutions, the major target of the models with sub-samples according to climate modes such as IPO and ENSO indices (discussed in section 4.2.2) was to preserve the SD of rainfall depths at monthly and multiyear resolutions. However, we found that these climate-based models also significantly underestimate the SD of rainfall depths at month and multiyear resolutions with performances similar to the DPMC, and are therefore not considered further.

### 6.5. Impact of Stochasticity of MC Parameters

Since the HMC significantly overestimates the SD of monthly wet periods (i.e. number of wet days and mean length of wet spell), the major target of the HMC2 and HMC3 models (with a lag–1 autocorrelation equation and a multivariate sampling system respectively; see section 4.4.1) was to better preserve the SD. However, these models also significantly overestimate the SD of monthly wet periods with performances similar to the HMC (negative Z scores less than -2 for all months). We conclude that the models with stochastic, yearly varied, parameters for the MC part of the model (i.e. HMC, HMC2 and HMC3) consistently overestimate the variability of monthly wet periods.

### 6.6. Overall Performances

Table 2 shows the average of the absolute values of Z scores (average of 12 values at daily and monthly resolutions and 10 values at multiyear resolution) for the distribution statistics of rainfall depths, and wet and dry periods at daily, monthly, annual and multiyear resolutions. It shows that models 1-4 (APMC, DPMC, CDMC and HMC) fail to preserve the following statistics:

- the APMC fails to preserve the SD and 95th percentile of rainfall depths and SD of number of wet days at multiyear resolution for Sydney,
- the DPMC fails to preserve the SD and 95th percentile of rainfall depths at multiyear resolution for Sydney,
- the CDMC fails to preserve the SD of number of wet days at multiyear resolution for Sydney,
- the HMC fails to preserve SD of mean length of wet spell at annual resolution for both Sydney and Adelaide.

However, model 5, DHMC, has preserved all of the statistics for both stations. We conclude that the DHMC is better than other four models at reproducing the distributions of rainfall depths, and wet and dry periods for resolutions varying from daily to multiyear.

### 7. Reproduction of Seasonal Autocorrelations

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Figure 13 compares how the five MC models reproduce the month-to-month autocorrelations of the monthly number of wet days and monthly rainfall depths. For Adelaide (Figure 13a), the lag–1 and lag–12 autocorrelations are strong with systematic seasonal variation, which have been reproduced very well in the corresponding APMC, DPMC, CDMC and DHMC simulations, while the HMC (the model with stochastic MC parameters) tends to underestimate the autocorrelations. For Sydney (Figure 13b), the month-to-month autocorrelations of monthly number of wet days and monthly rainfall depths in the data are weak and all models perform well.

## 8. Discussion

The primary motivation of this study was to develop a stochastic rainfall generation model that can reproduce not only the short resolution (daily) variability, but also the longer resolution (monthly to multiyear) variability of observed rainfall. Preserving long-term variability in rainfall models has been a difficult challenge for which a number of solutions have been proposed in the stochastic rainfall generation literature. The solutions developed and tested by this study are relatively simple MC models: two MC parameters ($P_{00}$ and $P_{11}$) of two-state, first-order processes defining the wet and dry days, and two Gamma-distribution parameters ($\mu$ and $\sigma$) defining the rainfall depths in wet days. For seasonal variability, the models operate at daily time step with monthly varying parameters for each of 12 months. Starting with the simplest MC-Gamma modelling approach with deterministic parameters (similar to Richardson, 1981), this study has developed and assessed four other variants of the MC-Gamma modelling approach with different parameterisations. The key finding is that if the parameters of the Gamma distribution and the MC model are randomly sampled from fitted distributions prior to simulating the rainfall for each year, the variability of rainfall depths at longer resolutions can be preserved, while the variability of wet periods (i.e. number of wet days and mean length of wet spell) can be preserved by decadally varying parameters for the MC model. This is a straightforward enhancement to the traditional simplest MC model, and the enhancement is both objective and parsimonious.

The overall comparative performances of the models to reproduce the distribution and autocorrelation characteristics of observed rainfall are as follows:

- For the simulation of the distribution of rainfall depths, the performances of the APMC and DPMC with deterministic Gamma parameters are similar, although DPMC with more parameters (e.g. the decadally varying MC parameters) performs slightly better. The performances of CDMC, HMC and DHMC are similar as they use the same stochastic sampling for the parameters of the Gamma distribution.
- For the mean and SD of daily rainfall depths, all five models perform satisfactorily. Good reproduction of daily statistics is expected as the model parameters are calibrated to daily timeseries. While the APMC and DPMC reproduce the statistics almost exactly, the CDMC, HMC and DHMC show a slight tendency to underestimate the SD. This indicates that the stochastic parameters of these three models slightly affected their performances at daily resolution compared to the APMC and DPMC with deterministic parameters.
- For the monthly to multiyear resolution, the APMC and DPMC reproduce the mean of rainfall depths well, but significantly underestimate the SD of rainfall depths. The underestimation of rainfall variability at monthly to multiyear resolutions by APMC-like models with deterministic parameters is a well-known limitation of these models (Wang and Nathan, 2007). Although the DPMC uses more parameters than the APMC, the DPMC has not significantly improved performance in reproducing the SD of rainfall depths at monthly to multiyear resolutions. Other models similar to DPMC (e.g. models with parameters varying for phases of IPO or ENSO) show similar performances to the DPMC and still systematically underestimate the SD of rainfall depths at monthly to multiyear

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resolutions. This suggests that the use of deterministic parameters in DPMC-like models might not be adequate to satisfactorily reproduce the SD of rainfall depths at longer resolutions.

- While the APMC and DPMC, with deterministic parameters for the Gamma distribution, significantly underestimate the SD of rainfall depths at monthly to multiyear resolutions, the CDMC, HMC and DHMC, with stochastic parameters for the Gamma distribution, preserve the SD of rainfall depths at monthly to multiyear resolutions. This indicates that the stochastic parameters for the Gamma distribution are useful to incorporate the longer-term variability of rainfall depths. However, these three models show a tendency to underestimate the mean of rainfall depths at all resolutions.

- The models that can preserve the SD of rainfall depths can also preserve the 95$^{th}$ percentile of rainfall depths.

- For the simulation of the distribution of wet periods, the performances of the APMC and CDMC are similar as both models use the same deterministic MC parameters. With a similar trend, the DPMC and DHMC perform better than the APMC and CDMC, while DPMC and DHMC use more deterministic MC parameters. The performance of the HMC, with stochastic MC parameters, is different (discussed below) from the other four models (that use deterministic MC parameters).

- For the mean of wet period statistics (e.g. number of wet days and mean length of wet spells) at monthly to multiyear resolutions, all models except HMC perform similarly and satisfactorily, while the HMC tends to overestimate the mean. We conclude that introducing stochasticity from year to year into the MC parameters, as in HMC, degrades the performance.

- For the SD of monthly wet period statistics, all models except HMC perform similarly and satisfactorily, while the HMC significantly overestimates the SD. This indicates that the stochastic MC parameters of the HMC introduce excessive variability in the wet period simulation at monthly resolution. This study has further examined two other variants of the HMC with different stochastic parameterisation of the MC process, but they did not perform better than the HMC. We conclude that introducing stochasticity from year to year into the MC parameters, as in HMC, degrades the ability to reproduce the variability about the mean of all of the wet period statistics.

- For the SD of wet period statistics at annual and multiyear resolutions, the APMC and CDMC tend to underestimate the SDs. This suggests that the APMC values of MC parameters (same monthly parameter values for each year of simulation) limits the reproduction of the wet period variability at multiyear resolutions. However, the APMC and CDMC preserved the multiyear SDs for Adelaide, where the inter-decadal variability of MC parameters is less variable. This suggests that for locations with less variability of wet-to-wet and dry-to-dry day transitions, a single set of deterministic MC parameters is adequate, however for locations with more transition variability, a single set of MC parameters (i.e. not varying with time) is insufficient, as it cannot introduce enough variability.

- The DPMC and DHMC with decade-varied MC parameters show a better ability to reproduce the SD of annual mean length of wet spells and SD of multiyear number of wet days. This suggests that the decade-varied MC parameters can significantly improve the simulation of wet period variability, although the decade-varied Gamma parameters cannot improve the simulation of rainfall depths variability. However, the HMC preserves the SD of multiyear number of wet days but overestimates the SD of annual mean length of wet spells. This suggests that the monthly and annually varying stochastic MC parameters can improve the simulation of wet period (i.e. number of wet days and mean length of wet spell) variability at multiyear resolutions, although they significantly overestimate the wet period variability at monthly and annual resolutions (i.e. they introduce too much variability).

- The models that can preserve the wet spells distributions can also preserve the dry spells distributions and vice-versa probably because the wet and dry days are modelled using similar transition probabilities of wet-to-wet and dry-to-dry days respectively.

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- The autocorrelation assessments have shown that the APMC, DPMC, CDMC and DHMC can satisfactorily reproduce the strong lag–1 and lag–12 monthly autocorrelations of monthly number of wet days and monthly rainfall depths. However, the HMC (the only model with monthly and annually varying MC parameter values) tends to underestimate the autocorrelations, which is possibly due to excessive variability in wet period simulation at monthly resolution.

## 9. Conclusions

Each model developed in this study has advantages and disadvantages. The APMC and DPMC with deterministic parameters significantly underestimate the variability of rainfall depths at monthly to multiyear resolutions. This systematic underestimation of the rainfall depths variability at monthly to multiyear resolutions is critical for using the models in urban water security assessment as the reservoir water levels usually vary at these longer resolutions. The CDMC, HMC and DHMC with stochastic parameters of the Gamma distribution preserve the rainfall depths variability at all resolutions, but the CDMC and HMC have limitations in reproducing the variability of wet periods. The CDMC with APMC values of MC parameters tends to underestimate the multiyear variability of wet periods, while the HMC with stochastic MC parameters tends to overestimate the monthly variability of wet periods. However, the DHMC with decade-varied MC parameters (same as DPMC) performs better than the CDMC and HMC, and preserves the wet period and dry period variability at monthly to multiyear resolutions.

Among the five MC models of this study, the overall performance of the DHMC is best. The DHMC model has (1) monthly varying MC parameters that vary from decade to decade, and (2) stochastic parameters for the Gamma rainfall distribution where the parameters are randomly varied from year to year using a probability distribution function that is derived for each month of the year. While the DHMC has great potential to be used in hydrological and agricultural impact studies (e.g. urban drought security assessment), there are two important limitations of the DHMC:

- The DHMC tends to underestimate the mean of multiyear rainfall depths, which is probably linked to the use of stochastic Gamma parameters. A more sophisticated stochastic sampling strategy for the Gamma parameters might overcome this limitation.
- The performance of the DHMC suggests that the use of decade-varied MC parameters are effective to incorporate the long-term variability of wet periods (although the use of decade-varied Gamma parameters in DPMC were not effective to incorporate the long-term variability of rainfall depths). However, other climate-based sub-samples (e.g. according to the ENSO phases) instead of decadal samples can be used for parameter calibration. This study tested the sub-samples according to the phases of IPO and ENSO climate modes with a focus on incorporating the long-term variability of rainfall depths, but has not incorporated climate-based sub sampling into DHMC because DHMC had not been developed at the time this analysis was performed. A more comprehensive assessment of such ideas might improve the wet period simulation of the DHMC.

In a subsequent paper, the performances of the CDMC, HMC and DHMC will be compared against the semi-parametric model of Mehrotra and Sharma (2007) using raingauge data from 30 stations around Sydney (those used in Mehrotra et al., 2015) and the 12 stations (Figure 1) around Australia.

## 10. Data Availability

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- Daily rainfall data used in this study can be obtained from the Bureau of Meteorology, Australia website link http://www.bom.gov.au/climate/data/index.shtml by using weather station number 66062 and 023034 for Observatory Hill and Adelaide Airport stations respectively.
- ONI and IPO indices used in this study can be obtained from the National Oceanic and Atmospheric Administration website link https://www.esrl.noaa.gov/psd/data/climateindices/list/ and Folland (2008) respectively.

## 11. Code Availability

Python codes for modelling and statistical analysis of this study are available from the first author.

## 12. Author Contributions

AFM Kamal Chowdhury has conducted the model development and statistical analysis of this study. Natalie Lockart and Garry Willgoose were the primary supervisors of this work and provided scientific oversight for the model development and statistical analysis. George Kuczera and Anthony Kiem provided more focussed advice on statistics and climatology. Nadeeka Parana Manage was involved in scientific discussions as a team member of our project team.

## 13. Acknowledgments

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Comments column:

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870 **15. Tables**

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<center>**Table 1: Methodological comparison of the five MC models.**</center>

|  | **Wet and dry day simulation** | **Wet day rainfall depth simulation** |
|---|---|---|
| **APMC** | Uses deterministic MC parameters, same set of parameters for each simulation year. | Uses deterministic Gamma parameters, same set of parameters for each simulation year. |
| **DPMC** | Uses decade-varied deterministic MC parameters. | Uses decade-varied deterministic Gamma parameters. |
| **CDMC** | Same as APMC. | Uses stochastic parameters (sampled from fitted bivariate-lognormal distribution) of Gamma distribution, parameters vary for each simulation year. |
| **HMC** | Uses stochastic MC parameters (sampled from fitted truncated normal distribution), parameters vary for each simulation year. | Same as CDMC. |
| **DHMC** | Same as DPMC. | Same as CDMC. |

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**Table 2: Average of the absolute values of Z scores (average of the Z scores for all 12 months at daily and monthly resolutions, and average of the Z scores for 1 to 10 years at multiyear resolution) for Sydney (SY) and Adelaide (AD). The averaged Z scores greater than 2 are shown in bold.**

| Variable | Resolution | Average of the Absolute Values of Z Scores | | | | | | | | | |
| | | APMC | | DPMC | | CDMC | | HMC | | DHMC | |
| | | SY | AD | SY | AD | SY | AD | SY | AD | SY | AD |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Mean of Rainfall Depth | Daily | 0.1 | 0.1 | 0.0 | 0.1 | 0.3 | 0.4 | 0.4 | 0.4 | 0.3 | 0.4 |
| | Monthly | 0.1 | 0.1 | 0.1 | 0.1 | 0.3 | 0.4 | 0.3 | 0.3 | 0.3 | 0.4 |
| | Multiyear | 0.1 | 0.1 | 0.1 | 0.2 | 0.9 | 0.9 | 0.7 | 0.3 | 1.0 | 0.9 |
| SD of Rainfall Depth | Daily | 0.1 | 0.2 | 0.1 | 0.2 | 0.6 | 0.4 | 0.7 | 0.4 | 0.7 | 0.4 |
| | Monthly | 1.9 | 0.9 | 1.4 | 0.7 | 0.5 | 0.6 | 0.6 | 0.8 | 0.5 | 0.6 |
| | Multiyear | **3.7** | 1.3 | **2.6** | 0.4 | 0.6 | 0.5 | 0.6 | 0.8 | 0.6 | 0.6 |
| 95th Percentile of Rainfall Depth | Daily | 0.8 | 0.7 | 0.5 | 0.6 | 0.5 | 0.6 | 0.5 | 0.5 | 0.5 | 0.6 |
| | Monthly | 1.7 | 0.8 | 1.3 | 0.6 | 0.5 | 0.6 | 0.5 | 0.7 | 0.5 | 0.6 |
| | Multiyear | **2.6** | 0.3 | **2.0** | 0.3 | 1.0 | 0.2 | 0.5 | 0.4 | 1.0 | 0.2 |
| Mean of Number of Wet Days | Monthly | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.2 | 0.2 | 0.3 | 0.1 | 0.1 |
| | Multiyear | 0.0 | 0.0 | 0.0 | 0.1 | 0.2 | 0.2 | 0.6 | 0.9 | 0.0 | 0.1 |
| SD of Number of Wet Days | Monthly | 1.0 | 0.9 | 0.8 | 0.8 | 1.0 | 0.8 | 1.7 | 1.8 | 0.8 | 0.8 |
| | Multiyear | **3.3** | 0.8 | 1.6 | 0.5 | **3.3** | 0.9 | 0.7 | 0.7 | 1.5 | 0.5 |
| Mean of Mean Wet Spell Length | Monthly | 0.4 | 0.5 | 0.3 | 0.4 | 0.4 | 0.5 | 0.4 | 0.5 | 0.3 | 0.4 |
| | Annual | 0.1 | 0.0 | 0.1 | 0.3 | 0.1 | 0.3 | 0.2 | 0.5 | 0.1 | 0.2 |
| SD of Mean Wet Spell Length | Monthly | 0.9 | 0.7 | 0.8 | 0.6 | 0.8 | 0.7 | 0.8 | 1.0 | 0.8 | 0.7 |
| | Annual | 0.2 | 0.2 | 0.3 | 0.6 | 0.3 | 0.4 | **2.6** | **2.3** | 0.3 | 0.6 |
| Mean of Mean Dry Spell Length | Monthly | 0.5 | 0.6 | 0.4 | 0.6 | 0.5 | 0.6 | 0.7 | 0.7 | 0.4 | 0.5 |
| | Annual | 0.1 | 0.0 | 0.2 | 0.0 | 0.1 | 0.0 | 0.3 | 0.4 | 0.2 | 0.1 |
| SD of Mean Dry Spell Length | Monthly | 0.9 | 1.3 | 0.8 | 1.2 | 0.9 | 1.3 | 1.1 | 1.3 | 0.8 | 1.3 |
| | Annual | 0.9 | 0.5 | 0.1 | 0.2 | 0.9 | 0.4 | 1.4 | 1.4 | 0.1 | 0.2 |

**16. Figures**

**Figure 1: Location map of 12 raingauge stations around Australia. This study has presented the assessment results of the developed models for Sydney and Adelaide stations (red circled) only. The shaded green, yellow and red colours indicate the coastal, inland and monsoonal areas respectively.**
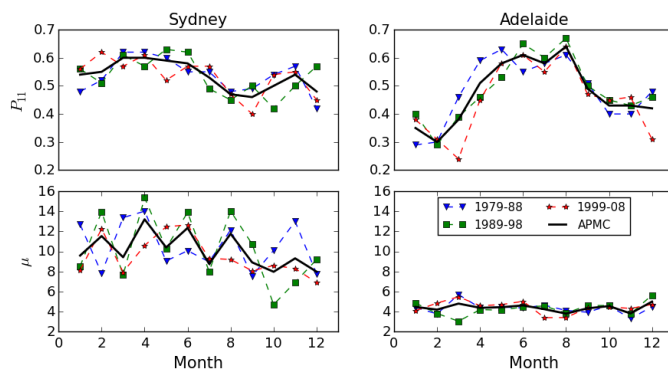
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885 **Figure 2: Comparison of the decadal variability of the DPMC parameters $P_{11}$ and $\mu$ (mm) with the APMC**
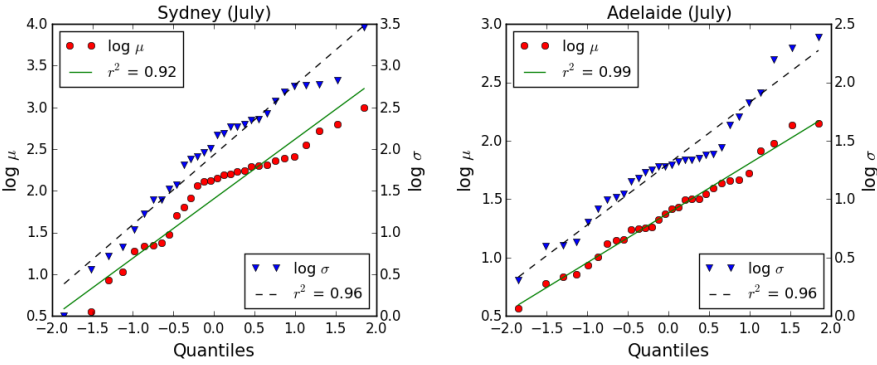886 **parameters.**

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891     **Figure 3: Lognormal probability plots of $\mu$ and $\sigma$ for July (typical of other months).**

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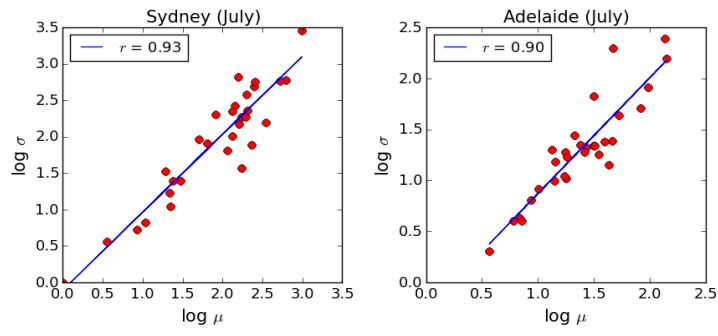
Figure 4: Correlation between *log μ* and *log σ* for July (typical of other months).

(a)



(b)
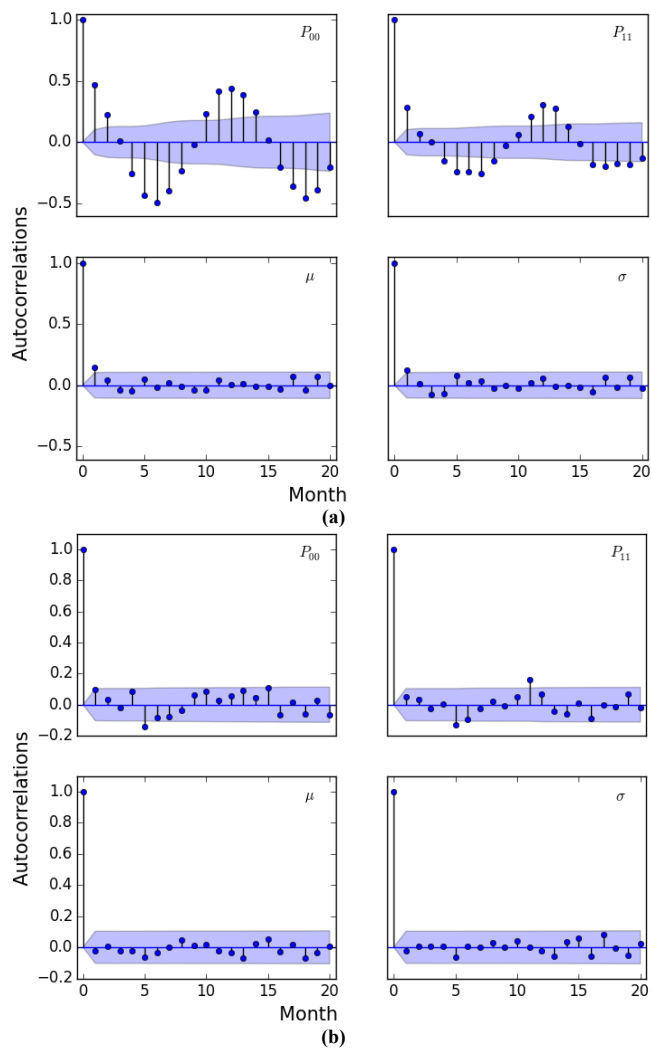
**Figure 5: Month-to-month autocorrelations of $P_{00}$, $P_{11}$, $\mu$ and $\sigma$ for (a) Adelaide and (b) Sydney. The shadings indicate 95% confidence limits.**
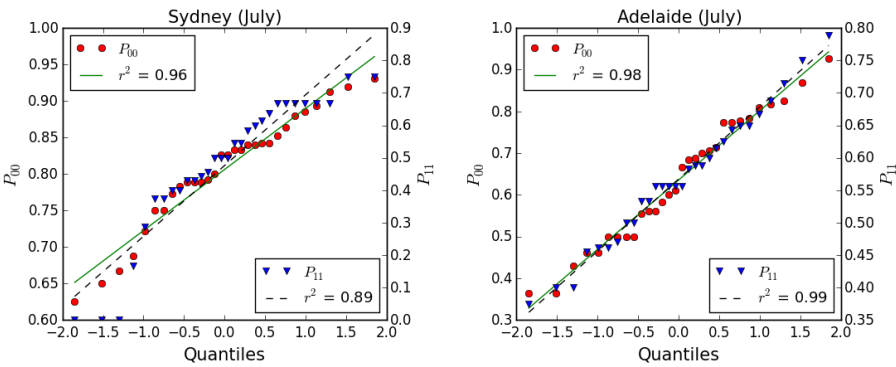
**Figure 6: Normal probability plots of $P_{00}$ and $P_{11}$ for July (typical of other months).**

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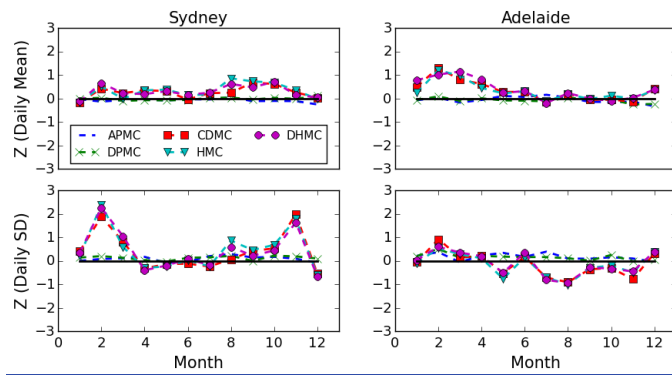
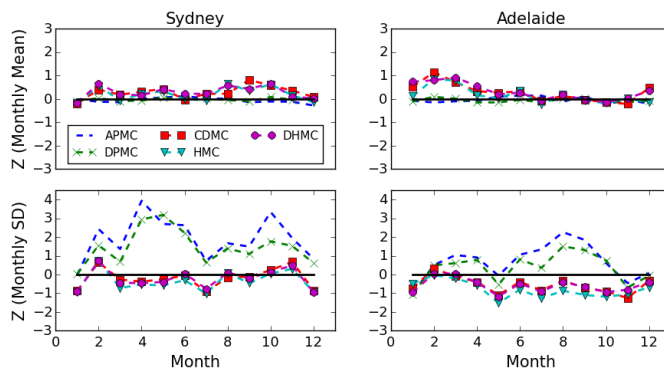**Figure 7: Comparison of the mean and SD of daily rainfall depths for the five MC models.**

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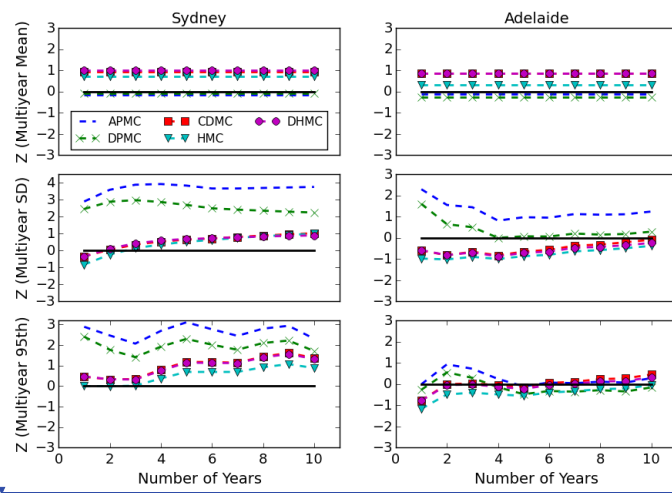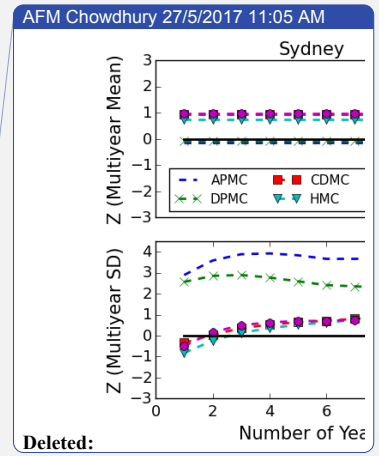**Figure 8: Comparison of the mean and SD of monthly rainfall depths for the five MC models.**

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**Figure 9: Comparison of the mean, SD and 95th percentiles of multiyear rainfall depths for the five MC models.**
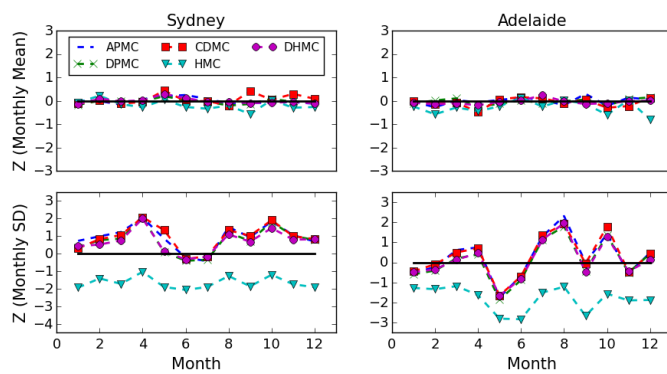
**Figure 10: Comparison of the mean and SD of monthly number of wet days for the five MC models.**

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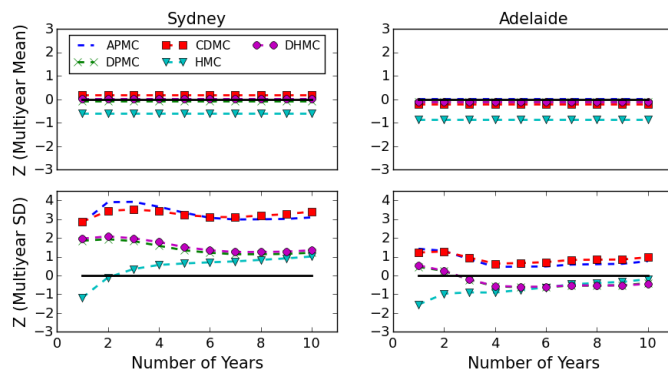
**927**     **Figure 11: Comparison of the mean and SD of multiyear number of wet days for the five MC models.**
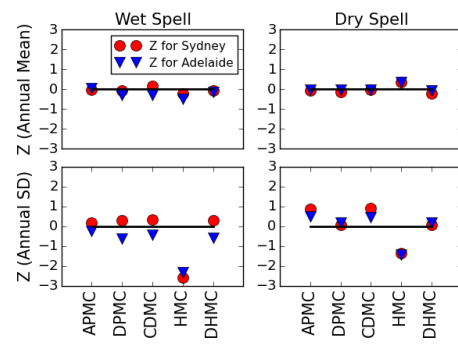
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**Figure 12: Comparison of the mean and SD of annual mean length of wet and dry spells for the five MC models.**

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**(a)**

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**(b)**

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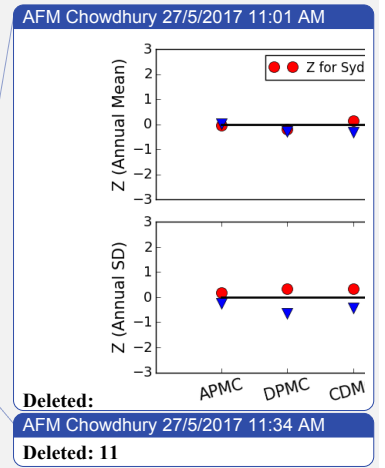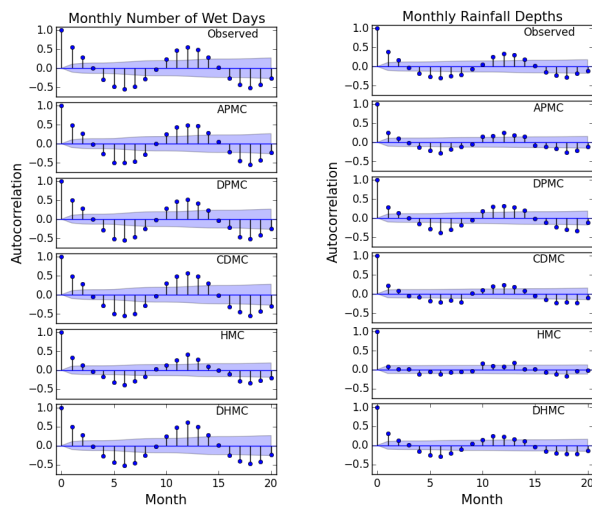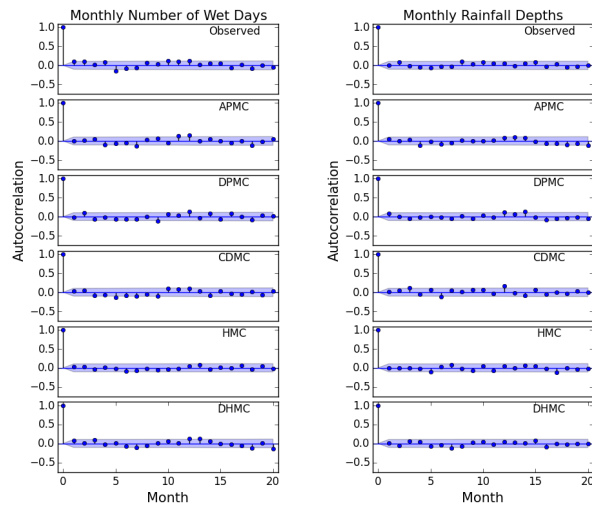
**Figure 13: Comparison of the autocorrelations of monthly number of wet days and monthly rainfall depths for the five MC models for (a) Adelaide and (b) Sydney. The shadings indicate 95% confidence limits.**

1. **In the abstract, only the variability at large aggregation levels is mentioned, but maybe it would be good to also reflect on the quality at a daily resolution.**

   - The model performances at daily resolution are indicated in the abstract (see line 35), while the relevant results have been added in section 6.1.

2. **Lines 49-51: a reference can be added to support this sentence.**

   - Added an appropriate reference. See line 51.

3. **Lines 57 to 62: maybe it would be good to expand a little on the alternative types of stochastic rainfall models. Now there's only a reference to Markov Chain models, while several other types of models, e.g. Neymann-Scott and Bartlett-Lewis models, are frequently used as well for this purpose.**

   - We have acknowledged other types of stochastic rainfall models in lines 62-63.

4. **Line 94: "…with modified Markov Chains" (plural).**

   - Corrected accordingly. See line 95.

5. **Line 169: "This paper also used the…"**

   - Corrected accordingly. See line 172.

6. **Line 178: introduce that SD stands for standard deviation.**

   - Line 87 has introduced that SD stands for standard deviation.

7. **Lines 182-184: how are wet spells defined? How much time should be between two wet instants to separate two spells? (is one day sufficient?)**

   - We defined wet spells as consecutive wet days of any length (i.e 1 or more consecutive wet days). Two wet spells are separated by a dry spell of 1 or more consecutive dry days. See line 187-190.

8. **Lines 234-236: setting rainfall below 0.3 mm to 0.3 mm: doesn't this lead to a bias in the total amount of rainfall?**

   - We have found that setting rainfall below 0.3 mm to 0.3 mm for lowest rainfall depths does not significantly affect the overall distribution of wet day rainfall depths. This issue has been tested and discussed in the PhD thesis of the first author (Chapter 4 of Chowdhury, 2016). See line 242-244.

9. **Lines 271 and 276: "For model according to…": not phrased well.**

   - Re-written the phrase as 'For IPO mode based model'. See lines 280 and 285.

10. **Lines 326-333: explain all the variables that are introduced in formula 4. In the current list, it is not clear how these variables are calculated.**

    - Comprehensive descriptions of the variables of formula 4 are given in lines 335-350.

11. **Lines 334-337: another alternative is mentioned: can this alternative get a name?**

    - Two alternative HMC models are referred as HMC2 and HMC3. See lines 330 and 353.

12. **Section 6: in the comparison using distribution statistics, it would be interesting to also investigate the tail behaviour, as mainly the tails are important when these stochastic models are used in designing hydraulic structures or for water management.**

- In section 6.1, we have added representative results of the performances of MC models for 95th percentiles of the rainfall depths.

**13. Section 6.3 focusses on wet spells, but a similar section on dry spells would be interesting as well, especially, if this stochastic model would be used in the frame of water management in drought conditions.**

- We found that the models that can preserve the wet spells distributions can also preserve the dry spells distributions and vice-versa as the wet and dry days are modelled using similar transition probabilities of wet-to-wet and dry-to-dry days respectively. We have added representative result in section 6.3 to support this statement.

**14. Lines 402-403: "… that can match not only the short resolution (daily) variability,…": this was not addressed in the paper.**

- We have added the results for daily rainfall depths in section 6.1.

**15. Line 417: "For the simulation" (add "the")**

- Corrected accordingly. See line 466.

**16. Line 421: "For the mean" (add "the")**

- Corrected accordingly. See line 470.

**17. Line 426: "At the monthly" (add "the")**

- Corrected accordingly. See line 475.

**18. Line 463: "… MC parameters show a better …" (add "a")**

- Corrected accordingly. See line 513.

**19. Line 519: "… was involved in scientific …" (add "in")**

- Corrected accordingly. See line 573.

**20. Reference list: for some references, all words in the title are capitalized (e.g. Bardossy and Plate, 1992), while for others, this is not the case (e.g. Chowdhury et al, 2015): please make this consistent.**

- The references are corrected to make consistent.

**21. Figure 5: explain the blue colour in the caption or in a legend in the figure.**

- Blue shade is explained in the caption of Figure 5.

**22. Figure 12: explain the blue colour in the caption or in a legend in the figure.**

- Blue shade is explained in the caption of Figure 12.

**Newly Added Reference**

Chowdhury, A.F.M.K.: Development and Evaluation of Stochastic Rainfall Models for Urban Drought Security Assesment, PhD Thesis, The University of Newcastle, Australia, 2016.

# Responses to the Comments of Referee-3

1. **Lines 155-156: It is meaningless to apply these single site models with dynamically downscaled data as these models do not have spatial correlation in them**

   - We have removed the sentence. The evaluation of the MC models using dynamically downscaled climate data will be presented in separate subsequent papers.

2. **Lines 185-198: This is not bootstrapping. In stochastic data generation, the usual practice is to generate a number of replicates and the mean values of the statistics from the replicates are compared with the observed values. The confidence limits can be obtained from the values from the replicates without making any assumption as to the distribution of the statistics. For instance, from 1000 replicates the 25$^{th}$ and 975$^{th}$ ranked values will give the 95% confidence limits.**

   - We have removed the term 'bootstrapping'.

3. **Lines 258-261: What is meant by" parametric bootstrapping"? Section 3.2 does not mention the word "parametric" at all. Equality of means and variances can be easily tested by t-test and F test respectively.**

   - We have replaced the term 'parametric bootstrapping' by 'Monte-Carlo simulation'.

4. **Line 277: What is "ONI" index?**

   - ONI indicate Oceanic Nino Index, which has been introduced in Line 171 of section 2.

5. **Lines 304-306: Not relevant to the subject matter of this manuscript**

   - We have removed this sentence.

6. **Line 364: "while the inter-decadal variabilities of parameters are less in Adelaide and high in Sydney (Figure 2)" - This is out of place and confusing. What are the authors trying to say here? The mean annual rainfall from the 1000 replicates is about one standard deviation away (Figure 8) from the observed value. What is the bias in terms of percentage of mean annual rainfall? One would expect the bias is less than 5% for a model to be considered as good.**

   - We have deleted that part of the sentence (see line 375). The key massage is that the APMC and DPMC underestimate the SD (Z score greater than +2) at monthly and multiyear resolutions for Sydney but preserve the SDs ( Z score between -2 and +2) for Adelaide.

7. **Lines 368-376: None of the models appear to preserve the SD of monthly number of wet days for all the months. Some of the sentences in this paragraph are either self-contradicting or clumsy. There is only one model (HMC) that has stochastic yearly varied parameters but it is mentioned as "…. models with stochastic, yearly varied, parameters for the MC part of the model ...." .This paragraph needs rewriting.**

   - We have corrected the paragraph for clarity and consistency. See section 6.2.

8. **Lines 378-382: The results for the monthly resolution are not presented. The first sentence is strange. This paragraph needs rewriting as well.**

   - We have corrected the paragraph for clarity and consistency. See section 6.3.

9. **Lines 395-400: I presume that the blue shading in Figure12 represents the 95% confidence band. For Sydney, none of the autocorrelation is significantly different from zero.**

   - Yes, the blue shading indicates 95% confidence limit. We have added this information in the caption.

10. **Lines 421-425: Mean and standard deviation of daily rainfall are neither presented nor discussed in the manuscript earlier. Provide these results to support this paragraph.**

    - We have added the results for mean and standard deviation of daily rainfall depths in section 6.1.