Reply to Reviewer 1

This paper represents a nice advancement of mathematical modeling of oscillatory pumping test. I have the following comments on the paper:

1. Provide some more background on Weber transform and its application in hydrology, including, but not limited to its advantages and disadvantages.

Response: Thanks for the comment. We add the following sentences in the revised manuscript.

"The Weber transform, defined in Eq. (B.1) of the supporting material, may be considered as a Hankel transform with a more general kernel function. It can be applied to diffusion-type problems with a radial-symmetric region from a finite distance to infinity. For groundwater flow problems, it can be used to develop the analytical solution for the flow equation with a boundary condition of Dirichlet, Neumann, or Robin type specified at the rim of a finite-radius well (e.g., Lin and Yeh, 2017; Povstenko, 2015)." (Pages 10 - 11, lines 218 - 223)

2. A great portion of the mathematical details may be moved into supplementary material, so the authors can concentrate on discussing the hydrogeological features of the problem.

Response: Thanks for the suggestion. The derivation of the present solution has been moved to the supplementary material, and then the Methodology section is shortened. Please refer to the revised manuscript as attached.

3. The mathematical modeling appears to be robust. The English is good too.

Response: Thanks.

4. Some associated literature using the similar approaches can be seen in Dr. Xiuyu Liang's recent publications (only one of them is cited here).

Response: The reference of Liang et al. (2018) is added in line 149, page 8 of the revised manuscript, and the citations there are then changed to (Latinopoulos, 1985; Liang et al., 2017; 2018).

The paper can be published after moderate revision.

Response: Many thanks.

References:

Latinopoulos, P.: Analytical solutions for periodic well recharge in rectangular aquifers with 3rd-kind boundary-conditions, J. Hydrol., 77(1–4), 293–306, 1985.

Liang, X., Zhan, H., Zhang, Y.-K., and Liu, J.: On the coupled unsaturated-saturated flow process induced by vertical, horizontal, and slant wells in unconfined aquifers, Hydrol. Earth Syst. Sci., 21, 1251–1262, 2017.

Liang, X., Zhan, H., Zhang, Y.-K., Liu, J.: Underdamped slug tests with unsaturated-saturated flows by considering effects of wellbore skins, Hydrol. Process., 32, 968 – 980, 2018.

Lin, Y.-C., Yeh, H.-D.: A lagging model for describing drawdown induced by a constant-rate pumping in a leaky confined aquifer, Water Resour. Res., 53, 8500 – 8511, 2017.

Povstenko, Y.: Linear fractional diffusion-wave equation for scientists and engineers. New York, Birkhäser, 2015.