We thank the colleague Hristos Tyralis for his comments and for having pointed out the importance of this research. We enormous appreciate his dedication. We hope that our answers successfully address his concerns and requirements.

Comment #1

In this manuscript, the authors model jointly the hydrological and error model parameters in a Bayesian framework. I am particularly interested in the mathematical theory of the model. Therefore, I took a closer look at the 2nd and 3rd section. In general trying to model the errors using varying parameters, which depend on the level of the hydrological model prediction is interesting. Furthermore, the derivations are far from trivial and the theoretical results are useful.

It seems that the authors first try to form the errors. As a result, some parameters of the model must be expressed as functions of the other parameters (e.g. implementing the law of total expectation and the law of total variance), with a simultaneous reduction of the parameter space. Here I think that the title of the manuscript is misleading, since satisfying the two laws must be mandatory. Therefore, the term "enforcement" seems unnecessary. I suggest that less emphasis is given to the enforcement and more to the investigation of various forms of the errors because the latter is the cause for further examining the two laws, while the former is the consequence.

Reply

Firstly, we agree with Dr. Tyralis: "*satisfying the two laws must be mandatory*". We also agree with him about the need of making research on the different error structures originated by the different sources of uncertainty, in hydrological modeling. However, the lumped via of error modeling which we are using makes difficult this task. In any case, our original target was making Bayesian inference of hydrological models rather than making research on errors. But we came across unexpected problems which forced us to change the problem to solve. The fact is that modeling the errors as in Schoups and Vrugt (2010), Evin et al. (2013, 2014), Scharnagl et al. (2015) or ourselves requires the enforcement of Total Laws because if not, they are not fulfilled. We will explain again why is this so, since it seems it has not been well understood with the original manuscript.

Commonly, the error modeling has been performed by using a likelihood function assuming a multivariate Gaussian distribution for the errors, with the definition of its covariance matrix, as for example in Del Giudice et al. (2013) or Reichert and Schuwirth (2012) among others: by using this approach we are already considering implicitly the relation between the marginal and conditional distributions of the multivariate Gaussian. However, Schoups and Vrugt (2010) introduced a different innovative and generalized approach for error modeling, which was also followed by Evin et al. (2013, 2014) and Scharnagl et al. (2015) among others. This more flexible approach is based on the modeling of the statistical features of the error conditional distributions: its variance, bias, kurtosis and skewness. This features are modeled each separately: we can hypothesize different functional expressions for each statistical feature. But one detail was neglected in this approach: we are only modeling the error conditional distributions, overlooking that they are conditionals within a multivariate distribution. **How are we sure that these conditional distributions are part of a multivariate distribution? How are we introducing in the inference framework the existence of this multivariate distribution?**

Explained with equations and following the notation proposed by Dr. Tyralis. We want to model the joint distribution of a simulated variable and its error, $p(\underline{e}, \underline{y}_s)$ for making inference. But instead, we are modeling the conditionals, $p(\underline{e} \mid \underline{y}_s)$, since this way to proceed can provide us more flexibility in the error modeling. Relation between them is established by the conditional probabilities law, as it follows:

$$p(\underline{e}, \underline{y}_s) = p(\underline{e} \mid \underline{y}_s) p(\underline{y}_s) \quad (1)$$

where $p(\underline{y}_s)$ is the marginal probability distribution of $\underline{y}_s$. On the other hand, the marginal distribution of the errors can be obtained by integrating this joint distribution as follows:

$$p(\underline{e}) = \int_{\underline{y}_s} p(\underline{e}, \underline{y}_s) d\underline{y}_s = \int_{\underline{y}_s} p(\underline{e} \mid \underline{y}_s) p(\underline{y}_s) d\underline{y}_s \quad (2)$$

But, it also holds that:

$$E\left[p(\underline{e} \mid \underline{y}_s) \right] = \int_{\underline{y}_s} p(\underline{e} \mid \underline{y}_s) p(\underline{y}_s) d\underline{y}_s \quad (3)$$

Therefore, from equations (2) and (3), we have:

$$p(\underline{e}) = \underset{\underline{y}_s}{E}\left[p(\underline{e} \mid \underline{y}_s) \right] \quad (4)$$

This is a **relationship between error marginal and conditional distributions which must always be maintained**. But by only defining the statistical features of $p(\underline{e} \mid \underline{y}_s)$ is not enough to ensure the fulfillment of this relationship.

Moreover, equality of Eq. (4) for distributions is also applicable for their expectations, hence we obtain the **Total Expectation Law (TEL)**:

$$E[\underline{e}] = \underset{\underline{y}_s}{E}\left[E\left[\underline{e} \mid \underline{y}_s \right] \right] \quad (5)$$

On the other hand, it can be demonstrated that Total Variance Law (TVL) is based on TEL. The steps of this demonstration are outlined in what follows. The marginal variance of the errors can be expressed as $V[\underline{e}] = E\left[\underline{e}^2 \right] - E^2[\underline{e}]$. By considering TEL on the two right terms of this equation, we can write:

$$V[\underline{e}] = \underset{\underline{y}_s}{E}\left[E\left[\underline{e}^2 \mid \underline{y}_s \right] \right] - \underset{\underline{y}_s}{E^2}\left[E\left[\underline{e} \mid \underline{y}_s \right] \right] \quad (6)$$

By expanding term inside the outer brackets, on the first right term of Eq. (6), we get:

$$V\left[\underline{e}\right] = \underset{\underline{y}_s}{E}\left[V\left[\underline{e} \mid \underline{y}_s\right] + E^2\left[\underline{e} \mid \underline{y}_s\right]\right] - \underset{\underline{y}_s}{E^2}\left[E\left[\underline{e} \mid \underline{y}_s\right]\right] \quad (7)$$

And by rearranging Eq. (7) and considering again the relation $V\left[\cdot\right] = E\left[\cdot^2\right] - E^2\left[\cdot\right]$, we reach the final **expression of TVL**:

$$V\left[\underline{e}\right] = \underset{\underline{y}_s}{E}\left[V\left[\underline{e} \mid \underline{y}_s\right]\right] - \underset{\underline{y}_s}{V}\left[E\left[\underline{e} \mid \underline{y}_s\right]\right] \quad (8)$$

Consequently, we have demonstrated from where TLs (TEL and TVL) arise. There is a relationship between the error marginal and conditional distributions, summarized by TLs, which is not considered by only modeling the error conditional variance or bias. Therefore, we must also establish (enforce) this relationship in an explicit way. We think nobody questioned these issues in the past: 1) with SLS error model it was not needed, since SLS meets TLs by its own hypotheses; 2) by using a multivariate Gaussian likelihood we are directly defining a joint distribution rather than modeling the conditional ones, so more considerations are not needed.

These are the reasons why we make a point in term "enforce" and we have included it in the Title.

Comment #2

Regarding the mathematical part of the manuscript, I recommend that the authors separate the notation for parameters and variables, to ease understanding of the framework and help the reviewing process. Please see the supplement for more details.

Reply

In case that manuscript be accepted, we will study which change in the notation could be more suitable, for the sake of both simplicity and better understanding.

Comment #3

Lastly, I do not understand why we should sample from eq. (19) of the manuscript rather than using the distribution of line 335. This would seem the straightforward approach, considering eq. (18) of the manuscript.

Reply

Dr. Tyralis is right. The Eq. (18) is the straightforward approach to obtain the predictive uncertainty. This approach, followed by many authors (e.g. Krzysztofowicz and Todini are good references), is related with the "operational point of view" of hydrology. But our research falls within the model calibration context. Therefore, we try to make a reliable parameter inference jointly with getting also reliable predictive uncertainty estimations. This problem is very different to the former one. We explain this in **Lines**

98-132 of the manuscript. The cornerstone of our approach is the error model. Once we have the error model, jointly inferred with the hydrological one, we have got, theoretically, well-estimated error and hydrological parameters. Therefore, by using Eq. (19) repeatedly, with several samples of the innovation distribution (SEP in our case) and several samples of the error and hydrological parameters, we can obtain samples from the predictive distribution expressed as in Eq. (18). It is important to point out (see **Lines 337-342**) that we want to obtain an approximation to the distribution of Eq. (18) and not to the distribution indicated in **Line 335**: this later is still conditional on parameters! It is also important to remark that the predictive distribution in Eq. (18) is conditional on observations, forcing inputs and initial conditions, all of them supposed known and certain quantities.

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