Exploring the relationship between warm season precipitation, potential evaporation, and “apparent” potential evaporation at site scale

Xi Chen¹ and Steven G. Buchberger²

¹College of Arts and Sciences, Department of Geography and Geographic Information Science, University of Cincinnati
²College of Engineering and Applied Science, Department of Civil Engineering and Architectural Engineering and Construction Management, University of Cincinnati

Corresponding author: Xi Chen (xi.chen@uc.edu)
Abstract

Bouchet’s complementary relationship and the Budyko hypothesis are two classic frameworks that are inter-connected. To systematically investigate the connections between the two frameworks, we analyze precipitation, pan evaporation and potential evaporation data at 259 weather stations across the United States. The precipitation and pan evaporation data are from field measurement and the potential evaporation data are computed from a remote-sensing dataset. We use pan evaporation to represent “apparent” potential evaporation, which is different from potential evaporation. With these data, we study the correlations between precipitation and potential evaporation, and between precipitation and “apparent” potential evaporation. The results show that 93% of the study weather stations exhibit a negative correlation between precipitation and “apparent” potential evaporation. Also, the aggregated data cloud of precipitation versus “apparent” potential evaporation with 5312 warm season data points from 259 weather stations shows a negative trend in which “apparent” potential evaporation decreases with increasing precipitation. On the other hand, no significant correlation is found in the data cloud of precipitation versus potential evaporation, indicating that precipitation and potential evaporation are independent. We combine a Budyko-type expression, the Turc-Pike equation, with the Bouchet’s complementary relationship to derive upper and lower Bouchet-Budyko curves, which display a complementary relationship between “apparent” potential evaporation and actual evaporation. The observed warm season data follow the Bouchet-Budyko curves well. Our study shows the consistency between Budyko’s framework and Bouchet’s complementary relationship, with the distinction between potential evaporation and “apparent” potential evaporation. The formulated complementary relationship can be used in quantitative modeling practices.
1. Introduction

Potential evaporation ($E_p$) is a widely used physical variable in hydrologic frameworks. It is the evaporation rate under two conditions: firstly, the land surface water supply is unlimited (Thornthwaite, 1948); secondly, the surface vapor pressure is saturated (Van Bavel, 1966; Brutsaert, 2015). Pan evaporation ($E_{pan}$) measurement is often used as a surrogate of potential evaporation. However, these two variables are not the same (Brutsaert and Parlange, 1998; Roderick, et al., 2009). The main difference between potential evaporation and pan evaporation is that pan evaporation is not measured under saturated surface vapor pressure. As a result, potential evaporation can be considered to depend only on the energy supply of climate while pan evaporation is driven by both energy supply and humidity deficit in the atmosphere (Rotstayn, et al., 2006). In Brutsaert and Parlange (1998), the term “apparent” potential evaporation ($E_{pa}$) is introduced to distinguish pan evaporation from potential evaporation. “Apparent” potential evaporation can be measured by evaporation pan, while potential evaporation cannot.

Because potential evaporation is energy-driven, it can be used as a physical variable to describe energy supply in a hydrologic system. For instance, the well-established Budyko framework (Budyko, 1958; 1974) uses the relationship between precipitation ($P$) and potential evaporation to represent the relationship between water supply and energy supply. The Budyko framework has been extensively used to analyze interactions between hydrology, climate, vegetation and other elements in watersheds (Milly, 1994; Zhang et al., 2001; Yang et al., 2007; Donohue et al., 2007; Yang, et al., 2011; Xu, et al., 2014; Zhou et al., 2015; Zhou et al., 2016). Several studies have made connections between the Budyko framework and the Bouchet’s complementary relationship (CR) (Bouchet, 1963). Yang et al. (2006) used the Fu’s equation
(Fu, 1981), which is one of the commonly used equations to represent the Budyko’s curve, to describe the relationship between actual evaporation and potential evaporation in the CR.

Roderick et al. (2009) presented a complementary relationship normalized by net irradiance and compared it with the Budyko framework. Lhomme and Moussa (2016) combined Turc-Pike equation (Turc, 1954; Pike, 1964), which is another commonly used Budyko-type equation, with the CR to show the dependence of Budyko curve on the drying power of the air.

When linking Budyko framework with the CR, it is crucial to have a clear definition of different types of evaporation used in these two frameworks. Brutsaert and Parlange (1998) and Brutsaert (2015) generalized the CR and provided definitions of the evaporation terms in the CR, namely actual evaporation ($E$), potential evaporation ($E_p$), and “apparent” potential evaporation ($E_{pa}$, see Fig. 1a). As described previously, potential evaporation, following the original potential evaporation definition (Thornthwaite, 1948; Van Bavel, 1966), is the evaporation rate under saturated surface vapor pressure and unlimited land surface water supply; while “apparent” potential evaporation, which can be measured by evaporation pan, is the evaporation rate under unlimited land surface water supply, but not under saturated vapor pressure. It is clarified in Brutsaert and Parlange (1998) that the complementary relationship is between actual evaporation and “apparent” potential evaporation, not between actual evaporation and potential evaporation.

In the Budyko framework (Fig. 1b), the definition of potential evaporation is also following the original potential evaporation definition that it is under unlimited land surface water supply and saturated vapor pressure (Budyko, 1974).
Our study investigates the relationship between precipitation and potential evaporation as well as between precipitation and “apparent” potential evaporation. We collect warm season precipitation, potential evaporation and pan evaporation data from 259 weather stations across the contiguous US. Studying the relationship between $P$, $E_p$, and $E_{pa}$, advances our understanding of the well-established classic Budyko framework and the CR. Furthermore, based on insights provided by previous studies (Yang et al., 2006; Roderick et al., 2009; Lhomme and Moussa, 2016), we use a Budyko-type expression to develop a new formulation for the CR.

2. Methodology

2.1 Theoretical development

2.1.1 Budyko framework
The Budyko curve (Fig. 1b) describes the relationship between long-term water partitioning, represented by the ratio of actual evaporation over precipitation, and long-term climate, represented by the ratio of potential evaporation over precipitation, namely aridity index (Budyko, 1958; 1974). In recent decades, the Budyko framework has been examined with annual data (e.g. Yang et al., 2007; Potter and Zhang, 2009; Cheng et al., 2011). A number of Budyko-type functions have been developed to mathematically describe the Budyko curve (Turc, 1954; Fu, 1981; Zhang, et al., 2001; Yang et al., 2008; Wang and Tang, 2014). Within these functions, the Turc-Pike equation is a parsimonious single parameter equation (Turc, 1954; Pike, 1964):

\[ \frac{E}{P} = \left[ 1 + \left( \frac{E_p}{P} \right)^{-\nu} \right]^{-\frac{1}{\nu}} \]  

(1)

where \( E \) is actual evaporation, \( E_p \) is potential evaporation, \( P \) is precipitation, and \( \nu \) is a parameter to represent landscape properties such as vegetation coverage and soil properties (Zhang, et al., 2001; Yang, et al., 2008). The parameter \( \nu \) needs to be a positive number, and its typical value is 2.0.

2.1.2 Generalized complementary relationship

Bouchet’s complementary relationship (Bouchet, 1963) is to describe the relationship between actual evaporation \( E \) and potential evaporation \( E_p \). Brutsaert and Parlange (1998) introduced the term “apparent” potential evaporation \( E_{pa} \) and clarified that the CR is between \( E \) and \( E_{pa} \), not \( E \) and \( E_p \) (Fig. 1a). They also proposed a generalized complementary relationship:

\[ bE + E_{pa} = (1 + b)E_p \quad 0 \leq E \leq E_p \leq E_{pa} \]  

(2)
where \( b \) is a proportionality parameter not less than one. When \( b \) is equal to one, Eq. (2) represents the original complementary relationship (Kahler and Brutsaert, 2006). “Apparent” potential evaporation will be higher than potential evaporation under dry condition; while it gradually approaches potential evaporation as ratio of \( E \) over \( E_{pa} \) increases (Fig. 1a). As suggested by Morton (1976) and Brutsaert and Stricker (1979), potential evaporation can be estimated using the Priestley-Taylor equation (Priestley and Taylor, 1972). “Apparent” potential evaporation can be estimated using the Penman equation (Penman, 1948; Linacre, 1994; Rotstayn et al., 2006) or using data measured at evaporation pans (Brutsaert, 1982; Brutsaert and Parlange, 1998):

\[
E_{pa} = aE_{pan}
\]  

(3)

where \( E_{pan} \) is the pan evaporation and \( a \) is the pan coefficient. The pan coefficient varies from location to location (Stanhill, 1976; Linacre, 1994). In Kahler and Brutsaert (2006), a pan coefficient of \( a = 1.0 \) is recommended for mixed natural vegetation, which will be used in this study. It should be noted that the linear relationship between \( E_{pa} \) and \( E_{pan} \) given in Eq. (3) and the choice of “\( a \)” value will not affect the correlations between \( P, E_p \) and \( E_{pa} \).

2.1.3 Relationships between \( P, E_p \) and \( E_{pa} \)

The x-axis of the complementary relationship is a ratio between \( E \) and \( E_{pa} \) (Bouchet, 1963). Ramírez et al. (2005) used the water-energy framework to link the CR with Budyko approach and changed the x-axis in the CR to moisture availability. Following this idea, several studies have used precipitation or wetness index (\( P/E_p \)) to represent moisture availability in the CR (Yang et al., 2006; Roderick et al., 2009). In this study, we also use \( P \) to represent moisture availability in the CR. The \( E_p \) is a horizontal line in the CR that is in parallel with the x-axis.
(Fig. 1a), which is now represented by $P$. Therefore, the modified CR is indicating that $P$ and $E_p$ are independent. On the other hand, the upper curve of the CR, which is representing “apparent” potential evaporation $E_{pa}$, is declining along the x-axis, indicating that $E_{pa}$ and $P$ are not independent. After changing the x-axis in the CR to $P$, to have a dimensionless CR, we normalize the x and y axes in the CR. The normalized CR describes the relationship between $E_{pa}$, $E_p$, and $P$ $(E_p)$ (Fig. 2).

To connect Budyko framework with the normalized CR, and therefore to formulate the Bouchet-Budyko curves, we first transform Eq. (1) into a relationship between $\frac{E}{E_p}$ and $\frac{P}{E_p}$:

$$\frac{E}{E_p} = \left( \frac{P}{E_p} \right)^{-v} + 1 \right]^{-\frac{1}{v}} \quad (4)$$

Yang et al. (2006) did similar transformation using Fu’s equation (Fu, 1981). Dividing both sides of Eq. (2) by $E_p$ yields:

$$b \frac{E}{E_p} + \frac{E_{pa}}{E_p} = 1 + b \quad (5)$$

Combining Eqs. (4) and (5), gives a relation between $\frac{P}{E_p}$ and $\frac{E_{pa}}{E_p}$:

$$\frac{E_{pa}}{E_p} = b + 1 - b \left[ \left( \frac{P}{E_p} \right)^{-v} + 1 \right]^{-1/v} \quad E_{pa} \geq E_p \quad (6)$$

Equations (4) and (6) represent the lower and upper curves of the normalized CR respectively (Fig. 2). Roderick et al. (2009) presented a similar framework, without the formulation of the curves. To verify the relationships between precipitation, potential evaporation and “apparent” potential evaporation and to examine the Bouchet-Budyko curves in Eqs. (4) and (6), we analyze climate data from 259 weather stations across the contiguous US.
2.2 Data sources

Monthly precipitation and pan evaporation are collected from the National Oceanic and Atmospheric Administration (NOAA) at the National Climatic Data Center (NCDC). The data can be downloaded at: https://www.ncdc.noaa.gov/IPS/cd/cd.html. The precipitation data are measured using standard rain gauge and the pan evaporation data using Class A evaporation pans. We collect data for the period 1984-2015 from a total of 259 weather stations (Fig. 3a). Since pan evaporation is collected only during warm months (when temperatures remain above freezing), the weather stations at cold regions have less than 12 months of pan readings in a year. We calculate the monthly average of pan evaporation and precipitation using only the warm months with available pan evaporation data for each year each weather station. The calculated warm month averages are used to represent warm season pan evaporation and precipitation in each year. For short, it is called warm season data (i.e., warm season pan evaporation, warm season precipitation). We also calculate the annually averaged warm season data to represent the

Fig. 2. Dimensionless Bouchet-Budyko curves in the normalized complementary relationship.
long term average level of pan evaporation and precipitation at each station. For short, it is called long term average data. Over the 259 selected stations, there is an average of seven months per year with warm season pan evaporation. As Fig. 3 shows, the number of available months decreases from Southern regions to Northern regions. On the other hand, not all 259 weather stations have the full record from 1984 to 2015, the average number of years with available data for each location is 18. A complete summary of the information available at all 259 weather station is provided in Table S1. In order to minimize the uncertainty from various warm periods in a year from station to station, we repeat the analysis using an alternative source of pan evaporation in the NCDC dataset containing homogenized warm month data from May to October (Hobbins, et al., 2017). A total of 93 weather stations overlap both sets of pan evaporation data for the period 1984 to 2001 (Fig. 3b). After data collection, we convert pan evaporation to “apparent” potential evaporation using Eq. (3). The potential evaporation $E_p$ data are generated using the Priestley-Taylor equation with remotely sensed net radiation (Zhang et al., 2010):

$$E_p = \alpha \frac{\Delta}{\Delta + \gamma} (R_n - G)$$  \hspace{1cm} (7)

where $\alpha$ is a coefficient to account for the effect of surface characteristics and vegetation, and is set to 1.26; $\Delta$ is the slope of the saturated vapor pressure curve; $\gamma$ is the psychometric constant; $R_n$ is the net radiation; and $G$ is the heat flux into the ground. The $E_p$ data cover the period 1983-2006. Similar with $P$ and $E_{pan}$, we calculate the warm season $E_p$ and long term annually averaged warm season $E_p$ based on the monthly $E_p$ data.
Fig. 3. (a) Map of 259 weather stations. The available month of a year of pan evaporation data for each weather station is presented using legends with different colors and shapes. (b) Map of 93 weather stations with homogenized pan evaporation data.

2.3 $P$, $E_p$, and $E_{pa}$ correlation analysis

Using the weather station data of precipitation and pan evaporation for the period 1984 to 2015, we first calculate the Pearson correlation coefficient between warm season $P$ and warm season $E_{pa}$ for each location (Fig. 3a). We then perform the same correlation analysis of $P$ and $E_{pa}$ using the homogenized pan evaporation dataset (Hobbins et al., 2017) (Fig. 3b). Secondly, we use data of warm season $P$ and warm season $E_p$ for the period of 1984 to 2006, which is the period both $P$ and $E_p$ data are available, to investigate the correlation between $P$ and $E_p$. Finally, to validate the newly derived Bouchet-Budyko curves, the relationship between $\frac{P}{E_p}$ and $\frac{E_{pa}}{E_p}$ is plotted using the collected data at both seasonal and long term average time scales.

3. Results

3.1 Correlations among $P$, $E_p$, and $E_{pa}$
In the 259 weather stations, 93% of the stations have a negative correlation between $P$ and $E_{pa}$ (Fig. 4a). However, only 43% of the stations have statistically significant correlation ($p<0.05$) between $P$ and $E_{pa}$ (Fig. 4b). All the weather stations with significant $P$, $E_{pa}$ correlation have negative correlation. The weather stations located in the western region are more likely to have a significant $P$, $E_{pa}$ negative correlation than in the east. This spatial difference may be related to climate characteristics that eastern region has higher precipitation and lower aridity index, while the western region has lower precipitation and higher aridity index. The Bouchet’s complementary relationship is more significant in arid regions (Ramírez et al., 2005), corresponding to the left side of the CR curves; while it is less significant in humid regions, corresponding to the right side of the CR curves (Fig. 1a). As a result, the negative correlation between precipitation and “apparent” potential evaporation is more significant in the west than in the east.

Fig. 4. Map of point scale annual $P$, $E_{pa}$ correlation at 259 weather stations, (a) $r$ value and (b) $p$ value.

We then plot all the warm season data of each year each station, totally 5312 data points, on a $P$ vs. $E_{pa}$ figure (Fig. 5a). The data cloud shows a negative trend in general. We also plot the long term annually averaged values of warm season $P$ and $E_{pa}$ of the 259 weather stations.
(Fig. 5b), which shows a similar negative trend. Hobbins et al. (2004) showed a similar negative trend between $P$ and $E_{pa}$ with watershed scale data. To represent the spatial distribution of the weather stations, we color code the data points based on their spatial coordinates of latitude and longitude. The climate in the eastern US is much wetter than the western US, and therefore the data cloud of $E_{pa}$ vs. $P$ is separated into two parts horizontally. The right side cloud is mostly green and brown, representing the northeastern and southeastern areas of the US, respectively; while the left side cloud is mostly yellow and red, representing the northwestern and southwestern areas, respectively. The left side cloud is more vertically oriented, indicating that the western region has higher $E_{pa}$ variability than $P$ variability. Southwestern region has the highest $E_{pa}$ in the US, represented by the red and orange points. Northwestern region has much lower $E_{pa}$, represented by the yellow points. On the other hand, the right side cloud is more horizontally oriented, indicating that the eastern region has higher $P$ variability than $E_{pa}$. Unlike the western region, the difference between the northeastern and southeastern regions are not very distinguishable. Southeastern region of the US has a wide range of precipitation; while points of the northeastern region are more concentrated.

As explained before, we also use an alternative pan evaporation dataset (Hobbins et al., 2017) to further validate our analysis result. This dataset is homogenized to have the same period of pan evaporation data record in each year from May to October and therefore to minimize the uncertainty from the various length of warm period from station to station. It is derived from the same dataset as our data, namely the NCDC dataset. Based on the homogenized pan evaporation data, 85 stations out of 93 (91%) have a negative correlation between $P$ and $E_{pa}$. Only 41% of the stations have a statistically significant correlation ($p<0.05$). All the significant correlations are negative. This result is consistent with the analysis result.
Based on our collected data from 259 weather stations. We also use the data cloud to show the relationship between $P$ and $E_{pa}$ in the warm period of May to October in each year at each of the 93 stations (Fig. 5c), as well as the relationship of long term annually averaged warm period $P$ and $E_{pa}$ (Fig. 5d). The trend of data cloud is similar with the data cloud trend using our collected data at both seasonal and long term average time scales. In other words, both datasets show a negative relationship between $P$ and $E_{pa}$.

We then plot the relationship between $P$ and $E_p$ (Fig. 5e and 5f), using the $E_p$ data generated by a remote-sensing algorithm based on the Priestley-Taylor equation as explained previously (Zhang et al., 2010). At both seasonal and long term average time scales, there is no clear relationship shown between $P$ and $E_p$, confirming the independence between $P$ and $E_p$ discussed in Section 2.1.3. This result shows the difference between $E_p$ and $E_{pa}$, that $E_p$ is independent from $P$ but $E_{pa}$ is not. Therefore, it is important to distinguish $E_{pa}$ from $E_p$ and to understand the different physical mechanisms of the two processes (Brutsaert, 2015).
Fig. 5. $P$ vs. $E_{pa}$ at 259 weather stations in the US for the period 1984 to 2015 for (a) warm season data (N=5312), and (b) long term annually averaged warm season data (N=259). The data points are color coded based on their latitudes and longitudes. $P$ vs. $E_{pa}$ at 93 weather stations in the US for the period 1984 to 2001 using the homogenized pan evaporation dataset for (c) warm period May-Oct in each year (N=1214), and (d) long term annual average warm period May-Oct data (N=93). $P$ vs. $E_p$ at the 259 weather stations for the period of 1984 to 2006 for (e) warm season data (N=5312) and (f) long term annual average warm season data (N=259).

To present the $P$, $E_p$ and $E_{pa}$ relationship at individual locations and therefore to further investigate the dependence between the three variables, we select four weather stations from the northwest, northeast, southwest, and southeast regions respectively to show the warm season $P$, $E_p$ and $E_{pa}$ in time series (Fig. 6). The two stations in the southern regions have data in all 12 months of a year; while the two stations in the northern regions only have pan evaporation data in warm months of 6 or 7 months of a year. All four stations show negative correlations between $P$ and $E_{pa}$. This negative correlation at the selected weather station in Florida is not statistically significant (Fig. 6d). As mentioned before, the $P$ and $E_{pa}$ correlation is less significant in the eastern region than in the west, because of the higher humidity in the east. On the other hand, at the other three locations, the warm season $P$ and $E_{pa}$ are relatively symmetric to each other (Fig. 6a, 6b, and 6c). During years when one series is above average, the other tends to be below average and vice versa. In terms of the relationship between $P$ and $E_p$, all four locations show no significant correlations between the two variables ($p>0.05$). This is consistent with the independence of $P$ and $E_p$ shown in Fig. 5e and 5f.
Fig. 6. Warm season $P$, $E_p$ and $E_{pa}$ time series of four example weather stations in the study period of 1984-2015: (a) Summer Lake 1 S, OR (N 42°58’, W 120°47’); (b) Geneva RSCH Farm, NY (N 42°53’, W 77°20’); (c) Cachuma Lake, CA (N 34°35’, W 119°59’); and (d) Moore Haven Lock 1, FL (N 26°50’, W 81°50’).

3.2 Bouchet-Budyko curves

There are two Bouchet-Budyko curves (Fig. 2). The upper curve describes the relationship between $E_{pa}$, $E_p$ and $P$ (Eq. 6) and the lower curve describes the relationship between $E$, $E_p$ and $P$. 

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(Eq. 4). The lower curve is derived from the Budyko curve based on Turc-Pike equation. This relationship between $E$, $P$ and $E_p$ has been studied extensively following the Budyko framework, and it is therefore not the focus of this study. This study investigates the relationship between $E_{pa}$, $E_p$ and $P$, which is represented by the upper Bouchet-Budyko curve. Since the collected weather station data of $P$ and $E_{pan}$ are available 1984 to 2015 and the $E_p$ data generated from remote-sensing algorithm are available 1983 to 2006, we study the relationship between $P/E_p$ and $E_{pa}/E_p$ in the overlapping period of 1984 to 2006. Based on warm season data of $P$, $E_p$ and $E_{pa}$, we plot the relationship between $P/E_p$ and $E_{pa}/E_p$ (Fig. 7). We draw three curves using Eq. (6) with different $b$ values of 1, 2, and 3. The $v$ value is set at 2, which is a commonly used value in the Budyko framework. When $b$ exceeds one, the two CR curves are asymmetric. This asymmetry is discussed in previous studies (Kahler and Brutsaert, 2006; Brutsaert, 2015).

Brutsaert (2015) reports $b$ values around 4.5. The horizontal solid black line in Fig. 7 is the boundary of the upper Bouchet-Budyko curve that $E_{pa} \geq E_p$.

![Diagram](image.png)

Fig. 7. $P/E_p$ vs. $E_{pa}/E_p$ at 259 weather stations in the US for the period 1984 to 2015 for (a) warm season data (N=5312), and (b) long term average data (N=259). The data points are color coded.
based on their latitudes and longitudes. The three upper Bouchet-Budyko curves are plotted with
different $b$ values of $b=1$, $b=2$, and $b=3$, and with the same $v$ value of $v=2$.

4. Discussion

4.1 Relationship between $P$ and $E_{pa}$, and between $P$ and $E_p$

With the weather station data, a negative correlation between warm season precipitation and
“apparent” potential evaporation is shown in 242 out of 259 weather stations (93%). The
negative correlation between $P$ and $E_{pa}$ is linked by the humidity deficit. The formation of
precipitation is positively related to the local level of humidity (Pal et al., 2000; Sheffield et al.,
2006; An et al., 2017) while “apparent” potential evaporation is inversely related to humidity or
positively related to the humidity deficit (Penman, 1948; Allen et al., 1998). As a result,
precipitation and “apparent” potential evaporation will tend to exhibit a negative correlation.

Similar with the Bouchet’s complementary relationship, this negative correlation between $P$ and
$E_{pa}$ is more significant in arid regions than in humid regions.

On the other hand, $P$ and $E_p$ shows no significant correlation at both seasonal and long
term average time scales. Potential evaporation is driven by energy supply, which is quantified
by the Priestley-Taylor equation using the remote-sensing data (Zhang, et al., 2010). As a result,
our study indicates that energy supply and precipitation, the representation of water supply, are
likely to be independent. This independence is currently under investigation with field data. It
should be noted that the relationship between $P$ and $E_p$ and between $P$ and $E_{pa}$ we find in this
study are not direct causal relationship, but rather the result of interactions between a number of
physical variables. We will collect more data and further investigate the physical mechanisms of these relationships in future studies.

4.2 The Bouchet-Budyko curve and its applications

Combining the Bouchet’s complementary relationship and the Budyko framework leads to two dimensionless CR curves, normalized by $E_p$ (Fig. 2). The upper Bouchet-Budyko curve is derived from the connection between Budyko framework and the CR, and the lower Bouchet-Budyko curve is directly derived from Budyko framework, based on the Turc-Pike equation. The two curves show that when the wetness index $P/E_p$ is lower than one, the complementary relationship between $E$ and $E_{pa}$ is more significant. In other words, the CR is more significant under water limited condition. As discussed in Ramírez et al. (2005), the CR can be considered as an extension of the Budyko framework. With the Bouchet-Budyko curves shown in Fig. 2, this connection can be quantitatively analyzed, which will be our future study direction.

The collected data of $P$, $E_p$ and $E_{pan}$ fits with the general trend of the upper Bouchet-Budyko curve (Fig. 7). The remote-sensing data of $E_p$ may not have the same level of accuracy as the field measured $P$ and $E_{pan}$ and the value of $\alpha$ in the Eq. (7) may vary from location to location (Chen and Brutsaert, 1995; Brutsaert and Chen, 1995). This may explain the deviation of some data points from the curve in Fig. 7. This upper Bouchet-Budyko curve can be used to estimate the “apparent” potential evaporation based on the data of precipitation and potential evaporation. The “apparent” potential evaporation can be measured by evaporation pan, but this measurement has its limitations. For example, it is only functional in warm period. The collected data with time averaged pan evaporation levels over weeks, months, and years may lead to systematic error in surface flux calculations (Brutsaert, 1982; Kahler and Brutsaert, 2006). The Bouchet-Budyko curve can help us to estimate $E_{pan}$ without the limitation of
evaporation pans. Comparing with more physically-based $E_{pa}$ quantification approaches, such as Penman equation (Penman, 1948) and “PenPan” model (Rotstayn et al., 2006), our model is derived from conceptual frameworks and therefore may provide top-down insights about the $E_{pa}$ level in hydrologic systems.

In addition, the lower Bouchet-Budyko curve is based on an alternative form of Budyko-type equation (Eq. 4), derived from the Turc-Pike equation. This curve can be used to show the relationship between $E$ and $E_p$ under varying climate characteristic. We will collect field evaporation data to investigate this curve in future studies.

Similar to the Budyko framework, the Bouchet-Budyko curves can be used in hydrologic models and climate models. Furthermore, the upper and lower curves can be used to estimate the trend of “apparent” potential evaporation and actual evaporation respectively, based on the level of precipitation and potential evaporation. These Bouchet-Budyko curves can also be used to examine the fidelity of simulated precipitation and evaporation sequences routinely produced by general circulation models to drive climate change investigations.

5. Conclusions

We collect warm season precipitation, potential evaporation, and pan evaporation data at 259 weather stations to investigate the correlation among these three physical variables. The results show a negative correlation between $P$ and $E_{pa}$ in 93% of the study locations. The physical reason of the $P$, $E_{pa}$ negative correlation could be related to the humidity variability. When humidity increases, the likelihood for precipitation increases while the rate of pan evaporation decreases. On the other hand, our study results on the relationship between warm season $P$ and
Ep support the assumption that P and Ep are independent. By combining the CR with a Budyko-type equation, we formulate the CR curves, showing the connection between the two frameworks. As a result, this research may encourage hydrologists to generate new ideas on the interpretation of the Budyko framework and the CR, promoting new ways of hydrologic modeling. Future work will investigate the physical mechanism behind the Bouchet-Budyko curves and explore the application of Bouchet-Budyko curves.

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The data of precipitation and pan evaporation measurements can be downloaded from the National Climatic Data Center website: https://www.ncdc.noaa.gov/IPS/cd/cd.html. The homogenized pan evaporation data can be downloaded from the USGS ScienceBase: https://www.sciencebase.gov/catalog/. The data of remote-sensing based potential evaporation is provided by the Numerical Terradynamic Simulation Group at University of Montana, based on the study of Zhang et al. (2010). The data can be downloaded from their website: http://www.ntsg.umt.edu/about/default.php.

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