Technical note: Analytical sensitivity analysis and uncertainty estimation of a two-component hydrograph separation method which uses conductivity as a tracer

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Abstract. The conductivity—two-component hydrograph separation method with conductivity as a tracer is favored by hydrologists owing to its low cost and easy application. It is cheap and easy to operate and is favored by hydrologists. This paper analyzes the sensitivity of the baseflow index (BFI, the long-term ratio of baseflow to streamflow) calculated using this method to errors or uncertainties of the two parameters (BFc, the conductivity of baseflow, and ROc, the conductivity of surface runoff) and the two variables (yũ, the specific streamflow, and SCũ, the specific conductivity of streamflow, where k is the time step), and then estimates the uncertainty of BFI. The analysis shows that when the time series is longer than 365 days, the random measurement errors of yũ or SCũ will cancel each other, and their influence on BFI can be neglected. Dimensionless sensitivity indices (the ratio of the relative error of BFI to the relative error of BFc or ROc) can well express the propagation of errors or uncertainties of BFc or ROc into BFI. Based on the sensitivity analysis, an uncertainty estimation method of BFI is derived on the basis of the sensitivity analysis. Representative sensitivity indices (the ratio of the relative error of BFI to that of BFc or ROc) and BFI uncertainties are determined by applying the resulting equations to 24 watersheds in the United States. These dimensionless sensitivity indices can well express the propagation of errors or uncertainties of BFc or ROc into BFI. The results indicate that BFI is more sensitive to BFc, and the conductivity two-component hydrograph separation method may be more suitable for the long time series in a small watershed. After considering the mutual offset of the measurement errors of conductivity and streamflow is considered, the uncertainty of BFI is reduced by half.

1 Introduction

Hydrograph separation (also called baseflow separation) aims to identify the proportion of water in different runoff pathways in a basin, the export flow of a basin, which helps in identifying the conversion relationship between groundwater and surface water. In addition, it is a necessary condition for optimal allocation of water resources (Cartwright et al., 2014; Miller et al., 2014; Costelloe et al., 2015). Some researchers indicated that tracer-based hydrograph separation methods yield the most realistic results. A tracer isotope (tracer) hydrograph separation method is considered to be the most effective separation method, because they are the most physically based methods which can reflect the actual characteristics of a basin (Miller et al., 2014; Mei and Anagnostou, 2015; Zhang et al., 2017). Many hydrologists have suggested that electrical conductivity can be used as a tracer to perform hydrograph separation (Stewart et al., 2007; Munyaneza et al., 2012; Cartwright et al., 2014; Lott and Stewart, 2016; Okello et al., 2018). The measurement of conductivity is a suitable tracer because its measurement is simple and inexpensive, and it has a distinct applicability in a long series of hydrograph separation (Okello et al., 2018).
The conductivity two-component hydrograph separation method, with conductivity as a tracer (also called conductivity mass balance method (CMB) (Stewart et al. 2007)), uses conductivity as a tracer to calculate baseflow through a two-component mass balance equation. The general equation is shown in Eq. (1), which is based on the following assumptions:

a) Contributions from end-members other than baseflow and surface runoff are negligible.

b) The specific conductance of runoff and baseflow are constant (or vary in a known manner) over the period of record.

c) Instream processes (such as evaporation) do not change the real uncertainty estimation of BFI. There is uncertainty in the determination of the parameters (BF$_C$, RO$_C$) of the conductivity two-component hydrograph separation method involves some uncertainties (Miller et al., 2014; Okello et al., 2018). Therefore, sensitivity analysis of parameters and the uncertainties of the separation results are helpful to improve the accuracy of hydrograph separation. Most of the existing parameter sensitivity analysis methods use experimental methods, sensitivity analysis method, which that usually substitute the fluctuation varying values of a certain parameter into the separation model, and then analyze the sensitivity of the parameters by comparing the range of the separation results produced by these fluctuation varying parameter values (Eckhardt, 2005; Miller et al., 2014; Okello et al., 2018). Eckhardt (2012) indicated that “An empirical sensitivity analysis is only a makeshift if an analytical sensitivity analysis, that is an analytical calculation of the error propagation through the model, is not feasible”. An empirical sensitivity analysis is only an analytical calculation of the error propagation through the model, is not feasible.”

Eckhardt (2012) derived the sensitivity indices of the equation parameters by the partial derivative of a two-parameter recursive digital baseflow separation filter equation. Until now, however, the parameters’ sensitivity indices of the conductivity two-component hydrograph separation equation have not been derived.

At present, the uncertainty of the separation results of the conductivity two-component hydrograph separation method is mainly estimated using an uncertainty transfer equation based on the uncertainty of BF$_C$, RO$_C$, and SO$_C$ (Genereux, 1998; Miller et al., 2014). See Sect. 3.1 for details. In this uncertainty estimation method, can only estimate the uncertainty of the baseflow ratio ($f_{Bf}$, the ratio of baseflow to streamflow in a single calculation process) is estimated, and then use the average uncertainty of multiple calculation processes is then used to estimate the uncertainty of the baseflow index (BFI, the long-term ratio of baseflow to total streamflow). This uncertainty estimation method can neither directly estimate the uncertainty of BFI nor consider the randomness and mutual offset of conductivity measurement errors, and thus, it does not provide accurate estimates of BFI uncertainty. The uncertainty estimation of BFI is not appropriate enough.
The main objectives of this study are as follows: (i) analyze the sensitivity of long-term series of baseflow separation results (BFI) to parameters and variables of the conductivity two-component hydrograph separation equation (Sect. 2); (ii) derive the uncertainty of BFI (Sect. 3). The purpose of this paper is to derive the parameters' sensitivity indices of the conductivity two-component hydrograph separation equation by calculating the partial derivative of Eq. (1) (Sect. 2), and further derive the direct estimation method of BFI uncertainty (Sect. 3). The derived solutions were applied to 24 basins in the United States, and the parameters' sensitivity indices and BFI uncertainty characteristics were analyzed (Sect. 4).

2 Analytical Sensitivity analysis

2.1 Parameters BF<sub>C</sub> and RO<sub>C</sub>

In order to calculate the sensitivity indices of the parameters, the partial derivatives of \( b_i \) in Eq. (1) with respect to BF<sub>C</sub> and RO<sub>C</sub> are required respectively (for the derivation process is expressed as, see Appendix Eq. (A1) and (A2)):

\[
\frac{\partial b_i}{\partial BF_C} = -y_k \frac{y_i C_i - y_i RO_C}{(y_i BF_C - y_i RO_C)^2}
\]

\[
\frac{\partial b_i}{\partial RO_C} = y_i \frac{y_i BF_C - y_i RO_C}{(y_i BF_C - y_i RO_C)^2}
\]

For the convenience of comparison, the baseflow index (BFI) is selected as the baseflow separation result for long time series to analyze the influence of parameter uncertainty on BFI.

\[
BFI = \frac{\sum_{k=1}^{n} b_k}{\sum_{k=1}^{n} y_k}
\]

where \( b \) denotes the total baseflow and \( y \) denote the total baseflow and the total streamflow, respectively, over the whole available streamflow sequences, and \( n \) is the number of available streamflow data.

Then, the partial derivatives of BFI to BF<sub>C</sub> and RO<sub>C</sub> should be calculated (for the derivation process, see Appendix is presented in Eq. (A3) and (A4)):

\[
\frac{\partial BFI}{\partial BF_C} = \frac{y_i RO_C - \sum_{k=1}^{n} y_i C_i - y_i RO_C}{y_i (BF_C - RO_C)^2}
\]

\[
\frac{\partial BFI}{\partial RO_C} = \frac{\sum_{k=1}^{n} y_i BF_C - y_i RO_C - y_i BF_C}{y_i (BF_C - RO_C)^2}
\]

It can be seen from the definition of the partial derivative suggests that the influence of the errors of the parameters (\( \Delta BF_C \) and \( \Delta RO_C \)) in Eq. (1) on the BFI can be expressed by the product of the errors and its partial derivatives. Then the errors of BFI caused by small errors of BF<sub>C</sub> and RO<sub>C</sub> can be approximated by the BFI errors caused by tiny errors of BF<sub>C</sub> and RO<sub>C</sub> can be expressed as:

\[
\Delta BF_C \rightarrow \Delta BF_C = \frac{\partial BFI}{\partial BF_C} \Delta BF_C = \frac{\sum_{k=1}^{n} y_i C_i - y_i RO_C}{y_i (BF_C - RO_C)^2} \Delta BF_C
\]

\[
\Delta RO_C \rightarrow \Delta RO_C = \frac{\partial BFI}{\partial RO_C} \Delta RO_C = \frac{\sum_{k=1}^{n} y_i BF_C - y_i RO_C - y_i BF_C}{y_i (BF_C - RO_C)^2} \Delta RO_C
\]

The dimensionless sensitivity indices (S) can be obtained by comparing the relative error of BFI caused by the smallest errors of BF<sub>C</sub> and RO<sub>C</sub> with that of BF<sub>C</sub> and RO<sub>C</sub> (see Appendix Eq. (B1), (B2)):

\[
S(BFI|BF<sub>C</sub>, RO<sub>C</sub> = \frac{\Delta BF_C}{BF_C} \frac{\Delta BF_C}{BF_C} = BF_C (BF_C - RO_C)^2
\]

\[
S(BFI|RO<sub>C</sub>, BF<sub>C</sub> = \frac{\Delta RO_C}{RO_C} \frac{\Delta RO_C}{RO_C} = RO_C (BF_C - RO_C)^2
\]
where \( S(\text{BFI}|\text{BF}, \text{BEL}/\text{BE}_c) \) represent the dimensionless sensitivity index of BFI (output) with \( \text{BF}_c \) (uncertain input), and \( S(\text{BFI}|\text{RO}_c, \text{BEL}/\text{BE}_c) \) with \( \text{RO}_c \).

The dimensionless sensitivity index is also called the “elasticity index,” and it reflects the proportional relationship between the relative error of BFI and the relative error of parameters (e.g., if \( S(\text{BFI}|\text{BF}, \text{BEL}/\text{BE}_c) = 1.5 \), and the relative error of \( \text{BF}_c \) is 5%, then the relative error of BFI should be 1.5 times 5% \( = 7.5\% \)). After determining the specific values of \( \text{BF}_c, \text{RO}_c, \text{BFI}, \gamma_k \) and \( \text{SC}_Q \), the sensitivity indices \( S(\text{BFI}|\text{BF}, \text{BEL}/\text{BE}_c) \) and \( S(\text{BFI}|\text{RO}_c, \text{BEL}/\text{BE}_c) \) can be calculated and compared.

### 2.2 Variables \( y_k \) and \( \text{SC}_Q \)

In addition to the two parameters, there are two variables (\( \text{SC}_Q \) and \( y_k \)) in Eq. (1). This section will analyze describes the sensitivity analysis of BFI to these two variables. Similar to Sect. 2.1, the partial derivatives of \( b_i \) in Eq. (1) to \( \text{SC}_Q \) and \( y_k \) are obtained (see Appendix Eq. (A5), (A6)), and the partial derivatives of BFI to \( \text{SC}_Q \) and \( y_k \) are further obtained (see Appendix Eq. (17), (AR)).

\[
\frac{\partial \text{BFI}}{\partial \text{SC}_Q} = \frac{1}{\text{BF}_c - \text{RO}_c} \\
\frac{\partial \text{BFI}}{\partial y_k} = \frac{\sum_{i=1}^{n}(\text{SC}_Q - \text{RO}_c)(\text{BF}_c - \text{RO}_c)}{\text{BF}_c - \text{RO}_c} \\
\text{(12)}
\]

According to previous studies (Munyaneza et al., 2012; Cartwright et al., 2014; Miller et al., 2014; Okello et al., 2018) and this study (Table 1), the difference between \( \text{BF}_c \) and \( \text{RO}_c \) is often greater than 100 \( \mu \text{m} \). Therefore, \( \partial \text{BFI}/\partial \text{SC}_Q \) is usually less than 0.01 cm/\( \mu \text{m} \). Appendix C shows that the value of \( \partial \text{BFI}/\partial y_k \) is usually far less than 1 cm/\( \mu \text{m} \).

Tiny Small errors in \( \text{SC}_Q \) and \( y_k \) cause errors in BFI.

\[
\Delta \text{SC}_Q = \Delta \text{BFI} = \frac{\partial \text{BFI}}{\partial \text{SC}_Q} \Delta \text{SC}_Q = \frac{\Delta \text{BFI}}{\text{BFI}_c - \text{RO}_c} \\
\text{(13)}
\]

\[
\Delta y_k = \frac{\partial \text{BFI}}{\partial y_k} \Delta y_k = \frac{\sum_{i=1}^{n}(\text{SC}_Q - \text{RO}_c)(\text{BF}_c - \text{RO}_c)}{\text{BF}_c - \text{RO}_c} \Delta y_k \\
\text{(14)}
\]

The errors of BFI caused by \( \text{SC}_Q \) and \( y_k \) are summed up to obtain the error of BFI caused by \( \sum_{i=1}^{n} \text{SC}_Q \) and \( \sum_{i=1}^{n} y_k \) in the whole time series:

\[
\Delta \text{BFI} = \sum_{i=1}^{n} \Delta \text{SC}_Q = \sum_{i=1}^{n} \frac{\Delta \text{BFI}}{\text{BFI}_c - \text{RO}_c} \\
\text{(15)}
\]

\[
\Delta y_k = \sum_{i=1}^{n} \Delta y_k = \sum_{i=1}^{n} \frac{\sum_{i=1}^{n}(\text{SC}_Q - \text{RO}_c)(\text{BF}_c - \text{RO}_c)}{\text{BF}_c - \text{RO}_c} \Delta y_k \\
\text{(16)}
\]

Wagner et al. (2006) reported that the tiny errors in \( \text{SC}_Q \) and \( y_k \) are mainly composed of random analysis errors. Random errors mostly follow a normal distribution or a uniform distribution. The magnitude and direction of the random errors are usually not fixed. As the number of measurements increases, the positive and negative errors can compensate each other, and the average value of the errors will gradually tend to zero (Huang and Chen, 2011).

The uncertainty of the instruments is usually less than \( 5\% \) for \( \text{SC}_Q \), less than \( \leq 100 \mu \text{m} \) and less than \( \leq 3\% \) for \( \text{BFI} \). According to Hamilton et al. (2012) streamflow data from USGS are often assumed by analysts to be accurate and precise within \( \pm 5\% \) at the 95% confidence interval. The measurement uncertainty of streamflow is usually \( \leq 3\% \) (Zhang, 2005). In this paper study, the error ranges of \( \text{SC}_Q \) and \( y_k \) are all considered to be \( \pm 5\% \) and \( \pm 3\% \), respectively. The errors in \( \text{SC}_Q \) and \( y_k \) mainly comprise random analysis errors which mostly follow a normal...
distribution or a uniform distribution (Huang and Chen, 2011). Considering the mutual offset of random errors, when the time series \( n \) is sufficiently long-enough, \( \sum_{k=1}^{n} \Delta C_{ik} \) in Eq. (15) and \( \sum_{k=1}^{n} \Delta y_{ik} \) in Eq. (16) will approach zero.

The analysis of \( \sum_{k=1}^{n} \Delta C_{ik} \) and \( \sum_{k=1}^{n} \Delta y_{ik} \) under different time series \( n \) and different error distributions (normal distribution or uniform distribution) of a surface water station (USGS site number 0297100) showed that the random errors of daily average conductivity and streamflow have a negligible effect on BFI when the time series is greater than 365 days (See Supplement S1 for detail). Therefore, when \( n \) is large enough, the error of BFI caused by the errors of \( Q_{ik} \) and \( y_{ik} \) can be neglected.

To verify this phenomenon, the study collected the daily average conductivity and daily average streamflow of the surface water station with the USGS site number 0297100 (Table 1) from 2001 to 2010 (2979 days in total). Then, office Excel was used to generate 10 sets (2979 per set) of random numbers between -0.05 and 0.05 that obey normal distribution and uniform distribution, respectively, to simulate the errors (%) of the daily average conductivity. And 10 sets (2979 per set) of random numbers obeying normal distribution and uniform distribution between -0.03 and 0.03, respectively, were used to simulate the errors (%) of the daily average streamflow. Finally, according to different time series \( n \) (e.g. 30, 60, 90, 120, 150, 180, 210, 240, 270, 300, 365, 730, 1095, ..., 2979, days) sum the errors value \( \sum_{k=1}^{n} \Delta C_{ik} \) and \( \sum_{k=1}^{n} \Delta y_{ik} \) and analyze the trend of the average error (%) with \( n \). The trend of the average error (%) of conductivity with \( n \) is shown in Fig. 1. The average errors of the uniform distribution (Fig. 1(a)) and the normal distribution (Fig. 1(b)) are all gradually approach zero with the increase of the time series \( n \), and the uniform distribution converges faster than the normal distribution. The average errors of the two distributions are between -2% and 2%, and the absolute value of the average errors are less than 0.49% when \( n \) is greater than 365.

Similar to the conductivity, the trend of the average error (%) of the streamflow with \( n \) is shown in Fig. 2. The average errors of the uniform distribution (Fig. 2(a)) and the normal distribution (Fig. 2(b)) all gradually approach zero as the time series \( n \) increases, and the uniform distribution converges faster than the normal distribution. The average errors of different \( n \) under the two distributions are between -2% and 2%, and the absolute value of the average errors are less than 0.67% when \( n \) is greater than 365.

From the above analysis, when the time series \( n \) is greater than 365 days (1 year), \( \Delta_{\text{BFI}} \) will be less than 0.01 (0.01 times 0.49%), and \( \Delta_{\text{BFI}} \) will be much less than 0.1% (1 times 0.76%). Therefore, the random errors of daily average conductivity and streamflow have a negligible effect on BFI.

**Figure 1.** Average conductivity error (%) with different distributions along the time series \( n \), (a) uniform distribution, (b) normal distribution.

**Figure 2.** Average streamflow error (%) with different distributions along the time series \( n \), (a) uniform distribution, (b) normal distribution.

### 3 Uncertainty estimation

#### 3.1 Previous attempts

According to previous studies, in the case where a parameter variable \( g \) is calculated as a function of several factors \( x_1, x_2, x_3, ..., x_n \), the transfer equation (also known as Gaussian error propagation) between the uncertainty of the independent factors and the uncertainty of \( g \) is (Taylor, 1982; Kline, 1985; Gourcuff, 1998):
\[ W_f = \sqrt{\left( \frac{8g}{a_{x1}} W_{x1} \right)^2 + \left( \frac{8g}{a_{x2}} W_{x2} \right)^2 + \cdots + \left( \frac{8g}{a_{x_n}} W_{x_n} \right)^2} \]  

(17)

where \( W_f, W_{x1}, W_{x2}, \) and \( W_{x_n} \) are the same type of uncertainty values (e.g. all average errors or all standard deviations) for \( g, x_1, x_2, \) and \( x_n \), respectively. A more detailed description of this equation can be found in Taylor (1982), Kline (1985), and Ernest (2005).

According to Genereux (1998), "While any set of consistent uncertainty (W) values may be propagated using Gaussian error propagation, using standard deviations multiplied by \( t \) values from the Student's \( t \) distribution (each \( t \) for the same confidence level, such as 95\%) has the advantage of providing a clear meaning (tied to a confidence interval) for the computed uncertainty Would correspond to, for example, 95\% confidence limits on BFI".

Based on the above principle, Genereux (1998) substituted Eq. (18) into Eq. (17) to derive the uncertainty estimation equation (Eq. (19)) of the two-component mass balance baseflow separation method:

\[
\begin{align*}
  f_{bf} &= \frac{\Delta C_{BF_c} \cdot \Delta C_{RO_c}}{\Delta C_{BF_c} \cdot \Delta C_{RO_c}} \\
  W_{bf} &= \sqrt{\left( \frac{f_{bf}}{\Delta C_{BF_c} \cdot \Delta C_{RO_c}} W_{BF_c \cdot RO_c} \right)^2 + \left( \frac{1-f_{bf}}{\Delta C_{BF_c} \cdot \Delta C_{RO_c}} W_{BF_c \cdot RO_c} \right)^2} 
\end{align*}
\]  

(18)

(19)

where \( f_{bf} \) is the ratio of baseflow to streamflow in a single calculation process, \( W_{bf} \) is the uncertainty in \( f_{bf} \) at the 95\% confidence interval, \( W_{BF_c \cdot RO_c} \) is the standard deviation of the \( BF_c \cdot RO_c \) highest 1\% of measured conductivity multiplied by the \( t \)-value (\( \alpha = 0.05; \) two-tail) from the Student’s distribution, \( W_{RO_c} \) is the standard deviation of the \( RO_c \) lowest 1\% of measured conductivity multiplied by the \( t \)-value (\( \alpha = 0.05; \) two-tail). (Miller et al., 2014) (Miller et al., 2014).

Equation (19) can better estimate the uncertainty of \( f_{bf} \) within a single calculation step can be obtained using Eq. (19).

Hydrologists usually approximate estimate the uncertainty of BFI approximately by averaging the uncertainty of all steps (Genereux, 1998; Miller et al., 2014). However, this method does not consider the mutual offset of the conductivity measurement errors, and cannot accurately reflect the uncertainty of BFI. In this paper, study, based on the parameter sensitivity analysis, the uncertainty estimation equation of BFI is derived on the basis of the parameter sensitivity analysis. See the next section for details.

3.2 Uncertainty estimation of BFI

BFI is a function of \( BF_c, RO_c, SCQ_{ab}, \) and \( y_k \). In addition, and the uncertainties of \( BF_c, RO_c, SCQ_{ab}, \) and \( y_k \) are independent of each other. As explained earlier (Sect. 2.2) Sect. 2.2 has explained that the random errors of daily average conductivity and streamflow have a negligible effect on BFI when the time series (n) is greater than 365 days (1 year). Therefore, the uncertainty of BFI can be expressed as:

\[
W_{BFI} = \sqrt{\left( \frac{\partial BFI}{\partial BF_c} W_{BF_c} \right)^2 + \left( \frac{\partial BFI}{\partial RO_c} W_{RO_c} \right)^2} \]  

(20)

where (see Eq. 5 and Eq. 9; Eq. 6 and Eq. 10)

\[
\begin{align*}
  \frac{\partial BFI}{\partial BF_c} &= S (BFI | BF_c, BF_c, BF_c) \\
  \frac{\partial BFI}{\partial RO_c} &= S (BFI | RO_c, BFI, BF_c) 
\end{align*}
\]  

(21)

(22)

Then, the Eq. (20) can be rewritten as:
where $W_{BFI}$, $W_{BFc}$, and $W_{ROC}$ are the same type of uncertainty values for BFI, BF$_C$, and RO$_C$, respectively. For instance, $W_{BFI}$ is the uncertainty in BFI at the 95% confidence interval, $W_{BFc}$ is the standard deviation of the highest 1% of measured conductivity BF$_C$ multiplied by the t-value ($α=0.05$; two-tail) from the Student’s distribution, and $W_{ROC}$ is the standard deviation of the lowest 1% of measured conductivity multiplied by the t-value ($α=0.05$; two-tail) from the Student’s distribution.

### 4 Application

#### 4.1 Data and processing

The above sensitivity analysis and uncertainty estimation methods were applied to 24 catchments in the United States (Table 1). All basins used in this study are perennial streams, with drainage areas ranging from 10 km$^2$ to 1258481 km$^2$. Each gage has about at least 1 year of continuous streamflow and conductivity at the same period of records. All streamflow and conductivity data are daily average values retrieved from the United States Geological Survey’s (USGS) National Water Information System (NWIS) website, http://waterdata.usgs.gov/nwis.

The daily baseflow of each basin was calculated using Eq. (1). The 99th percentile of the conductivity of each year was used as BF$_C$, and linear variation of baseflow conductivity between years was assumed. The 1st percentile 99th percentile of the conductivity of the whole series of streamflow in each basin was used as the BF$_C$ and the 1st percentile as the RO$_C$. The total baseflow $b$, the total streamflow $y$ and the baseflow index BFI of each watershed were then calculated. According to the results of the hydrograph separation, the parameter sensitivity indices of BFI for mean BF$_C$ ($S(BFI|BF_C,BF/BF_C)$) and RO$_C$ ($S(BFI|RO_C,BF/BF_C)$) were calculated using Eq. (9) and Eq. (10), respectively.

Finally, the uncertainty of $f_{bf}$ in each step was calculated using Eq. (19) and averaged to obtain the mean $W_{bf}$ for each basin. The uncertainty ($W_{bf}$) of BFI was directly calculated using Eq. (23), and then the values of mean $W_{bf}$ and $W_{BFc}$ were compared. For each basin, $W_{BFc}$ is the standard deviation of the BF$_C$ of the whole series highest 1% of measured conductivity multiplied by the t-value ($α=0.05$; two-tail) from the Student’s distribution, $W_{ROC}$ is the standard deviation of the lowest 1% of measured conductivity multiplied by the t-value ($α=0.05$; two-tail) from the Student’s distribution, and $W_{BFc}$ is the analytical error in the conductivity (5%) multiplied by the t-value ($α=0.05$; two-tail).

#### 4.2 Results and discussion

The calculation results are shown in Table 1. The average baseflow index of the 24 watersheds is 0.29, the average sensitivity index of BFI for mean BF$_C$ ($S(BFI|BF_C,BF/BF_C)$) is -1.4, and the average sensitivity index of BFI for RO$_C$ ($S(BFI|RO_C,BF/BF_C)$) is -0.9. The negative sensitivity indices indicate a negative correlation between BFI and BF$_C$, RO$_C$. The absolute value of the sensitivity index for BF$_C$ is generally greater than that for RO$_C$, indicating that BFI is more affected by BF$_C$ (for example, if there are 10% uncertainties in both BF$_C$ and RO$_C$, then BF$_C$ leads to -1.35 times 10% of uncertainty in BFI (-1.40% 14.0%), while RO$_C$ leads to -0.38 times 10% (4.0-8.9%)). Therefore, the determination of BF$_C$ requires more caution, and any small error may lead to greater uncertainty in BFI. Miller et al. (2014) indicated that anthropogenic activities over long periods of time, or year to year changes in the elevation of the water table may...
result in temporal changes by changing in the BFc. They recommended taking different BFc values per year based on the conductivity values during low flow periods to avoid the effects of temporal fluctuations in BFc. Table 1 shows a decreasing trend with the increase of time series (n) (Fig. 14(a)) and has an increasing trend with the increasing watershed area (Fig. 14(b)), with correlation coefficients of 0.4698 and 0.4463, respectively. Although the correlations are not obvious, they have important guiding significance. In the United States, footnotes “a” in the “Area” column indicates that the values are estimated based on data from adjacent sites.

Table 1. Basic information, parameter sensitivity analysis, and uncertainty estimation results for 24 basins in the United States. Footnote “a” in the “Area” column indicates that the values are estimated based on data from adjacent sites.

The sensitivity index of BFI for BFc shows a decreasing trend with the increase of time series (n) (Fig. 14(a)) and has an increasing trend with the increasing watershed area (Fig. 14(b)), with correlation coefficients of 0.4698 and 0.4463, respectively. Although the correlations are not obvious, they have important guiding significance. In the United States, footnotes “a” in the “Area” column indicates that the values are estimated based on data from adjacent sites.

The sensitivity index of BFI for ROc did not change significantly with the increase of time series and watershed area (Fig. 14(c), Fig. 14(d)). During rainfall, the water level of the stream rises sharply, the subsurface flow is suppressed, and the streamflow is almost entirely from the rainfall runoff. At this time, the conductivity of the stream became similar to the conductivity of the local rainfall (Stewart et al., 2007). The electrical conductivity of regional rainfall varies slightly, usually at a fixed value, and it has no significant relationship with the basin area and year (Munyanza et al., 2012). Therefore, the temporal and spatial variation characteristics of BFI for ROc are not obvious.

Figure 13. Scatter plots of sensitivity indices vs. time series (n) and drainage area of the 24 US basins. The watershed area uses a logarithmic axis, while the others are linear-normal axes.

Generex’s method (Eq. 19) estimates the average uncertainty of BFI in the 24 basins (average of mMean Wuf) to be 0.1220, whereas the average uncertainty of BFI (average of Wuf) calculated directly using the proposed methodology (Eq. 23) is 0.0611 (Table 1). Mean Wuf in each basin is generally larger than Wuf (Wuf is about 0.51 times of mMean Wuf), and there is a significant linear correlation (Fig. 24). This shows that the two methods have the same volatility characteristics for BFI uncertainty estimation results, but Generex’s method (Eq. 19) often overestimates the uncertainty of BFI. This also means that when the time series is longer than 365 days, the measurement errors of conductivity and streamflow will cancel each other and thus reduce the uncertainty of BFI (about half of the original).

Figure 24. Scatter plot of uncertainty in BFI (Wuf) and mean uncertainty in fuf (mMean Wuf).

5 Conclusions

This study analyzed the sensitivity of BFI calculated using the conductivity two-component hydrograph separation method to errors or uncertainties of parameters BFc and ROc and variables yk and SCc. In addition, the uncertainty of BFI was calculated. The equations derived in this study (Equation Eq. (9) and Eq. (10)) can well calculate the sensitivity indices of BFI for BFc and ROc. For time series longer than 365 days, the measurement errors of conductivity and streamflow exhibited an obvious mutual offset effect, and their influence on BFI could be neglected. Considering the mutual offset, the uncertainty of BFI would be reduced to half. From this viewpoint, Eq. (23) can estimate the uncertainty of BFI when the time series is longer than 365 days, taking into account the mutual cancellation of conductivity measurement errors. The application of the method to
Applications in 24 basins in the United States showed that BFI is more sensitive to BF<sub>C</sub>, and future studies should dedicate more effort to determining the value of BF<sub>C</sub>. In addition, the conductivity two-component hydrograph separation method may be more suitable for the long-term series in small watersheds. When the time series is greater than 365 days, the measurement errors of conductivity and streamflow have obvious mutual offset, and its influence on BFI can be neglected. After considering the mutual offset of random errors, the uncertainty of BFI will be reduced to half. Systematic errors in specific conductance and streamflow as well as temporal and spatial variations in baseflow conductivity may be the main sources of BFI uncertainty. Better rating curves are probably more important than better loggers, and more work on understanding the specific conductance of baseflow is more important than understanding that of surface runoff.

The above conclusions are only from the average of the 24 basins in the United States, and further research is needed in other countries or in more watersheds. This study research in this paper only focused on the two-component hydrograph separation method with conductivity as a tracer, but the parameter sensitivity analysis and uncertainty analysis methods involving other tracers are very similar to this paper, and therefore, it is easy to derive similar equations can be easily derived by referring to the findings of this study.

### Appendix A

Calculation of the partial derivatives

\[
\frac{\partial \Delta f}{\partial \Delta f} = \frac{\partial}{\partial \Delta f} \left( \frac{y_k SCQ_{A,k} - RO_k}{y_k BF_{C,k} - RO_k} \right) = \frac{y_k SCQ_{A,k} - RO_k}{y_k BF_{C,k} - RO_k^2}
\]

\[
\frac{\partial \Delta f}{\partial \Delta f} = \frac{ \frac{\partial}{\partial \Delta f} y_k SCQ_{A,k} - RO_k}{\frac{\partial}{\partial \Delta f} y_k BF_{C,k} - RO_k} = \frac{y_k SCQ_{A,k} - RO_k}{y_k BF_{C,k} - RO_k}
\]

\[
\frac{\partial \Delta f}{\partial \Delta f} = \frac{\partial}{\partial \Delta f} \left( \frac{y_k SCQ_{A,k} - RO_k}{y_k BF_{C,k} - RO_k} \right) = \frac{y_k SCQ_{A,k} - RO_k}{y_k BF_{C,k} - RO_k^2}
\]

\[
\frac{\partial \Delta f}{\partial \Delta f} = \frac{\partial}{\partial \Delta f} \left( \frac{y_k SCQ_{A,k} - RO_k}{y_k BF_{C,k} - RO_k} \right) = \frac{y_k SCQ_{A,k} - RO_k}{y_k BF_{C,k} - RO_k^2}
\]

\[
\frac{\partial \Delta f}{\partial \Delta f} = \frac{\partial}{\partial \Delta f} \left( \frac{y_k SCQ_{A,k} - RO_k}{y_k BF_{C,k} - RO_k} \right) = \frac{y_k SCQ_{A,k} - RO_k}{y_k BF_{C,k} - RO_k^2}
\]

\[
\frac{\partial \Delta f}{\partial \Delta f} = \frac{\partial}{\partial \Delta f} \left( \frac{y_k SCQ_{A,k} - RO_k}{y_k BF_{C,k} - RO_k} \right) = \frac{y_k SCQ_{A,k} - RO_k}{y_k BF_{C,k} - RO_k^2}
\]

\[
\frac{\partial \Delta f}{\partial \Delta f} = \frac{\partial}{\partial \Delta f} \left( \frac{y_k SCQ_{A,k} - RO_k}{y_k BF_{C,k} - RO_k} \right) = \frac{y_k SCQ_{A,k} - RO_k}{y_k BF_{C,k} - RO_k^2}
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\[
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\]

\[
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\]

\[
\frac{\partial \Delta f}{\partial \Delta f} = \frac{\partial}{\partial \Delta f} \left( \frac{y_k SCQ_{A,k} - RO_k}{y_k BF_{C,k} - RO_k} \right) = \frac{y_k SCQ_{A,k} - RO_k}{y_k BF_{C,k} - RO_k^2}
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\[
\frac{\partial \Delta f}{\partial \Delta f} = \frac{\partial}{\partial \Delta f} \left( \frac{y_k SCQ_{A,k} - RO_k}{y_k BF_{C,k} - RO_k} \right) = \frac{y_k SCQ_{A,k} - RO_k}{y_k BF_{C,k} - RO_k^2}
\]

\[
\frac{\partial \Delta f}{\partial \Delta f} = \frac{\partial}{\partial \Delta f} \left( \frac{y_k SCQ_{A,k} - RO_k}{y_k BF_{C,k} - RO_k} \right) = \frac{y_k SCQ_{A,k} - RO_k}{y_k BF_{C,k} - RO_k^2}
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\[
\frac{\partial \Delta f}{\partial \Delta f} = \frac{\partial}{\partial \Delta f} \left( \frac{y_k SCQ_{A,k} - RO_k}{y_k BF_{C,k} - RO_k} \right) = \frac{y_k SCQ_{A,k} - RO_k}{y_k BF_{C,k} - RO_k^2}
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\frac{\partial \Delta f}{\partial \Delta f} = \frac{\partial}{\partial \Delta f} \left( \frac{y_k SCQ_{A,k} - RO_k}{y_k BF_{C,k} - RO_k} \right) = \frac{y_k SCQ_{A,k} - RO_k}{y_k BF_{C,k} - RO_k^2}
\]

\[
\frac{\partial \Delta f}{\partial \Delta f} = \frac{\partial}{\partial \Delta f} \left( \frac{y_k SCQ_{A,k} - RO_k}{y_k BF_{C,k} - RO_k} \right) = \frac{y_k SCQ_{A,k} - RO_k}{y_k BF_{C,k} - RO_k^2}
\]
Appendix B

Calculation of the sensitivity indices

\[
S(BF|BF, \text{BEI}/BF) = \frac{\Delta BF}{BF} \cdot \frac{\Delta BF}{BF} = \frac{y_{RO}}{y_{(BF-RQ)}} \cdot \frac{\sum_{l=1}^{n} y_{B_{l}} SC_{Q_{l}} B_{l}}{y_{(BF-RQ)^2}} \cdot \Delta BF (\text{see Eq. 7}) = \frac{BF(\sum_{l=1}^{n} y_{B_{l}} SC_{Q_{l}} B_{l})}{y_{BF(R-RQ)^2}}
\]

(B1)

\[
S(BF|RO, \text{BEI}/RO) = \frac{\Delta RO}{RO} \cdot \frac{\Delta RO}{RO} = \frac{\sum_{l=1}^{n} y_{B_{l}} SC_{Q_{l}} B_{l}}{y_{(BF-RQ)^2}} \cdot \Delta RO (\text{see Eq. 8}) = \frac{RO(\sum_{l=1}^{n} y_{B_{l}} SC_{Q_{l}} B_{l})}{y_{BF(R-RQ)^2}}
\]

(B2)

Appendix C

Prove that \( \frac{\partial B F I}{\partial y_{B}} \) is far less than \( \frac{d m^3}{d} \)

\[
\frac{\partial B F I}{\partial y_{B}} = \frac{\sum_{l=1}^{n} y_{B_{l}} SC_{Q_{l}} B_{l}}{y_{(BF-RQ)^2}} (\text{see Eq. A8}) \quad (C1)
\]

Because of \( n > 0, B F I > 0 \), \( (BF-I-RQ) > 0 \), the above formula can be simplified:

\[
\frac{\partial B F I}{\partial y_{B}} < \frac{\sum_{l=1}^{n} y_{B_{l}} SC_{Q_{l}} B_{l}}{y_{(BF-RQ)^2}} \quad (C2)
\]

Since \( BF_{C} \) is usually much larger than \( SC_{Q_{l}} \), the above formula can be rewritten as:

\[
\frac{\partial B F I}{\partial y_{B}} < \frac{\sum_{l=1}^{n} y_{B_{l}} SC_{Q_{l}} B_{l}}{y_{(BF-RQ)^2}} = \frac{n(BF-I-RQ)}{y_{BF(I-RQ)^2}} = \frac{n}{y} = \frac{1}{y} \quad (C3)
\]

The daily average streamflow \( (\bar{y}) \) is usually much larger than \( 1 \ m^3/d \), so \( \frac{\partial B F I}{\partial y_{B}} \) is far less than \( \frac{d m^3}{d} \).

Data availability

All streamflow and conductivity data can be retrieved from the United States Geological Survey’s (USGS) National Water Information System (NWIS) website use the special gauge number, http://waterdata.usgs.gov/nwis.

Author contributions

Weifei Yang, Changlai Xiao and Xiujuan Liang designed the research train of thought. Weifei Yang and Changlai Xiao completed the parameters’ sensitivity analysis. Xiujuan Liang completed the uncertainty estimate of BFI. Weifei Yang carried out most of the data analysis and prepared the manuscript with contributions from all co-authors.

Competing interests

The authors declare that they have no conflict of interest.

Acknowledgements

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References


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Tables

Table 1. Basic information, parameter sensitivity analysis, and uncertainty estimation results for 24 basins in the United States. Footnote “a” in the “Area” column indicates that the values are estimated based on data from adjacent sites.

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Figures

Figure 1. Average conductivity error (%) with different distributions along the time series (n), (a) uniform distribution, (b) normal distribution.
Figure 2. Average streamflow error (%) with different distributions along the time series (n), (a) uniform distribution, (b) normal distribution.
Figure 13. Scatter plots of sensitivity indices vs. time series (n) and drainage area of the 24 US basins. The watershed area uses a logarithmic axis, while the others are linear normal axes.
Figure 24. Scatter plot of uncertainty in BFI ($W_{BFI}$) and mean uncertainty in $f_B$ (Mean $W_{fb}$).