Response to Reviewer' Comments on:
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The authors thank the reviewer for the quick response. The manuscript has been improved substantially based on the constructive comments of the reviewer.

Response to Comments of Reviewer #1

General Comment: The main point of the manuscript is the computational efficiency of the proposed ROM methodology. It is claimed to be computationally more efficient than simulating MT3DMS, however, the manuscript does not provide any comparison of computation time. An appropriate comparison between computing time would include time required for the predictive MT3DMS model, versus time required for ROM analyses that would include calculations to generate eigenvalues and vectors, matrix computations, and predictive analyses.

Response: We agree and have now added a new section in the manuscript that provides a comparison between the running times of the developed ROM and MT3DMS {Track Change Version (Page 10: Lines 19 – 27)}.

> *"Considering the computational costs, the developed nitrate ROM was superior (it ran about ten times faster than MT3DMS). More specifically, the computational time for the predictive MT3DMS model was about 89 seconds. This value reduced to nine seconds when we applied the developed ROM for simulation of nitrate in the aquifer using the same computer. Note that the computational time for running the developed ROM model includes calculations to generate matrix computations, eigenvalues and eigenvectors, and predictive analyses."*

General Comment: It would be beneficial to the reader to also include in the manuscript a discussion about the general applicability and suitability of the ROM methodology, limitations, and the robustness of the ROM predictions. For example, is ROM suitable for predictive scenarios to examine remediation options by adding pumping wells?

Response: We agree and have added a new section in the manuscript that provides useful information about the general applicability, suitability, limitations, and the robustness of the ROM {Track Change Version (Page 10: Lines 19 – 31; Page 11: Lines 1 and 2)}.

> *"Considering the computational costs, the developed nitrate ROM was superior (it ran about ten times faster than MT3DMS). Also, it is noteworthy that the quality of analyses resulting from ROM was as good as the quality of the underlying numerical model. Thus, it can be concluded that the developed ROM model with a simple structure and considerably less running time than MT3DMS appropriately simulated the nitrate concentration in the Karaj Aquifer. Regarding the ROM applications for the future times, all time dependent factors in the aquifer responses, such as hydro-meteorological, hydrogeological, and well operation strategies in the simulation period is expressed by temporal component τ(t) in the ROM. Thus, the model is applicable for predictive scenarios to examine the options that already existed during the simulation period of the modelling process. This may limit the application of the developed ROM model for scenarios not experienced during the simulation period (e.g., examine remediation options by adding pumping wells). In other words, the model can appropriately memorize historical processes experienced during the simulation period. Time dependent factors are shown in the temporal component τ(t) (Fig. 5B) although the spatial component $\boldsymbol{\theta}(\boldsymbol{x})$ is not*

time dependent. Also, it is noteworthy that the ROM is developed based on a historic numerical model simulated using MT3DMS and therefore, (1) the quality of analyses resulting from this model is as good as the quality of the underlying numerical model, (2) one needs to have the results of a historic numerical model to develop ROM model although it can be independently applied for the prediction of nitrate."

General Comment: It is noteworthy that the ROM methodology is based on a historic numerical model simulated using MT3DMS and therefore, the quality of analyses resulting from ROM is expected to be just as good as the quality of the underlying numerical model, based on which eigen vectors are generated for ROM calculations.
Response: Thanks for the comment. We have now added a new section in the manuscript that provides useful information about the general applicability, suitability, limitations, and the robustness of the ROM {Track Change Version (Page 10: Lines 24 – 27)}.

"Also, it is noteworthy that the ROM is developed based on a historic numerical model simulated using MT3DMS and therefore, (1) the quality of analyses resulting from this model is as good as the quality of the underlying numerical model, (2) one needs to have the results of a historic numerical model to develop ROM model although it can be independently applied for the prediction of nitrate."

Comment: Page 1, line 22: the "simpler structure" of ROM computation is based on matrix calculations but the results are primarily based on MT3DMS computations. I suggest deleting "simpler structure" from the text as that description is misleading.
Response: We agree and have deleted it and updated the statement {Track Change Version (Page 1: Line 21)}.

Comment: Page 1, line 22: provide some numbers to demonstrate "shorter calculation times".
Response: We have now provided some numbers about the running times of the models {Track Change Version (Page 1: Line 22)}.

Comment: Page 2, line 9: "information produced by the models is confusing", is an inappropriate statement. It is the modeler's job to understand the meaning of the output that a model generates. Again, the numerical model output is what is used for ROM, which makes it further "confusing", doesn't it?
Response: Thanks for the comment. We meant the difficult management of large data produced by these models. However, we have now updated this statement {Track Change Version (Page 2: Lines 6 to 8)}.

Comment: Page 2, line 31: "complex mathematical form" and "complex solving methods" for numerical models is presented as a limitation, however, ROM is based on the output from these very "complex" numerical models; ROM, in my opinion adds one more layer. of mathematical complexity to the system. If complexity of numerical models is being criticized, ROM stands to be criticized even more. I suggest deleting this line entirely.
Response: Thanks for the comment. We have now deleted this statement entirely {Track Change Version (Page 2: Lines 30 – 33)}.

Comment: Page 5, line 1: the impact of river recharge is not seen in the head contours; may be the recharge amount is small? I am only stating an observation; this need not be addressed in the manuscript.

Response: In arid and semi-arid regions like Iran, most of rivers are not permanent and the Karaj river is no exception. In our groundwater modeling, recharge of river is a small amount compared to the deep groundwater flow. In addition, it is varied during the year although it is considered indirectly in the model.

Comment: Page 5, line 30: calibration is discussed in detail. My suggestion would be to either shorten the calibration discussion as the focus of the manuscript is the ROM methodology, or include a plot showing the goodness of fit, comparing observed values and simulated results using a scatter plot one for heads and one for concentrations, to complete the calibration discussion.
Response: Thanks for the comment. We have now shortened this section {Track Change Version (Page 5: Lines 16 – 31; Page 6: Lines 2 – 6)}.

Comment: Page 6, line 4: incorrect statement, TVD scheme is not a combination of four other methods. I suggest modifying this statement.
Response: Thanks for the comment. We have now modified this statement {Track Change Version (Page 6: Lines 2 – 6)}.

Comment: Page 7, line 8-14: the explanation seems unclear. This paragraph is the main feature of this manuscript and needs to be explained better.
Response: We have now updated this paragraph. Noted all steps for the introduced methodology are given in Figure 2. However, we did our best to properly describe this section by referring to Figure 2 in the revised manuscript {Track Change Version (Page 7: Lines 8 – 27)}.

Note that if the ROM developed by few first selected modes (Eq. (8)) didn't satisfy the defined criteria for evaluating the model performance (such as RMSE and MAE), one need to increase the number of modes so that to reach the defined criteria. This procedure is properly shown in Figure 2.
However, Eq. (8) has a simple structure with a few modes. Thus, one needs the few first significant modes developed based on simulated historic nitrate concentrations using MT3DMS to regenerate this pollutant in the aquifer. In other words, the ROM is applicable for regeneration of nitrate on the solution domain of MT3DMS and it is not applicable to make prediction in the future. In this study, we aimed to enhance the ROM to make predictions into the future, for time steps with no nitrate simulations from the MT3DMS model. This step is shown in subsection "Model Verification" of Fig. 2. For this purpose, we developed a methodology that enables the ROM to independently simulate nitrate concentrations. According to this, it is necessary to calculate both the spatiotemporal components $\boldsymbol{\theta}(\boldsymbol{x})$ *and* $\tau(t)$ *for the future time interval* $(t + n)$*. Since the component* $\boldsymbol{\theta}$ *is a function of space, it does not change in time. Therefore, it is necessary to calculate* τ *for future time steps. For this purpose, a regression equation was used to estimate the time variation of this component* $(\tau(t+n))$*. Having* $\tau(t+n)$*, the nitrate concentration for future time steps* $t+n$ *can be calculated by:*

$$\boldsymbol{\Omega}(\boldsymbol{x}, t+n) \cong \sum_{i=1}^{l} \tau_i(t+n)\boldsymbol{\theta}_i(\boldsymbol{x}) \quad , \quad l \ll N \qquad (9)$$

Eq. (9) simulates the nitrate concentration in different parts of the aquifer for future time $t+n$ *using the first few significant modes. According to Fig. 2, if the ROM developed didn't satisfy the defined criteria for evaluating the model performance, one need to increase the number of modes by estimating the trend of more numbers of the time dependent components so that the developed ROM satisfies the defined criteria.*

Comment: Page 7, line 28: there are several aspects to examine before calling the model well calibrated. It also depends on the objective of the model. In this case, since solute transport is important, getting the gradients and velocities correct becomes important. I am simply pointing

this out and the authors may have already examined this aspect but not reported it. This point need not be addressed in the manuscript.
Response: Thanks for the comment. Due to main focus of this submission is solute transport, we have ignored to provide more discussion on the model calibration.

Comment: Page 8, line 5: looking at only the difference can be misleading. Examining timeseries is also important to assess trends.
Response: We agree and have now added the trends of measured nitrate concentrations and simulated ones by MT3DMS model. We have also added another figure that properly shows the trends of RMSE and MAE averaged over the study area during the calibration period {Track Change Version (Page 8: Lines 17 – 22; Figure 3D to 3G)}.

Comment: Page 8, line 8: Nitrate distribution seems locally contained? Is it because the movement is slow with respect to the simulation period? Again, just an observation. This point need not be addressed in the manuscript.
Response: Thanks for the comment. Yes.

Comment: Page 10, line 4: the limitations of numerical models listed in the manuscript are arbitrary, as pointed out in some of my previous comments. The only relevant limitation of numerical models, in context of this manuscript, could be the computation time, but that analyses is not presented in the manuscript.
Response: Thanks for the comment. We have now added a new section in the manuscript that provides useful information about the running times of the models, i.e. ROM and MT3DMS {Track Change Version (Page 10: Lines 19 – 27)}.

> *"Considering the computational costs, the developed nitrate ROM was superior where it was run about ten times faster than MT3DMS. More specifically, the computational time for the predictive MT3DMS model was about 89 seconds. This value reduced to nine seconds when we applied the developed ROM for simulation of nitrate in the aquifer using the same computer. Note that the computational time for running the developed ROM model includes calculations to generate matrix computations, eigenvalues and eigenvectors, and predictive analyses."*

Comment: Page 10, line 15: the claim that "ROM was superior than MT3DMS" is incorrect. ROM is based on the results generated by MT3DMS. How would that make ROM superior than MT3DMS in terms of quality of results? I suggest deleting this line from the manuscript.
Response: Thanks for the comment. We have now deleted the mentioned line {Track Change Version (Page 11: Lines 13 and 14)}.

Comment: Page 1, line 13: delete "Please", the first word in the Abstract.
Response: Thanks for the comment. Agreed and amended {Track Change Version (Page 1: Line 13)}.

Comment: Page 1, line 13: high computational cost is a result of long simulation times, not the other way around.
Response: Thanks for the comment. We have now updated this statement {Track Change Version (Page 1: Line 14)}.

Comment: Page 1, line 16: replace "presents a solution for the problem in ROMs" with "was.

Response: Thanks for the comment. We have now updated this statement {Track Change Version (Page 1: Line 16)}.

Comment: Page 1, line 20: insert code or program or simulator before "(MT3DMS)".
Response: Thanks for the comment. We have now updated this statement {Track Change Version (Page 1: Line 20)}.

Comment: Page 1, line 26: consider rearranging the sentence as: ". . . and activities have resulted in spreading pollution in the aquifers that result in groundwater quality deterioration.".
Response: Thanks for the comment. We have now updated this statement {Track Change Version (Page 1: Lines 27 and 28)}.

Comment: Page 1, line 28: nitrate is not "often the main concern", but is one of the common contaminants.
Response: Thanks for the comment. We have now updated this statement {Track Change Version (Page 1: Line 29)}.

Comment: Page 1, line 31: abstract uses GQM, not GQSM. Search the remainder of manuscript and use a consistent acronym.
Response: Thanks for the comment. We have now check all text of manuscript and corrected this acronym.

Comment: Page 3, line 11: "annual evaporation" or "annual potential evaporation"?
Response: Thanks for the comment. We have now updated this statement {Track Change Version (Page 3: Line 10)}.

Comment: Page 3, line 28: in equation 1, "∂" is missing in the denominator in 2 places.
Response: Thanks for the comment. We have now corrected this equation {Track Change Version (Page 3: Line 28)}.

Comment: Page 4, line 22: "distribution of hydraulic", the word 'of' is missing.
Response: Thanks for the comment. We have now updated this statement {Track Change Version (Page 4: Line 22)}.

Comment: Page 4, line 24: consider replacing "are" with "were".
Response: Thanks for the comment. We have now updated this statement {Track Change Version (Page 4: Line 24)}.

Comment: Page 5, line 11: consider replacing "including" with "the availability of".
Response: Thanks for the comment. We have now updated this statement {Track Change Version (Page 5: Line 11)}.

Comment: Page 5, line 16: what is the difference between gridded network and mesh dimension? Consider clarifying in the text.
Response: Thanks for the comment. It was a typos error. We have now updated this statement {Track Change Version (Page 5: Lines 16 and 17)}.

A novel model for simulation of nitrate in aquifers

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Abstract. ~~Please~~ Numerical groundwater quality models (GQMs) often run at high computational cost resulting in long simulation times ~~and complex parameter calibration~~ that limit their practical applications. In this study, a novel reduced-order model (ROM) was developed for nitrate simulation in groundwater including a simple structure and with similar accuracy as more extensive GQMs. The proposed methodology for the ROM ~~presents a solution for the problem in ROMs~~was developed with eigenvectors, to make predictions into the future. The model performance was investigated by simulation of nitrate in the Karaj Aquifer, Iran. The dominant modes of spatiotemporal variation of nitrate during a five-year period were calculated by the model. The results revealed an excellent agreement between nitrate simulated by the ROM and the well-known Modular Transport 3D Multi Species code (MT3DMS). The absolute error between the ROM and the MT3DMS was less than 0.5 mg/l in most parts of the aquifer. Thus, results confirm that the use of ROM ~~has~~ has advantages through ~~a much simpler structure and~~ shorter calculation time (about ten times faster than MT3DMS). Observed spatiotemporal variation of nitrate in the aquifer was well represented by the ROM simulations. The simplicity of the model makes it highly interesting also to other water resources problems.

1 Introduction

High dependency on groundwater resources and excessive water withdrawal from aquifers have led to drastic drop in groundwater levels in many parts of the world. In addition, agricultural, industrial, and urban activities ~~may~~ have~~be~~ resulted in spread of pollution ~~spread~~ in aquifers ~~finally~~ resulting in groundwater quality deterioration. In this regard, nitrate is a ~~often~~ common contaminant~~a main concern~~ for most aquifers. Studying the spatiotemporal variation (STV) of nitrate in aquifers provides important information for effective management of the contaminated groundwater resources and possibilities for future improvement. Due to lack of proper groundwater monitoring systems as well as small research budgets especially in developing countries, groundwater quality ~~simulation~~ models (GQ~~S~~Ms) are essential for evaluation of the STV of nitrate. The

models can provide important information regarding the present status and implications of future scenarios regarding the STV of nitrate contamination.

A literature review clearly demonstrates the successful applications of one of the most popular GQSMs, the Modular Transport 3D Multi Species (MT3DMS) (Zheng and Wang, 1999; Peña-Haro et al., 2009 and 2010; Rasmussen et al., 2013; Gusyev et al., 2014; Abdelaziz and Merkel, 2015; Pulido-Velazquez et al., 2015). Three Two important challenges, however, as discussed below, influence such models regarding management decisions based on their results: (A) In many cases (especially the use of three-dimensional models with long simulation times for pollutants in extended large aquifers), the computational costs are very high and the large amount of information produced by the models is difficult to manage (Li et al., 2004; Vermeulen et al., 2005; Stanko et al., 2016). This poses challenges regarding decision making and proper management of the aquifers. In these cases, application of alternative methods such as reduced-order models (ROMs) based on proper orthogonal decomposition (POD) may be an alternative (Esfahanian and Ashrafi, 2009). The accuracy of ROMs has been shown to be compatible with GQSMs while ROMs pose a much simpler structure (Cardoso et al., 2009; Ushijima and Yeh, 2017). In ROMs based on eigenvectors the dominant spatiotemporal modes of the target are calculated. Using the first significant modes, a simple model is developed to simulate the target. Successful application of this approach in the field of water resources management (beyond the laboratory scale in which many simplifying assumptions may be employed) has been reported for simulation of nitrate and water temperature in the Karkheh Dam Reservoir located in Iran (Noori et al., 2015 and 2018). Some other studies have aimed at applying ROMs in subsurface flow (Vermeulen et al., 2004, 2005 and 2006; McPhee and Yeh, 2008; Siade et al., 2010; Pasetto et al., 2011, 2013 and 2014; Stanko et al., 2016), however, no applications have been reported using them to quantify pollutants in aquifers. Due to the difficulty of ROMs to make predictions into the future as described by Noori et al. (2017), the above work (Noori et al., 2015) only aimed at regeneration of nitrate in the reservoir during the simulation period. Therefore, the most important goal of this study was to develop ROM based eigenvectors that can predict the future concentration of nitrate in the aquifer (beyond the observation period). (B) The second challenge in application of GQSMs is their limitation in providing the dominant modes of the pollutant variation for the simulation period. GQSMs can provide STV of pollutants in aquifers and deliver useful information for water quality managers. But, when the aim is determining the dominant modes of the pollutant, the user needs to use an alternative solution like POD linked to the GQSMs (Kostas et al., 2005; Bennacer and Sefiane, 2016). The POD can use snapshots of pollutant distribution simulated by GQSMs, thus, determining the dominant modes of pollutant variation. These dominant modes appropriately present the most representative patterns of STV of pollutants for the simulation period (Kang et al., 2015). To the best of the authors' knowledge, no application has yet been reported aiming to present dominant modes of STV for pollutants in groundwater systems. (C) Finally, the third challenge for application of QGSMs is the complex mathematical form of the governing equations as well as their complicated solving methods (Mehl, 2006; Stanko et al., 2016; Pasetto et al., 2017). Fortunately, the ROMs overcome this problem to a large extent by providing a model with much simpler mathematical form than the QGSMs whilst they keep an acceptable level of accuracy (Boyce et al., 2015).

In view of the above, in the present study, we modify the developed nitrate ROM with eigenvectors so that it enables prediction of nitrate STV in the Karaj Aquifer. The dominant modes of the nitrate variation in the aquifer are calculated and presented. In this regard, the groundwater flow in the study area was firstly simulated by MODFLOW (Harbaugh et al., 2000). Thereafter, STV of nitrate was calculated by MT3DMS model. Finally, the nitrate ROM was developed for the aquifer. The presented methodology in this study is novel and it has broad applications in other fields of science and engineering.

2 Methods

2.1 Study area

The study area is the large unconfined Karaj groundwater aquifer with an area of 175.6 km^2 located in the southern parts the Alborz Mountain chain (Figure 1). The mean annual rainfall from 1967 to 2012 was 285.5 mm, with a coefficient of variation of 30%. The minimum and maximum annual rainfall were 93.3 and 452.6 mm, respectively, whilst the average annual potential evaporation was about 2430 mm. The only permanent river in the study area is the Karaj River with an average annual flow of 470 million cubic meters (MCM) that flows through the eastern region of the aquifer. The northern part is mountainous including high-altitude areas that are generally covered with fine-grain alluvial deposits, while the low-altitude areas in the southern part are coarse-grained alluvial deposits. In general, the study area has a relatively sharp slope with north and northeast to south and southwest direction. It results in variation of land surface topography in the study area from 1381 m in the north-east to 1200 m in the south-west of the plain.

Due to its proximity to the Tehran Metropolis, Karaj City has a fast increasing water demand. To meet the water demand, groundwater withdrawal has increased resulting in a significant drop in water level as well as groundwater quality degradation especially in recent years. There are 12 observation wells (piezometers) in the study area (Figure S1). The water level monitoring in these wells, clearly shows that the aquifer is experiencing a decreasing trend (typically 1.14 m per year water level decrease from April 2006 to April 2012).

2.2 Groundwater flow and nitrate transport models

To simulate the groundwater flow and nitrate transport in the Karaj Aquifer, MODFLOW and the Modular Transport 3D Multi Species (MT3DMS) models (Harbaugh et al., 2000; Zheng and Wang, 1999), respectively, were used. Figure 2 shows the different steps of conducting the present study. MODFLOW is a three-dimensional (3D) model originally developed by the US Geological Survey that uses a block-centered finite difference technique to solve the groundwater flow equations in the saturated aquifers. The general governing equation solved by MODFLOW is:

$$\frac{\partial}{\partial x_i}\left(K_i \frac{\partial h}{\partial x_i}\right) + q_s = S_s \frac{\partial h}{\partial t} \quad (1)$$

where K_i is hydraulic conductivity along i axis (m/s), x_i is the distance along the respective Cartesian coordinate axis (m), q_s is volumetric flux per unit volume (1/s), S_s is specific storage (1/m), h is the potentiometric head (m), and t is time (s) (Abdelaziz and Zambrano-Bigiarini, 2014; Abdelaziz and Le, 2014).

The flow pattern simulated by MODFLOW was used as input to MT3DMS, developed by Zheng and Wang (1999), to simulate the nitrate concentration in the aquifer. The partial differential equation describing the transport of contaminants in 3D transient groundwater flow systems can be written as:

$$\frac{\partial(nC)}{\partial t} = \frac{\partial}{\partial x_i}\left(nD_{ij}\frac{\partial C}{\partial x_i}\right) - \frac{\partial}{\partial x_i}(nv_iC) + q_sC_s + \sum R_n \quad (2)$$

where C is dissolved concentration of nitrate (g/l), n is porosity of the medium, D_{ij} is hydrodynamic dispersion coefficient tensor (m^2/s), v_i is the seepage or linear pore water velocity (m/s) calculated according to Eq. (3), C_s is concentration of the source or sink flux (g/l), and $\sum R_n$ is chemical reaction term (g\l-s).

$$v_i = -\frac{K_i}{n}\frac{\partial h}{\partial x_i} \quad (3)$$

MODFLOW requires aquifer geometry, initial estimate of hydrodynamic parameters (hydraulic conductivity and storage coefficients), discharge of operational wells and their returned flow, amount of recharge, piezometers' information, and boundary and initial conditions as inputs. MT3DMS, in addition to the MODFLOW outputs, needs dispersion coefficients and nitrate loads as initial and boundary condition.

According to geological studies, the Karaj Aquifer is a single-layer unconfined aquifer. The aquifer's thickness at different locations is illustrated in Figure S2. It varies from 86 to 347 m as presented in the figure. Evaluation of the water table in the piezometers reveals that the groundwater flows from north and northwest to the south and southwest of the aquifer. Thus, the boundary conditions are defined as two boundary lines including given heads at the north-western part of the aquifer for the entrance and at the southern part of the aquifer for the outflow (red lines in Figure S1). For instance, the iso-water table for April 2011 is shown in Figure S1. For the other boundaries there is no interaction between the adjacent aquifers.

The spatial distribution of hydraulic conductivity was obtained by the Ministry of Energy that estimated these parameters from pumping tests. Initial values of specific yield were chosen on the basis of the guideline suggested by Johnson (2012). The final values of the hydraulic conductivity and specific yield ~~are~~ were determined during the calibration stage.

There are 1698 production wells used for different purposes such as industrial and agricultural activities (Figure S1). Using the information from these wells and land use (Figure S1), the groundwater discharge and water return coefficients for each land use type were calculated. The evaporation from groundwater was considered negligible since the groundwater table is deeper than three meter in all parts of the aquifer.

Besides natural rainfall infiltration, seepage from pit latrines is one of the sources of aquifer recharge due to vast use of this system for wastewater disposal in the residential areas. The recharge amount from pit latrines was estimated based on population density and land use maps included in the calibration procedure. Karaj River is the other source of groundwater

recharge in the study area. Note that at the entrance point of the river into the plain, part of water is diverted and transferred by a channel to supply drinking water of the surrounding towns. According to available data, mean annual depth of the river at the entry point to the plain for 2006-2008, 2008-2010, and 2010-2012 was 76.7, 92.8, and 66.7 cm, respectively. At the outlet of the river from the plain, the river had typically insignificant flow for all years.

The spatial distribution of nitrate in April 2006 was used as initial conditions for the transport model. By using estimates of water recharge from different agricultural, industrial, and residential land uses and nitrate concentrations in recharged water, the nitrate loads into the aquifer were calculated. The nitrate loads were then calibrated according to those measured in the piezometers.

2.3 Calibration and verification

A six-year period from April 2006 to March 2012 was selected for calibration of the flow (MODFLOW) and nitrate transport (MT3DMS) models under unsteady conditions. This period was selected due to ~~including~~ the availability of the most accurate and complete data such as meteorological, hydrological, geological, hydrogeological, and water quality measurements. In addition, April 2011 was considered for calibration of the groundwater flow model under steady state condition. To verify the unsteady models' robustness, monthly data from April 2012 to March 2013 were employed.

2.3.1 Groundwater flow model (MODFLOW)

The aquifer was divided into 4691 active cells with a dimension of ~~a~~ 100×100 m. ~~gridded network with 217.9×183 m mesh dimension. The number of active and inactive cells in the model were 4,691 and 5,309, respectively.~~ In order to calibrate the model under steady state conditions, firstly, the piezometer data in April 2011 were used in the model as initial values. ~~Note that spatially distributed groundwater levels for the aquifer were generated from piezometer data by kriging. In the next step,~~ The model's calibration was performed manually by adjustment of hydraulic conductivities. Input recharge was affected by great uncertainty, thus, they were simultaneously modified in the calibration process. ~~The calibration process was continued until the Mandle (2002) criterion was met (difference between simulated and observed hydraulic heads less than 10% of the domain variation). The maximum domain variation was considered 10 m.~~ Spatial variation of calibrated hydraulic conductivity and recharge are shown in Figures S3 and S4, respectively. ~~According to the figure, highest hydraulic conductivity is more than 45 m/day in the southwest part, while minimum is about 4 m/day in the northwest. Spatial distribution of the calibrated recharge indicates that the urban densely populated areas experience high recharge values of more than 10 mm/day. The widespread using of pit latrines and wells for wastewater disposal may contribute the high rate of recharge in the urban areas.~~ In the next step the specific yields (Fig. S5) were calibrated under unsteady state condition. In this process, monthly time series of the calibrated recharge, piezometer, and operational well data were used as model input. ~~Considering 10 m accuracy of hydraulic head in the model domain, the spatial variation of calibrated specific yields is shown in Figure S5. According to the figure, the specific yield varies from 0.15 in the south to 0.01 in the north of the aquifer.~~

After calibration, the model was verified for the period April 2012 to March 2013.

2.3.2 Contaminant transport model (MT3DMS)

~~In MT3DMS model, there are five methods for solving the governing partial differential equations: method of characteristics , modified method of characteristics, hybrid method, finite difference, and ultimate methods.~~ In the present study, the ~~ultimate method, which is a combination of four other methods~~hybrid method~~,~~ was used~~,~~ ~~to discretize the advection-diffusion equation~~which is a combination of method of characteristics and modified method of characteristics, to solve the governing equations on MT3DMS model. The model was calibrated with trial and error. For this purpose, nitrate concentrations in April 2006 were used as initial conditions. Then, calibration of the model during the six-year period was done by modifying dispersion coefficients and nitrate input loads. Note that as the aquifer is single layer and unconfined, only longitudinal and transverse dispersion coefficients were calibrated in the model. Due to the fact that porosity is almost equal to the specific yield in unconfined course-graded aquifers, the calibrated specific yield was used for porosity in the calibration of the MT3DMS model. ~~A threshold of 10% of nitrate variation during the simulation period was attained (here 10 mg/l).~~
After calibration of the MT3DMS, verification was carried out for a one-year period (April 2012 to March 2013).

2.4 Nitrate ROM

After calibration and verification of the flow and nitrate transport models for the Karaj Aquifer, the nitrate ROM was developed. A POD model was used in a first step to determine dominant modes of nitrate variation for the Karaj Aquifer as shown in Fig. 2. The POD was applied to nitrate concentration simulations by MT3DMS for the aquifer ($\boldsymbol{\Omega}$) resulting in spatial components of dominant modes of nitrate concentrations ($\boldsymbol{\Theta}$) given by:

$$\boldsymbol{\Theta} = \sum_{i=1}^{N} \xi \boldsymbol{\Omega}^{(i)}(\mathbf{x}) \tag{4}$$

In Eq. (4), N is the number of snapshots, $\mathbf{x}$ is the location matrix of cells, and ξ is the eigenvectors (Ashrafi, 2012). This relationship is a linear combination of snapshots taken from nitrate simulated in the Karaj Aquifer. To increase the similarity between functions $\boldsymbol{\Theta}$ and $\boldsymbol{\Omega}$, it is necessary to determine ξ in such a way that the following expression is optimized (Noori et al., 2015):

$$\frac{1}{N}\sum_{i=1}^{N}\left\{\left|\left(\boldsymbol{\Omega}^{(i)}, \boldsymbol{\Theta}\right)\right|^2 / (\boldsymbol{\Theta}, \boldsymbol{\Theta})\right\} \quad if \quad (\boldsymbol{\Theta}, \boldsymbol{\Theta}) = \|\boldsymbol{\Theta}\|^2 = 1 \tag{5}$$

The optimization of Eq. (5) is done by solving the following eigenvalue problem:

$$\sum_{j=1}^{N} \mathbf{M}_{ij}\xi_j = \lambda\xi_i \tag{6}$$

where $\mathbf{M}$, λ, and ξ are Hermitian matric, eigenvalues, and their corresponding eigenvectors, respectively (Noori et al., 2015). Having the function $\boldsymbol{\Theta}$ and using Eq. (7), the temporal components of dominant modes of nitrate in the Karaj Aquifer can be calculated (Ashrafi, 2012):

$$\tau_i(t) = (\boldsymbol{\Omega}(\mathbf{x}, t), \boldsymbol{\Theta}_i(\mathbf{x})) \tag{7}$$

It is noteworthy that all eigenvalues calculated by Eq. (6) are positive. Only the few first calculated eigenvalues are large and the rest are close to zero. Since the calculated modes are corresponding to eigenvalues, only the few first modes are important in practical problems so that they represent most of the system variation (STV of nitrate concentration in the aquifer). Thus, it is possible to regenerate the STV of nitrate concentration in the aquifer by means of the first significant modes and application of the following equation (Noori et al., 2017):

$$\boldsymbol{\Omega}(\mathbf{x}, t) \cong \sum_{i}^{l} \tau_i(t)\boldsymbol{\Theta}_i(\mathbf{x}) \quad , \quad l \ll N \tag{8}$$

where l is the number of modes.

Note that if the ROM developed by few first selected modes (Eq. (8)) didn't satisfy the defined criteria for evaluating the model performance (such as RMSE and MAE), one need to increase the number of modes so that it satisfies the defined criteria. This procedure is properly shown in Figure 2.

However, Eq. (8) has a simple structure with a few modes. Thus, one needs the few first significant modes developed based on simulated historic nitrate concentrations using MT3DMS to regenerate this pollutant in the aquifer. In other words, the ROM is applicable for regeneration of nitrate on the solution domain of MT3DMS and it is not applicable to make prediction in the future. In this study, we aimed to enhance the ROM to make predictions into the future, for time steps with no nitrate simulations from the MT3DMS model. This step is shown in subsection "Model Verification" of Fig. 2. ~~Note that to be able to develop the nitrate concentration ROM by application of Eq. (8), one need simulated nitrate concentrations from the MT3DMS. In order to circumvent this problem,~~For this purpose, we developed a methodology that enables the ROM to independently simulate nitrate concentrations. ~~For this purpose~~According to this, it is necessary to calculate both the spatiotemporal components $\boldsymbol{\Theta}(\mathbf{x})$ and $\tau(t)$ for the future time interval $(t + n)$. Since the component $\boldsymbol{\Theta}$ is a function of space, it does not change in time. Therefore, it is necessary to calculate τ for future time steps. For this purpose, a regression equation was used to estimate the time variation of this component ($\tau(t+n)$). Having $\tau(t+n)$, the nitrate concentration for future time steps $t+n$ can be calculated by:

$$\boldsymbol{\Omega}(\mathbf{x}, t + n) \cong \sum_{i=1}^{l} \tau_i(t + n)\boldsymbol{\Theta}_i(\mathbf{x}) \quad , \quad l \ll N \tag{9}$$

Eq. (9) simulates the nitrate concentration in different parts of the aquifer for future time t+n using the first few significant modes. According to Fig. 2, if the ROM developed didn't satisfy the defined criteria for evaluating the model performance, one need to increase the number of modes by estimating the trend of more numbers of the time dependent components so that the developed ROM satisfies the defined criteria.

3 Results and discussion

3.1 Results of groundwater flow model

~~The~~ C~~c~~alibration of MODFLOW was ~~continued~~ done by optimizing *K* and *S*~~,~~ for steady and unsteady conditions till the Mandle (2002) criterion was met. According to this criterion, ~~by six-year data from April 2006 to March 2012 until~~ the difference between simulated and observed groundwater levels ~~for steady state condition~~ in all piezometers ~~was~~ should be less than 10% of the hydraulic load variation ~~within the domain~~ (i.e., ~~<~~10 m). ~~In addition, the~~The difference between simulated and measured groundwater levels during the groundwater calibration for unsteady conditions is shown in Figs. 3A and 3B for two piezometers. As seen from the figures, simulated piezometer levels match the observations well. ~~Errors during calibration period were less than 10% of the hydraulic head variation (i.e., ±10 m) as suggested by Mandle (2002).~~

~~After successful calibration of the groundwater flow model, its performance was verified using~~For the verification step, data from April 2012 to March 2013 were used and the simulated results were compared to the observed. ~~as shown in Figure 3C.~~ According to ~~the~~ Fig. 3C~~figure~~, the temporal variation of mean absolute error (MAE) and root mean square error (RMSE) ~~is~~ are between 2.6 and 4.2 m. ~~Therefore, it can be concluded that the groundwater flow model was reliably calibrated and verified for the aquifer.~~

3.2 Results of nitrate transport model

To properly tune the model, a threshold value of 10% of nitrate concentration variation during the simulation period was chosen (10 mg/l). Figures 3D to 3F show the ~~difference~~ trends of simulated and measured nitrate concentration during the calibration ~~between simulated vs measured nitrate concentration during the calibration~~ of MT3DMS under unsteady condition~~s~~ for three selected piezometers for the Karaj Aquifer. Also, Fig. 3G shows the trends of RMSE and MAE averaged over the study area during the calibration period. According to ~~figures~~Figs. 3D to 3F, MT3DMS calibration was well performed so that the error between simulated vs measured nitrate concentrations is less than 10 mg/l. Besides, Fig. 3G indicates that the temporal variations of MAE and RMSE are between 0.6 and 6 mg/l of nitrate concentration.

Verification of the MT3DMS model was carried out using one-year data from April 2012 to March 2013. In this step, mean values of MAE and RMSE were equal to 2.3 and 2.9 mg/l for nitrate concentration, respectively. The spatial distribution of nitrate concentration in the Karaj Aquifer for six selected months is illustrated in Figs. 4A to 4F. The figures show that the northern part of the aquifer are more exposed to risk of nitrate contamination. Nitrate concentration in some parts exceeds the maximum permissible limit 50 mg/l suggested by World Health Organization. A look at the land use map (Figure S1) indicates that these parts include densely populated urban areas that typically use pit latrines for wastewater disposal.

3.3 Dominant modes of nitrate STV

To calculate the dominant modes of nitrate STV in the Karaj Aquifer, snapshots of nitrate concentrations simulated by MT3DMS were used from the calibration period. Due to the low variation of nitrate in groundwater compared to surface water,

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one-day intervals were selected for capturing variation. Consequently, according to the six-year simulation period, 1772 snapshots of nitrate concentration were obtained for 4691 active cells of the Karaj Aquifer. In other words, a matrix $\boldsymbol{\Omega}$ was formed with the dimension 1772×4691 as described in "Methods" section. By applying the POD on the matrix $\boldsymbol{\Omega}$, firstly the Hermitian matrix was formed with dimension equal the number of snapshots, i.e., 1772×1772. Thereafter, the eigenvalues and their corresponding eigenvectors were calculated. The results for the first ten eigenvalues are shown in Fig. 5A. The energy of the system conserved (STV of nitrate concentration in the aquifer) for the first ten eigenvalues is shown in Figure 5A. The results show that except the first eigenvalue that conserves about 96.2% of the energy of the system, the others are negligible. Thus, only the first mode may represent the STV of nitrate concentration in the Karaj Aquifer. Temporal and spatial components of the first mode ($\tau_1(t)$ and $\boldsymbol{\Theta_1(x)}$) are shown in Figs. 5B and 5C, respectively. According to Fig. 5C, it is clear that the largest variation of nitrate occurs in the northern parts of the aquifer (shown in orange and blue color) in connection to urban areas as mentioned above. Indeed, high nitrate loads are introduced into the groundwater from residential areas due to the lack of sewage collection systems and using pit latrines for wastewater disposal. According to Fig. 5B, the first temporal component ($\tau_1(t)$) indicates a gradually increasing trend for nitrate concentration in the aquifer. This is due to the increase in population and development of agricultural lands, especially in recent years, and consequently increasing nitrate loads into the Karaj Aquifer. Figure 5B also indicates that the temporal variation of the other components is relatively insignificant.

3.4 Results of nitrate ROM

3.4.1 Developing ROM for nitrate calculation

In this study, two important principles were considered for the development of the nitrate ROM: (A) The accuracy should be similar to the results of MT3DMS, and (B) the model structure should be kept simple. For this purpose, a trade-off between the model's accuracy and its complexity was performed. Figure 5D shows the RMSE for simulated nitrate concentrations by the MT3DMS and different ROMs developed with different number of modes. According to the figure, it is clear that increase in number of modes is accompanied by an increase in accuracy of the ROM. The increase in the models' accuracy is meaningful until the ROM developed by the first ten modes. Therefore, the nitrate ROM developed by the first ten out of 1772 modes were used for simulation of nitrate in the aquifer. These ten modes together represent more than 99.999% of the STV of nitrate in the aquifer. The developed nitrate ROM was applied to simulate the nitrate concentrations for the aquifer during the calibration period. The spatial distribution of the simulated nitrate concentration by ROM and MT3DMS, as well as the difference between results of the two models for four selected days are presented in Figs. 6A to 6D. According to the figures, the difference between results of the two models in different parts of the aquifer is less than 0.1 mg/l of nitrate. This value is very small as compared to the observed large variation in nitrate concentration in the Karaj Aquifer (maximum nitrate concentration is about 116 mg/l). The mean error for each cell during the other four selected days are shown in Figs. 6E to 6H. The graphs show that the error varies between of -0.4 and 0.4 mg/l nitrate concentration. To better evaluate the accuracy of the ROM, nitrate concentrations simulated by ROM vs MT3DMS for four different days are shown in Figs. 6I to 6L. According

to the figures, both models perform similar giving a correlation of about 1 for each of the days. Thus, it can be concluded that both models are equal in terms of accuracy.

3.4.2 Developing ROM for prediction purposes

In order to develop ROM for prediction of nitrate concentration beyond the simulation period, it is necessary to calculate the temporal components of the first ten modes for future time step $t+n$. In this regard, one-year data from April 2012 to March 2013 were used. However, given the high importance of the temporal component of the first four modes that represent more than 99.999% of STV of nitrate in the aquifer, only the trend of these components are estimated in the future times $t+n$. According to Figure 5B, for each component the best models were fitted. The fitted models with R^2 equal to 0.999, 0.995, 0.959, and 0.892 for the first four temporal components, respectively, can be used to properly reconstruct these components for future time steps with good accuracy. The other temporal components were ignored due to their small contribution. Thereafter, having the spatial components of the first four modes, ROM was developed by application of Eq. (9) in the "Methods" section. Figures 7A to 7C show the spatial distribution of nitrate concentration simulated by ROM and MT3DMS, respectively, as well as the difference between the model results for three selected days in 2012. According to the figures, maximum spatial difference between the models is less than 3 mg/l nitrate concentration. The results indicate that the absolute error is less than 0.5 mg/l nitrate concentration in the most parts of the aquifer. Thus, both models behave in a similar manner. It is noteworthy that the accuracy of the prediction is slightly lower than that obtained by ROM developed by the first ten modes. This is due to the use of the only first four out of first ten modes for development of ROM that conserved a lower percentage of the system energy compared to the first ten modes.

3.5 Computational time comparison

Considering the computational costs, the developed nitrate ROM was superior by running about ten times faster than MT3DMS. More specifically, the computational time for the predictive MT3DMS model was about 89 seconds. This value reduced to nine seconds when we applied the developed ROM for simulation of nitrate in the aquifer using the same computer. Note that the computational time for running the developed ROM model includes calculations to generate matrix computations, eigenvalues and eigenvectors, and predictive analyses. Also, it is noteworthy that the ROM was based on a historic numerical model simulated using MT3DMS and therefore, the quality of analyses resulting from this model was as good as the quality of the underlying numerical model. Thus, it can be concluded that the developed ROM model with a simple structure and considerably less running time than MT3DMS appropriately simulated the nitrate concentration in the Karaj Aquifer. Regarding the ROM applications for the future times, all time dependent factors in the aquifer responses, such as hydro-meteorological, hydrogeological, and well operation strategies in the simulation period are expressed by temporal component $\tau(t)$ in the ROM. Thus, this model is just applicable for predictive scenarios to examine the options that already existed in the simulation period of the modelling process. In other words, the model appropriately memorized the historical situation that

existed during the simulation periods. This is shown in temporal component $\tau(t)$ (Fig. 5B) although the spatial component $\boldsymbol{\theta}(\mathbf{x})$ is not time dependent.

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4 Conclusions

Some limitations of GQSMs were the main motivation for us to explore and present an alternative method to overcome these problems. In this regard, a ROM was developed by linking MT3DMS and POD models and its performance was checked by simulation of nitrate concentration in the Karaj Aquifer, Iran. The developed nitrate ROM provided good results that matched the nitrate simulated by MT3DMS model well. More specifically, dominant variation of nitrate, based on the results obtained by spatial distribution of the first mode of nitrate, revealed the largest variations of nitrate in the northern parts of the aquifer occupied by dense population and using septic wells for wastewater disposal. Also, the temporal component of the first mode indicated a gradually increasing trend of nitrate during the simulation period in the aquifer. The absolute error between the ROM and the MT3DMS was very small in most parts of the aquifer (about 0.5 mg/l nitrate concentration) compared to the large observed variation in nitrate concentration (maximum nitrate concentration was about 116 mg/l). In fact, the developed ROM with a much simpler structure had the same performance as MT3DMS. ~~Therefore, it can be concluded that the developed ROM was superior than MT3DMS.~~ Indeed, the introduced methodology is general and can be applied for simulation of target parameters in other fields of study.

Author contribution

Roohollah Noori and Mehrnaz Dodangeh conceived the study, collated the data, and ran the model. Ronny Berndtsson, Farhad Hooshyaripor, Jan Franklin Adamowski, Saman Javadi, and Akbar Baghvand supervised the result analyses. Also, all authors contributed to the writing and reviewing of the manuscript.

Supporting information

SI includes: Figure S1: observation wells, land use map, production wells, and groundwater table in the aquifer, Figure S2: thickness of the aquifer, Figure S3: spatial variation of calibrated hydraulic conductivity in the aquifer, Figure S4: spatial variation of recharge in the aquifer, and Figure S5: spatial variation of calibrated specific yields in the aquifer.

Conflict of interest

The authors declare no conflict of interest.

Acknowledgments

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Figure Captions

Figure 1: The Karaj Aquifer located in northern Iran

Figure 2: Methodology used for the present study

Figure 3: Difference between simulated and observed groundwater levels during the calibration of MODFLOW: (**A**) under unsteady state condition for Ghezel-hesar piezometer located in the northwestern part of the aquifer, (**B**) under unsteady state condition for Marlik piezometer located in the southern part of the aquifer; (**C**) Temporal variation of mean absolute error (MAE) and root mean square error (RMSE) during the one-year verification of MODFLOW; ~~Difference~~ Trends of ~~between~~ simulated and observed nitrate concentration during the calibration of MT3DMS under unsteady state condition in: (**D**) Golshahr piezometer, (**E**) Mehrshahr piezometer, and (**F**) Meshkin piezometer, (G) trends of RMSE and MAE averaged over the study area during the calibration period

Figure 4: Spatial distribution of nitrate concentration in the Karaj Aquifer for six selected days (**A**) 15th September 2006, (**B**) 15th April 2007, (**C**) 15th July 2008, (**D**) 15th November 2009, (**E**) 15th August 2010, (**F**) 15th February 2011

Figure 5: (**A**) Eigenvalues and corresponding eigenvectors with energy of the system conserved for the first ten eigenvalues calculated by application of POD, (**B**) temporal components of first to fourth modes {$\tau_1(t)$ to $\tau_1(t)$}, (**C**) spatial component of the first mode {$\Theta_1(x)$}, (**D**) root mean square error (RMSE) for nitrate concentrations simulated by MT3DMS and ROMs with different number of modes

Figure 6: Spatial distribution of nitrate concentrations simulated by ROM, MT3DMS, and difference between the two model results for four selected days: (**A**) 15th September 2006, (**B**) 10th June 2007, (**C**) 10th April 2008, (**D**) 10th January 2010; Mean error in each cell for four selected days: (**E**) 15th November 2006, (**F**) 12th May 2008, (**G**) 14th September 2009, (**H**) 08th February 2010; Nitrate concentrations simulated by ROM vs MT3DMS for four different days: (**I**) 05th December 2006, (**J**) 26th August 2007, (**K**) 20th April 2009, (**L**) 12th May 2010

Figure 7: Spatial distribution of nitrate concentrations simulated by ROM and MT3DMS, and difference between the models for ~~three~~ two selected days beyond the simulation period: (**A**) 21th May 2012 and, (**B**) 18th September 2012~~, (C) 04th February 2013~~

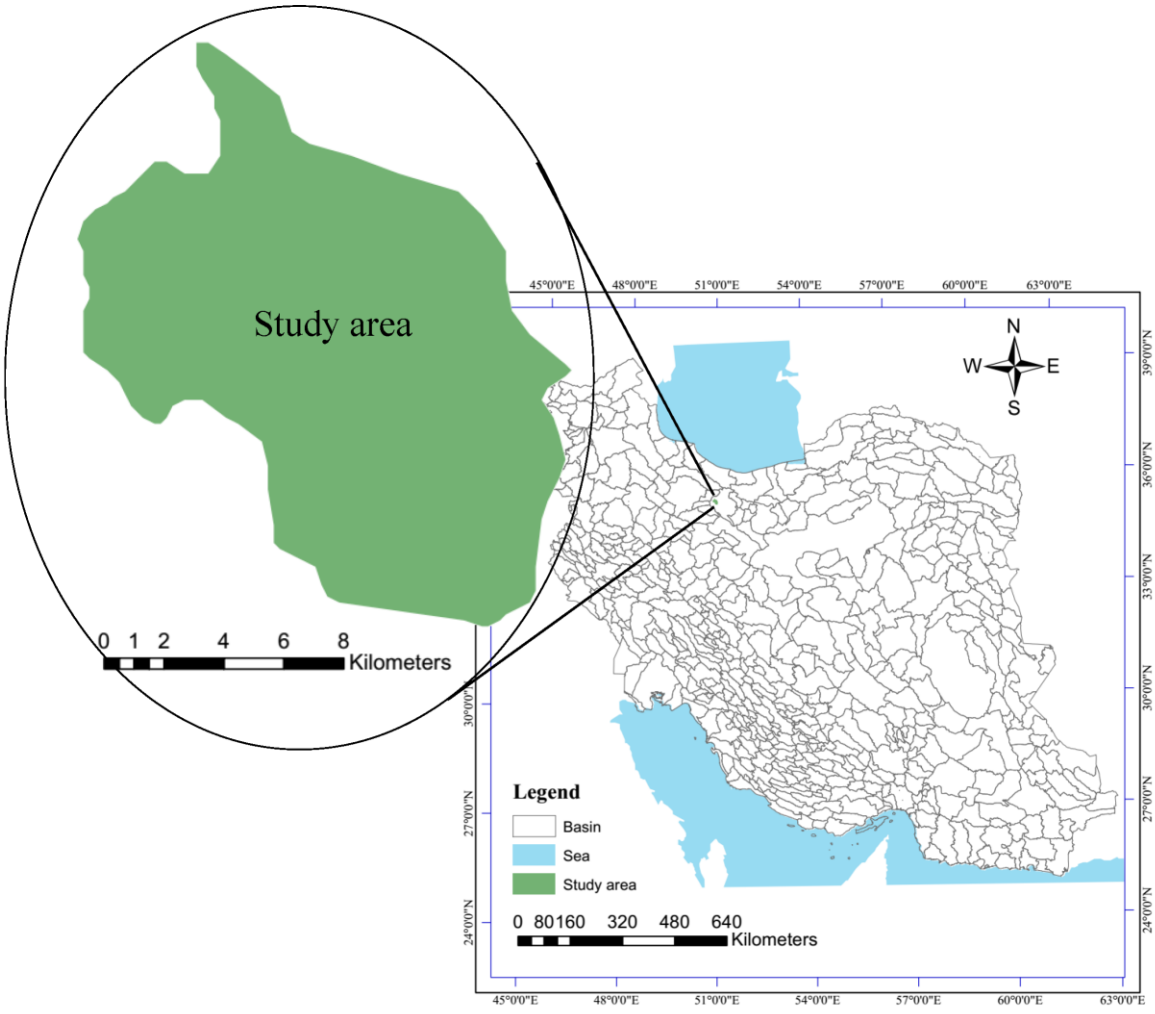
Study area
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Figure 1

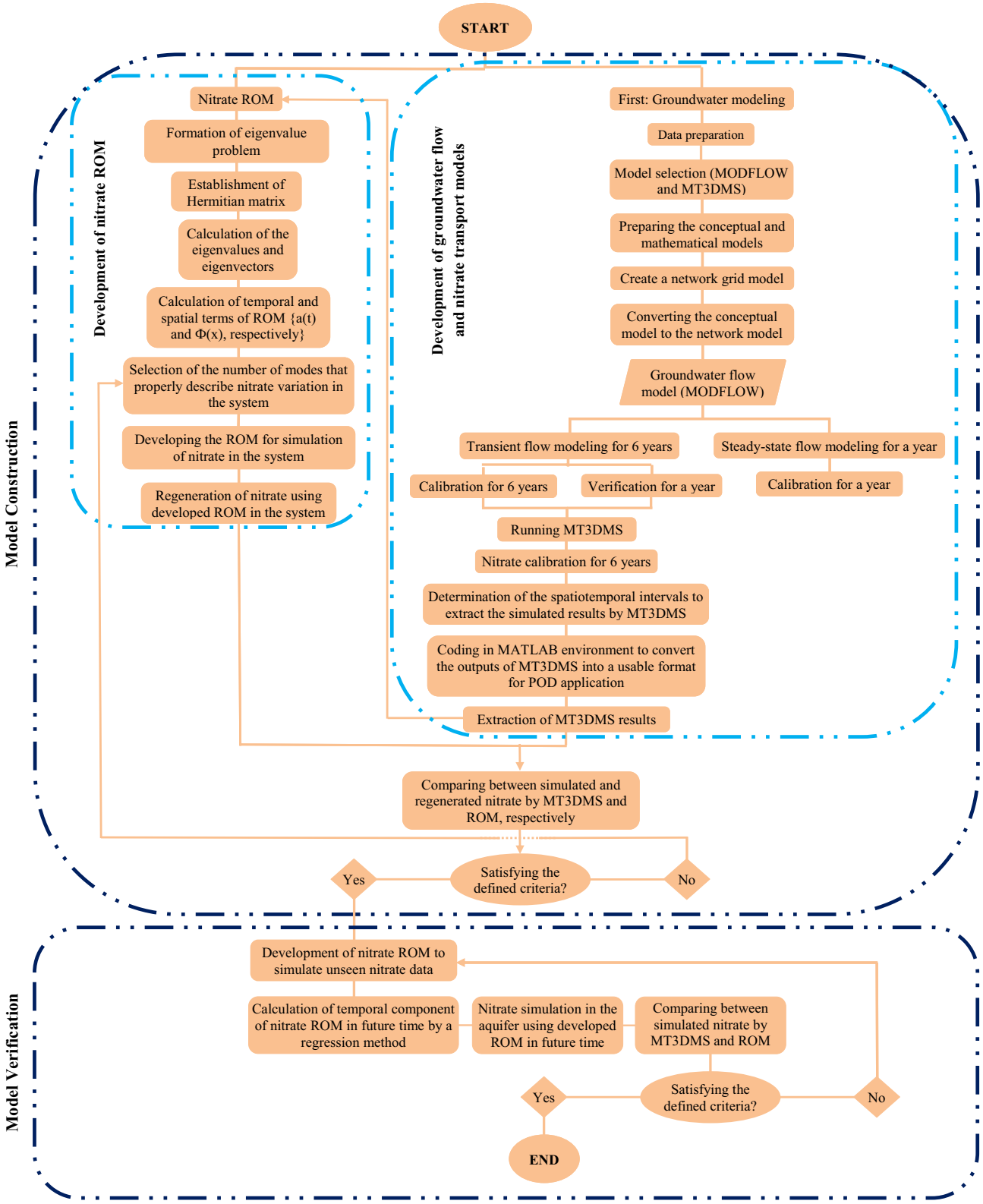
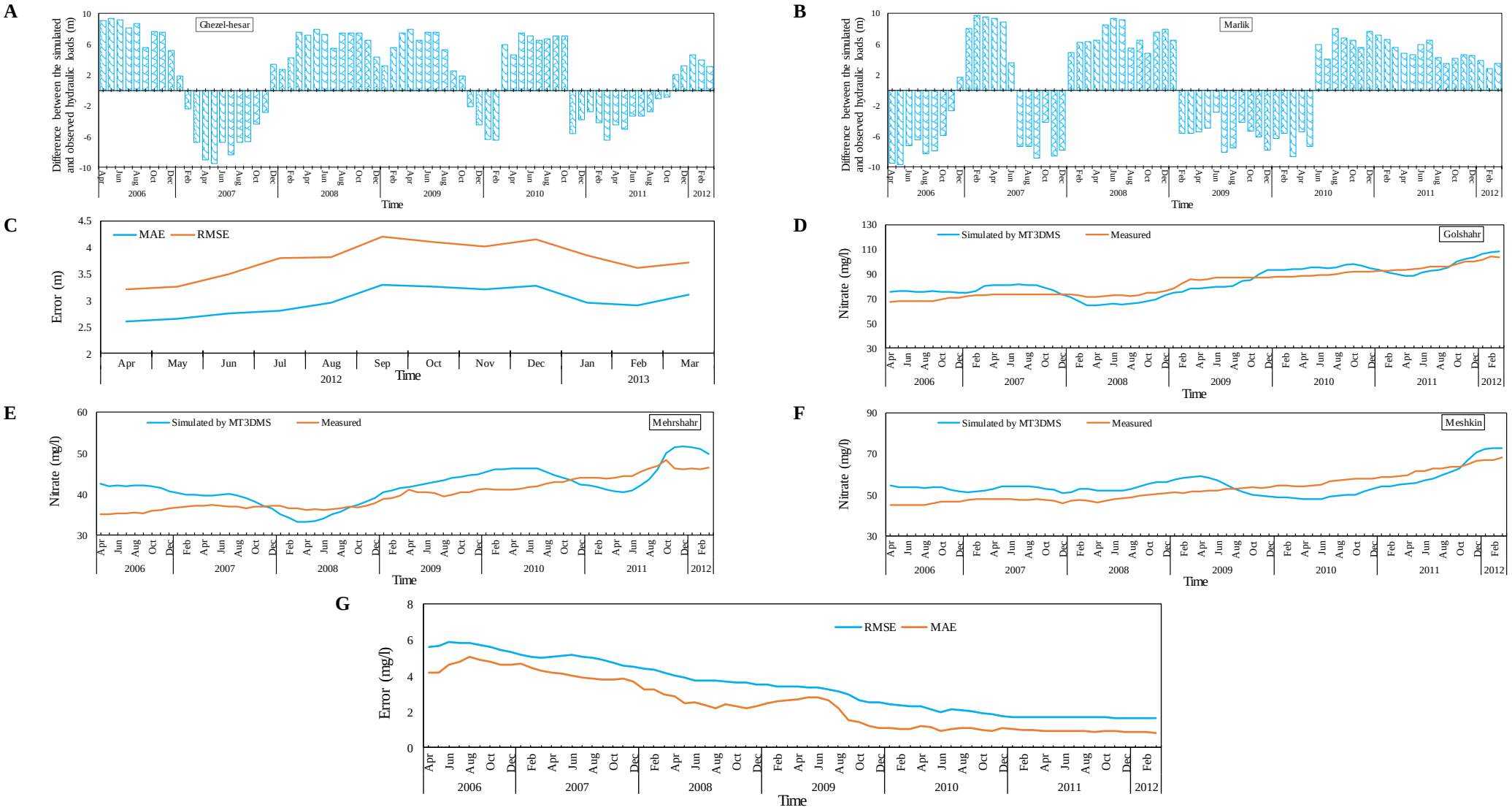

START
Model Construction
Development of nitrate ROM
Nitrate ROM
Formation of eigenvalue problem
Establishment of Hermitian matrix
Calculation of the eigenvalues and eigenvectors
Calculation of temporal and spatial terms of ROM {a(t) and Φ(x), respectively}
Selection of the number of modes that properly describe nitrate variation in the system
Developing the ROM for simulation of nitrate in the system
Regeneration of nitrate using developed ROM in the system
Development of groundwater flow and nitrate transport models
First: Groundwater modeling
Data preparation
Model selection (MODFLOW and MT3DMS)
Preparing the conceptual and mathematical models
Create a network grid model
Converting the conceptual model to the network model
Groundwater flow model (MODFLOW)
Transient flow modeling for 6 years
Steady-state flow modeling for a year
Calibration for 6 years
Verification for a year
Calibration for a year
Running MT3DMS
Nitrate calibration for 6 years
Determination of the spatiotemporal intervals to extract the simulated results by MT3DMS
Coding in MATLAB environment to convert the outputs of MT3DMS into a usable format for POD application
Extraction of MT3DMS results
Comparing between simulated and regenerated nitrate by MT3DMS and ROM, respectively
Satisfying the defined criteria?
Yes
No
Model Verification
Development of nitrate ROM to simulate unseen nitrate data
Calculation of temporal component of nitrate ROM in future time by a regression method
Nitrate simulation in the aquifer using developed ROM in future time
Comparing between simulated nitrate by MT3DMS and ROM
Satisfying the defined criteria?
Yes
No
END

Figure 2


A
Ghezel-hesar
Difference between the simulated and observed hydraulic loads (m)
Time
B
Marlik
Difference between the simulated and observed hydraulic loads (m)
Time
C
MAE
RMSE
Error (m)
Time
D
Simulated by MT3DMS
Measured
Golshahr
Nitrate (mg/l)
Time
E
Simulated by MT3DMS
Measured
Mehrshahr
Nitrate (mg/l)
Time
F
Simulated by MT3DMS
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Time
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RMSE
MAE
Error (mg/l)
Time

Figure 3

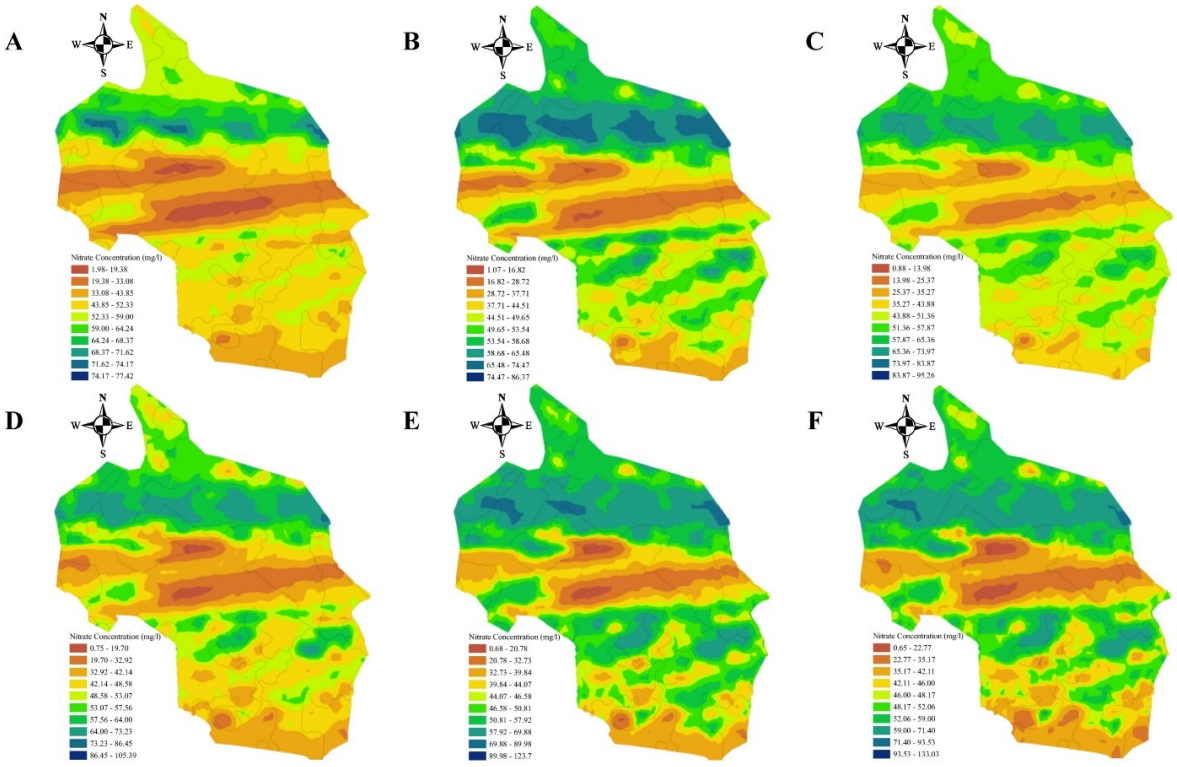

A
N
W
E
S
Nitrate Concentration (mg/l)
1.98- 19.38
19.38 - 33.08
33.08 - 43.85
43.85 - 52.33
52.33 - 59.00
59.00 - 64.24
64.24 - 68.37
68.37 - 71.62
71.62 - 74.17
74.17 - 77.42
B
N
W
E
S
Nitrate Concentration (mg/l)
1.07 - 16.82
16.82 - 28.72
28.72 - 37.71
37.71 - 44.51
44.51 - 49.65
49.65 - 53.54
53.54 - 58.68
58.68 - 65.48
65.48 - 74.47
74.47 - 86.37
C
N
W
E
S
Nitrate Concentration (mg/l)
0.88 - 13.98
13.98 - 25.37
25.37 - 35.27
35.27 - 43.88
43.88 - 51.36
51.36 - 57.87
57.87 - 65.36
65.36 - 73.97
73.97 - 83.87
83.87 - 95.26
D
N
W
E
S
Nitrate Concentration (mg/l)
0.75 - 19.70
19.70 - 32.92
32.92 - 42.14
42.14 - 48.58
48.58 - 53.07
53.07 - 57.56
57.56 - 64.00
64.00 - 73.23
73.23 - 86.45
86.45 - 105.39
E
N
W
E
S
Nitrate Concentration (mg/l)
0.68 - 20.78
20.78 - 32.73
32.73 - 39.84
39.84 - 44.07
44.07 - 46.58
46.58 - 50.81
50.81 - 57.92
57.92 - 69.88
69.88 - 89.98
89.98 - 123.7
F
N
W
E
S
Nitrate Concentration (mg/l)
0.65 - 22.77
22.77 - 35.17
35.17 - 42.11
42.11 - 46.00
46.00 - 48.17
48.17 - 52.06
52.06 - 59.00
59.00 - 71.40
71.40 - 93.53
93.53 - 133.03

Figure 4

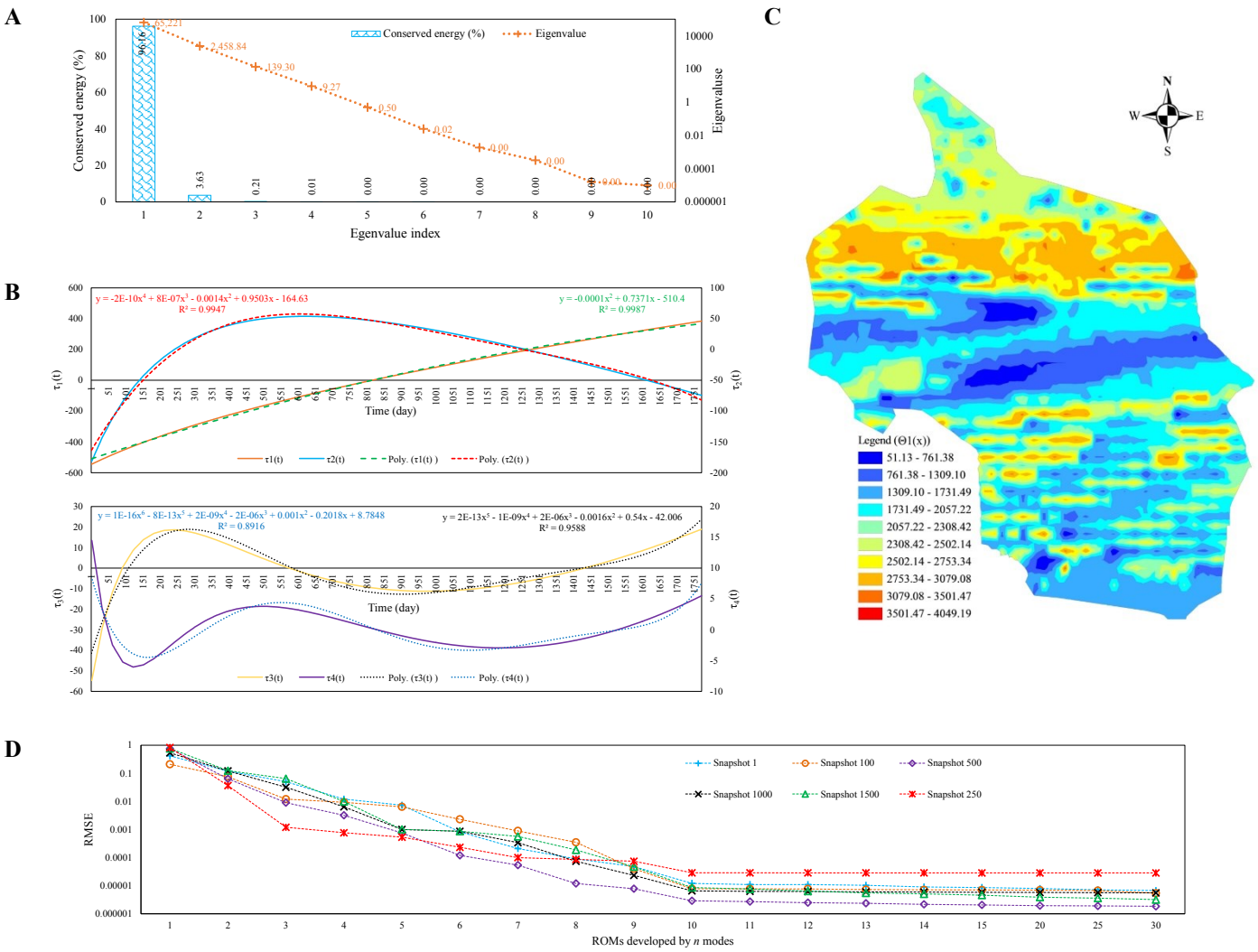

A
Conserved energy (%)
Eigenvalue
Eigenvaluse
Egenvalue index
B
Time (day)
C
Legend (Θ1(x))
51.13 - 761.38
761.38 - 1309.10
1309.10 - 1731.49
1731.49 - 2057.22
2057.22 - 2308.42
2308.42 - 2502.14
2502.14 - 2753.34
2753.34 - 3079.08
3079.08 - 3501.47
3501.47 - 4049.19
N
W
E
S
D
RMSE
Snapshot 1
Snapshot 100
Snapshot 500
Snapshot 1000
Snapshot 1500
Snapshot 250
ROMs developed by n modes

Figure 5

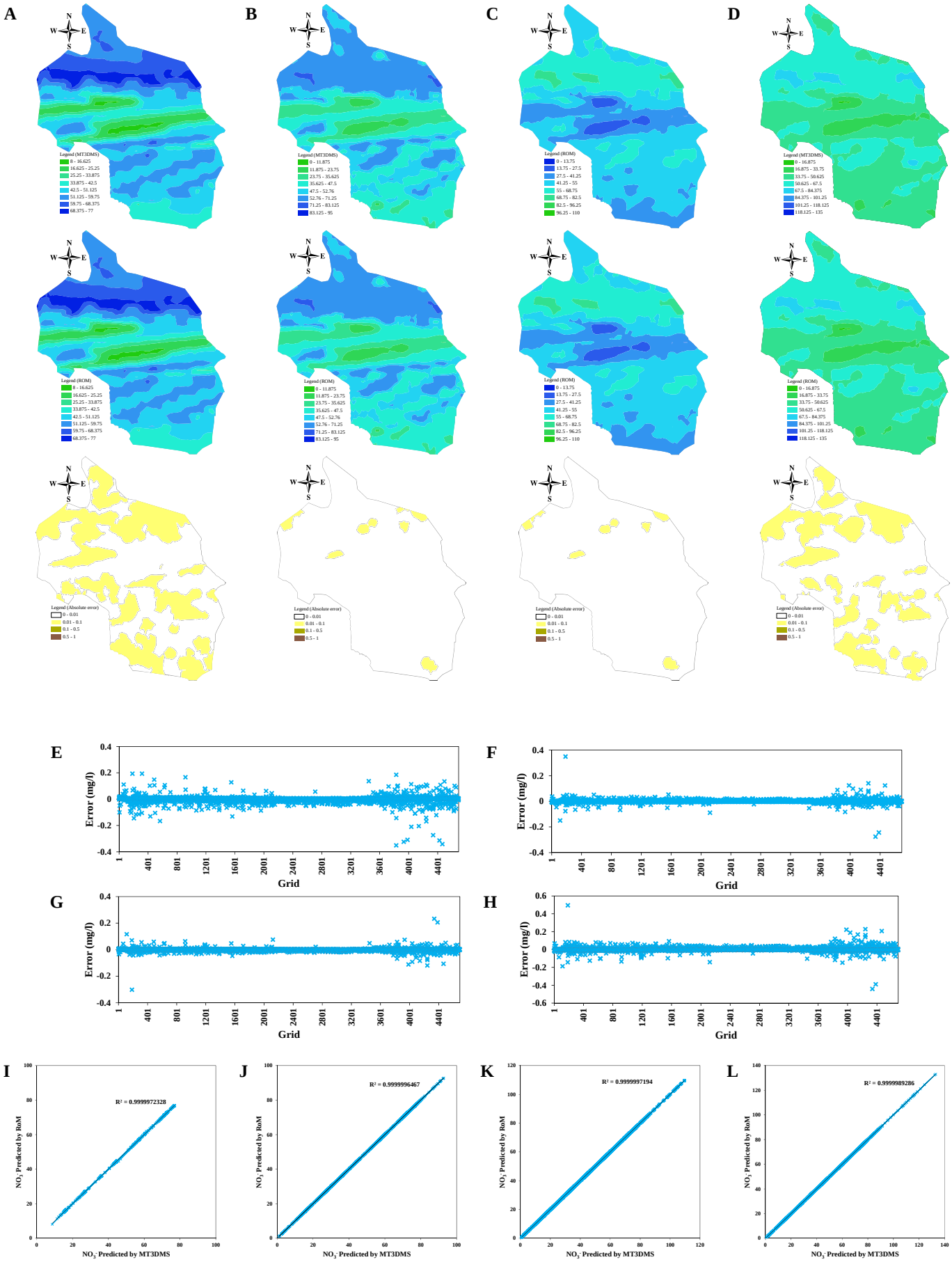
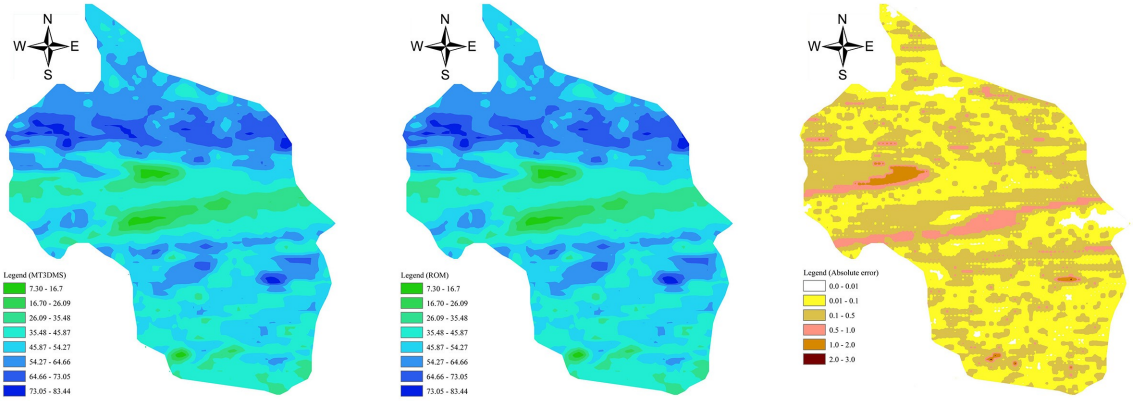
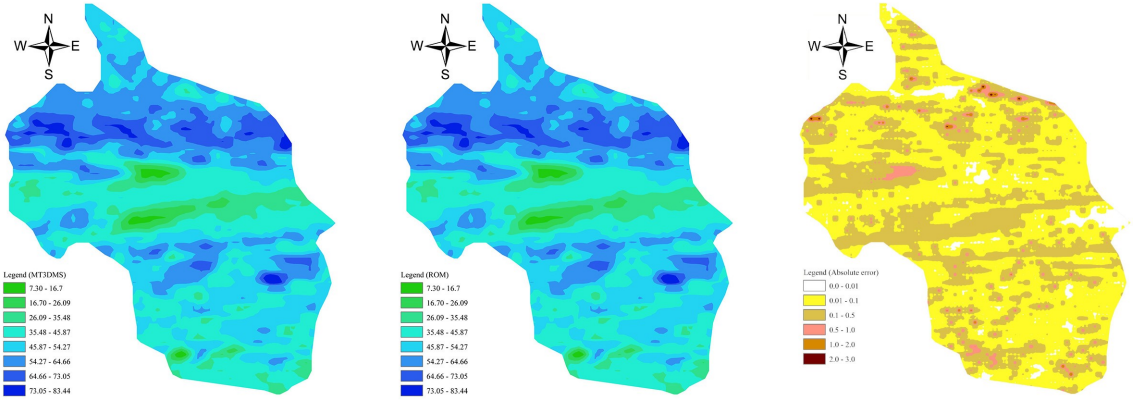

A
B
C
D
E
F
G
H
I
J
K
L
Error (mg/l)
Grid
R² = 0.9999972328
R² = 0.9999996467
R² = 0.9999997194
R² = 0.9999989286
NO3- Predicted by RoM
NO3- Predicted by MT3DMS

Figure 6

(A)


N
W
E
S
Legend (MT3DMS)
7.30 - 16.7
16.70 - 26.09
26.09 - 35.48
35.48 - 45.87
45.87 - 54.27
54.27 - 64.66
64.66 - 73.05
73.05 - 83.44
Legend (ROM)
7.30 - 16.7
16.70 - 26.09
26.09 - 35.48
35.48 - 45.87
45.87 - 54.27
54.27 - 64.66
64.66 - 73.05
73.05 - 83.44
Legend (Absolute error)
0.0 - 0.01
0.01 - 0.1
0.1 - 0.5
0.5 - 1.0
1.0 - 2.0
2.0 - 3.0

(B)


N
W
E
S
Legend (MT3DMS)
7.30 - 16.7
16.70 - 26.09
26.09 - 35.48
35.48 - 45.87
45.87 - 54.27
54.27 - 64.66
64.66 - 73.05
73.05 - 83.44
Legend (ROM)
7.30 - 16.7
16.70 - 26.09
26.09 - 35.48
35.48 - 45.87
45.87 - 54.27
54.27 - 64.66
64.66 - 73.05
73.05 - 83.44
Legend (Absolute error)
0.0 - 0.01
0.01 - 0.1
0.1 - 0.5
0.5 - 1.0
1.0 - 2.0
2.0 - 3.0

Supplementary Information for:

A novel model for simulation of nitrate in aquifers

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SI Figure Captions

Figure S1. Location of 12 observation wells (piezometers), land use map, production wells used for industrial and agricultural activities, and groundwater table in the Karaj Aquifer for April 2011 (the two red lines indicate boundary conditions containing given heads)

Figure S2. Thickness of the Karaj Aquifer

Figure. S3. Spatial variation of calibrated hydraulic conductivity in the Karaj Aquifer

Figure S4. Spatial variation of recharge in the Karaj Aquifer

Figure S5. Spatial variation of calibrated specific yields in the Karaj Aquifer

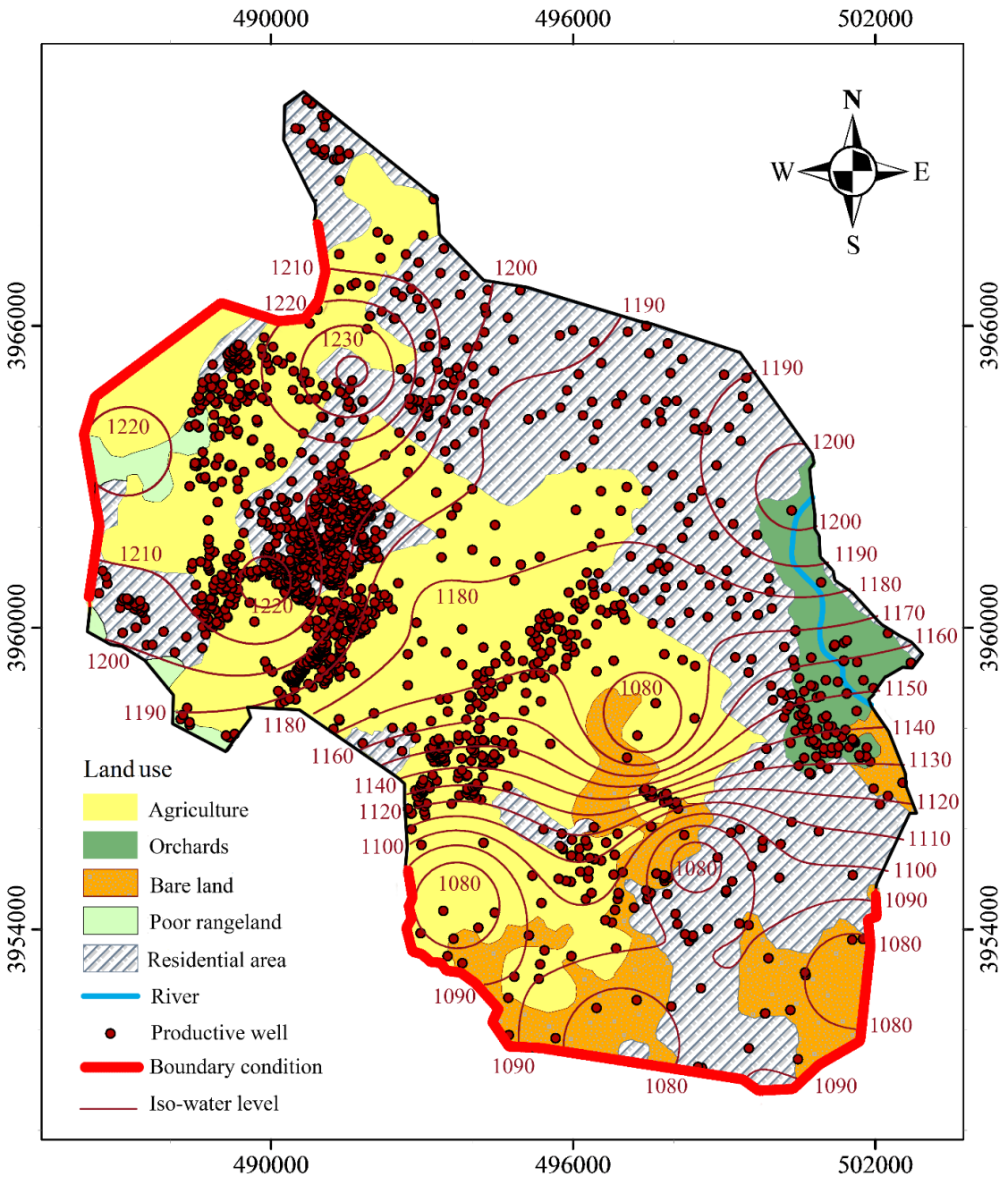
490000
496000
502000
3966000
3960000
3954000
N
W
E
S
Land use
Agriculture
Orchards
Bare land
Poor rangeland
Residential area
River
Productive well
Boundary condition
Iso-water level

Figure S1

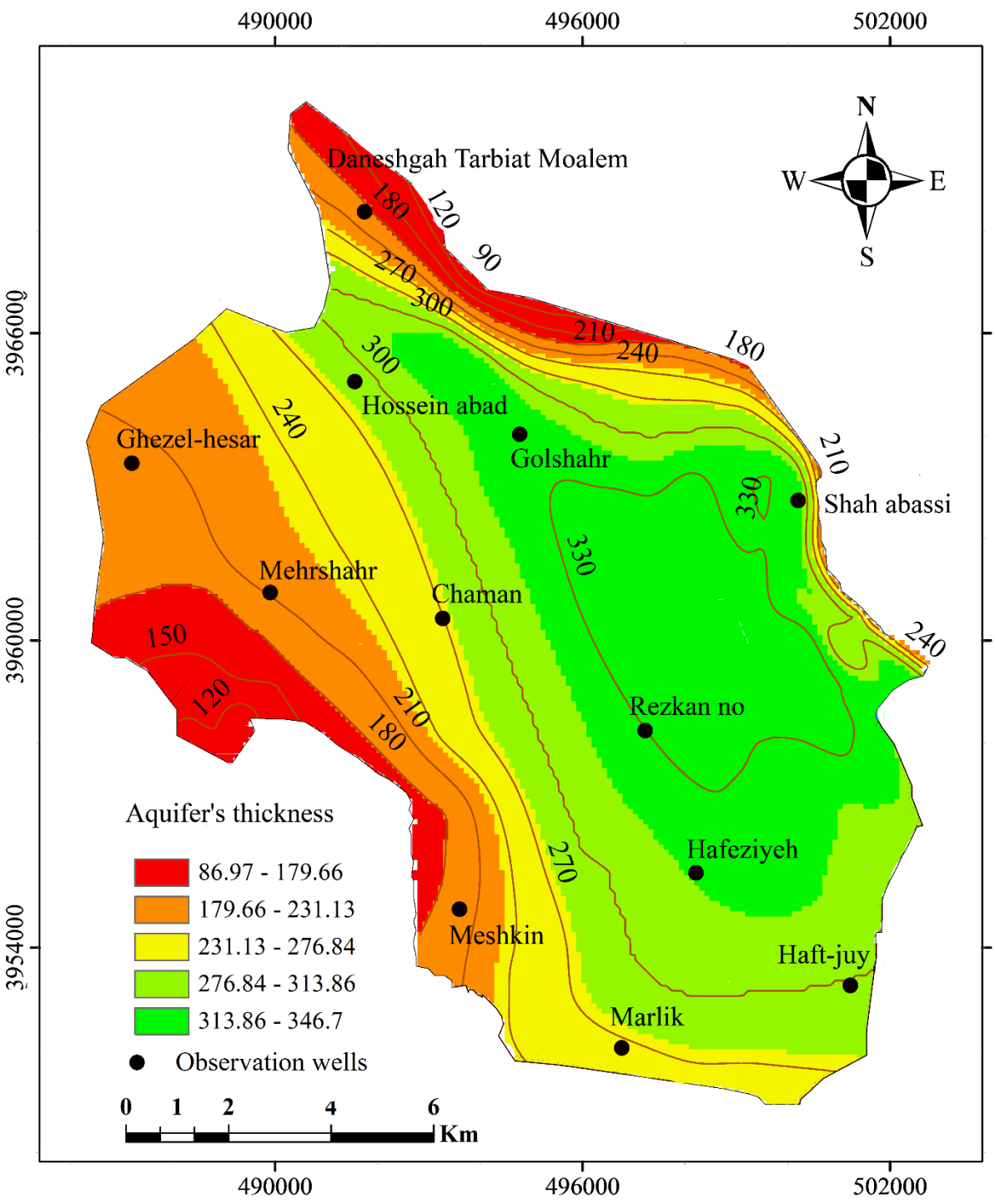
490000
496000
502000
3966000
3960000
3954000
N
W
E
S
Daneshgah Tarbiat Moalem
Hossein abad
Golshahr
Shah abassi
Ghezel-hesar
Mehrshahr
Chaman
Rezkan no
Hafeziyeh
Meshkin
Haft-juy
Marlik
180
120
90
270
300
210
240
330
150
Aquifer's thickness
86.97 - 179.66
179.66 - 231.13
231.13 - 276.84
276.84 - 313.86
313.86 - 346.7
Observation wells
0 1 2 4 6
Km

Figure S2

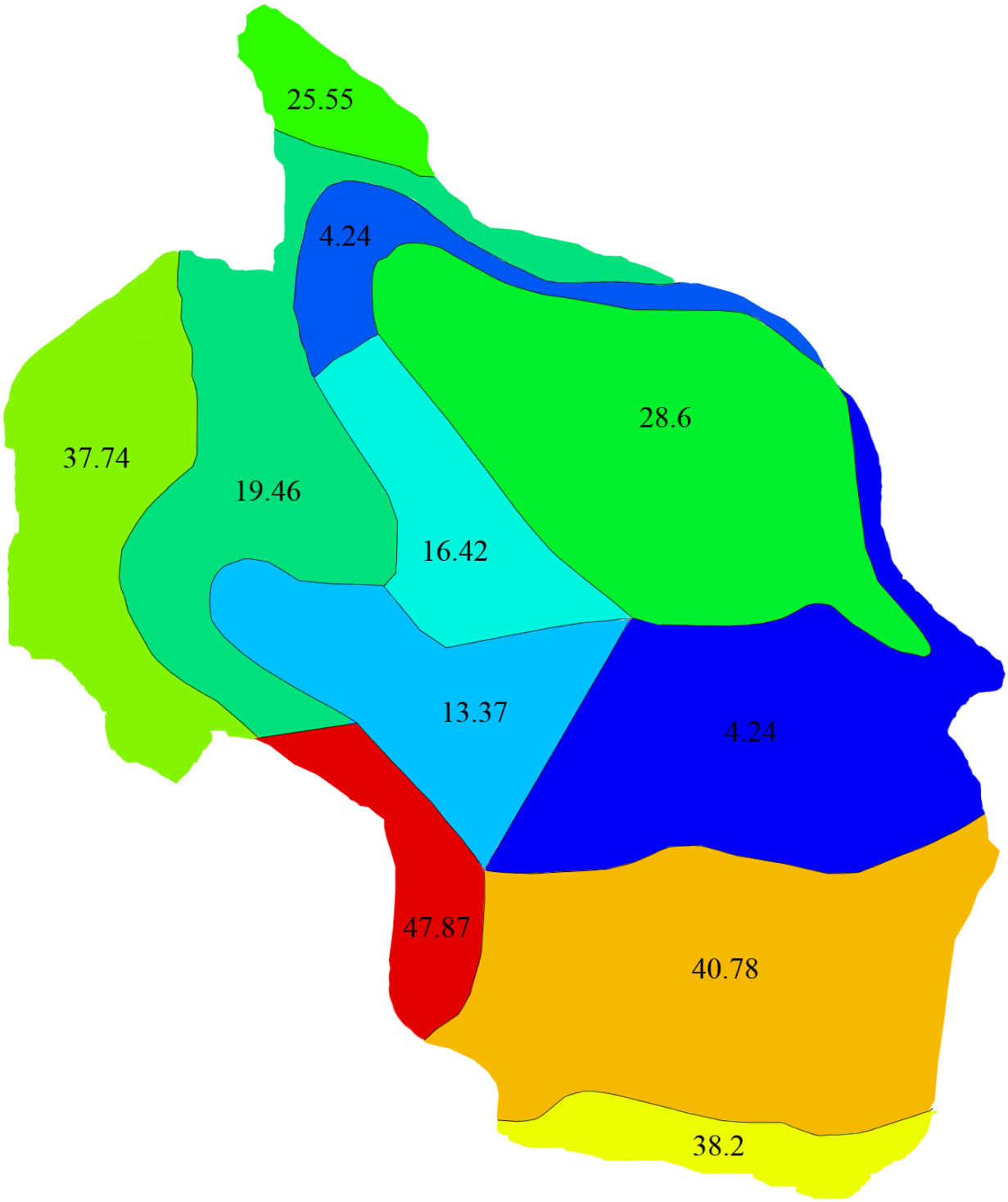

Figure S3

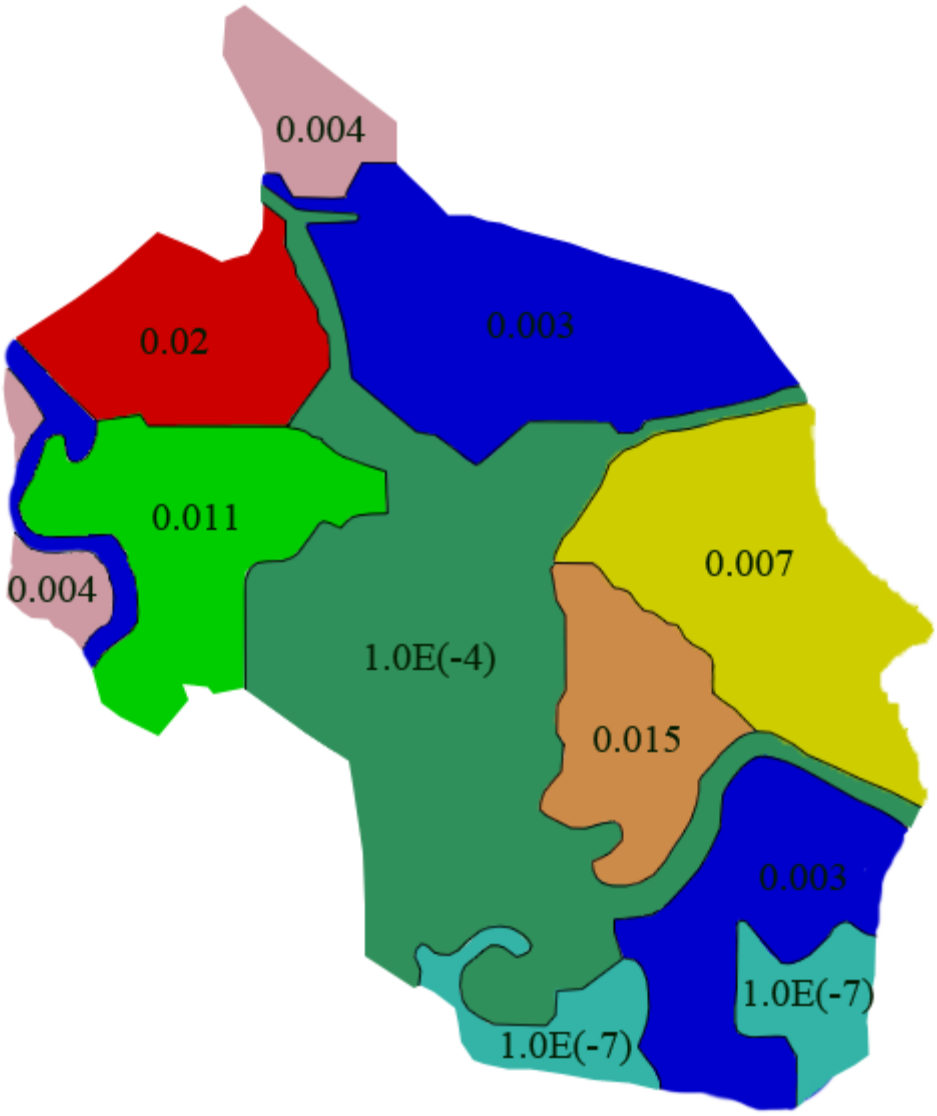

Figure S4

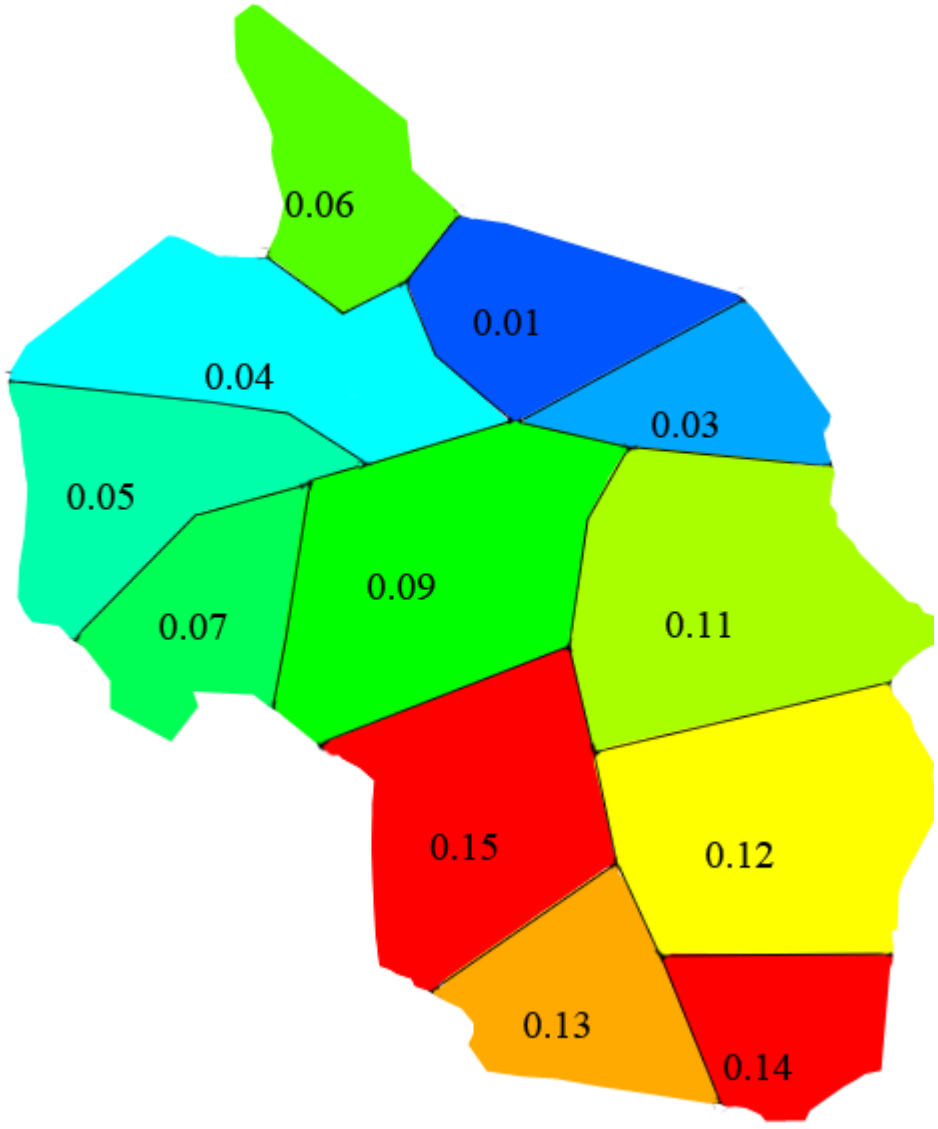

Figure S5