Probabilistic hydrological mODEL MARCS
(MARkov Chain System)

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The mODEL overview

**Aim:** to support the Advanced Frequency Analysis approach on a climate scale hydrological prediction of river runoff in changing environment

**Evolution steps:** 2015: Shevnina E., Doctor of Science Thesis, Russian State Hydrometeorological Institute, St. Petersburg, Russia

2017: Shevnina E. and Gaidukova E., doi:10.1007/978-3-319-57532-2_8


**Applications:** 2017: Shevnina et al., doi: 10.5194/hess-21-2559-2017


**Location:** https://github.com/ElenaShe000/MARCS

**Code:** mix of python, perl, netCDF, QGIS guided by Bash (Linux)

**Challenges:** the approach, the names of the model, the developing community

**Support:** Academy of Finland (TWASE project, contract 283101) and Ministry of Education and Science of the Russian Federation (01 2009 52622).
The Advanced Frequency Analysis: to estimate runoff extremes for a Civil Engineering

River runoff is a random value: the values of river runoff ($RF_{n\%}$) together with their exceedance probabilities ($n\%$).

River runoff extremes: the runoff value of low/high exceedance probability expressed as river streamflow discharge, $[m^3 s^{-1}]$. 

OBSERVATIONS >
The Advanced Frequency Analysis (AFA): to estimate runoff extremes in the changing climate

FA: Statistical estimators (moments, L-moments, CV, CS) from observed time series + Pearson System Distributions (Type III and IV)

Statistical moments estimates + Pearson System

AFA: FA + Probabilistic hydrological model

Simulated statistical moments estimates + Pearson System

FM: Rainfall Intensity Duration Curves + Physically-based hydrological models

Simulated time series of river discharges (seasonal, monthly or daily)

Observations + Climate Change Projections
The theoretical basis of the AFA approach


\[ a_n(t) \frac{d^n Q}{dt^n} + a_{n-1}(t) \frac{d^{n-1} Q}{dt^{n-1}} + \ldots + a_1(t) \frac{d Q}{dt} + a_0(t) Q = \]
\[ = b_n(t) \frac{d^n P}{dt^n} + b_{n-1}(t) \frac{d^{n-1} P}{dt^{n-1}} + \ldots + b_1(t) \frac{d P}{dt} + b_0(t) P \]

where, \( Q \) is model output (runoff), \( P \) is model input (precipitation), and \( a_n, b_n, \ldots, a_1, b_1 \) are parameters connected to catchment physiography and meteorology.


\[ \frac{dY}{dt} = f(Y, t) + g(Y, t, \xi(t)) + h(Y, t, \eta(t)) \]

where, \( Y \) model output, \( \xi(t) \) and \( \eta(t) \) are Gaussian noise signals with zero means and intensities \( D_\xi \) and \( D_\eta \); the signals are mutually correlated and their correlation function:

\[ K_{\xi\eta}(\tau) = E[\xi(t)\eta(t+\tau)] = D_{\xi\eta} \delta(\tau) \]

“Pearson Distributions” Model (System), Pearson: 1895.

\[ \frac{dp(Q)}{dQ} = \frac{Q-a}{b_0 + b_1 Q + b_2 Q^2} p(Q) \]

where: \( a, b_0, b_1, \) and \( b_2 \) are parameters of Pearson System.


\[ m_{n-1} b_n n + m_n [b_1 (n+1) - a] + m_{n+1} [(n+2) b_2 + 1] = 0 \]


First challenge of the MARCS model:

It is not easy to understand the approach, which used methods and terms mainly from two scientific disciplines: Hydrology and Statistical Radiophysics.

The probabilistic hydrological model MARCS (MARkov Chain System): the theoretical basis for the core version 0.2

Elena Shevnina and Andrey Silaev
The hydrological mODEL MARCS: the core version 0.2

\[ m_{n-1} b_0 n + m_n [b_1 (n+1) - a] + m_{n+1} [(n+2) b_2 + 1] = 0 \]

\[ a = \frac{G_{\bar{c}\bar{N}} + 2 \bar{N}}{2 \bar{c} + G_{\bar{c}}} \quad b_0 = \frac{G_{\bar{N}}}{2 \bar{c} + G_{\bar{c}}} \quad b_1 = \frac{2 G_{\bar{c}\bar{N}}}{2 \bar{c} + G_{\bar{c}}} \quad b_2 = -\frac{G_{\bar{c}}}{2 \bar{c} + G_{\bar{c}}} \]

\[-k(c - 0.5 k G_{\bar{c}}) m_k + k N m_{k-1} - k k - 0.5 k G_{\bar{c}\bar{N}} m_{k-1} + 0.5 k k - 1 G_{\bar{N}} m_{k-2} = 0 \]

\[ m_i \left| 2b_2 + 1 \right| a + b_1 = 0 \]
\[ \left| 3b_2 + 1 \right| m_2 + 2b_1 - a \left| m_1 + b_0 = 0 \right| \]
\[ \left| 4b_2 + 1 \right| m_3 + 3b_1 - a \left| m_2 + 2b_0 m_1 = 0 \right| \]

\[ G_{\bar{c}} = \frac{\left| 5b_2 + 1 \right| m_4 + 4b_1 - a \left| m_3 + 3b_0 m_2 = 0 \right|}{4b_2 + 1} \approx 0 \quad \left| 2b_2 + 1 \right| \approx 0 \]

Core Model: Linear filter with stochastic component

Regime type: quasi-stationary period

Distribution type: the Pearson Type III

\[ b_2 = -\frac{G_{\bar{c}}}{2 \bar{c} + G_{\bar{c}}} \approx 0 \quad \left| 2b_2 + 1 \right| \approx 1 \]

G\_\bar{c} \ll \bar{c}

\[ -a + b_1 = -m_1 \]
\[ b_0 + 2m_1 b_1 - a m_1 = -m_2 \]
\[ 2m_1 b_0 + 3m_2 b_1 - a m_2 = -m_3 \]

Statistical estimator for runoff: initial moments

\[ m_1 \Rightarrow \text{Norm, } Q = m_1 \]
\[ m_1, m_2 \Rightarrow \text{Coefficient of variation, } CV = \sqrt{\frac{m_2 - m_1^2}{m_1}} \]
\[ m_1, m_2, m_3 \Rightarrow \text{Coefficient of skewness, } CS = \frac{(m_3 - 3m_1 m_2 + 2m_1^3)}{(CV^3 m_1^3)} \]

Options: No changes on the variability of catchment physiography (VCP), No changes on the variability of precipitation (VP), No changes on the mutual variability of VCP and VP.

Climate change will increase potential hydropower production in six Arctic Council member countries based on probabilistic hydrological projections

Elena Shevnina et al.

Interactive discussion

RC1: 'Review of study into climate impacts on hydropower in Arctic Council countries', Anonymous Referee #1, 20 Oct 2018


RC3: 'Review comments', Anonymous Referee #3, 21 Oct 2018

Status: final response (author comments only)
The mODEL cross-validation procedure


$$D = \sup \left| \frac{P(h)^* - P(h)}{P(h)} \right|$$

$$\chi^2 = N \sum_{i=1}^{N} \frac{P(h)_i^* - P(h)_i}{P(h)_i}$$
The mODEL MARCS structure

- **Observations**
  - Statistical moments estimates (runoff) and mean values of precipitation and air temperature

- **Climate projections**
  - Projected statistics of precipitation and air temperature

- **Cross-validation**

- **Regional parameterization**
  - Projected statistical moments estimates (runoff) + Pearson System

- **Basic parameterization**

Civil Engineering and Risks Assessment

More details in Shevnina E. and Gaidukova E., 2017
doi:10.1007/978-3-319-57532-2_8
Aim: to define the regions, where the probability to get extreme floods is expected to change significantly. The future climate projections from the CMIP4/CMIP5 are used to generate the set of maps showing the risks of floods.

### Risks Assessment:

**the potential hydropower production in the Arctic Council countries**

<table>
<thead>
<tr>
<th><strong>Climatology</strong></th>
<th><strong>Hydrology</strong></th>
<th><strong>Hydropower</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Observations + projections for precipitation</td>
<td>MARCS: core 0.1: Projections of annual runoff with 10/50/90% exceedance probabilities</td>
<td>Simple relation of a potential hydropower production and annual runoff</td>
</tr>
<tr>
<td><em>FA:</em> 326 sites of national hydrological networks of the six countries: annual runoff time series with length 33-151 year; 11 projections of the future mean of precipitation classified into 2 types: &quot;wet&quot; and &quot;dry&quot;</td>
<td>AFA: 176 river catchments are aggregated on a country level (averaged)&gt; Exceedance Probability Curves (EPC) of annual runoff: Reference + 2 Projected EPC for the period of 2020-2050</td>
<td>The ratio of reference/future water resources to the potential hydropower production of 10/50/90% exceedance probabilities</td>
</tr>
</tbody>
</table>

Second challenge of the mODEL MARCS

The name of the model is MARCS (MARkov Chain System), and the model with the same name is already exist.

A grid of MARCS model atmospheres for late-type stars

I. Methods and general properties

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Solutions:

1. the name of the hydrological probabilistic mODEL may be specify as the MARCS:hydro;

2. the mODEL MARCS may be incorporated to the HOPS hydrological model (in-house developed in the FMI).
Third challenge of the mODEL MARCS

Developing community is too small and too busy...

Andrey M. Silaev
Head of department of Mathematical Economics, Professor

Alexander I. Krasikov
Professional programmer with 15+ experience in the development

Elena Shevnina
Researcher, group of Polar Meteorology and Climatology, Finnish Meteorological Institute, PhD in Hydrology
Thank you for your attention!

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Linux to catch...

The Schirmacher oasis, East Antarctica, 2018. photo by D. Emelyanov