Exploring the relationship between warm-season precipitation, potential evaporation, and “apparent” potential evaporation at site scale

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Abstract

Bouchet’s complementary relationship and the Budyko hypothesis are two classic frameworks that are inter-connected. To systematically investigate the connections between the two frameworks, we analyze precipitation, pan evaporation and potential evaporation data at 259 weather stations across the United States. The precipitation and pan evaporation data are from field measurement and the potential evaporation data are collected from a remote-sensing dataset. We use pan evaporation to represent “apparent” potential evaporation, which is different from potential evaporation. With these data, we study the correlations between precipitation and potential evaporation, and between precipitation and “apparent” potential evaporation. The results show that 93% of the study weather stations exhibit a negative correlation between precipitation and “apparent” potential evaporation. Also, the aggregated data cloud of precipitation versus “apparent” potential evaporation with 5312 warm-season data points from 259 weather stations shows a negative trend in which “apparent” potential evaporation decreases with increasing precipitation. On the other hand, no significant correlation is found in the data cloud of precipitation versus potential evaporation, indicating that precipitation and potential evaporation are independent. We combine a Budyko-type expression, the Turc-Pike equation, with Bouchet’s complementary relationship to derive upper and lower Bouchet-Budyko curves, which display a complementary relationship between “apparent” potential evaporation and actual evaporation. The observed warm-season data follow the trend of the Bouchet-Budyko curves. Our study shows the consistency between Budyko’s framework and Bouchet’s complementary relationship, with the distinction between potential evaporation and “apparent” potential evaporation. The formulated complementary relationship can be used in quantitative modeling practices.
1. Introduction

Potential evaporation ($E_p$) is a widely used physical variable in hydrologic frameworks. It is the evaporation rate under unlimited land surface water supply (Thornthwaite, 1948). Pan evaporation ($E_{pan}$) measurement is often used as a surrogate of potential evaporation. However, these two variables are not the same (Brutsaert and Parlange, 1998; Roderick et al., 2009). A stipulation is added in the potential evaporation definition in Van Bavel (1966) and further clarified in Brutsaert (2015) that: “the surface vapor pressure be saturated, so that it can be found from the surface temperature.” Therefore, the main difference between potential evaporation and pan evaporation is that pan evaporation is not measured under saturated surface vapor pressure.

As a result, potential evaporation can be considered to depend only on the energy supply of climate while pan evaporation is driven by both energy supply and humidity deficit in the atmosphere (Rotstayn et al., 2006). In Brutsaert and Parlange (1998), the term “apparent” potential evaporation ($E_{pa}$) is introduced to distinguish pan evaporation from potential evaporation. “Apparent” potential evaporation can be measured by an evaporation pan, while potential evaporation cannot. We acknowledge that there are different definitions of potential evaporation in the literature (Aminzadeh et al., 2016). Our study follows the definition of potential evaporation in Brutsaert and Parlange (1998) and Brutsaert (2015).

Because potential evaporation is energy-driven, it can be used as a physical variable to describe the energy supply in a hydrologic system. For instance, the well-established Budyko framework (Budyko, 1958; 1974) uses precipitation ($P$) and potential evaporation to represent the relationship between water supply and energy supply, and therefore to describe the impact of long-term climate on the hydrologic cycle. The Budyko framework has been extensively used to analyze interactions between hydrology, climate, vegetation and other elements in watersheds.
Several studies have made connections between the Budyko framework and Bouchet’s complementary relationship (CR) (Bouchet, 1963). Yang et al. (2006) used the Fu equation (Fu, 1981), which is one of the commonly used equations to represent the Budyko curve, to describe the relationship between actual evaporation and potential evaporation in the CR. Roderick et al. (2009) presented a complementary relationship normalized by net irradiance and compared it with the Budyko framework. Lhomme and Moussa (2016) combined Turc-Pike equation (Turc, 1954; Pike, 1964), which is another commonly used Budyko-type equation, with the CR to show the dependence of Budyko curve on the drying power of the air.

When linking the Budyko framework with the CR, it is crucial to have a clear definition of different types of evaporation used in these two frameworks. Brutsaert and Parlange (1998) and Brutsaert (2015) generalized the CR and provided definitions of the evaporation terms in the CR, namely actual evaporation ($E$), potential evaporation ($E_p$), and “apparent” potential evaporation ($E_{pa}$, see Fig. 1a). Brutsaert and Parlange (1998) point out that the complementary relationship is between actual evaporation and “apparent” potential evaporation, not between actual evaporation and potential evaporation. In the Budyko framework (Fig. 1b), the definition of potential evaporation follows Van Bavel (1966)’s potential evaporation definition that it is under unlimited land surface water supply without the effect of humidity deficit (Budyko, 1974), which is the same as the $E_p$ definition in the generalized CR. The definitions of evaporation,
potential evaporation and “apparent” potential evaporation in these different frameworks are summarized in Table 1.

Process-based speaking, the CR suggests a connection between evaporation and “apparent” potential evaporation (Fig. 1a), which is driven by the energy feedbacks between atmosphere and land surface. During the drying process at the land surface, the excessive energy that is not used for evaporation will be available for the increase of sensible heat. The rise in air temperature will lead to an increase in the rate of “apparent” potential evaporation (Brutsaert and Parlange, 1998; Brutsaert, 2005; Aminzadeh et al., 2016). This connection between $E_{pa}$ and $E$ also suggests a connection between $E_{pa}$ and $P$, since the water supply from precipitation will affect the rate of evaporation. In terms of the Budyko framework, $E_p$ and $P$ are used as the representations of energy supply and water supply respectively. The ratio between $E_p$ and $P$ is the primary controlling factor of the ratio of $E$ over $P$ in watersheds at long-term mean annual time scale (Fig. 1b). The ratio of $E_p$ over $P$ is also called the aridity index, which represents the dryness of the climate in a watershed. The ratio of $E$ over $P$ increases with the increase of aridity index, indicating that more water from precipitation will become evaporation rather than runoff under drier climate (Arora, 2002). No connection between $E_p$ and $P$ is suggested in the Budyko framework.
Fig. 1. Conceptual representations of (a) the complementary relationship and (b) Budyko framework.

Table 1. Types of evaporation in the Budyko framework and the original CR, and their redefined evaporation type based on generalized CR. The last column refers to the definitions of the three types of evaporation in the generalized CR provided in Brutsaert (2015).

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In order to explore the connections between the Budyko framework and the CR, our study investigates the relationships between precipitation and potential evaporation as well as between precipitation and “apparent” potential evaporation. We collect warm-season precipitation, potential evaporation and pan evaporation data from 259 weather stations across the contiguous US. Studying the relationships between \(P, E_p\) and \(E_{pa}\), advances our
understanding of the well-established classic Budyko framework and the CR. Furthermore, based on insights provided by previous studies (Yang et al., 2006; Roderick et al., 2009; Lhomme and Moussa, 2016), we use a Budyko-type expression to develop a new formulation for the CR.

2. Methodology

2.1 Theoretical development

2.1.1 Budyko framework

The Budyko curve (Fig. 1b) describes the relationship between long-term water partitioning, represented by the ratio of actual evaporation over precipitation, and long-term climate, represented by the ratio of potential evaporation over precipitation, namely aridity index (Budyko, 1958; 1974). In recent decades, the Budyko framework has been examined with annual data (e.g. Yang et al., 2007; Potter and Zhang, 2009; Cheng et al., 2011). A number of Budyko-type functions have been developed to mathematically describe the Budyko curve (Turc, 1954; Fu, 1981; Zhang et al., 2001; Yang et al., 2008; Wang and Tang, 2014). Within these functions, the Turc-Pike equation is a parsimonious single parameter equation (Turc, 1954; Pike, 1964):

\[ \frac{E}{P} = \left[ 1 + \left( \frac{E_p}{P} \right)^{-\nu} \right]^{-\frac{1}{\nu}} \]  

(1)

where \( E \) is actual evaporation, \( E_p \) is potential evaporation, \( P \) is precipitation, and \( \nu \) is a parameter to represent landscape properties such as vegetation coverage and soil properties (Zhang et al.,
The parameter $\nu$ needs to be a positive number, and its typical value is 2.0.

2.1.2 Generalized complementary relationship

Bouchet’s complementary relationship (Bouchet, 1963) describes the relationship between actual evaporation $E$ and potential evaporation $E_p$. Brutsaert and Parlange (1998) introduced the term “apparent” potential evaporation $E_{pa}$ and clarified that the CR is between $E$ and $E_{pa}$, not $E$ and $E_p$ (Fig. 1a). They also proposed a generalized complementary relationship:

$$bE + E_{pa} = (1 + b)E_p \quad 0 \leq E \leq E_p \leq E_{pa} \quad (2)$$

where $b$ is a proportionality parameter not less than one. When $b$ is equal to one, Eq. (2) represents the original complementary relationship (Kahler and Brutsaert, 2006). “Apparent” potential evaporation will be higher than potential evaporation, especially under dry conditions; while it gradually approaches potential evaporation as the ratio of $E$ over $E_{pa}$ increases (Fig. 1a).

As suggested by Morton (1976) and Brutsaert and Stricker (1979), potential evaporation can be estimated using the Priestley-Taylor equation (Priestley and Taylor, 1972), which is also called equilibrium evaporation (Brutsaert and Chen, 1995; Jiang and Islam, 2001). “Apparent” potential evaporation can be estimated using the Penman equation (Penman, 1948; Linacre, 1994; Rotstayn et al., 2006) or using data measured at evaporation pans (Brutsaert, 1982; Brutsaert and Parlange, 1998):

$$E_{pa} = aE_{pan} \quad (3)$$

where $E_{pan}$ is the pan evaporation and $a$ is the pan coefficient. The pan coefficient varies from location to location (Stanhill, 1976; Linacre, 1994). In Kahler and Brutsaert (2006), a pan
coefficient of $a = 1.0$ is recommended for mixed natural vegetation, which will be used in this study. It should be noted that the linear relationship between $E_{pa}$ and $E_{pan}$ given in Eq. (3) and the choice of “$a$” value will not affect the correlations between $P$, $E_p$ and $E_{pa}$.

2.1.3 Relationships between $P$, $E_p$ and $E_{pa}$

The x-axis of the complementary relationship is a ratio between $E$ and $E_{pa}$ (Bouchet, 1963). Ramírez et al. (2005) used the water-energy framework to link the CR with Budyko approach and changed the x-axis in the CR to moisture availability. Following this idea, several studies have used precipitation or wetness index ($P/E_p$) to represent moisture availability in the CR (Yang et al., 2006; Roderick et al., 2009). In this study, we also use $P$ to represent moisture availability in the CR. $E_p$ is a horizontal line in the CR that is parallel to the x-axis (Fig. 1a). Therefore, the modified CR indicates that $P$ and $E_p$ are independent. On the other hand, the upper curve of the CR, representing “apparent” potential evaporation $E_{pa}$, declines along the x-axis, indicating that $E_{pa}$ and $P$ are not independent. For a dimensionless CR, we normalize the x and y axes. The normalized CR describes the relationship between $\frac{E_{pa}}{E_p}$, $\frac{E}{E_p}$, and $\frac{P}{E_p}$ (Fig. 2).

To connect the Budyko framework with the normalized CR toward formulating the Bouchet-Budyko curves, we first transform Eq. (1) into a relationship between $\frac{E}{E_p}$ and $\frac{P}{E_p}$:

$$\frac{E}{E_p} = \left[ \left( \frac{P}{E_p} \right) - v + 1 \right]^{-\frac{1}{v}} \quad (4)$$

$$b \frac{E}{E_p} + \frac{E_{pa}}{E_p} = 1 + b \quad (5)$$
Combining Eqs. (4) and (5), gives a relation between $\frac{P}{E_p}$ and $\frac{E_{pa}}{E_p}$:

$$\frac{E_{pa}}{E_p} = b + 1 - b\left[\left(\frac{P}{E_p}\right)^{-\nu} + 1\right]^{-1/\nu} \quad E_{pa} \geq E_p \quad (6)$$

Equations (4) and (6) represent the lower and upper curves of the normalized CR respectively (Fig. 2). Roderick et al. (2009) presented a similar framework, without the formulation of the curves. To verify the relationships between $P$, $E_p$, and $E_{pa}$, and to examine the Bouchet-Budyko curves in Eqs. (4) and (6), we analyze climate data from 259 weather stations across the contiguous US.

![Dimensionless Bouchet-Budyko curves in the normalized complementary relationship](image)

**Fig. 2.** Dimensionless Bouchet-Budyko curves in the normalized complementary relationship.

### 2.2 Data sources

Monthly precipitation and pan evaporation are collected from the National Oceanic and Atmospheric Administration (NOAA) at the National Climatic Data Center (NCDC). The data can be downloaded at: [https://www.ncdc.noaa.gov/IPS/cd/cd.html](https://www.ncdc.noaa.gov/IPS/cd/cd.html). The precipitation data are
measured using standard rain gauge and the pan evaporation data using Class A evaporation pans. We collect data for the period 1984-2015 from a total of 259 weather stations (Fig. 3a). Since pan evaporation is collected only during warm months (when temperatures remain above freezing), the weather stations at cold regions have less than 12 months of pan readings in a year. We call the period of warm months in a year “warm-season”. We calculate the monthly average pan evaporation and precipitation using only the warm months for each year at each weather station. For short, it is called warm-season data (i.e., warm-season pan evaporation, warm-season precipitation). We also calculate the annually averaged warm-season data to represent the long-term average level of pan evaporation and precipitation at each station. For short, it is called long-term average data. Over the 259 selected stations, there is an average of seven months per year with available pan evaporation data. As Fig. 3 shows, the number of available months decreases from the southern regions to the northern regions. For stations in the southern states with all 12 months of available data in a year, the full year will be considered as a warm-season. The northern state stations have fewer warm months, and, accordingly, the warm-season is much shorter. On the other hand, not all 259 weather stations have the full record from 1984 to 2015, the average number of years with available data for each location is 18. A complete summary of the information available at all 259 weather station is provided in Table S1. In order to minimize the uncertainty from various warm periods in a year from station to station, we repeat the analysis using an alternative source of pan evaporation in the NCDC dataset containing homogenized warm month data from May to October (Hobbins, et al., 2017). A total of 93 weather stations overlap both sets of pan evaporation data for the period 1984 to 2001 (Fig. 3b). We convert pan evaporation in the NCDC dataset to “apparent” potential evaporation using Eq. (3).
The $E_p$ data are collected from a remote-sensing dataset (Zhang et al., 2010), which is generated using the Priestley-Taylor equation with remotely sensed net radiation:

$$\lambda E_p = \alpha \frac{\Delta}{\Delta + \gamma} (R_n - G)$$

(7)

where $\lambda$ (J/kg) is the latent heat of vaporization; $\lambda E_p$ (W/m$^2$) is the latent heat flux; $\alpha$ is a coefficient to account for the effect of surface characteristics and vegetation, and is set to 1.26; $\Delta$ (Pa/°C) is the slope of the saturated vapor pressure curve; $\gamma$ (Pa/°C) is the psychometric constant; $R_n$ (W/m$^2$) is the net radiation; and $G$ (W/m$^2$) is the heat flux into the ground. The $E_p$ data cover the period 1983-2006. Similar with $P$ and $E_{pa}$, we calculate the warm-season $E_p$ and long-term annually averaged $E_p$ based on the monthly $E_p$ data.

Fig. 3. (a) Map of 259 weather stations. The available month of a year of pan evaporation data for each weather station is presented using legends with different colors and shapes. Four representative weather stations are selected from the four quadrants of the US respectively, which are highlighted with red circles. (b) Map of 93 weather stations with homogenized pan evaporation data that overlap the 259-station dataset.

2.3 $P$, $E_p$ and $E_{pa}$ correlation analysis
Using the collected weather station data of precipitation and pan evaporation for the period 1984 to 2015, we first calculate the Pearson correlation coefficient between warm-season $P$ and warm-season $E_{pa}$ for each location (Fig. 3a). We then perform the same correlation analysis of $P$ and $E_{pa}$ using the homogenized pan evaporation dataset (Hobbins et al., 2017) (Fig. 3b). Secondly, we use data of warm-season $P$ and warm-season $E_p$ for the period of 1984 to 2006, which is the period both $P$ and $E_p$ data are available, to investigate the correlation between $P$ and $E_p$. Finally, to validate the newly derived Bouchet-Budyko curves, the relationship between $\frac{P}{E_p}$ and $\frac{E_{pa}}{E_p}$ is plotted using the collected data at both seasonal and long-term average time scales.

3. Results

3.1 Correlations among $P$, $E_p$, and $E_{pa}$

In the 259 weather stations, 93% of the stations have a negative correlation between $P$ and $E_{pa}$ (Fig. 4a), but only 43% of the stations are statistically significant ($p<0.05$; Fig. 4b). All significant $P$, $E_{pa}$ correlations are negative. The weather stations located in the western region (regions with longitude higher than the weather station average longitude of W 94.81°) are more likely to have a significant $P$, $E_{pa}$ negative correlation than those located in the east (regions with longitude lower than W 94.81°). This spatial difference may be related to climate characteristics: the eastern region has higher precipitation (averagely 105.5 mm/month) and lower “apparent” potential evaporation (averagely 145.3 mm/month), while the western region has lower precipitation (averagely 44.6 mm/month) and higher “apparent” potential evaporation (averagely 203.5 mm/month). The Bouchet’s complementary relationship is more significant in arid regions (Ramírez et al., 2005), corresponding to the left side of the CR curves; while it is less significant.
in humid regions, corresponding to the right side of the CR curves (Fig. 1a). As a result, the negative correlation between precipitation and “apparent” potential evaporation is more significant in the west than in the east.

Fig. 4. Map of point-scale annual $P, E_{pa}$ correlation at 259 weather stations, (a) $r$ value and (b) $p$ value.

All the warm-season $P$ vs. $E_{pa}$ relations (i.e., all years, all seasons, for a total of 5312 data points) are shown in Fig. 5a. The data cloud shows a negative trend in general. We also plot the long-term annually averaged values of warm-season $P$ and $E_{pa}$ of the 259 weather stations (Fig. 5b), which shows a similar negative trend. Hobbins et al. (2004) showed a similar negative trend between precipitation and pan evaporation with watershed scale data. To represent the spatial distribution of the weather stations, we color code the data points based on their spatial coordinates of latitude and longitude. The climate in the eastern US is much wetter than the western US, and therefore the data cloud of $E_{pa}$ vs. $P$ is separated into two parts horizontally. The right side of the cloud represents the northeastern and southeastern US (green and brown, respectively); while the left side of the cloud generally represents the northwestern and southwestern US (yellow and red, respectively).
As explained before, we also use an alternative pan evaporation dataset (Hobbins et al., 2017) to further validate our analysis result. This dataset is homogenized to have the same period of pan evaporation data record in each year from May to October. In order to minimize the data heterogeneity caused by station move and human errors, this dataset compiled pan evaporation data from 247 stations across the US with thorough quality control. It is derived from the same dataset as our data, namely the NCDC dataset. Based on the homogenized pan evaporation data, 85 stations out of 93 (91%) have a negative correlation between $P$ and $E_{pa}$. Of these, 41% of the stations have a statistically significant relationship ($p<0.05$); all negative. This result is consistent with the analysis result based on our collected data from 259 weather stations.

We also use the data cloud to show the relationship between $P$ and $E_{pa}$ in the warm period of May to October in each year at each of the 93 stations (Fig. 5c), as well as the relationship of long-term annually averaged warm period $P$ and $E_{pa}$ (Fig. 5d). The trend of data cloud is similar with the data cloud trend using our collected data at both seasonal and long-term average time scales. In other words, both datasets show a negative relationship between $P$ and $E_{pa}$.

The $P$ and $E_p$ data are shown in Figures 5e and 5f. At both seasonal and long-term average time scales, there is no clear relationship shown between $P$ and $E_p$, confirming the independence between $P$ and $E_p$ discussed in Section 2.1.3. This result shows the difference between $E_p$ and $E_{pa}$, that $E_p$ is independent from $P$ but $E_{pa}$ is not. Therefore, it is important to distinguish $E_{pa}$ from $E_p$ and to understand the different physical mechanisms of the two processes (Brutsaert, 2015).
Fig. 5. $P$ vs. $E_{pa}$ at 259 weather stations in the US for the period 1984 to 2015 for (a) warm-season data (N=5312), and (b) long-term annually averaged warm-season data (N=259). The data points are color coded based on their latitudes and longitudes. $P$ vs. $E_{pa}$ at 93 weather stations in the US for the period 1984 to 2001 using the homogenized pan evaporation dataset for (c) warm period May-Oct in each year (N=1214), and (d) long-term annual average warm period May-Oct data (N=93). $P$ vs. $E_p$ at the 259 weather stations for the period of 1984 to 2006 for (e) warm-season data (N=5312) and (f) long-term annual average warm-season data (N=259).

To present the $P$, $E_p$ and $E_{pa}$ relationships at individual locations and therefore to further investigate the dependence between the three variables, we select four weather stations from the four quadrants of the contiguous US (Fig. 3a), to show the warm-season $P$, $E_p$ and $E_{pa}$ in time series (Fig. 6). The two stations in the southern regions have data in all 12 months of a year; while the two stations in the northern regions only have $E_{pa}$ data for six months of each year. All four stations show negative correlations between $P$ and $E_{pa}$. This negative correlation at the weather station in Florida is not statistically significant (Figs. 6g and 6h). As mentioned before, the $P$ and $E_{pa}$ correlation is less significant in the eastern region than in the west, because of the wetter climate in the east. On the other hand, at the other three locations, the warm-season $P$ and $E_{pa}$ are relatively symmetric to each other (Figs. 6a to 6f). During years when one series is above average, the other tends to be below average and vice versa. In terms of the relationship between $P$ and $E_p$, all four locations show no significant correlations between the two variables ($p>0.05$). This is consistent with the independence of $P$ and $E_p$ shown in Fig. 5e and 5f.
Fig. 6. Warm-season $P$, $E_p$ and $E_{pa}$ time series of four example weather stations in the study period of 1984-2015: (a) Summer Lake 1 S, OR (N 42°58’, W 120°47’); (c) Geneva RSCH Farm, NY (N 42°53’, W 77°20’); (e) Cachuma Lake, CA (N 34°35’, W 119°59’); (g) Moore Haven Lock 1, FL (N 26°50’, W 81°50’); and the scatterplots of $P$ vs $E_{pa}$ at the four example stations (b, d, f, h).

3.2 Bouchet-Budyko curves

There are two Bouchet-Budyko curves (Fig. 2). The upper curve describes the relationship between $E_{pa}$, $E_p$ and $P$ (Eq. 6) and the lower curve describes the relationship between $E$, $E_p$ and $P$ (Eq. 4). The lower curve is derived from the Budyko curve based on Turc-Pike equation. This relationship between $E$, $E_p$ and $P$ has been studied extensively following the Budyko framework and, therefore, it is not the focus of this study. This study investigates the relationship between $E_{pa}$, $E_p$ and $P$, which is represented by the upper Bouchet-Budyko curve. Since the collected weather station data of $P$ and $E_{pa}$ are available 1984 to 2015 and the $E_p$ data collected from the remote-sensing dataset are available 1983 to 2006, we examine the relationship between $P/E_p$ and $E_{pa}/E_p$ in the overlapping period of 1984 to 2006 (Fig. 7). Using Eq. (6), three curves with
different $b$ values (1, 2, and 3) are shown in Figure 7. The $v$ value is set at 2, which is commonly used in the Budyko framework. When $b$ equals one, the two CR curves are symmetric. When $b$ exceeds one, the two CR curves are asymmetric. This asymmetry is discussed in previous studies (Kahler and Brutsaert, 2006; Brutsaert, 2015). One explanation of this asymmetry between $E$ and $E_{pa}$ is that the evaporation pan will receive more heat than the surrounding area (Kahler and Brutsaert, 2006). Brutsaert (2015) reports an even higher $b$ values of 4.5. The horizontal solid black line in Fig. 7 is the boundary of the upper Bouchet-Budyko curve, above which $E_{pa}$ exceeds $E_p$.

Fig. 7. $P/E_p$ vs. $E_{pal}E_p$ at 259 weather stations in the US for the period 1984 to 2015 for (a) warm-season data (N=5312), and (b) long-term average data (N=259). The data points are color coded based on their latitudes and longitudes. The three upper Bouchet-Budyko curves are plotted with different $b$ values of $b=1$, $b=2$, and $b=3$, and with the same $v$ value of $v=2$. The dashed line is the lower Bouchet-Budyko curve with $v=2$.

4. Discussion
4.1 Relationship between $P$ and $E_{pa}$, and between $P$ and $E_p$

With the weather station data, a negative correlation between warm-season $P$ and $E_{pa}$ is shown in 242 out of 259 weather stations (93%). The negative correlation between $P$ and $E_{pa}$ is linked by the humidity deficit. The formation of precipitation is positively related to the local level of humidity (Pal et al., 2000; Sheffield et al., 2006; An et al., 2017) while “apparent” potential evaporation is inversely related to humidity or positively related to the humidity deficit (Penman, 1948; Allen et al., 1998). As a result, precipitation and “apparent” potential evaporation will tend to exhibit a negative correlation. According to the Bouchet’s complementary relationship, this negative correlation between $P$ and $E_{pa}$ is more pronounced in arid regions than in humid regions.

On the other hand, $P$ and $E_p$ shows no significant correlation at both the seasonal and the long-term average time scales. As a result, our study indicates that potential evaporation and precipitation, the representations of energy supply and water supply, are likely to be independent. This independence is currently under investigation with field data. It should be noted that the relationship between $P$ and $E_p$ and between $P$ and $E_{pa}$ found in this study are not direct causal relationships, but rather the result of interactions between a number of physical variables, such as net radiation, wind speed, humidity, and so forth. Further investigation into the physical mechanisms connecting these variables is underway.

4.2 The Bouchet-Budyko curve and its applications

Combining the Bouchet’s complementary relationship and the Budyko framework leads to two dimensionless CR curves, normalized by $E_p$ (Fig. 2). The upper Bouchet-Budyko curve is derived from the connection between Budyko framework and the CR, and the lower Bouchet-
Budyko curve is derived directly from the Budyko framework, based on the Turc-Pike equation. The companion CR curves show that as the wetness index $P/E_p$ decreases, the difference between $E$ and $E_{pa}$ grows. This indicates the complementary relationship between $E$ and $E_{pa}$ is most pronounced in arid environments; that is, the CR is more significant under water-limited condition. As discussed in Ramírez et al. (2005), the CR can be considered as an extension of the Budyko framework.

The $P$, $E_p$, and $E_{pa}$ collected in this study are following the general trend of the upper Bouchet-Budyko curve (Fig. 7). The remote-sensing data of $E_p$ may not have the same level of accuracy as the field measured $P$ and $E_{pa}$. The value of $\alpha$ in the Eq. (7) may vary from location to location (Chen and Brutsaert, 1995; Brutsaert and Chen, 1995). Such factors may explain the deviation of some data points from the CR curve in Fig. 7.

This upper Bouchet-Budyko curve can be used to estimate the $E_{pa}$ based on the data of $P$ and $E_p$. The “apparent” potential evaporation can be measured by evaporation pan, but this measurement has its limitations. For example, it is only available for warm periods. The collected data with time averaged pan evaporation levels over weeks, months, and years may lead to systematic error in surface flux calculations (Brutsaert, 1982; Kahler and Brutsaert, 2006). The Bouchet-Budyko curve can help us to estimate $E_{pa}$ without the limitations of evaporation pans. Comparing with more physically based $E_{pa}$ quantification approaches, such as Penman equation (Penman, 1948) and “PenPan” model (Rotstayn et al., 2006), our model is derived from conceptual frameworks and therefore may provide top-down insights about the $E_{pa}$ level in hydrologic systems.

Similar to the Budyko framework, the Bouchet-Budyko curves can be used in hydrologic models and climate models. These Bouchet-Budyko curves can be used to examine the fidelity
of simulated precipitation and evaporation sequences routinely produced by general circulation
models to drive climate change investigations.

5. Conclusions

We collected warm-season precipitation, potential evaporation, and “apparent” potential
evaporation data at 259 weather stations in the US to investigate the correlation among these
three physical variables. The results showed a negative correlation between $P$ and $E_{pa}$ at 93% of
the stations. The physical reason for the $P$, $E_{pa}$ negative correlation could be related to the
humidity variability. When humidity increases, the likelihood for precipitation increases while
the rate of “apparent” potential evaporation decreases. On the other hand, our study results
supported the assumption that $P$ and $E_p$ are independent. Combining the CR with a Budyko-type
equation, we formulated the companion CR curves, showing the connection between the Bouchet
and Budyko frameworks. These insights may encourage hydrologists to further explore the
strong link between the Budyko framework and the CR, promoting new ways of hydrologic
modeling. Future work will investigate the physical mechanisms behind the newly-derived
Bouchet-Budyko curves and explore the application of these companion curves.

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comments and valuable suggestions. The data of precipitation and pan evaporation
measurements can be downloaded from the National Climatic Data Center website:
https://www.ncdc.noaa.gov/IPS/cd/cd.html. The homogenized pan evaporation data can be
downloaded from the USGS ScienceBase: https://www.sciencebase.gov/catalog/. The data of remote-sensing based potential evaporation are provided by the Numerical Terradynamic Simulation Group at University of Montana, based on the study of Zhang et al. (2010). The data can be downloaded from their website: http://www.ntsg.umt.edu/about/default.php.

References


